Bell inequalities for entangled kaons and their unitary time evolution

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We investigate Bell inequalities for neutral kaon systems from Φ resonance decay to test local realism versus quantum mechanics. We emphasize the unitary time evolution of the states, which means we also include *all* decay product states, in contrast to other authors. Only this guarantees the use of the complete Hilbert space. We develop a general formalism for Bell inequalities including both arbitrary ''quasispin'' states and different times; finally, we analyze Wigner-type inequalities. They contain an additional term, a correction function *h*, as compared to the spin 1/2 or photon case, which changes considerably the possibility of quantum mechanics to violate the Bell inequality. Examples for special ''quasispin'' states are given, especially those that are sensitive to the CP parameters ε and ε'.

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I. INTRODUCTION

Schrödinger in 1935 [1] pointed to the peculiar features of what he called entangled states (''verschränkte Zustände'' in his original words). It was Einstein, Podolsky, and Rosen (EPR) who, in their famous paper [2], tried to show the incompleteness of quantum mechanics (QM) by considering a quantum system of two particles. Also, Furry [3] emphasized, inspired by EPR and Schrödinger, the differences between the predictions of QM of nonfactorizable systems and models with spontaneous factorization.

Much later, in 1964, this subject was brought up again by John S. Bell [4] who reanalyzed the ''EPR paradox.'' He discovered, via an inequality, that the predictions of QM differ from those of (all) local realistic theories (LRT); inequalities of this type are now named, quite generally, ''Bell inequalities.''

It is the nonlocality, the ''spooky action at a distance,'' which is the basic feature of quantum physics and is so contrary to our intuition, or more precise, the nonlocal correlations between the spatially separated EPR pair, which occur due to the quantum entanglement.

Many beautiful experiments have been carried out over the years (see, e.g., Refs. [5–8]) by using the entanglement of the polarization of two photons; all confirm impressively this very peculiar quantum feature.

The nonlocality does not conflict with Einstein's relativity, so it cannot be used for superluminal communication, nevertheless, it is the basis for such physics as quantum cryptography [9–12] and quantum teleportation [13,14], and it triggered a technology such as: quantum information [15,16].

Of course, it is of great interest to test the EPR-Bell correlations also for massive systems in particle physics. Already in 1960, Lee and Yang [17] and several other authors [18–20] emphasized the EPR-like features of a $K^0\bar{K}^0$pair in a $J^{PC}=1^{--}$ state. Indeed, many authors [21–26] suggested an investigation the $K^0\bar{K}^0$pairs that are produced at the Φ resonance—for instance in the e^+e^--machine DAΦNE at Frascati. The nonseparability of the neutral kaon system—created in $p\bar{p}$ collisions—has been analyzed by the authors [27–29].

Similar systems are the entangled $B^0\bar{B}^0$ pairs produced at the Υ (4S) resonance (see, e.g., Refs. [30–35]), which we do not consider here.

Specific realistic theories have been constructed [36–38], which describe the $K^0\bar{K}^0$pairs, as tests versus quantum mechanics. However, the general test of QM versus LRT relies on Bell inequalities (BI), where the different kaon detection times play the role of the different angles in the photon or spin-1/2 case. On the other hand, the free choice of the kaon ''quasi spin'' state is also of importance. Furthermore an interesting feature of kaons is the CP violation and indeed, it turns out that Bell inequalities imply bounds on the physical CP violation parameters ε and ε'. In this connection also, a bound on the degree of decoherence of the wave function can be found [39], which turns out to be very strong for a distinction of QM versus LRT.

The important difference of the kaon systems as compared to photons is their decay. Focusing, therefore, just on some particular ''quasispin'' states and not accounting for the decay states restricts the investigation to a subset of the total Hilbert space and will limit the validity of the physical theories.

Therefore, we allow in our work the freedom of choosing arbitrary ''quasispin'' states and we emphasize the importance of including *all* decay product states into the BI, in contrast to other authors, so we use a unitary time evolution. Only this guarantees the use of the complete Hilbert space. It may very well happen that in a particular subspace, QM violates indeed the BI for certain times $t>0$, and thus, contradicts with the assumptions of reality and locality, but in the total Hilbert space the violation will disappear. We show cases where this will happen. Note that for entangled spin-1/2 particles or photon systems, all operations are already defined on the total Hilbert space, since the photon does not decay and its polarization is conserved, whereas in the kaon systems, strangeness is not conserved due to the weak interactions.

The paper is organized as follows. In Sec. II we give an

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introduction to neutral kaons and explain the ''quasispin'' picture. In Sec. III the unitary time evolution is worked out in detail and it is shown how one has to calculate the probabilities in quantum mechanics. In Sec. IV we review briefly the Bell inequalities for spin-1/2 particles. Our main part is contained in Sec. V, there we derive the generalized Bell inequalities for entangled kaons and analyze three different examples that can be found in the literature. Section VI summarizes our results and conclusions are drawn. Finally, some useful formulas can be found in the Appendix.

II. NEUTRAL *K* MESONS

Let us start with a discussion of the properties of the neutral kaons, which we need in the following. The neutral K mesons are characterized by their strangeness quantum number S

$$S|K^0\rangle = +|K^0\rangle,$$
$$S|\overline{K^0}\rangle = -|\overline{K^0}\rangle. \tag{2.1}$$

As the K mesons are pseudoscalars, their parity P is minus and charge conjugation C transforms K^0 and $\overline{K}^0$ into each other so that we have for the combined transformation CP (in our choice of phases)

$$CP|K^0\rangle = -|\overline{K^0}\rangle,$$
$$CP|\overline{K^0}\rangle = -|K^0\rangle. \tag{2.2}$$

From this, it follows that the orthogonal linear combinations

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle - |\overline{K^0}\rangle\},$$
$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle + |\overline{K^0}\rangle\}, \tag{2.3}$$

are eigenstates of CP

$$CP|K_1^0\rangle = +|K_1^0\rangle,$$
$$CP|K_2^0\rangle = -|K_2^0\rangle, \tag{2.4}$$

a quantum number conserved in strong interactions.

Due to weak interactions, which are CP violating, the kaons decay and the physical states, having the mass m_S and m_L, are the short- and long-lived states

$$|K_S\rangle = \frac{1}{N}\{p|K^0\rangle - q|\overline{K^0}\rangle\},$$
$$|K_L\rangle = \frac{1}{N}\{p|K^0\rangle + q|\overline{K^0}\rangle\}, \tag{2.5}$$

with $p=1+\varepsilon$, $q=1-\varepsilon$, $N^2=|p|^2+|q|^2$, and ε being the complex CP violating parameter (CPT invariance is assumed; thus the short- and long-lived states contain the same CP violating parameter $\varepsilon_S=\varepsilon_L=\varepsilon$). They are eigenstates of the non-Hermitian ''effective mass'' Hamiltonian

$$H = M - \frac{i}{2}\Gamma, \tag{2.6}$$

satisfying

$$H|K_{S,L}\rangle = \lambda_{S,L}|K_{S,L}\rangle, \tag{2.7}$$

with

$$\lambda_{S,L} = m_{S,L} - \frac{i}{2}\Gamma_{S,L}. \tag{2.8}$$

Both mesons K^0 and $\overline{K}^0$ have transitions to common states (due to CP violation) therefore they mix, that means they oscillate between K^0 and $\overline{K}^0$ before decaying. Since the decaying states evolve—according to the Wigner-Weisskopf approximation—exponentially in time

$$|K_{S,L}(t)\rangle = e^{-i\lambda_{S,L}t}|K_{S,L}\rangle, \tag{2.9}$$

the subsequent time evolution for K^0 and $\overline{K}^0$ is given by

$$|K^0(t)\rangle = g_+(t)|K^0\rangle + \frac{q}{p}g_-(t)|\overline{K^0}\rangle,$$
$$|\overline{K^0}(t)\rangle = \frac{p}{q}g_-(t)|K^0\rangle + g_+(t)|\overline{K^0}\rangle, \tag{2.10}$$

with

$$g_\pm(t) = \tfrac{1}{2}[\pm e^{-i\lambda_S t} + e^{-i\lambda_L t}]. \tag{2.11}$$

Supposing that at $t=0$, a K^0 beam is produced, e.g., by strong interactions, then the probability for finding a K^0 or $\overline{K}^0$ in the beam is calculated by

$$|\langle K^0|K^0(t)\rangle|^2 = \frac{1}{4}\{e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\Gamma t}\cos(\Delta m t)\},$$
$$|\langle \overline{K^0}|K^0(t)\rangle|^2 = \frac{1}{4}\frac{|q|^2}{|p|^2}\{e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\Gamma t}\cos(\Delta m t)\}, \tag{2.12}$$

with $\Delta m = m_L - m_S$ and $\Gamma = \frac{1}{2}(\Gamma_L + \Gamma_S)$. The K^0 beam oscillates with frequency $\Delta m/2\pi$, the oscillation being clearly visible at times of the order of a few τ_S, before all K_S have died out, leaving only the K_L in the beam. So, in a beam which contains only K_0 mesons at the time $t=0$, the $\overline{K}_0$ will appear far from the production source through its presence in the K_L meson with equal probability as the K_0 meson. A similar feature occurs when starting with a $\overline{K}^0$ beam.

In comparison with spin-1/2 particles, or with photons having the polarization directions vertical and horizontal, it is especially useful to work with the ''quasispin'' picture for kaons introduced by Lee and Wu [40] and Lipkin [20]. The two states $|K^0\rangle$ and $|\overline{K^0}\rangle$ are regarded as the quasispin states up $|\Uparrow\rangle$ and down $|\Downarrow\rangle$ and the operators acting in this quasispin space are expressed by Pauli matrices. So the strangeness operator S can be identified with the Pauli matrix σ_3, the CP operator with $(-\sigma_1)$ and CP violation is proportional to σ_2. In fact, the Hamiltonian (2.6) can be written as

$$H = a\cdot\mathbf{1} + \vec{b}\cdot\vec{\sigma}, \tag{2.13}$$

with

$$b_1 = b\cos\alpha, \quad b_2 = b\sin\alpha, \quad b_3 = 0,$$

$$a = \tfrac{1}{2}(\lambda_L + \lambda_S), \quad b = \tfrac{1}{2}(\lambda_L - \lambda_S) \tag{2.14}$$

($b_3=0$ due to CPT invariance), and the phase α is related to the CP parameter ε by

$$e^{i\alpha} = \frac{1-\varepsilon}{1+\varepsilon}. \tag{2.15}$$

Now, what we are actually interested in are entangled states of $K^0\overline{K^0}$pairs, in analogy to the entangled spin-up and spin-down pairs, or photon pairs. Such states are produced by e^+e^--machines through the reaction $e^+e^- \to \Phi \to K^0\overline{K^0}$, in particular at DA Φ NE, or they are produced in $p\bar{p}$ collisions such as, e.g., at LEAR. There, a $K^0\overline{K^0}$pair is created in a $J^{PC}=1^{--}$ quantum state, and thus, antisymmetric under C and P, and is described at the time $t=0$ by the entangled state

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle_l \otimes |\overline{K^0}\rangle_r - |\overline{K^0}\rangle_l \otimes |K^0\rangle_r\}, \tag{2.16}$$

which can be rewritten in the K_SK_L-basis

$$|\psi(t=0)\rangle = \frac{N_{SL}}{\sqrt{2}}\{|K_S\rangle_l \otimes |K_L\rangle_r - |K_L\rangle_l \otimes |K_S\rangle_r\}, \tag{2.17}$$

with $N_{SL}=N^2/2pq$. Then the neutral kaons fly apart and will be detected on the left (l) and right (r) side of the source. Of course, during their propagation, the $K^0\overline{K^0}$oscillate and K_S, K_L decays will take place. This is an important difference to the case of spin-1/2 particles or photons.

III. TIME EVOLUTION—UNITARITY

Now let us discuss more closely the time evolution of the kaon states [41] . At any instant t, the state $|K^0(t)\rangle$ decays to a specific final state $|f\rangle$ with a probability proportional to the absolute squared of the transition-matrix element. Because of unitarity of the time evolution, the norm of the total state must be conserved. This means that the decrease in the norm of the state $|K^0(t)\rangle$ must be compensated by the increase in the norm of the final states. So, starting at $t=0$ with a K^0 meson, the state we have to consider for a complete t evolution is given by

$$|K^0\rangle \to a(t)|K^0\rangle + b(t)|\overline{K^0}\rangle + \sum_f c_f(t)|f\rangle, \tag{3.1}$$

with

$$a(t) = g_+(t) \quad \text{and} \quad b(t) = \frac{q}{p}g_-(t), \tag{3.2}$$

and the functions $g_\pm(t)$ are defined in Eq. (2.11). Denoting the amplitudes of the decays of the $K^0,\overline{K^0}$ to a specific final state f by

$$\mathcal{A}(K^0\to f) \equiv \mathcal{A}_f \quad \text{and} \quad \mathcal{A}(\overline{K^0}\to f) \equiv \overline{\mathcal{A}}_f, \tag{3.3}$$

we have

$$\frac{d}{dt}|c_f(t)|^2 = |a(t)\mathcal{A}_f + b(t)\overline{\mathcal{A}}_f|^2, \tag{3.4}$$

and for the probability of the decay $K_0\to f$ at a certain time τ

$$P_{K^0\to f}(\tau) = \int_0^\tau \frac{d}{dt}|c_f(t)|^2 dt. \tag{3.5}$$

Since the state $|K^0(t)\rangle$ evolves according to a Schrödinger equation with ‘‘effective mass’’ Hamiltonian (2.6) the decay amplitudes are related to the Γ matrix by

$$\Gamma_{11} = \sum_f |\mathcal{A}_f|^2, \quad \Gamma_{22} = \sum_f |\overline{\mathcal{A}}_f|^2, \quad \Gamma_{12} = \sum_f \mathcal{A}_f^*\overline{\mathcal{A}}_f. \tag{3.6}$$

These are the Bell-Steinberger unitarity relations [41]; they are a consequence of probability conservation, and play an important role.

For our purpose, the formalism used by Ghirardi, Grassi, and Weber [42] is quite convenient, and we generalize it to arbitrary quasispin states. So we describe the complete evolution of the mass eigenstates by a unitary operator $U(t,0)$ whose effect can be written as

$$U(t,0)|K_{S,L}\rangle = e^{-i\lambda_{S,L}t}|K_{S,L}\rangle + |\Omega_{S,L}(t)\rangle, \tag{3.7}$$

where $|\Omega_{S,L}(t)\rangle$ denotes the state of all decay products. For the transition amplitudes of the decay product states, we then have

$$\langle\Omega_S(t)|\Omega_S(t)\rangle = 1 - e^{-\Gamma_S t}, \tag{3.8}$$

$$\langle\Omega_L(t)|\Omega_L(t)\rangle = 1 - e^{-\Gamma_L t}, \tag{3.9}$$

$$\langle\Omega_L(t)|\Omega_S(t)\rangle = \langle K_L|K_S\rangle(1 - e^{i\Delta m t}e^{-\Gamma t}), \tag{3.10}$$

$$\langle K_{S,L}|\Omega_S(t)\rangle = \langle K_{S,L}|\Omega_L(t)\rangle = 0. \tag{3.11}$$

Note that the mass eigenstates (2.5) are normalized but due to CP violation not orthogonal

$$\langle K_L|K_S\rangle = \frac{2\,\mathrm{Re}\{\varepsilon\}}{1+|\varepsilon|^2} =: \delta. \tag{3.12}$$

Now we consider entangled states of kaon pairs, and we start at time $t=0$ from the entangled state (2.16) given in the K_SK_L basis choice

$$|\psi(t=0)\rangle = \frac{N^2}{2\sqrt{2}pq}\{|K_S\rangle_l\otimes|K_L\rangle_r - |K_L\rangle_l\otimes|K_S\rangle_r\}. \tag{3.13}$$

Then we get the state at time t from Eq. (3.13) by applying the unitary operator

$$U(t,0) = U_l(t,0)U_r(t,0), \tag{3.14}$$

where the operators $U_l(t,0)$ and $U_r(t,0)$ act on the space of the left and of the right mesons according to the time evolution (3.7).

What we are finally interested in are the quantum mechanical probabilities for detecting, or not detecting, a specific quasispin state on the left side $|k_n\rangle_l$ and on the right side $|k_n\rangle_r$ of the source. For that we need the projection operators $P_{l,r}(k_n)$ on the left, right quasi-spin states $|k_n\rangle_{l,r}$ together with the projection operators that act onto the orthogonal states $Q_{l,r}(k_n)$

$$P_l(k_n) = |k_n\rangle_{ll}\langle k_n| \quad \text{and} \quad P_r(k_n) = |k_n\rangle_{rr}\langle k_n|, \tag{3.15}$$

$$Q_l(k_n) = \mathbf{1} - P_l(k_n) \quad \text{and} \quad Q_r(k_n) = \mathbf{1} - P_r(k_n). \tag{3.16}$$

So, starting from the initial state (3.13) the unitary time evolution (3.14) gives the state at a time t_r

$$|\psi(t_r)\rangle = U(t_r,0)|\psi(t=0)\rangle = U_l(t_r,0)U_r(t_r,0)|\psi(t=0)\rangle. \tag{3.17}$$

If we now measure a k_m at t_r on the right side, that means we project onto the state

$$|\tilde{\psi}(t_r)\rangle = P_r(k_m)|\psi(t_r)\rangle. \tag{3.18}$$

This state, which is now a one-particle state of the left-moving particle, evolves until t_l when we measure a k_n on the left side and we get

$$|\tilde{\psi}(t_l,t_r)\rangle = P_l(k_n)U_l(t_l,t_r)P_r(k_m)|\psi(t_r)\rangle. \tag{3.19}$$

The probability of the joint measurement is given by the squared norm of the state (3.19). It coincides (due to unitarity, composition laws, and commutation properties of l,r operators) with the state

$$|\psi(t_l,t_r)\rangle = P_l(k_n)P_r(k_m)U_l(t_l,0)U_r(t_r,0)|\psi(t=0)\rangle, \tag{3.20}$$

which corresponds to a factorization of the time into an eigentime t_l on the left side and into an eigentime t_r on the right side.

Then we can calculate the quantum-mechanical probability $P_{n,m}(Y,t_l;Y,t_r)$ for finding a k_n at t_l on the left side *and* a k_m at t_r on the right side and the probability $P_{n,m}(N,t_l;N,t_l)$ for finding *no* such kaons by the following norms; and similarly, the probability $P_{n,m}(Y,t_l;N,t_r)$ when a k_n at t_l is detected on the left but *no* k_m at t_r on the right

$$P_{n,m}(Y,t_l;Y,t_r) = ||P_l(k_n)P_r(k_m)U_l(t_l,0)U_r(t_r,0)|\psi(t=0)\rangle||^2, \tag{3.21}$$

$$P_{n,m}(N,t_l;N,t_r) = ||Q_l(k_n)Q_r(k_m)U_l(t_l,0)U_r(t_r,0)|\psi(t=0)\rangle||^2, \tag{3.22}$$

$$P_{n,m}(Y,t_l;N,t_r) = ||P_l(k_n)Q_r(k_m)U_l(t_l,0)U_r(t_r,0)|\psi(t=0)\rangle||^2. \tag{3.23}$$

IV. BELL INEQUALITIES FOR SPIN-1/2 PARTICLES

In this section, we will review briefly the well-known derivation of Bell inequalities [43]. Our intention is to draw the readers attention to the analogies, but more importantly to the differences of the spin/photon correlations as compared to the quasispin correlations discussed in the following sections.

We want to start with the derivation a general Bell inequality, the CHSH inequality, named after Clauser, Horne, Shimony, and Holt [44], and then we derive from that inequality—with two further assumptions—the original Bell inequality and the Wigner-type inequality.

Let $A(n,\lambda)$ and $B(m,\lambda)$ be the definite values of two quantum observables $A^{QM}(n)$ and $B^{QM}(m)$, λ denoting the hidden variables that are not accessible to an experimenter but carry the additional information needed in a LRT. The measurement result of one observable is $A(n,\lambda)=\pm 1$ corresponding to the spin measurement ''spin up'' and ''spin down'' along the quantization direction n of particle 1; and $A(n,\lambda)=0$ if no particle was detected at all. The analogue holds for the result $B(m,\lambda)$ of particle 2.

Assuming now Bell's locality hypothesis $[A(n,\lambda)$ depends only on the direction n, but not on m, the analogue holds for $B(m,\lambda)]$—which is the crucial point—we have for the combined spin measurement the following expectation value

$$M(n,m) = \int d\lambda\,\rho(\lambda)A(n,\lambda)B(m,\lambda), \tag{4.1}$$

with the normalized probability distribution

$$\int d\lambda\,\rho(\lambda) = 1. \tag{4.2}$$

This quantity $M(n,m)$ correspond to the quantum-mechanical mean value $M^{QM}(n,m) = \langle A^{QM}(n)B^{QM}(m)\rangle$.

A straight forward calculation (for example [44,45,46]) gives the estimate of the absolut value of the difference of two mean values

$$|M(n,m)-M(n,m')| \leqslant \int d\lambda \rho(\lambda)\{1 \pm A(n',\lambda)B(m',\lambda)\} + \int d\lambda \rho(\lambda)\{1 \pm A(n',\lambda)B(m,\lambda)\}, \tag{4.3}$$

and using normalization (4.2) we get

$$|M(n,m)-M(n,m')| \leqslant 2 \pm |M(n',m')+M(n',m)|, \tag{4.4}$$

and more symmetrically

$$|M(n,m)-M(n,m')|+|M(n',m')+M(n',m)| \leqslant 2. \tag{4.5}$$

This is the familiar CHSH inequality, derived by Clauser, Horne, Shimony, and Holt [44] in 1969. Every local realistic hidden variable theory must obey that inequality.

Inserting the quantum-mechanical expectation values $M^{QM}(n,m)$ for $M(n,m)$, we get, with $\phi_{n,m}$ being the angle between the two quantization directions n and m,

$$S(n,m,n',m') = |\cos(\phi_{n,m})-\cos(\phi_{n,m'})| + |\cos(\phi_{n',m'})+\cos(\phi_{n',m})| \leqslant 2, \tag{4.6}$$

which is for some choices of the angles ϕ violated; the maximal value of the left-hand side is $2\sqrt{2}$, with for instance $\phi_{n,m'}=3\pi/4$ and $\phi_{n,m}=\phi_{n',m'}=\phi_{n',m}=\pi/4$. Experimentally, for entangled photon pairs inequality (4.6) is violated under strict Einstein locality conditions in an impressive way, with a result close in agreement with QM [8], confirming such previous experimental results on similar inequalities [5–7].

In order to come to the original Bell inequality or to the Wigner inequality we make two assumptions, first we assume always perfect anticorrelation $M(n,n)=-1$, and second, the measurement of the state of the particles has to be perfect, so there are no omitted events that were interpreted in the CHSH derivation as 0 results.

Considering now just three different quantization directions, choosing, e.g., $n'=m'$, inequality (4.4) gives

$$|M(n,m)-M(n,n')| \leqslant 2 \pm \{\underbrace{M(n',n')}_{-1\forall n'}+M(n',m)\},$$

or

$$|M(n,m)-M(n,n')| \leqslant 1+M(n',m). \tag{4.7}$$

This is the famous original inequality derived by J.S. Bell [4] in 1964. Note, that this derivation is already true for the entangled kaon system where the different kaon quasispin eigenstates on the left and right side, measured at equal times, play the role of the different angles, see Sec. V C.

Finally, we rewrite the expectation value for two spin-1/2 particles in terms of probabilities

$$M(n,m) = P(\vec{n}\Uparrow;\vec{m}\Uparrow)+P(\vec{n}\Downarrow;\vec{m}\Downarrow)-P(\vec{n}\Uparrow;\vec{m}\Downarrow) - P(\vec{n}\Downarrow;\vec{m}\Uparrow) = -1+4P(\vec{n}\Uparrow;\vec{m}\Uparrow), \tag{4.8}$$

where we used $\Sigma P=1$. Then Bell's original inequality (4.7) provides the Wigner inequality

$$P(\vec{n};\vec{m}) \leqslant P(\vec{n};\vec{n}')+P(\vec{n}';\vec{m}), \tag{4.9}$$

where the P can be the measurement of all spins up or spins down on both sides, or spin up on one side and spin down on the other side, or vice versa. Note, that the Wigner inequality has been originally derived by a set-theoretical approach.

V. GENERALIZED BELL INEQUALITIES FOR *K* MESONS

Let us consider again the entangled state $|\psi(t=0)\rangle$ (2.16) of a $K^0\bar{K}^0$pair and its time evolution $U(t,0)|\psi(0)\rangle$, then we find the following situation: Performing two measurements to detect the kaons at the same time at the left side and at the right side of the source the probability of finding two mesons with the same strangeness K^0K^0 or $\bar{K}^0\bar{K}^0$ is zero. If we measure at time t a $\bar{K}^0$ meson on the left side, we will find with certainty at the same time t $no\bar{K}^0$ on the right side. This is an EPR-Bell correlation analogously to the spin 1/2 or photon (e.g., with polarization vertical horizontal) case. The analogy would be perfect, if the kaons were stable ($\Gamma_S=\Gamma_L=0$); then the quantum probabilities become

$$P(Y,t_l;Y,t_r) = P(N,t_l;N,t_r) = \tfrac{1}{4}\{1-\cos[\Delta m(t_l-t_r)]\},$$
$$P(Y,t_l;N,t_r) = P(N,t_l;Y,t_r) = \tfrac{1}{4}\{1+\cos[\Delta m(t_l-t_r)]\}. \tag{5.1}$$

They coincide with the probabilities of finding simultaneously two entangled spin-1/2 particles in spin directions $\Uparrow\Uparrow$ or $\Uparrow\Downarrow$ along two chosen directions $\vec{n}$ and $\vec{m}$

$$P(\vec{n},\Uparrow;\vec{m},\Uparrow) = P(\vec{n},\Downarrow;\vec{m},\Downarrow) = \tfrac{1}{4}\{1-\cos\theta\},$$
$$P(\vec{n},\Uparrow;\vec{m},\Downarrow) = P(\vec{n},\Downarrow;\vec{m},\Uparrow) = \tfrac{1}{4}\{1+\cos\theta\}. \tag{5.2}$$

The time differences $\Delta m(t_l-t_r)$ in the kaon case play the role of the angle differences θ in the spin-1/2 case.

Nevertheless, there are important physical differences between kaon and spin-1/2 states (for an experimenter's point of view, see Ref. [47]).

(1) While in the spin-1/2 or photon case one can test whether a system is in an arbitrary spin state $\alpha|\Uparrow\rangle+\beta|\Downarrow\rangle$ one cannot test it for an arbitrary superposition $\alpha|K^0\rangle+\beta|\bar{K}^0\rangle$.

(2) For entangled spin-1/2 particles or photons it is sufficient to consider the direct product space $H^l_{spin}\otimes H^r_{spin}$, however, this is not so for kaons. The unitary time evolution of a kaon state also involves the decay product states (see

Sec. III), therefore one has to include the decay product spaces which are orthogonal to the product space $H^r_{kaon} \otimes H^l_{kaon}$.

So by measuring a $\bar{K}^0$ at the left side we can predict with certainty to find at the same time $no\bar{K}^0$ at the right side. In any LRT, this property $no\bar{K}^0$ must be present at the right side independent of having the measurement performed or not. In order to discriminate between QM and LRT, we set up a Bell inequality for the kaon system where now the different times play the role of the different angles in the spin-1/2 case. But, in addition, we use the freedom of choosing a particular quasispin state of the kaon, the strangeness eigenstate, the mass eigenstate, or the CP eigenstate.

A. Expectation values and locality

As discussed before in kaon systems, we have the freedom of choosing the time, when a measurement takes place *and* the freedom to choose which particular quasispin state we want to measure.

The locality hypothesis then requires that the results of measurement on the left side are completely independent of the chosen time and chosen quasispin state in the measurement on the right side.

Let us consider an observable $O(k_n,t_a)$ on each side of the source, which gets the value $+1$ if in a measurement at time t_a the quasispin state k_n is found, and the value -1 if not. Then we can define a correlation function $O(k_n,t_a;k_m,t_b)$ which gets the value $+1$, both when at the left side, a k_n at t_a was detected and at the right side, a k_m at t_b was, or when nok_n and nok_m was found. In the case when only one of the desired quasispin eigenstates has been found, no matter at which side, the correlation function has the value -1.

Locality hypothesis: Locality in the sense of Bell means that the correlation function $O(k_n,t_a;k_m,t_b)$ is equal to the product of the observables on each side

$$O(k_n,t_a;k_m,t_b)=O^l(k_n,t_a)O^r(k_m,t_b). \tag{5.3}$$

Then the following relation holds

$$|O(k_n,t_a;k_m,t_b)-O(k_n,t_a;k_{m'},t_d)| + |O(k_{n'},t_c;k_{m'},t_d)+O(k_{n'},t_c;k_m,t_b)|=2, \tag{5.4}$$

with $k_n,k_m,k_{n'}$ and $k_{m'}$ being arbitrary quasispin eigenstates of the meson and t_a, t_b, t_c, and t_d four different times.

Now we consider a series of N identical measurements and we denote by O_i the value of O in the ith experiment. The average is given by

$$M(k_n,t_a;k_m,t_b)=\frac{1}{N}\sum_{i=1}^{N} O_i(k_n,t_a;k_m,t_b). \tag{5.5}$$

Taking the absolute values of differences and sums of such averages and inserting relation (5.4), we obtain the Bell-CHSH inequality for the expectation values

$$|M(k_n,t_a;k_m,t_b)-M(k_n,t_a;k_{m'},t_d)| + |M(k_{n'},t_c;k_{m'},t_d)+M(k_{n'},t_c;k_m,t_b)|\le 2. \tag{5.6}$$

If we identify $M(k_n,t_a;k_m,t_b)\equiv M(n,m)$ we are back at the inequality (4.5) for the spin-1/2 case.

B. Probabilities

Now we consider the expectation value (5.5) for the series of identical measurements in terms of the probabilities, where we denote by $P_{n,m}(Y,t_a;Y,t_b)$ the probability for finding a k_n at t_a on the left side and finding a k_m at t_b on the right side and by $P_{n,m}(N,t_a;N,t_b)$ the probability for finding *no* such kaons; similarly, $P_{n,m}(Y,t_a;N,t_b)$ denotes the case when a k_n at t_a is detected on the left but nok_m at t_b on the right. Then we can re-express the expectation value by the following linear combination

$$M(k_n,t_a;k_m,t_b)=P_{n,m}(Y,t_a;Y,t_b)+P_{n,m}(N,t_a;N,t_b) - P_{n,m}(Y,t_a;N,t_b)-P_{n,m}(N,t_a;Y,t_b). \tag{5.7}$$

Since the sum of the probabilities for (Y,Y), (N,N), (Y,N), and (N,Y) must be unity we get

$$M(k_n,t_a;k_m,t_b)=-1+2\{P_{n,m}(Y,t_a;Y,t_b) + P_{n,m}(N,t_a;N,t_b)\}. \tag{5.8}$$

Note that relation (5.7) between the expectation value and the probabilities is satisfied for QM and LRT as well.

Setting this expression into the Bell-CHSH inequality (5.6) we finally arrive at the following inequality for the probabilities

$$\begin{aligned}&|P_{n,m}(Y,t_a;Y,t_b)+P_{n,m}(N,t_a;N,t_b)-P_{n,m'}(Y,t_a;Y,t_d)\\ &\quad -P_{n,m'}(N,t_a;N,t_d)|\\ &\quad \le 1\pm\{-1+P_{n',m}(Y,t_c;Y,t_b)+P_{n',m}(N,t_c;N,t_b)\\ &\quad +P_{n',m'}(Y,t_c;Y,t_d+P_{n',m'}(N,t_c;N,t_d)\},\end{aligned} \tag{5.9}$$

or

$$\begin{aligned}&S(k_n,k_m,k_{n'},k_{m'};t_a,t_b,t_c,t_d)\\ &\quad =|P_{n,m}(Y,t_a;Y,t_b)+P_{n,m}(N,t_a;N,t_b)\\ &\qquad -P_{n,m'}(Y,t_a;Y,t_d)-P_{n,m'}(N,t_a;N,t_d)|\\ &\qquad +|-1+P_{n',m}(Y,t_c;Y,t_b)+P_{n',m}(N,t_c;N,t_b)\\ &\qquad +P_{n',m'}(Y,t_c;Y,t_d)+P_{n',m'}(N,t_c;N,t_d)|\le 1.\end{aligned} \tag{5.10}$$

C. Wigner-type inequalities

What we aim is to find Wigner-type inequalities. The most general one we get from above inequality (5.9) by choosing the upper sign +

$$P_{n,m}(Y,t_a;Y,t_b) \leqslant P_{n,m'}(Y,t_a;Y,t_d) + P_{m',n'}(Y,t_d;Y,t_c) + P_{n',m}(Y,t_c;Y,t_b) + h(n,m,n',m';t_a,t_b,t_c,t_d), \tag{5.11}$$

where

$$h(n,m,n',m';t_a,t_b,t_c,t_d) = -P_{n,m}(N,t_a;N,t_b) + P_{n,m'}(N,t_a;N,t_d) + P_{n',m}(N,t_c;N,t_b) + P_{n',m'}(N,t_c;N,t_d), \tag{5.12}$$

is a correction function to the usual set-theoretical result, see Section IV. It arises because for a unitary time evolution we also have to include the decay states [see Eq. (3.7)], contributing to the *no kaon* states, thus, the decay product spaces that are orthogonal to the product space $H^l_{kaon} \otimes H^r_{kaon}$.

For zero times $t_{a,b} \to 0$, when we have no decays, the probabilities for (N,N) become the ones for (Y,Y)

$$P_{n,m}(N,t_a;N,t_b)|_{t_{a,b}=0} \equiv P_{n,m}(Y,t_a;Y,t_b)|_{t_{a,b}=0}, \tag{5.13}$$

the correction function (for $t_a = t_b = t_c = t_d = t = 0$) is then equal to

$$h(n,m,n',m';t=0) = -P_{n,m}(Y,Y)|_{t=0} + P_{n,m'}(Y,Y)|_{t=0} + P_{n',m}(Y,Y)|_{t=0} + P_{n',m'}(Y,Y)|_{t=0}, \tag{5.14}$$

and just adds up to the inequality (5.11) in such a way that we obtain the usual set-theoretical result

$$P_{n,m}(Y,Y)|_{t=0} \leqslant P_{n,m'}(Y,Y)|_{t=0} + P_{m',n'}(Y,Y)|_{t=0} + P_{n',m}(Y,Y,)|_{t=0}. \tag{5.15}$$

Of course, the case we are interested in contains only three different states, so we put $n' = m'$ and $t_c = t_d$, then the probability for (Y,Y) vanishes $P_{n',n'}(Y,t_c;Y,t_c) = 0$ due to the EPR-Bell anticorrelation, but certainly not the probability for (N,N), $P_{n',n'}(N,t_c;N,t_c) \neq 0$ (it vanishes only for $t_c \to 0$).

So we obtain the following Wigner-type inequality for three different quasispin states

$$P_{n,m}(Y,t_a;Y,t_b) \leqslant P_{n,n'}(Y,t_a;Y,t_c) + P_{n',m}(Y,t_c;Y,t_b) + h(n,m,n';t_a,t_b,t_c), \tag{5.16}$$

with the correction function

$$h(n,m,n';t_a,t_b,t_c) = -P_{n,m}(N,t_a;N,t_b) + P_{n,n'}(N,t_a;N,t_c) + P_{n',m}(N,t_c;N,t_b) + P_{n',n'}(N,t_c;N,t_c). \tag{5.17}$$

Again, in the limit of zero times $t \to 0$, we arrive at the familiar Wigner-type inequality

$$P_{n,m}(Y,Y)|_{t=0} \leqslant P_{n,n'}(Y,Y)|_{t=0} + P_{n',m}(Y,Y)|_{t=0}. \tag{5.18}$$

We certainly can also achieve Bell's original case (4.7), which is more restrictive since we have to require perfect anticorrelation

$$M(k_n,t;k_n,t) = -1. \tag{5.19}$$

Then the general CHSH relation, Eq. (5.6), implies the specific inequality of Bell

$$|M(k_n,t;k_m,t) - M(k_n,t;k_{n'},t)| \leqslant 1 + M(k_{n'},t;k_m,t). \tag{5.20}$$

Converting it into a Wigner-type we come back to inequality (5.16), but with a smaller correction function

$$h_{Bell}(t) = h_{CHSH}(t) - P_{n',n'}(N,t;N,t), \tag{5.21}$$

which is more restrictive.

D. The choice sensitive to the *CP* parameter ε

Choosing the quasispin states

$$|k_n\rangle = |K_S\rangle,$$
$$|k_m\rangle = |\bar{K}^0\rangle,$$
$$|k_{n'}\rangle = |K_1^0\rangle, \tag{5.22}$$

and denoting the probabilities $P_{K_S,\bar{K}^0}(Y,Y)|_{t=0} \equiv P(K_S,\bar{K}^0)$ etc., we recover Uchiyama's inequality [48]

$$P(K_S,\bar{K}^0) \leqslant P(K_S,K_1^0) + P(K_1^0,\bar{K}^0), \tag{5.23}$$

which he derived by a set-theoretical approach. The interesting point here is its connection to a physical parameter, the *CP* violating parameter ε. As Uchiyama has shown, his inequality can be turned into an inequality for ε

$$\mathrm{Re}\{\varepsilon\} \leqslant |\varepsilon|^2, \tag{5.24}$$

which is obviously violated by the experimental value of ε, having an absolute value of about 10^{-3} and a phase of about $45°$ [49].

Another meaningful choice would be the replacement of the short-lived state $|K_S\rangle$ by the long-lived state $|K_L\rangle$ and the *CP* eigenstate $|K_1\rangle$ by $|K_2\rangle$ in Eq. (5.22) then we arrive at the same inequality (5.24).

Our Wigner-type inequality (5.16) differs from the ones discussed in the literature [24,25,26,47,50,51]; in the sense that we have an additional term h (5.17) due to the unitary time evolution of the considered states. Since h is positive, it worsens the possibility for quantum mechanics to violate the Bell inequality.

This can be clearly seen in case of equal times $t_a=t_b=t_c=t$, when the exponential t dependence factorizes in the (Y,Y) probabilities but not in the (N,N) ones. Then we have for the choice (5.22) the following Wigner-type inequality

$$e^{-2\Gamma t}P(K_S,\overline{K^0})\leqslant e^{-2\Gamma t}P(K_S,K_1^0)+e^{-2\Gamma t}P(K_1^0,\overline{K^0}) + h(K_S,\overline{K^0},K_1^0;t), \tag{5.25}$$

where the probabilities and the correction function h can be found explicitly in the Appendix . As we can see, due to the fast damping of the probabilities (and $h\rightarrow 2$) a violation of inequality (5.25) by QM is only possible for very small times, in fact, only for times $t\leqslant 8\cdot 10^{-4}\tau_S$.

But fortunately there exist certain cases where the situation is better. We can avoid a fast increase of the correction function h by taking the times $t_a=t_c$ and $t_a\leqslant t_b$. Then a violation of the Wigner-type inequality (5.16) occurs, which is strongest for $t_a\approx 0$; and in this case, t_b can be chosen up to $t_b\leqslant 4\tau_S$, which is already quite large.

E. The choice sensitive to the direct *CP* parameter ε'

As shown by Benatti and Floreanini in Refs. [50,51], the case has been also discussed carefully in Refs. [25,26], some decay end products can be identified with the quasispin eigenstates. For example, the two neutral pions or the two charged pions can be associated with the quasispin eigenstates:

$$|K_{00}\rangle=\frac{1}{\sqrt{1+|\varepsilon_{00}|}}\{|K_1^0\rangle+\varepsilon_{00}|K_2^0\rangle\}\rightarrow|\pi^0\pi^0\rangle,$$

$$|K_{+-}\rangle=\frac{1}{\sqrt{1+|\varepsilon_{+-}|}}\{|K_1^0\rangle+\varepsilon_{+-}|K_2^0\rangle\}\rightarrow|\pi^+\pi^-\rangle, \tag{5.26}$$

with

$$\varepsilon_{00}=-2\varepsilon'+i\frac{\mathrm{Im}\{\mathcal{A}_0\}}{\mathrm{Re}\{\mathcal{A}_0\}},$$

$$\varepsilon_{+-}=\varepsilon'+i\frac{\mathrm{Im}\{\mathcal{A}_0\}}{\mathrm{Re}\{\mathcal{A}_0\}}. \tag{5.27}$$

Here, $\mathcal{A}_0\equiv\langle\pi\pi,I=0|H_w|K^0\rangle$ is the weak decay amplitude with I being the isospin (for further information, see Refs. [52–54]) and ε' being the direct CP violation parameter; the third order and higher orders in ε and $\varepsilon_{00},\varepsilon_{+-}$ are already neglected.

We choose—analogously to previous section—the quasispin states

$$|k_n\rangle=|K^0\rangle,$$

$$|k_m\rangle=|K_{00}\rangle,$$

$$|k_{n'}\rangle=|K_{+-}\rangle, \tag{5.28}$$

and we get the following Wigner-type inequality for $t=0$

$$P(K^0,K_{00})\leqslant P(K^0,K_{+-})+P(K_{+-},K_{00}). \tag{5.29}$$

The calculation of the probabilities gives an inequality

$$|-\mathrm{Re}\{\varepsilon_{00}\}(1+|\varepsilon_{+-}|^2)+\mathrm{Re}\{\varepsilon_{+-}\}(1+|\varepsilon_{00}|^2)| \leqslant|\varepsilon_{00}|^2+|\varepsilon_{+-}|^2-2\,\mathrm{Re}\{\varepsilon_{00}^*\varepsilon_{+-}\}, \tag{5.30}$$

which, when the results (5.27) for ε_{00} and ε_{+-} are inserted, turns into an inequality in the direct CP violating parameter ε' (third-order terms neglected)

$$\mathrm{Re}\{\varepsilon'\}\leqslant 3|\varepsilon'|^2, \tag{5.31}$$

the inequality of Refs. [50,51].

This inequality is clearly violated by the experimental value of ε', $|\varepsilon'|\lesssim 10^{-6}$ and has a phase of about 45° [49].

Again, for times $t>0$, we have to include the correction function h. Choosing all four times equal $t_a=t_b=t_c=t_d=t$, the inequality (5.16) with the choice (5.28) cannot be violated for times larger than $t=3.7\cdot 10^{-6}\tau_S$.

Varying all four times, unfortunately, does not improve the test QM versus LRT, we only find a violation in the region where all times are smaller than $10^{-6}\tau_S$.

F. The choice of the strangeness eigenstate

Finally, we also can reproduce the case of Ghirardi, Grassi, and Weber [42], we just have to consider the same quasispin states

$$k_n=k_m=k_{n'}=k_{m'}=\overline{K^0}. \tag{5.32}$$

Evaluating the Bell-CHSH inequality (5.6) by the quantum-mechanical probabilities, neglecting CP violation, the result is [42]

$$|e^{-(\Gamma_S/2)(t_a+t_c)}\cos[\Delta m(t_a-t_c)]-e^{-(\Gamma_S/2)(t_a+t_d)} \times\cos[\Delta m(t_a-t_d)]|+|e^{-(\Gamma_S/2)(t_b+t_c)}\cos[\Delta m(t_b-t_c)] +e^{-(\Gamma_S/2)(t_b+t_d)}\cos[\Delta m(t_b-t_d)]|\leqslant 2. \tag{5.33}$$

Unfortunately, inequality (5.33) *cannot* be violated [42,55] for any choice of the four (positive) times t_a,t_b,t_c,t_d due to the interplay between the kaon decay and strangeness oscillations. As demonstrated in Ref. [56], a possible violation depends very much on the kaon parameter $x=\Delta m/\Gamma$; if we had $x=4.3$ instead of the experimental $x\approx 1$, this Bell-CHSH inequality (5.33) would be broken. Note, that in this case, the CHSH inequality maximizes at different time values than expected from the corresponding photon CHSH inequality (4.6).

VI. SUMMARY AND CONCLUSIONS

A. Quantum theory

We consider the time evolution of neutral kaons and emphasize the unitary time evolution that includes the decay states. Starting at $t=0$, with a K^0, after a certain time t, one gets a superposition of the strangeness eigenstates due to strangeness oscillations *and* the decay states. In this way we consider the total Hilbert space—analogously to the photon case.

Then we treat entangled states and derive their quantum-mechanical probabilities of finding or not finding arbitrary quasispin states at arbitrary times. With these QM probabilities we calculate the quantum-mechanical expectation value.

B. LRT

We derive the general Bell-CHSH inequality (5.6) based on a local realistic hidden variable theory. From this general Bell inequality follows a Wigner-type inequality (5.11) and an inequality analogously to Bell's original version.

C. QM versus LRT

Next we compare the quantum theory with LRT, which means we insert the quantum-mechanical expectation value into the general Bell-CHSH inequality. Expressing the expectation value in terms of probabilities we arrive at a Wigner-type inequality (5.11) that contains an additional term due to the unitary time evolution, the correction function h (5.12).This function h is missing in the inequalities of other authors [24,50,51] since they restrict themselves to a subset of the Hilbert space.

D. Results

This correction function h makes it rather difficult for QM to violate the Bell inequality (in order to show the nonlocal character of QM). In case of Ghirardi, Grassi, and Weber [42], where only $\bar{K}^0$ or *no* $\bar{K}^0$ is detected, it is impossible for any choice of the times that QM violates the BI. On the other hand, if we consider, in addition to the choice of time, the freedom of choosing particular quasispin eigenstates, then we find cases were QM does violate the BI for certain times. For example, in the choice (5.22), the Bell inequality is violated for $t_a=t_c\approx 0$ and $t_b\leqslant 4\tau_S$. Considering another choice (5.28) we find no violation at all, except for $t=0$.

E. Comments

The authors of Refs. [24,50,51,47] restrict their analysis to a subset of the Hilbert space; tests on such subspaces, however, probe only a restricted class of LRT. In such subspaces Bell inequalities may be violated, but this need not be the case in the total Hilbert space.

We, on the other hand, aim to exclude the largest class of LRT, therefore we work with a unitary time evolution, a point of view we share with Refs. [42,55].

F. Outlook

In cases where QM does not violate the Bell inequality, we trace it back to the specific value of the internal parameter $x=\Delta m/\Gamma$, given by nature. And it does not indicate that these massive systems *have* real properties independent of the act of measurement. However, some of these quasispin eigenstates are difficult to detect experimentally, in this connection, the idea of the ''quasispin rotations,'' introducing appropriate kaon ''regenerators'' along the kaon flight paths, and the resulting Bell inequalities is of special interest (see, e.g., Refs. [25,26]).

An interesting feature of the neutral kaon systems in comparison with photon is that this system has CP violation. Although the Bell inequalities themselves are hard to check experimentally, they imply an inequality on the physical CP violation parameter ε or ε', which is experimentally testable.

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APPENDIX

1. Formula for the choice sensitive to the CP parameter ε

The (Y,Y) probabilities:

$$P_{K_S,\bar{K}^0}(Y,t_l;Y,t_r)=N_{SL}^2\frac{1}{4}(1-\delta)\{e^{-\Gamma_S t_l-\Gamma_L t_r}+\delta^2 e^{-\Gamma_L t_l-\Gamma_S t_r}+2\delta\cos(\Delta m\Delta t)e^{-\Gamma(t_l+t_r)}\},\tag{A1}$$

$$P_{K_S,K_1^0}(Y,t_l;Y,t_r)=N_{SL}^2\frac{1}{2}\frac{1}{1+|\varepsilon|^2}\{|\varepsilon|^2 e^{-\Gamma_S t_l-\Gamma_L t_r}+\delta^2 e^{-\Gamma_S t_r-\Gamma_L t_l}-2\delta\,\mathrm{Re}\{\varepsilon^* e^{-i\Delta m\Delta t}\}e^{\Gamma(t_l+t_r)}\},\tag{A2}$$

$$P_{K_1^0,\bar{K}^0}(Y,t_l;Y,t_r)=N_{SL}^2\frac{1}{4}\frac{1-\delta}{1+|\varepsilon|^2}\{e^{-\Gamma_S t_l-\Gamma_L t_r}+|\varepsilon|^2 e^{-\Gamma_L t_l-\Gamma_S t_r}+2\,\mathrm{Re}\{\varepsilon e^{-i\Delta m\Delta t}\}e^{-\Gamma(t_l+t_r)}\},\tag{A3}$$

$$P_{K_1^0,K_1^0}(Y,t_l;Y,t_r)=N_{SL}^2\frac{1}{2}\frac{|\varepsilon|^2}{(1+|\varepsilon|^2)^2}\{e^{-\Gamma_S t_l-\Gamma_L t_r}+e^{-\Gamma_L t_l-\Gamma_S t_r}-2\cos(\Delta m\Delta t)e^{-\Gamma(t_l+t_r)}\},\tag{A4}$$

The correction function:

$$h(K_S,t_a;\bar{K}^0,t_b;K_1^0,t_c;K_1^0,t_d) = -P_{K_S,\bar{K}^0}(Y,t_a;Y,t_b)+P_{K_S,K_1^0}(Y,t_a;Y,t_c) + P_{K_1^0,K_1^0}(Y,t_d;Y,t_b)+P_{K_1^0,\bar{K}^0}(Y,t_d;Y,t_c)+3 - N_{SL}^2\Big\{ e^{-\Gamma_S t_a}+\delta^2 e^{-\Gamma_L t_a}-2\delta^2\cos(\Delta m t_a)e^{-\Gamma t_a} + \frac{1}{1+|\varepsilon|^2}(e^{-\Gamma_S t_d}+|\varepsilon|^2 e^{-\Gamma_L t_d} - 2\delta\,\mathrm{Re}\{\varepsilon e^{-i\Delta m t_d}\}e^{-\Gamma t_d})+\frac{1-\delta}{2}(e^{-\Gamma_S t_b}+e^{-\Gamma_L t_b} + 2\delta\cos(\Delta m t_b)e^{-\Gamma t_b})+\frac{1}{1+|\varepsilon|^2}(e^{-\Gamma_S t_c}+|\varepsilon|^2 e^{-\Gamma_L t_c} - 2\delta\,\mathrm{Re}\{\varepsilon e^{-i\Delta m t_c}\}e^{-\Gamma t_c})\Big\}. \quad \text{(A5)}$$

2. Formula for the choice sensitive to the direct *CP* parameter ε'

The (Y,Y) probabilities:

$$P_{K^0,K_{00}}(Y,t_l;Y,t_r)=\frac{N_{SL}}{2}\frac{1+\delta}{2}\frac{1}{1+|\varepsilon|^2}\frac{1}{1+|r_{00}|^2} \times\{|\varepsilon_{00}^*+\varepsilon|^2 e^{-\Gamma_S t_l-\Gamma_L t_r}+|1+\varepsilon\varepsilon_{00}^*|^2 \times e^{-\Gamma_L t_l-\Gamma_S t_r}-2\,\mathrm{Re}\{(\varepsilon_{00}+\varepsilon^*) \times(1+\varepsilon\varepsilon_{00}^*)e^{-i\Delta m\Delta t}\}e^{-\Gamma(t_l+t_r)}\}, \quad \text{(A6)}$$

$$P_{K^0,K_{+-}}(Y,t_l;Y,t_r)=\frac{N_{SL}}{2}\frac{1+\delta}{2}\frac{1}{1+|\varepsilon|^2}\frac{1}{1+|r_{+-}|^2}\{|\varepsilon_{+-}^* +\varepsilon|^2 e^{-\Gamma_S t_l-\Gamma_L t_r}+|1+\varepsilon\varepsilon_{+-}^*|^2 \times e^{-\Gamma_L t_l-\Gamma_S t_r}-2\,\mathrm{Re}\{(\varepsilon_{+-}+\varepsilon^*) \times(1+\varepsilon\varepsilon_{+-}^*)e^{-i\Delta m\Delta t}\}e^{-\Gamma(t_l+t_r)}\}, \quad \text{(A7)}$$

$$P_{K_{+-},K_{00}}(Y,t_l;Y,t_r)=\frac{N_{SL}}{2}\frac{1}{(1+|\varepsilon|^2)^2}\frac{1}{1+|r_{00}|^2}\frac{1}{1+|r_{+-}|^2} \times\{|1+\varepsilon\varepsilon_{+-}^*|^2|\varepsilon_{00}^*+\varepsilon|^2 e^{-\Gamma_S t_l-\Gamma_L t_r} +|\varepsilon_{+-}^*+\varepsilon|^2|1+\varepsilon\varepsilon_{00}^*|^2 e^{-\Gamma_L t_l-\Gamma_S t_r} -2\,\mathrm{Re}\{(1+\varepsilon^*\varepsilon_{+-})(\varepsilon_{00}+\varepsilon^*) \times(\varepsilon_{+-}^*+\varepsilon)(1+\varepsilon\varepsilon_{00}^*)e^{-i\Delta m\Delta t}\} \times e^{-\Gamma(t_l+t_r)}\}, \quad \text{(A8)}$$

$$P_{K_{+-},K_{+-}}(Y,t_l;Y,t_r)=\frac{N_{SL}}{2}\frac{1}{(1+|\varepsilon|^2)^2}\frac{1}{(1+|r_{+-}|^2)^2} \times|1+\varepsilon\varepsilon_{+-}^*|^2|\varepsilon_{+-}^*+\varepsilon|^2 \times\{e^{-\Gamma_S t_l-\Gamma_L t_r}+e^{-\Gamma_L t_l-\Gamma_S t_r} -2\cos(\Delta m\Delta t)e^{-\Gamma(t_l+t_r)}\}. \quad \text{(A9)}$$

The correction function:

$$h(K^0,t_a;K_{00},t_b;K_{+-},t_c;K_{+-},t_d) = -P_{K^0,K_{00}}(Y,t_a;Y,t_b)+P_{K^0,K_{+-}}(Y,t_a;Y,t_c)+P_{K_{+-},K_{+-}}(Y,t_d;Y,t_b)+P_{K_{+-},K_{00}}(Y,t_d;Y,t_c)+3 - N_{SL}^2\Big\{\frac{1+\delta}{2}[e^{-\Gamma_S t_a}+e^{-\Gamma_L t_a}-2\delta\cos(\Delta m t_a)e^{-\Gamma t_a}]+\frac{1}{1+|\varepsilon|^2}\frac{1}{1+|\varepsilon_{+-}|^2}[|1+\varepsilon_{+-}^*\varepsilon|^2 e^{-\Gamma_S t_d}+|\varepsilon+\varepsilon_{+-}^*|^2 e^{-\Gamma_L t_d} -2\delta\,\mathrm{Re}\{(1+\varepsilon_{+-}\varepsilon^*)(\varepsilon+\varepsilon_{+-}^*)e^{-i\Delta m t_d}\}e^{-\Gamma t_d}]+\frac{1}{1+|\varepsilon|^2}\frac{1}{1+|\varepsilon_{00}|^2}[|1+\varepsilon_{00}^*\varepsilon|^2 e^{-\Gamma_S t_b}+|\varepsilon+\varepsilon_{00}^*|^2 e^{-\Gamma_L t_b} -2\delta\,\mathrm{Re}\{(1+\varepsilon_{00}\varepsilon^*)(\varepsilon+\varepsilon_{00}^*)e^{-i\Delta m t_b}\}e^{-\Gamma t_b}]+\frac{1}{1+|\varepsilon|^2}\frac{1}{1+|\varepsilon_{+-}|^2}[|1+\varepsilon_{+-}^*\varepsilon|^2 e^{-\Gamma_S t_c}+|\varepsilon+\varepsilon_{+-}^*|^2 e^{-\Gamma_L t_c} -2\delta\,\mathrm{Re}\{(1+\varepsilon_{+-}\varepsilon^*)(\varepsilon+\varepsilon_{+-}^*)e^{-i\Delta m t_c}\}e^{-\Gamma t_c}]\Big\}. \quad \text{(A10)}$$

[1] E. Schrödinger, Naturwissenschaften **23**, 807 (1935); **23**, 823 (1935); **23**, 844 (1935).
[2] A. Einstein, B. Podolski, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[3] W. H. Furry, Phys. Rev. **49**, 393 (1936); *ibid.* **49**, 476 (1936).
[4] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1964).
[5] S. J. Freedman and J. F. Clauser, Phys. Rev. Lett. **28**, 938 (1972).
[6] E. S. Fry and R. C. Thompson, Phys. Rev. Lett. **37**, 465 (1976).
[7] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 91 (1982); A. Aspect, J. Dalibard, and G. Roger, *ibid.* **49**, 1804 (1982).
[8] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **81**, 5039 (1998).
[9] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
[10] D. Deutsch and A. K. Ekert, Phys. World **11**, 47 (March 1998).
[11] R. J. Hughes, Contemp. Phys. **36**, 149 (1995).
[12] W. Tittel, G. Ribordy, and N. Gisin, Phys. World **11**, 41 (March 1998); W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, Phys. Rev. Lett. **81**, 3563 (1998).
[13] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[14] J-W. Pan, D. Bouwmeester, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).
[15] A. Zeilinger, Phys. World **11**, 35 (March 1998).
[16] D. Bouwmeester, A. Ekert, and A. Zeilinger, *The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computations* (Springer-Verlag, Berlin, 2000).
[17] T. D. Lee and N. C. Yang (unpublished).
[18] D. R. Inglis, Rev. Mod. Phys. **33**, 1 (1961).
[19] T. B. Day, Phys. Rev. **121**, 1204 (1961).
[20] H. J. Lipkin, Phys. Rev. **176**, 1715 (1968).
[21] P. H. Eberhard, in *The Second DaΦne Physics Handbook*, edited by L. Maiani, G. Pancheri, and N. Paver (SIS–Pubblicazioni dei Laboratori di Frascati, Italy, 1995), Vol. I, p. 99.
[22] P. H. Eberhard, Nucl. Phys. B **398**, 155 (1992).
[23] I. I. Bigi, Nucl. Phys. B (Proc. Suppl.) **24A**, 24 (1991).
[24] A. Di Domenico, Nucl. Phys. B **450**, 293 (1995).
[25] A. Bramon and M. Nowakowski, Phys. Rev. Lett. **83**, 1 (1999).
[26] B. Ancochea, A. Bramon, and M. Nowakowski, Phys. Rev. D **60**, 094 008 (1999).
[27] J. Six, Phys. Lett. B **114**, 200 (1982).
[28] A. Apostolakis *et al.*, Phys. Lett. B **422**, 339 (1998).
[29] R. A. Bertlmann, W. Grimus, and B. C. Hiesmayr, Phys. Rev. D **60**, 114 032 (1999).
[30] R. A. Bertlmann and W. Grimus, Phys. Lett. B **392**, 426 (1997).
[31] G. V. Dass and K. V. L. Sarma, Eur. Phys. J. C **5**, 283 (1998).
[32] R. A. Bertlmann and W. Grimus, Phys. Rev. D **58**, 034 014 (1998).
[33] A. Datta and D. Home, Phys. Lett. A **119**, 3 (1986).
[34] A. Pompili and F. Selleri, Eur. Phys. J. C **14**, 469 (2000).
[35] R. A. Bertlmann and W. Grimus, e-print hep-ph/0101160.
[36] F. Selleri, Lett. Nuovo Cim. **36**, 521 (1983); P. Privitera and F. Selleri, Phys. Lett. B **296**, 261 (1992); R. Foadi and F. Selleri, *ibid.* **461**, 123 (1999).
[37] A. Afriat and F. Selleri, *The Einstein, Podolsky, and Rosen Paradox in Atomic, Nuclear and Particle Physics* (Plenum Press, New York, 1999).
[38] J. Six, Phys. Lett. A **150**, 243 (1990).
[39] B. C. Hiesmayr, Found Phys. Lett. **14**, 231 (2001).
[40] T. D. Lee and C. S. Wu, Annu. Rev. Nucl. Sci. **16**, 511 (1966).
[41] J.S. Bell and J. Steinberger, *Weak Interactions of Kaons*, Proceedings of the Oxford International Conference, Sept. 10–25, 1965, p. 147 (unpublished).
[42] G. C. Ghirardi, R. Grassi, and T. Weber, in *Proceedings of Workshop on Physics and Detectors for DAΦNE, the Frascati Φ Factory*, April 9–12th, 1991, edited by G. Pancheri (unpublished).
[43] J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, England, 1987).
[44] J. E. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. D **10**, 880 (1969).
[45] J. S. Bell (Ref. [43]).
[46] J. E. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978).
[47] N. Gisin and A. Go, Am. J. Phys. **69**, 264 (2001).
[48] F. Uchiyama, Phys. Lett. A **231**, 295 (1997).
[49] Eur. Phys. J. C **3**, 1 (1998).
[50] F. Benatti and R. Floreanini, Phys. Rev. D **57**, 1332 (1998).
[51] F. Benatti and R. Floreanini, Eur. Phys. J. C **13**, 267 (2000).
[52] O. Nachtmann, *Phänomene und Konzepte der Elementarteilchenphysik* (Vieweg, Braunschweig, 1986).
[53] Q. Ho-Kim and P. X. Yem, *Elementary Particles and Their Interactions* (Springer Verlag, Berlin, 1998).
[54] G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation* (Oxford University Press, New York, 1999).
[55] G. C. Ghirardi, R. Grassi, and R. Ragazzon, in *The DAΦNE Physics Handbook*, edited by L. Maiani, G. Pancheri, and N. Paver (Servizio Documentazione dei Laboratori Nazionale di Frascati, Italy, 1992), Vol. I, p. 283.
[56] B. C. Hiesmayr, Diploma thesis, University of Vienna, 1999.