Quantum Zeno and anti-Zeno effects in the Friedrichs model

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We analyze the short-time behavior of the survival probability in the frame of the Friedrichs model for different form factors. We have shown that this probability is not necessarily analytic at the time origin. The time when the quantum-Zeno effect could be observed is found to be much smaller than usually estimated. We have also studied the anti-Zeno period and have estimated its duration.

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I. INTRODUCTION

Since the very beginning of quantum mechanics, the measurement process has been a most fundamental issue. The main characteristic feature of the quantum measurement is that the measurement changes the dynamical evolution. This is the main difference in the quantum measurement compared to its classical analogue. In this framework, Misra and Sudarshan pointed out [1] that repeated measurements can prevent an unstable system from decaying. Indeed, as the survival probability is in most cases proportional to the square of the time for short times (see, however, the discussion below), the measurement effectively projects the evolved state back to the initial state with such a high probability that the sequence of the measurements ''freezes'' the initial state. Led by analogy with the Zeno paradox, this effect has been called the quantum-Zeno effect (QZE).

Cook [2] suggested an experiment on the QZE that was realized by Itano *et al.* [3]. In this experiment, the Rabi oscillations have been used in order to demonstrate that the repeated observations slow down the transition process. However, the detailed analysis [4–6] has shown that the results of this experiment could equally well be understood using a density matrix approach for the whole system. Recently, an experiment similar to Ref. [3] has been performed by Balzer *et al.* [7] on a single trapped ion. This experiment has removed some drawbacks usually associated with the experiment of Itano *et al.* [3], for example, dephasing a system's wave function caused by a large ensemble and the nonrecording of the results of the intermediate measurements pulses. We refer to recent reviews [8,9] for detailed discussions of these and related questions.

Both experiments [3,7] demonstrate the perturbed evolution of a coherent dynamics, as opposed to spontaneous decay. So, the demonstration of the QZE for an unstable system with exponential decay, as originally proposed in Ref. [1], is still an open question. The main problem in such an experimental observation of the Zeno effect is the very short time when the quadratic behavior of the transition amplitude is valid [10,11]. On the other hand, the Zeno-type experiment could also reveal deviations from the exponential decay law and the magnitude of these deviations.

The QZE has been discussed for many physical systems including atomic physics [10–13], radioactive decay [14], and mesoscopic physics [15–18], and has even been proposed as a way to control decoherence for effective quantum computations [19]. Recently, however, a quantum anti-Zeno effect has been found [20,21]. Under some conditions the repeated observations could speed up the decay of the quantum system. The anti-Zeno effect has been further analyzed in [18,22–25].

We carefully analyze here the short-time behavior of the survival probability in the frame of the Friedrichs model [26]. We have shown that this probability is not necessarily analytic at zero time. Furthermore, the probability may not even be quadratic for the short times while the QZE still exists in such a case [20,27]. We have shown (see also Kofman and Kurizki [24]) that the time period within which the QZE could be observed is much smaller than previously believed. Hence, we conclude that the experimental observation/realization of the QZE is quite challenging.

We have also analyzed the anti-Zeno period. While it seems that most decaying systems exhibit anti-Zeno behavior, our examples contradict the estimations of Lewenstein and Rzazewski [22]. We have studied the duration of the anti-Zeno period and have estimated this duration when possible.

II. MODEL AND EXACT SOLUTION

The Hamiltonian of the second quantized formulation of the Friedrichs model [26] is

$$H = H_0 + \lambda V, \tag{1}$$

where the unperturbed Hamiltonian is defined as

$$H_0 = \omega_1 a^\dagger a + \int_0^\infty d\omega \omega b_\omega^\dagger b_\omega ,$$

and the interaction is

$$V = \int_0^\infty d\omega f(\omega)(a b_\omega^\dagger + a^\dagger b_\omega). \tag{2}$$

Here, $a^\dagger$, a are creation and annihilation boson operators of the atom excitation, $b^\dagger_\omega$, b_ω are creation and annihilation boson operators of the photon with frequency ω, $f(\omega)$ is the form factor, λ is the coupling parameter, and the vacuum energy is chosen to be zero. The creation and annihilation operators satisfy the following commutation relations:

$$[a,a^\dagger]=1, \quad [b_\omega,b^\dagger_{\omega'}]=\delta(\omega-\omega'). \tag{3}$$

All other commutators vanish.

The Hamiltonian H_0 has continuous spectrum $[0,\infty)$ of uniform multiplicity, and the discrete spectrum $n\omega_1$ (with integer n) is embedded in the continuum. The space of the wave functions is the direct sum of the Hilbert space of the oscillator and the Fock space of the field.

For $\omega_1>0$, the oscillator excitations are unstable due to the resonance between the oscillator energy levels and the energy of a photon. Therefore, the total evolution leads to the decay of a wave packet corresponding to the bare atom $|1\rangle$. Decay is described by the survival probability $p(t)$ to find, after time t, the bare atom evolving according to the evolution $\exp(-iHt)$ in its excited state [11]:

$$p(t)\equiv|\langle 1|e^{-iHt}|1\rangle|^2. \tag{4}$$

The survival probability can be easily calculated in the second quantized representation:

$$\begin{aligned} p(t)&=|\langle 0|a(0)e^{-iHt}a^\dagger(0)|0\rangle|^2\\ &=|\langle 0|e^{-iHt}e^{iHt}a(0)e^{-iHt}a^\dagger(0)|0\rangle|^2\\ &=|\langle 0|a(t)a^\dagger(0)|0\rangle|^2, \end{aligned}$$

where $a^\dagger(0)=a^\dagger$. The time evolution of $a(t)$ in the Heisenberg representation is presented in Appendix A. Using Eq. (A12) we obtain

$$p(t)=|A(t)|^2,$$

where the survival amplitude $A(t)$ is given by Eq. (A14).

Due to the dimension argument, we can write the form factor $f(\omega)$ in the form

$$f^2(\omega)=\Lambda\varphi\left(\frac{\omega}{\Lambda}\right),$$

where $\varphi(x)$ is a dimensionless function. Here, Λ is a parameter with the dimension of ω. The survival amplitude $A(t)$ in the dimensionless representation is

$$A(t)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}dy\frac{e^{iy\Lambda t}}{\eta^-_\Lambda(y)}, \tag{5}$$

where

$$\eta^-_\Lambda(z)=\omega_\Lambda-z-\lambda^2\int_0^\infty dx\frac{\varphi(x)}{x-z+i0}, \tag{6}$$

and $\omega_\Lambda=\omega_1/\Lambda$.

III. SHORT-TIME BEHAVIOR AND THE ZENO REGION

The short time evolution of the model (1) depends essentially on the form factor. In order to illustrate different types of evolution, we shall consider two form factors, namely

$$f_1^2(\omega)=\Lambda\frac{\sqrt{\frac{\omega}{\Lambda}}}{1+\frac{\omega}{\Lambda}}, \quad \varphi_1(x)=\frac{\sqrt{x}}{1+x}, \tag{7}$$

and

$$f_2^2(\omega)=\Lambda\frac{\frac{\omega}{\Lambda}}{\left[1+\left(\frac{\omega}{\Lambda}\right)^2\right]^2}, \quad \varphi_2(x)=\frac{x}{(1+x^2)^2}. \tag{8}$$

The form factor f_1 permits exact calculations [28,29]. It turns out that the short-time behavior is not quadratic [20,27] as anticipated by [22]. We shall also use, for comparison the results presented in [10,11] for the form factor $\varphi_3(x)=x/(1+x^2)^4$ [30]. To get a first impression about the time scales, we associate with each form factor a physical system: the photodetachement process for $\varphi_1(x)$ [22,28,31], the quantum dot for $\varphi_2(x)$ [32], and the hydrogen atom for $\varphi_3(x)$ [10]. The corresponding numerical values of the parameters Λ, ω_1, and λ^2 are listed in Table I. We would like to emphasize that these values (as well as the model itself) are approximate estimations of the corresponding effects.

Let us discuss the short-time behavior of the survival probability $p(t)$. We shall assume here the existence of all necessary matrix elements, and denote $\langle\cdot\rangle=\langle 1|\cdot|1\rangle$.

$$\begin{aligned} p(t)&=\langle e^{-iHt}\rangle\\ &=\left\langle 1-iHt-\frac{1}{2}H^2t^2+\frac{i}{6}H^3t^3+\frac{1}{24}H^4t^4+O(t^5)\right\rangle\\ &=\left(1-\frac{t^2}{2}\langle H^2\rangle+\frac{t^4}{24}\langle H^4\rangle\right)^2+\left(t\langle H\rangle-\frac{t^3}{6}\langle H^3\rangle\right)^2\\ &\quad+O(t^6)\\ &=1-t^2(\langle H^2\rangle-\langle H\rangle^2)+t^4\left(\frac{1}{4}\langle H^2\rangle^2+\frac{1}{12}\langle H^4\rangle\right.\\ &\quad\left.-\frac{1}{3}\langle H\rangle\langle H^3\rangle\right)+O(t^6)\\ &=1-\frac{t^2}{t_a^2}+\frac{t^4}{t_b^4}+O(t^6). \end{aligned} \tag{9}$$

In order to calculate the parameters t_a and t_b, we need to calculate the averages of the powers of $H=H_0+\lambda V$:

$$\langle H\rangle=\omega_1,$$

$$\langle H^2\rangle=\omega_1^2+\lambda^2\langle V^2\rangle,$$

TABLE I. The Zeno time t_Z, the time t_a, the decay time t_d, and the time t_{ep} of the transition from the exponential to power law decay for different model of interactions and for different physical systems. Numerical values are given in seconds and in units of t_d.

Form factor $\varphi(x)$	$\frac{\sqrt{x}}{1+x}$	$\frac{x}{(1+x^2)^2}$	$\frac{x}{(1+x^2)^4}$
t_Z	$\frac{32}{9\pi}\frac{1}{\Lambda}$	$\frac{\sqrt{6}}{\Lambda\sqrt{\left\lvert\ln\left(\frac{2\sqrt{6}\omega_1}{\Lambda}\right)\right\rvert}}$	$\frac{2\sqrt{6}}{\Lambda}$
t_a	$\frac{\left(\frac{3}{4\sqrt{2\pi}}\right)^{2/3}}{\lambda^{4/3}\Lambda}$	$\frac{\sqrt{2}}{\lambda\Lambda}$	$\frac{\sqrt{6}}{\lambda\Lambda}$
t_d	$\frac{1}{\pi\lambda^2\sqrt{\Lambda\tilde{\omega}_1}}$	$\frac{1}{2\pi\lambda^2\omega_1}$	$\frac{1}{2\pi\lambda^2\omega_1}$
t_{ep}	$-\frac{5\ln\left(\lambda^4\frac{\Lambda}{\tilde{\omega}_1}\right)}{4\pi\lambda^2\sqrt{\Lambda\tilde{\omega}_1}}$	$-\frac{2\ln(2\pi\lambda^3)}{\pi\lambda^2\omega_1}$	$-\frac{2\ln(2\pi\lambda^3)}{\pi\lambda^2\omega_1}$
System	Photodetachement	Quantum dot	Hydrogen atom
$\Lambda(\mathrm{s}^{-1})$	1.0×10^{10}	1.67×10^{16}	8.498×10^{18}
$\omega_1(\mathrm{s}^{-1})$	2.0×10^{4}	7.25×10^{12}	1.55×10^{16}
λ^2	3.18×10^{-7}	3.58×10^{-6}	6.43×10^{-9}
t_Z(s) $[t_d]$	1.1×10^{-10} $[1.1\times10^{-9}]$	5.9×10^{-17} $[9.7\times10^{-9}]$	5.76×10^{-19} $[3.6\times10^{-10}]$
t_a(s) $[t_d]$	9.6×10^{-7} $[9.6\times10^{-6}]$	4.5×10^{-14} $[7.4\times10^{-6}]$	3.59×10^{-15} $[2.2\times10^{-6}]$
t_d(s) $[t_d]$	0.1 [1]	6.1×10^{-9} [1]	1.60×10^{-9} [1]
t_ep(s) $[t_d]$	1.7 [17]	4.2×10^{-7} [69]	1.69×10^{-7} [110]

$$\langle H^3\rangle=\omega_1^3+2\lambda^2\omega_1\langle V^2\rangle+\lambda^2\langle VH_0V\rangle,$$

$$\langle H^4\rangle=\omega_1^4+\lambda^2(3\omega_1^2\langle V^2\rangle+2\omega_1\langle VH_0V\rangle+\langle VH_0^2V\rangle)+\lambda^4\langle V^4\rangle. \quad (10)$$

These expressions are valid in our model because of the special structure of the potential V (2). Now we can find

$$\frac{1}{t_a^2}=\lambda^2\langle V^2\rangle=\lambda^2\Lambda^2 I_0,$$

$$\frac{1}{t_b^4}=\lambda^2\left(\frac{\omega_1^2}{12}\Lambda^2 I_0-\frac{\omega_1}{6}\Lambda^3 I_1+\frac{\Lambda^4}{12}I_2\right)+\lambda^4\Lambda^4\left(\frac{I_0^2}{4}+\frac{\int_0^\infty\varphi^2(x)dx}{12}\right), \quad (11)$$

where

$$I_k=\int_0^\infty x^k\varphi(x)dx.$$

In the weak-coupling models, the following inequalities are satisfied (see Table I):

$$\lambda^2\ll 1 \quad \text{and} \quad \Lambda\gg\omega_1. \quad (12)$$

In this approximation we can simplify the expression for t_b:

$$\frac{1}{t_b^4}\approx\frac{\lambda^2\Lambda^4}{12}I_2. \quad (13)$$

The parameter t_a has been called Zeno time [10,11] because it has been conjectured to be related to the Zeno region, i.e., the region where the decay is slower than the exponential one and the Zeno effect can manifest. On the other hand, a more precise estimation reveals that the Zeno region is, in fact, orders of magnitude shorter than t_a. We illustrate this in Fig. 1 where the survival probabilities for the form factors $\varphi_1(x)$ and $\varphi_2(x)$ are plotted. The corresponding analytical expressions and the numerical values for different time scales are presented in Table I. We see that $p(t)$ is not convex, already at times much shorter than the time t_a.

In view of this fact, we propose another definition for the Zeno time. As one refers in discussions about the Zeno effect on the expansion of survival probability for small times, and specifically on the second term, we shall define the Zeno time t_Z as corresponding to the region where the second term

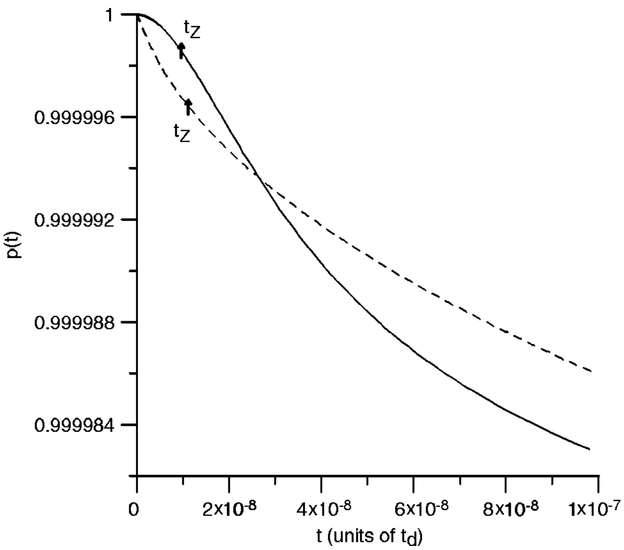

FIG. 1. The survival probability $p(t)$ for the photodetachement model [$\varphi_1(x)=\sqrt{x}/(1+x)$, the dashed line], and for the quantum dot model [$\varphi_2(x)=x/(1+x^2)^2$, the solid line]. The Zeno time t_Z is indicated. Time is in units of the decay time t_d.

dominates. Hence, the introduced time t_Z is a natural boundary where the second and third terms have the same amplitude:

$$\frac{t_Z^2}{t_a^2}=\frac{t_Z^4}{t_b^4}, \quad \text{so} \quad t_Z=t_b^2/t_a. \tag{14}$$

In the weak-coupling models,

$$t_Z=\frac{1}{\Lambda}\sqrt{\frac{12I_0}{I_2}},$$

that agrees with the estimation in [24]. This time is much shorter than $t_a\sim 1/\lambda\Lambda$ and agrees much better with the numerical estimations. For example, for the interaction $\varphi_3(x)$ we find $t_Z=2\sqrt{6}/\Lambda\approx 5.8\times 10^{-19}$ s while $t_a=\sqrt{6}/\lambda\Lambda\approx 3.6\times 10^{-16}$ s [10].

Our conclusions are in fact valid for a rather wide class of interactions. Namely, they are valid if the matrix elements (10) exist and conditions (12) are satisfied. For example, any bounded locally integrable interaction $\varphi(x)$ decreasing as $\varphi(x)\sim C/x^{1.5+\epsilon}$, $\epsilon>0$ at $x\to\infty$, gives finite matrix elements (10). Furthermore, we show below that the relation $t_R\ll t_Z$ may be valid even when the matrix elements (10) do not exist.

For the form factor φ_1, already the matrix element $\langle V^2\rangle$ does not exist, and the short time expansion is written (see Appendix B) as

$$p(t)=1-\left(\frac{t}{t_a}\right)^{1.5}+\left(\frac{t}{t_b}\right)^2+O(t^{5/2}),$$

where $t_a=(3/(4\sqrt{2\pi}))^{2/3}/(\lambda^{4/3}\Lambda)$, and $t_b=1/(\sqrt{\pi}\lambda\Lambda)$. In fact, from the representation (10) one can easily deduce that for any form factor decreasing according to the power law when $x\to\infty$, $p(t)$ is not analytic at $t=0$. Specifically, if the form factor decreases as $\varphi(x)\sim x^{\alpha}$ when $x\to\infty$, only the Taylor coefficients up to t^n with $n<1+|\alpha|$ can be defined.

Following the previous discussion, for the form factor $\varphi_1(x)$ the Zeno time t_Z can be estimated by the condition

$$\left(\frac{t_Z}{t_a}\right)^{1.5}=\left(\frac{t_Z}{t_b}\right)^2, \quad \text{so} \quad t_Z=\frac{32}{9\pi\Lambda}.$$

For this case, one can see that the time t_a has scaling properties which differ from Eq. (11) while the Zeno time t_Z has a value similar to Eq. (14).

For φ_2, the matrix element $\langle V^2\rangle$ exists so the usual time t_a can be introduced: $t_a=\sqrt{2}/\lambda\Lambda$. However, $\langle VH_0^2V\rangle$ does not exist and the asymptotic behavior of $p(t)$ is (see Appendix C)

$$p(t)=1-\left(\frac{t}{t_a}\right)^2-\frac{\lambda^2}{12}\ln(2\omega_1 t)\Lambda^4t^4+O(t^4).$$

Repeating the arguments concerning the Zeno region, we find $t_Z=\sqrt{6}/\Lambda\sqrt{|\ln(2\sqrt{6}\omega_1/\Lambda)|}$. In this case one can see again that the Zeno time t_Z has a value similar to Eq. (14), and the inequality $t_Z\ll t_a$ is satisfied.

IV. ZENO AND ANTI-ZENO EFFECTS

The probability that the state $|1\rangle$ after N equally spaced measurements during the time interval $[0,T]$ has not decayed, is given by [1]

$$\begin{aligned} p_N(T)&=\langle 1|(|1\rangle\langle 1|e^{-iHT/N})^N|1\rangle\\ &=p^N(T/N)\langle 1|1\rangle=p^N(T/N). \end{aligned} \tag{15}$$

Expression (15) is only correct for the ideal von Neumann measurements [1]. We are interested in the behavior of $p_N(T)$ as $N\to\infty$ or, equally, when the time interval between the measurements $\tau=T/N$ goes to zero:

$$\begin{aligned} \lim_{\tau\to 0} p_N(T)&=\lim_{\tau\to 0} p(\tau)^{T/\tau}\\ &=\left[\lim_{\tau\to 0}\left(1-\frac{\frac{1-p(\tau)}{\tau}}{\frac{1}{\tau}}\right)^{1/\tau}\right]^T\\ &=\begin{cases} 0, & \text{when} \quad p'(0)=-\infty,\\ e^{-cT}, & \text{when} \quad p'(0)=-c,\\ 1, & \text{when} \quad p'(0)=0. \end{cases} \end{aligned} \tag{16}$$

Hence, for the case $p(t)=1-ct^{\alpha}$ one has the Zeno effect for all $\alpha>1$ [20,27]. We should notice that in case of the linear asymptotics of $p(t)$ at short times (in particular, for the purely exponential decay) there is no Zeno effect, and the probability to find the system in the initial state $|1\rangle$ decreases exponentially with the time of observation. The results (16) are found in case of continuously ongoing measurements during the entire time interval $[0,T]$. Obviously, this is an idealization. In practice, we have a manifestation of the Zeno effect, if the probability (15) increases as the time interval τ between measurements decreases. Formula (16) may be accepted as an approximation for a very short time interval $\tau \ll t_Z$. For longer times, we cannot use the Taylor expansion, therefore Eq. (16) is not valid. It appears that in order to analyze longer time behavior, the long time asymptotics of the $p(t)$ are more convenient. These asymptotics can be summarized as follows (see Appendicies B, C):

$$p(t) \approx |A_1|^2 e^{-4\gamma\sqrt{\omega_1 \Lambda t}} + \frac{\pi\lambda^4\Lambda}{4\omega_1^4 t^3} h_1^2(t) - \frac{\sqrt{\pi}\lambda^2\Lambda^{1/2}}{\omega_1^2 t^{3/2}} |A_1| h_1(t) e^{-2\gamma\sqrt{\omega_1\Lambda t}} \cos(\omega_1 t - \pi/4), \tag{17}$$

when $t \gg 24/\omega_1$ for the $\varphi_1(x)$, and

$$p(t) \approx |A_2|^2 e^{-\gamma_1 \Lambda t} + \frac{\lambda^4}{\omega_1^4 t^4} h_2^2(t) - \frac{2\lambda^2 e^{-\gamma_1\Lambda t/2}}{\omega_1^2 t^2} |A_2| h_2(t) \cos(\omega_1 t), \tag{18}$$

when $t \gg 4/\omega_1$ for the $\varphi_2(x)$. Here, the constants A_1, A_2 satisfy the inequality $|1-|A_k|^2| \ll 1$, $k=1,2$. The functions h_1, h_2 have the following asymptotic properties:

$$\lim_{t\to\infty} h_1(t) = 1; \quad \lim_{t\to 0} \frac{h_1(t)}{t^{3/2}} = \text{const};$$

$$\lim_{t\to\infty} h_2(t) = 1; \quad \lim_{t\to 0} \frac{h_2(t)}{t^2} = \text{const}.$$

In paper [10], an expression very similar to Eq. (18) was found for the form factor $\varphi_3(x)$. Expressions (17),(18) are analytically established only in the region $t \gg C/\omega_1$. However, the numerical investigation shows that for our choice of parameters, we can use Eqs. (17),(18) for a qualitative description already in the region $t \sim 1/\omega_1$. Then one can see that the oscillation with the frequency ω_1 starts always with the negative cosine wave. Therefore, the survival probability (4) turns out to be less than purely exponential, and one can expect decreasing of the probability $p_N(T)$ as well. We illustrate this effect in Fig. 2 for both the photodetachement process and the quantum dot. The anti-Zeno region (AZ region), i.e., the region where the probability $p_N(T)$ is less than purely exponential, is clearly seen for both systems. For $\tau \to 0$, $p_N(T)$ approaches 1 according to Eq. (16).

We should stress that the above-described behavior shows that the initial quadratic behavior is not just a beginning of the first wave of oscillation as stated in Ref. [10]. This is true because the time t_a is actually not the time within which $p(t)$ has quadratic behavior. In fact, the quadratic behavior is only valid for $t \ll t_Z$ and has nothing in common with the oscillations in Eqs. (17),(18).

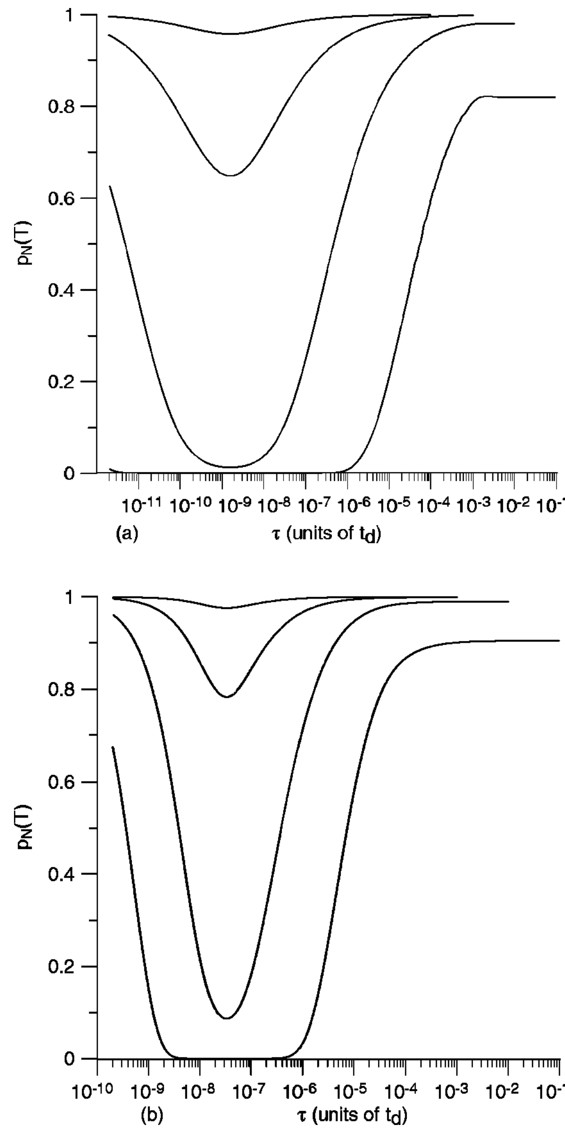

FIG. 2. The probability $p_N(T)$ [Eq. (15)] as a function of the duration τ between measurements. From above, the curves correspond to the time of observation $T=10^{-4}$, 10^{-3}, 10^{-2}, and 10^{-1}, respectively. T and τ are in units of the decay time t_d. The photodetachement model $[\varphi_1(x)=\sqrt{x}/(1+x)]$ [Fig. 2(a)] and the quantum dot model $[\varphi_2(x)=x/(1+x^2)^2]$ [Fig. 2(b)] are presented.

On the basis of Fig. 2 we would like to make some additional remarks. First of all, for lager observation time T, the AZ region is wider and the probability $p_N(T)$ in the AZ region is lower. This is natural: the bigger the time of observation is, the harder to restore the initial state of the system. Second, one can see that the value t_Z describes very well a minimum of the probability in the AZ region. For shorter times, $p_N(T)$ increases, but it still may be much less compared to the $p_N(T)$ in the purely exponential region. Hence, the classical Zeno effect [1] could be observed only when $t \ll t_Z$. Finally, we also notice that one can sometimes observe the second wave of oscillation in Eqs. (17,18) [see Fig. 2(a)]. However, its amplitude is much less than the amplitude of the first wave.

A general consideration of the AZ region is presented in

Ref. [22]. The authors conclude that the AZ region exists for all generic weakly coupled decaying systems. Under some assumptions, they have found that

$$|A_k|^2 < 1, \tag{19}$$

and use this condition for the explanation of the existence of the AZ region. However, some assumptions made in Ref. [22] for the derivation of Eq. (19) are not always valid. For example, for the model $\varphi_1(x)$ (the form factor used in Ref. [22]) one calculates $|A_1|^2 \approx 1 + 1.1\times 10^{-6} > 1$ for our choice of the parameters. For the model $\varphi_2(x)$ we have $|A_2|^2 \approx 1 + \lambda^2(3 + 2\log \omega_\Lambda) < 1$, but this effect is of the second order in the coupling while in Ref. [22] the fourth order was found. Hence, the above-mentioned results cannot be considered as a proof of the existence of the AZ region.

Indeed, our results show that there exist two different types of the AZ region. The first case takes place as the amplitude of oscillations in Eqs. (17,18) is less than $|1 - |A|^2|$, and $|A|^2 < 1$. This corresponds to the arguments of Ref. [22]. In this situation, the survival probability is always less than the ''ideal'' one corresponding to the pure exponential decay (except for the very short times $t \ll t_Z$). The second case arises when the amplitude of oscillations in Eqs. (17,18) is bigger than $|1 - |A|^2|$ (for any $|A^2|$), or when $|A^2| > 1$. In this case, the survival probability may be lower or higher than the ''ideal'' one, that may result in oscillations of the probability $p_N(T)$. This is exactly the situation in Fig. 2(a).

It would be very interesting to find an estimation for the duration of the AZ region. We have found that the minimum of the $p_N(T)$ is reached at t_Z, however the whole region is much wider. Unfortunately, we can present this estimation only for the second type of the AZ region. In order to illustrate this, we plot in Fig. 3 the value $N_\varepsilon(T)$ defined as

$$p_{N_\varepsilon(T)}(T) = (1-\varepsilon)p_1(T). \tag{20}$$

This value gives the maximum number of repeated observation such that the probability $p_N(T)$ would not be less than $p(T)$ with accuracy ε. The difference between two types of the AZ region is very pronounced. For the first type [$\varphi_2(x)$ interaction] $N_\varepsilon(T) \sim C\varepsilon$ and is almost independent of the time T of observation. It means that the anti-Zeno region t_{AZ} should be described as $t_{AZ} \sim cT/\varepsilon$. So the duration depends critically on the time of observation and the accuracy, and cannot be attributed to the properties of the system itself.

For the second type [$\varphi_1(x)$ interaction] $N_\varepsilon(T) \sim CT$ and is almost independent of the accuracy ε. This means that t_{AZ} is independent of the time of the observation and the accuracy, so it can be correctly introduced. In fact, in this case, t_{AZ} is defined by the oscillations of the survival probability and can be estimated as $1/\omega_1$.

The estimation $t_{AZ} \ll 1/\omega_1$ was given by Kofman and Kurizki [24]. While this estimation obviously holds, it is necessary to establish more precise boundaries for t_{AZ}. We have found the boundary $t_{AZ} \sim 1/\omega_1$ for the $\varphi_1(x)$ interaction. However, from the results presented in Fig. 3, one can see

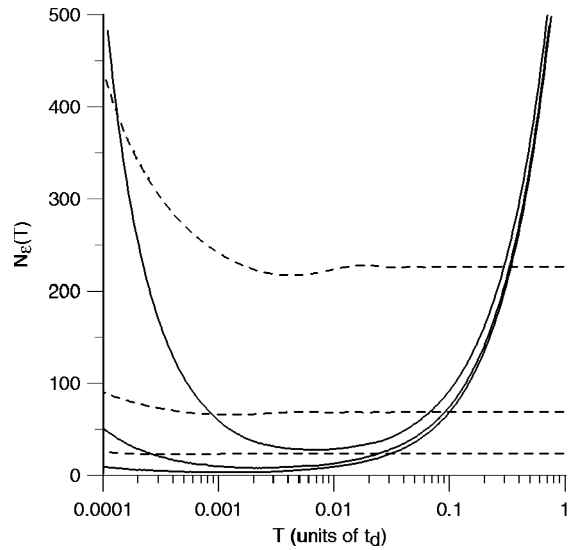

FIG. 3. The value $N_\varepsilon(T)$ [Eq. (20)] as a function of observation time T. From above, the curves correspond to the accuracy $\varepsilon = 10^{-2}$, 3×10^{-3}, and 10^{-3}, respectively. The solid lines are for the photodetachement model [$\varphi_1(x) = \sqrt{x}/(1+x)$], and the dashed lines are for the quantum dot model [$\varphi_2(x) = x/(1+x^2)^2$]. T is in units of the decay time t_d.

that for the interactions $\varphi_2(x)$ and $\varphi_3(x)$ the estimation $1/\omega_1$ can hardly be used, contrary to the results of [24].

We would like to mention that the estimation $t_{DC} = 1/\omega_1$ has been obtained by Petrosky and Barsegov [33] as an upper boundary of the decoherence time marking the onset of the exponential era. As the Zeno effect cannot be realized for times larger than t_{DC}, Petrosky and Barsegov called t_{DC} the Zeno time. In fact, this is a rough estimation of the real Zeno time t_Z.

V. CONCLUSION

Let us summarize the short-time behavior of the survival probability. We introduce two regions: the very short Zeno region t_Z with the scale $1/\Lambda$ and the much longer anti-Zeno region t_{AZ}. If one performs a Zeno-type experiment, and the time between measurements is much shorter than t_Z, then the Zeno effect – increasing of the survival probability – can be observed. In the time range between t_Z and t_{AZ}, the anti-Zeno effect exists, i.e., decay is accelerated by repeated measurements. That is why the Zeno time cannot be longer than t_Z. The previous estimations of the Zeno time t_a [10,11] and t_{DC} [33] are much longer than our estimation t_Z for physically relevant systems (12).

While the acceleration of decay is clearly seen in all cases, it is not always possible to introduce the value t_{AZ}. The reason is the possible dependence of t_{AZ} on the moment of the observation and on the accuracy of the observation. When this dependence is absent, one finds $t_{AZ} \sim 1/\omega_1$. Hence, the anti-Zeno region is, for typical values of parameters, a few orders of magnitude longer than the Zeno region. It would be very important from the experimental point of view, to find an estimation for the anti-Zeno region in terms of the initial parameters without any reference to the constant A_k.

It is possible, in principle, that the oscillations in Eqs. (17,18) may give a few successive Zeno and anti-Zeno regions. However, as the amplitude of the oscillations decreases exponentially with time, these regions are hardly visible. After the anti-Zeno region, the system decays exponentially up to the time t_{ep} when the long-tail asymptotics substitutes the exponential decay.

In accordance with this picture, the experimental observation of the Zeno effect is very difficult. Indeed, the Zeno region appears to be considerably shorter than previously believed. The acceleration of the decay should be observed before the deceleration will be possible. In this connection, the proposals for using the Zeno effect for increasing of the decoherence time [19] should be critically analyzed. We conclude that the Zeno effect may not be very appropriate for decoherence control desired for quantum computations.

There seems to be no place for the usual estimations of the Zeno time by t_a. There are no physical effects that can be associated with this time scale. In our opinion, the widespread expectation that the time t_a describes the Zeno region, is based on a naive perturbation theory. One could assume that $p(t)=1-\Sigma_{k=2}^{\infty}c_k(\lambda t)^k$, where c_k is defined in terms of the matrix elements of the interactions and are independent of λ. In this case, all terms in the series for $p(t)$ have the same order at t_a. However, this assumption is not true as H_0 and V do not commute, hence $\langle 1|e^{-iHt}|1\rangle \neq \langle 1|e^{-i\lambda Vt}|1\rangle$.

We would like to mention a few interesting problems related to the Zeno effect. (1) A better characterization of the anti-Zeno region. This problem is relevant to the experimental demonstration of the (anti-) Zeno behavior of the survival probability. (2) How the nonideal measurements influence the Zeno effect? (3) Is the asymptotic quantum-Zeno dynamics $\lim_{N\to\infty}p_N(T)$ governed by a unitary group or a semigroup of isometries or contractions [34]? This question defines if the quantum-Zeno dynamics introduces irreversibility in the evolution of a system.

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APPENDIX A: TIME EVOLUTION IN THE HEISENBERG REPRESENTATION

The second quantized form of the well-known Friedrichs model [26] is given by the Hamiltonian (1). For $\omega_1>0$, the oscillator excitations are unstable due to the resonance between the oscillator energy levels and the energy of a photon. Strong interaction, however, may lead to the emergence of a bound state. In weak-coupling cases discussed here, bound states do not arise [see Eq. (A5) below].

The solution of the eigenvalue problem

$$[H,B_\omega^\dagger]=\omega B_\omega^\dagger \quad \text{and} \quad [H,B_\omega]=-\omega B_\omega, \tag{A1}$$

obtained with the usual procedure of the Bogolubov transformation [35,36] is

$$(B_\omega^\dagger)_{\substack{\text{in}\\ \text{out}}}=b_\omega^\dagger+\frac{\lambda f(\omega)}{\eta^{\pm}(\omega)}\int_0^\infty d\omega'\lambda f(\omega')\left(\frac{b_{\omega'}^\dagger}{\omega'-\omega\mp i0}-a^\dagger\right), \tag{A2}$$

$$(B_\omega)_{\substack{\text{in}\\ \text{out}}}=b_\omega+\frac{\lambda f(\omega)}{\eta^{\mp}(\omega)}\int_0^\infty d\omega'\lambda f(\omega')\left(\frac{b_{\omega'}}{\omega'-\omega\pm i0}-a\right). \tag{A3}$$

In Eqs. (A2), (A3) we used the notation $1/\eta^{\pm}(\omega)\equiv 1/\eta(\omega \pm i0)$ where the function $\eta(z)$ of the complex argument z is

$$\eta(z)=\omega_1-z-\int_0^\infty d\omega\frac{\lambda^2 f^2(\omega)}{\omega-z}. \tag{A4}$$

The following condition on the form factor $f(\omega)$

$$\omega_1-\int_0^\infty d\omega\frac{\lambda^2 f^2(\omega)}{\omega}>0, \tag{A5}$$

guarantees that the function $1/\eta(z)$ is analytic everywhere on the first sheet of the Riemann manifold except for the cut $[0,\infty)$. Therefore, the total Hamiltonian H has no discrete spectrum and there are no bound states.

The incoming and outgoing operators $(B_\omega^\dagger)_{\substack{\text{in}\\ \text{out}}}$, $(B_\omega)_{\substack{\text{in}\\ \text{out}}}$ satisfy the following commutation relation

$$[(B_\omega)_{\substack{\text{in}\\ \text{out}}},(B_{\omega'}^\dagger)_{\substack{\text{in}\\ \text{out}}}]=\delta(\omega-\omega'). \tag{A6}$$

The other commutators vanish. The bare vacuum state $|0\rangle$ satisfying

$$a_1|0\rangle=b_\omega|0\rangle=0,$$

is also the vacuum state for the new operators:

$$(B_\omega)_{\substack{\text{in}\\ \text{out}}}|0\rangle=0.$$

Therefore, the new operators diagonalize the total Hamiltonian (1) as

$$H=\int_0^\infty d\omega\,\omega(B_\omega^\dagger)_{\substack{\text{in}\\ \text{out}}}(B_\omega)_{\substack{\text{in}\\ \text{out}}}. \tag{A7}$$

Using the inverse relations

$$b_\omega^\dagger=(B_\omega^\dagger)_{\text{in}}-\lambda f(\omega)\int_0^\infty d\omega'\frac{\lambda f(\omega')}{\eta^{-}(\omega')}\frac{(B_{\omega'}^\dagger)_{\text{in}}}{\omega'-\omega-i0}, \tag{A8}$$

$$b_\omega=(B_\omega)_{\text{in}}-\lambda f(\omega)\int_0^\infty d\omega'\frac{\lambda f(\omega')}{\eta^{+}(\omega')}\frac{(B_{\omega'})_{\text{in}}}{\omega'-\omega+i0}, \tag{A9}$$

$$a^{\dagger}=-\int_0^{\infty}d\omega\frac{\lambda f(\omega)}{\eta^{-}(\omega)}(B_{\omega}^{\dagger})_{\rm in}, \tag{A10}$$

$$a=-\int_0^{\infty}d\omega\frac{\lambda f(\omega)}{\eta^{+}(\omega)}(B_{\omega})_{\rm in}, \tag{A11}$$

we obtain the time evolution of the bare creation and annihilation operators in the Heisenberg representation:

$$b_{\omega}^{\dagger}(t)=b_{\omega}^{\dagger}e^{i\omega t}+\lambda f(\omega)\left\{\int_0^{\infty}d\omega'\lambda f(\omega')\times\frac{g(\omega',t)-g(\omega,t)}{\omega'-\omega}b_{\omega'}^{\dagger}-g(\omega,t)a^{\dagger}\right\},$$

$$b_{\omega}(t)=b_{\omega}e^{-i\omega t}+\lambda f(\omega)\left\{\int_0^{\infty}d\omega'\lambda f(\omega')\times\frac{g^*(\omega',t)-g^*(\omega,t)}{\omega'-\omega}b_{\omega'}-g^*(\omega,t)a\right\},$$

$$a^{\dagger}(t)=\int_0^{\infty}d\omega\lambda f(\omega)g(\omega',t)b_{\omega'}^{\dagger}+A(t)a^{\dagger},$$

$$a(t)=\int_0^{\infty}d\omega\lambda f(\omega)g^*(\omega',t)b_{\omega'}+A^*(t)a. \tag{A12}$$

Except for the oscillating exponent, all time dependence of the field operators is described by the functions $g(\omega,t)$ and $A(t)$:

$$g(\omega,t)=-\frac{1}{2\pi i}\int_{-\infty}^{\infty}d\omega'\frac{1}{\eta^{-}(\omega')}\frac{e^{i\omega' t}}{\omega'-\omega-i0}, \tag{A13}$$

$$A(t)=\left(i\frac{\partial}{\partial t}+\omega\right)g(\omega,t)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}d\omega'\frac{e^{i\omega' t}}{\eta^{-}(\omega')}. \tag{A14}$$

APPENDIX B: TYPE 1 FORM FACTOR

For the form factor $\varphi_1(x)=\sqrt{x}/(1+x)$, we have:

$$\eta_{\Lambda}(z)=\omega_{\Lambda}-z-\frac{\pi\lambda^2}{1-i\sqrt{z}}, \tag{B1}$$

where the first sheet of the complex z plane corresponds to the upper half of the complex $\sqrt{z}$ plane. The exact expression for the survival amplitude is known [28,29,37]:

$$A(t)=\frac{i\gamma+\sqrt{\tilde{\omega}_{\Lambda}}}{2\gamma\sqrt{\tilde{\omega}_{\Lambda}}}\frac{\pi\lambda^2}{z_3-z_2}e^{iz_2\Lambda t}+\pi e^{i\pi/4}\lambda^2\sum_{k=1}^{3}\left(\prod_{\substack{m=1\\ m\neq k}}^{3}\frac{1}{z_k-z_m}\right)\times\sqrt{iz_k}e^{iz_k\Lambda t}[-1+\mathrm{erf}(\sqrt{iz_k\Lambda t})]. \tag{B2}$$

Here, z_k is the root of $\eta_{\Lambda}(z)$ on the second sheet of z plane, and $\tilde{\omega}_{\Lambda}$, γ are expressed in terms of z_k. If conditions (12) are satisfied, we have the following approximate expressions:

$$\gamma\approx\frac{\pi}{2}\lambda^2,\quad \tilde{\omega}_{\Lambda}\approx\omega_{\Lambda}. \tag{B3}$$

In order to analyze the survival probability for large times, we need the asymptotics of the $A(t)$ as $t\to\infty$:

$$A(t)=\frac{2it^{-3/2}}{z_1z_2z_3}\left[1-\frac{12}{t}\sum_{k=1}^{3}\frac{1}{iz_k}+O(1/t^2)\right].$$

In the last expression, we can use the first term only when $t\gg 12|\Sigma_{k=1}^{3}1/iz_k|\approx 24/\omega_1$. We have in fact checked numerically, that this is valid even on shorter times.

Using Eq. (B3), we can now calculate the survival probability:

$$p(t)\approx e^{-4\gamma t\sqrt{\omega_1\Lambda}}+\frac{\pi\lambda^4\Lambda}{4\omega_1^4t^3}-\frac{\sqrt{\pi}\lambda^2\Lambda^{1/2}}{\omega_1^2t^{3/2}}\times e^{-2\gamma t\sqrt{\omega_1\Lambda}}\cos(\omega_1t-\pi/4)\quad \text{when}\quad t\gg\frac{24}{\omega_1}. \tag{B4}$$

One can see that the survival probability decays exponentially for intermediate times, while for large times, there is a power law. We can calculate the transition time t_{ep} when the exponential decay is replaced by the power law. This happens when these two terms in the expression for $p(t)$ are equal. This condition leads to a transcendental equation that can be approximately solved

$$t_{ep}\approx-\frac{5\ln\left[(2\pi^4)^{0.4}\lambda^4\frac{\Lambda}{\omega_1}\right]}{4\pi\lambda^2\sqrt{\Lambda\omega_1}}.$$

We should notice that in the vicinity of t_{ep}, the survival probability oscillates with the frequency ω_1.

Let us now discuss the asymptotics of Eq. (B2) for small times $t\sim 0$. From the definition of the survival probability $p(t)$ we know that $|A(0)|=1$. As the evolution is unitary, we know that a linear term in the expansion of $p(t)$ vanishes in the vicinity of $t=0$. Expanding Eq. (B2) at small times, we find for the survival probability

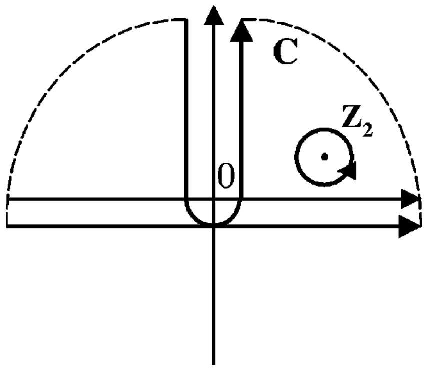

FIG. 4. The contour of integration.

$$p(t)=1-\left(\frac{t}{t_a}\right)^{1.5}+\left(\frac{t}{t_b}\right)^2+O(t^{5/2}). \tag{B5}$$

where $t_a=[3/(4\sqrt{2\pi})]^{2/3}/(\lambda^{4/3}\Lambda)$, and $t_b=1/(\sqrt{\pi}\lambda\Lambda)$.

APPENDIX C: TYPE 2 FORM FACTOR

For the form factor $\varphi_2(x)=x/(1+x^2)^2$ the dimensionless function $\eta_\Lambda(z)$ is

$$\eta_\Lambda(z)=\omega_\Lambda-z-\lambda^2\frac{\pi-2z}{4(1+z^2)}+\lambda^2\frac{\pi z^2+2z(\ln z-i\pi)}{2(1+z^2)^2}. \tag{C1}$$

This function has no roots on the first Riemann sheet. The roots on the second sheet are defined by the equation

$$\eta_\Lambda(z)+\frac{2\pi iz\lambda^2}{(1+z^2)^2}=0. \tag{C2}$$

Inserting Eq. (C1) into Eq. (5), we can see that the integrand vanishes at infinity at the upper half of the complex z plane and we can change the contour of the integration as it is shown in Fig. 4. Hence, only two roots of Eq. (C2) contribute to $A(t)$:

$$z_1=\omega_1+i\frac{\gamma_1}{2}\approx\omega_\Lambda+i\pi\lambda^2\omega_\Lambda,$$

$$z_2\approx\frac{\sqrt{\pi}}{2}\lambda+i.$$

It is interesting to notice that the root z_2 does not approach the continuous spectrum when $\lambda\to0$. Instead, z_2 ''annihilates'' with the root $z_3\approx-\sqrt{\pi}\lambda/2+i$, which, however, does not contribute to the survival amplitude.

Combining the pole contributions with the background integral, we have for the survival amplitude

$$\begin{aligned}A(t)&=\sum_{k=1}^{2}R(z_k)e^{iz_k\Lambda t}\\&\quad+\lambda^2\int_0^\infty dx\frac{x(1-x^2)^2e^{-x\Lambda t}}{[Q(x)+\frac{1}{2}\lambda^2\pi x](Q(x)-\frac{3}{2}\lambda^2\pi x)}\\&=\sum_{k=1}^{2}R(z_k)e^{iz_k\Lambda t}+\lambda^2I(t),\end{aligned} \tag{C3}$$

where

$$Q(x)=(\omega_\Lambda-ix)(1-x^2)^2-\frac{\lambda^2}{4}(\pi-2ix)(1-x^2)-\frac{\lambda^2}{2}(\pi x^2-2ix\ln x),$$

and

$$R(z)=-\left[1-\frac{\lambda^2}{2}\left(\frac{3-z^2+2\pi z}{(1+z^2)^2}+\frac{1-3z^2}{(1+z^2)^3}\times(\pi z+2\ln z+2i\pi)\right)\right]^{-1}.$$

It is worth noticing that we have two exponential terms in representation (C3). The first corresponds to the usual exponential decay of the system. The second decays very fast, with the time constant $1/2\Lambda$. However, this term is very important for the description of the survival amplitude at times $t\sim1/\Lambda$. As shown in Sec. IV, in this region, the Taylor expansion at $t=0$ already cannot be used, hence the representation (C3) is the only way to get results. We would like to notice that for the interaction $\varphi(x)=x/(1+x^2)^4$ there are three roots contributing to the survival amplitude: $z_1\approx\omega_\Lambda+i\pi\lambda^2\omega_\Lambda$, $z_2\approx i(1-\sqrt[4]{\lambda\pi/8}e^{\pi i/8}$, and $z_3\approx i(1-\sqrt[4]{\lambda\pi/8}$ $e^{5\pi i/8}$. Hence, the expressions for the survival amplitude previously obtained [10,11] cannot be used for arbitrary time t and should be corrected for $t\sim1/\Lambda$ by adding two additional exponential terms.

Let us calculate first the long-time asymptotics. For the integral term in the $A(t)$ we have

$$I(t)=\frac{1}{Q^2(0)\Lambda^2t^2}\left[1+\frac{4i}{t\Lambda Q(0)}+O(1/(\Lambda t)^2)\right].$$

As in Appendix B, we can use only one term of the asymptotics when $t\gg4/\omega_1$. In this region, the survival probability can be written as

$$p(t)\approx e^{-\gamma_1\Lambda t}+\frac{\lambda^4}{Q^4(0)\Lambda^4t^4}-\frac{2\lambda^2e^{-\gamma_1\Lambda t/2}}{Q^2(0)\Lambda^2t^2}\cos(\omega_1t). \tag{C4}$$

Here, again, we can see two regions: intermediate with exponential behavior and long tail with the power-law decay. The transition time t_{ep} can also be calculated:

$$t_{ep}=-\frac{4}{\gamma_1}\ln\frac{\lambda\gamma_1}{Q(0)\Lambda}.$$

In order to calculate the short-time asymptotics we expand $I(t)$ into the series at $t=0$:

$$I(t)\approx C_0+C_1t+C_2t^2+C_3t^3+\int_0^\infty dx\frac{x(-x\Lambda)^4(1-x^2)^2e^{-x\Lambda t}}{[Q(x)-\frac{1}{2}\lambda^2\pi x][Q(x)+\frac{3}{2}\lambda^2\pi x]},\tag{C5}$$

where C_i are constants. The asymptotics of the integral term in the last expression can be easily found [38]:

$$I^{(4)}(t)\approx-\Lambda^4\int_0^\infty dx\frac{e^{-x\Lambda t}}{x+2i\omega_\Lambda}=\Lambda^4\ln(2i\omega_\Lambda t)+O(1).$$

Combining these results with Eq. (9), we get

$$p(t)=1-\left(\frac{t}{t_a}\right)^2-\frac{\lambda^2}{12}\ln(2\omega_1 t)\Lambda^4t^4+O(t^4),\tag{C6}$$

where $t_a=\sqrt{2}/\lambda\Lambda$.

[1] B. Misra and E.C.G. Sudarshan, J. Math. Phys. **18**, 756 (1977).
[2] R. Cook, Phys. Scr., T **21**, 49 (1988).
[3] W.M. Itano, D.J. Heinzen, J.J. Bollinger, and D.J. Wineland, Phys. Rev. A **41**, 2295 (1990).
[4] T. Petrosky, S. Tasaki, and I. Prigogine, Phys. Lett. A **151**, 109 (1990); T. Petrosky, S. Tasaki, and I. Prigogine, Physica A **170**, 306 (1991).
[5] V. Frerichs and A. Schenzle, Phys. Rev. A **44**, 1962 (1991).
[6] E. Block and P.R. Berman, Phys. Rev. A **44**, 1466 (1991).
[7] Chr. Balzer, R. Huesmann, W. Neuhauser, and P.E. Toschek, Opt. Commun. **180**, 115 (2000).
[8] D. Home and M.A.B. Whitaker, Ann. Phys. (N.Y.) **258**, 237 (1997).
[9] P.E. Toschek and C. Wunderlich, LANL e-print quant-ph/0009021 2000.
[10] P. Facchi and S. Pascazio, Phys. Lett. A **241**, 139 (1998).
[11] P. Facchi and S. Pascazio, Physica A **271**, 133 (1999).
[12] A. Beige and G.C. Hegerfeldt, J. Phys. A **30**, 1323 (1997).
[13] W.L. Power and P.L. Knight, Phys. Rev. A **53**, 1052 (1996).
[14] A.D. Panov, Ann. Phys. (N.Y.) **249**, 1 (1996).
[15] G. Hackenbroich, B. Rosenow, and H.A. Weidenmller, Phys. Rev. Lett. **81**, 5896 (1998).
[16] B. Elattari and S.A. Gurvitz, Phys. Rev. Lett. **84**, 2047 (2000).
[17] S.A. Gurvitz, Phys. Rev. B **56**, 15 215 (1997).
[18] B. Elattari and S.A. Gurvitz, Phys. Rev. A **62**, 032102 (2000).
[19] L. Vaidman, L. Goldenberg, and S. Wiesner, Phys. Rev. A **54**, R1745 (1996).
[20] A.G. Kofman and G. Kurizki, Phys. Rev. A **54**, R3750 (1996).
[21] B. Kaulakys and V. Gontis, Phys. Rev. A **56**, 1131 (1997).
[22] M. Lewenstein and K. Rzazewski, Phys. Rev. A **61**, 022105 (2000).
[23] A. Marchewka and Z. Schuss, Phys. Rev. A **61**, 052107 (2000).
[24] A.G. Kofman and G. Kurizki, Nature (London) **405**, 546 (2000).
[25] P. Facchi, H. Nakazato, and S. Pascazio, LANL e-print quant-ph/0006094 2000.
[26] K. Friedrichs, Commun. Pure Appl. Math. **1**, 361 (1948).
[27] C.B. Chiu, E.C.G. Sudarshan, and B. Misra, Phys. Rev. D **16**, 520 (1977).
[28] K. Rzazewski, M. Lewenstein, and J.H. Eberly, J. Phys. B **15**, L661 (1982).
[29] A.G. Kofman, G. Kurizki, and B. Sherman, J. Mod. Opt. **41**, 353 (1994).
[30] H.E. Moses, Lett. Nuovo Cimento Soc. Ital. Fis. **4**, 51 (1972); Phys. Rev. A **8**, 1710 (1973); J. Seke, Physica A **203**, 269 (1994).
[31] S.L. Haan and J. Cooper, J. Phys. B **17**, 3481 (1984).
[32] L. Jacak, P. Hawrylak, A. Wojs, *Quantum Dots* (Springer, Berlin, 1998); D. Steinbach *et al.*, Phys. Rev. B **60**, 12079 (1999).
[33] T. Petrosky and V. Barsegov, in *The Chaotic Universe, Proceedings of the Second International Center for Relativistic Astrophysics Network Workshop, Rome, Pescara, Italy, February*, 1999, edited by V. G. Gurzadyan and R. Ruffini (World Scientific, Singapore, 1999), p. 143.
[34] P. Facchi, V. Gorini, G. Marmo, S. Pascazio, and E.C.G. Sudarshan, Phys. Lett. A **275**, 12 (2000).
[35] I. Antoniou, M. Gadella, I. Prigogine, and G. Pronko, J. Math. Phys. **39**, 2995 (1998).
[36] E. Karpov, I. Prigogine, T. Petrosky, and G. Pronko, J. Math. Phys. **41**, 118 (2000).
[37] A. Likhoded and G. Pronko, Int. J. Theor. Phys. **36**, 2335 (1997).
[38] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products* (Academic, London, 1980).