Bell-state preparation using pulsed nondegenerate two-photon entanglement

Yoon-Ho Kim,* Sergei P. Kulik,† and Yanhua Shih

Department of Physics, University of Maryland, Baltimore County, Baltimore, Maryland 21250

We report a Bell-state preparation experiment. High-purity Bell states are prepared by using femtosecond pulse pumped *nondegenerate* collinear spontaneous parametric down-conversion. The use of a femtosecond pump pulse *does not* result in a reduction of quantum interference visibility in our scheme in which the postselection of amplitudes and other traditional mechanisms, such as using thin nonlinear crystals or narrow-band spectral filters are not used. Another distinct feature of this scheme is that the pump, the signal, and the idler wavelengths are all distinguishable, which is very useful for quantum communications.

The preparation and measurement of the Bell states are two important issues in modern quantum optics, especially for quantum communications, quantum teleportation, etc. [1]. For photons, such states can be realized by using the entangled photon pairs generated in spontaneous parametric down-conversion (SPDC). By making appropriate local operations on the SPDC photon pairs, one can prepare all four Bell states.

The polarization Bell states, for photons, can be written as

$$|\Phi^{\pm}\rangle = |X_1, X_2\rangle \pm |Y_1, Y_2\rangle,$$
$$|\Psi^{\pm}\rangle = |X_1, Y_2\rangle \pm |Y_1, X_2\rangle,$$
(1)

where the subscripts 1 and 2 refer to two different photons, photon 1 and photon 2, respectively, and they can be arbitrarily far apart from each other. $|X\rangle$ and $|Y\rangle$ form the orthogonal basis for the polarization states of a photon; for example, it can be a horizontal ($|H\rangle$) and vertical ($|V\rangle$) polarization state, as well as $|45°\rangle$ and $|-45°\rangle$, respectively. This means that the quantum interference should be independent of the choice of the bases.

Such an experiment was first performed by Shih and Alley, in which noncollinear type-I SPDC and a beam splitter were used to prepare a Bell state [2], but it is very difficult to align such a system. Collinear type-II SPDC is thus developed [3]. There is, however, a common problem: the entangled photon pairs have a 50% chance of leaving at the same output ports of the beam splitter. Therefore, the state prepared after the beam splitter may not be considered as a Bell state without amplitude postselection [4]. Only when one considers the coincidence contributing terms by throwing away two out of four amplitudes (postselection of 50% of the amplitudes), is the state then said to be a Bell state. This problem is later solved by using noncollinear type-II SPDC or using two noncollinear type-I SPDCs [5].

In the cw pumped SPDC, entangled photon pairs occur randomly since the process is ''spontaneous,'' so the whereabouts of the photon pair is completely uncertain within the

coherence length of the pump laser beam. This huge time uncertainty makes it difficult for applications such as generation of the multiphoton entangled state, quantum teleportation, etc., as interactions between entangled photon pairs generated from different sources are required. This difficulty was thought to be solved by using a femtosecond pulse laser as a pump. Unfortunately, femtosecond pulse pumped type-II SPDC shows poor quantum interference visibility due to the very different (compared to the cw case) behavior of the two-photon effective wave function [6]. One has to utilize special experimental schemes to achieve complete overlap of the two-photon amplitudes. Traditionally, the following methods have been used to restore the quantum interference visibility in femtosecond pulse pumped type-II SPDC: (i) using a thin nonlinear crystal (≈ 100 μm) [8] or (ii) using narrow-band spectral filters in front of detectors [6,7]. Both methods, however, reduce the available flux of the entangled photon pair significantly [9] and cannot achieve complete overlap of the wave functions in principle [6].

The first attempt to achieve high-visibility quantum interference in femtosecond pulse pumped type-II SPDC without using narrow-band filters and a thin crystal was reported in Ref. [10]. The observed visibility, however, was rather low, and keeping the phase coherence over a long term would be very difficult since a Michelson interferometer is used. Also, such a scheme cannot be used to prepare a Bell-state. Recently, we reported a high-visibility quantum interference experiment in which photon pairs are entangled both in polarization and space-time using femtosecond pulse pumped type-I SPDC [11]. However, it cannot be considered as a true Bell-state preparation since postselecting 50% of the amplitudes was still necessary.

In this Rapid Communication, we report a Bell-state preparation experiment in which we effectively eliminate *any postselection* in femtosecond pulse pumped SPDC. Other features in our scheme include: (i) collinear SPDC makes the alignment much easier, (ii) Alice and Bob share photons of different frequencies entangled in both space-time and polarization, (iii) phase coherence is automatically kept and the visibility as high as 92% is observed, (iv) thick crystals can be used to increase the intensity (without losing the visibility), and (v) the spectral bandwidth is reduced significantly (compared to type-I degenerate SPDC) by the use of *nondegenerate* SPDC. These features make our scheme a good source of Bell states for quantum information experiments.

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*Email address: yokim@umbc.edu

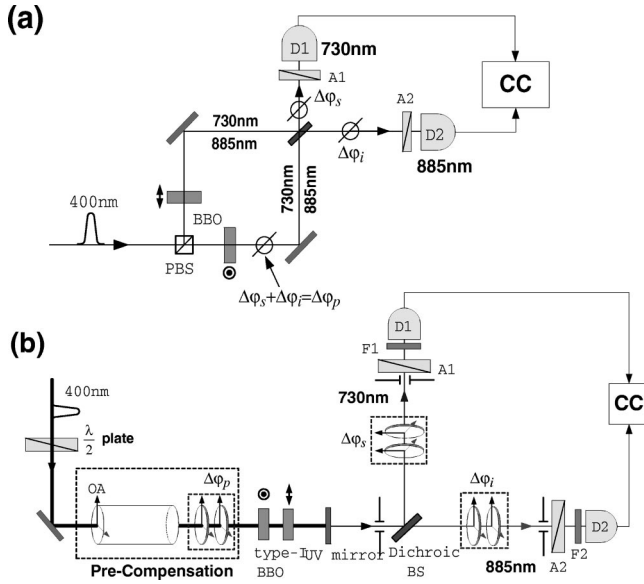†Permanent address: Department of Physics, Moscow State University, Moscow, 119899, Russia.

FIG. 1. (a) Principle schematic of the experiment. The pump pulse is polarized at 45°. Nondegenerate collinear type-I SPDC occurs at the nonlinear crystal placed in each arm of the MZI. (b) Schematic of the experimental setup. Note that three different phases can be observed. Interference filters F_1 and F_2 are used to cut the pump noise.

The basic idea of the experiment is illustrated in Fig. 1(a). A 45° polarized femtosecond laser pulse (central wavelength $\lambda_p = 400$ nm and pulse duration $\sigma = 80$ fsec.) enters the Mach-Zehnder interferometer (MZI), which contains a type-I nonlinear crystal in each arm. One crystal has its optic axis oriented vertically ($\updownarrow$) and another horizontally ($\odot$). The polarizing beam splitter (PBS) splits the 45° polarized pump pulse into the vertical and horizontal polarized pulses propagating along different arms of the MZI. Then nondegenerate collinear type-I SPDC occurs, with equal probability, at each crystal (signal wavelength $= 730$ nm and idler wavelength $= 885$ nm) and they are mixed at the dichroic beam splitter, which directs 730 nm photons to detector D_1 and 885 nm to detector D_2. In the simplified single-mode approximation, the quantum state generated from the vertically oriented crystal ($\updownarrow$) is $|\psi\rangle_1 = |H_{730}\rangle|H_{885}\rangle$ and that from the horizontally oriented one ($\odot$) is $|\psi\rangle_2 = |V_{730}\rangle|V_{885}\rangle$. H and V represent the horizontal and vertical polarization state of a single photon, respectively. Subscripts 730 and 885 refer to the wavelengths 730 and 885 nm, respectively. When the MZI is balanced, the quantum state after the MZI is (*without* throwing away any amplitudes)

$$|\Phi\rangle = |V_{730}\rangle_1|V_{885}\rangle_2 + e^{i\Delta\varphi}|H_{730}\rangle_1|H_{885}\rangle_2, \qquad (2)$$

where $\Delta\varphi$ is the relative phase between the two amplitudes and can easily be varied by scanning one of the mirror of the MZI.

The coincidence counting rate (R_c) is calculated as [12,13]

$$R_c = \int\int dt_+ dt_{12} |\mathcal{A}(t_+, t_{12})|^2, \qquad (3)$$

where $t_{12} = t_1 - t_2$ and $t_+ = (t_1 + t_2)/2$. $t_i = T_i - l_i/c$, where T_i is the time at which detector i fires and l_i is the optical path length from the surface of the crystal to the detector i. $\mathcal{A}(t_+, t_{12})$ is the amplitude of the biphoton as explicitly calculated in Ref. [6].

For the scheme shown in Fig. 1(a), $\mathcal{A}(t_+, t_{12})$ is the sum of the two amplitudes originated from the crystal in each arm of the MZI:

$$\mathcal{A}(t_+, t_{12}) = \mathcal{A}_a(t_+, t_{12}) + \mathcal{A}_b(t_+, t_{12}), \qquad (4)$$

where the subscripts a and b refer to the crystal from which the amplitudes are created. The delay T introduced in one arm modifies the amplitude $\mathcal{A}_b(t_+, t_{12}) \rightarrow \mathcal{A}_b(t_+ + T, t_{12})$ and determines the additional phase shift for the biphoton amplitudes $\Delta\varphi = \Omega_p T = K_p \Delta x$, where $K_p = 2\pi/\Lambda_p$, $\Omega_p(\Lambda_p)$ the central frequency (wavelength) of the pump, and Δx the spatial delay. Due to the energy conservation and negligibly small dispersion of the air, the phase shift depends only on the pump wavelength $\Delta\varphi = K_s\Delta x + K_i\Delta x = K_p\Delta x = \Delta\varphi_p$, although the delay is introduced to the SPDC field [11,14]. If the crystals are the same and the pump fields in different arms of the MZI are identical,

$$|\mathcal{A}_a(t_+, t_{12})| = |\mathcal{A}_b(t_+, t_{12})|. \qquad (5)$$

The coincidence counting rate is then calculated to be

$$R_c = 1 + V\cos(\Omega_p T), \qquad (6)$$

where $V \approx 1$ in this experiment [15,16]. Note that the angles of the analyzers A_1 and A_2 are assumed to be 45°. From Eq. (6), we expect that the coincidence counting rate will be modulated in the pump central wavelength when T is varied.

There are also two more ways to vary the phases of interference by introducing relative delays (using a piece of birefringent material, such as a quartz plate) after the output beam splitter, i.e., in the signal ($\Delta\varphi_s$) and/or in the idler ($\Delta\varphi_i$) channels. Therefore we obtain

$$R_c = 1 + V\cos(\Delta\varphi_p - \Delta\varphi_i - \Delta\varphi_s), \qquad (7)$$

where $\Delta\varphi_p$, $\Delta\varphi_i$, and $\Delta\varphi_s$ refer to the relative phases introduced into the pump, the idler, and the signal, respectively.

As we have shown so far, one can eliminate the possibility of the entangled photon pairs leaving at the same output ports of the beam splitter by employing nondegenerate two-photon entanglement. In this scheme, high-visibility quantum interference can be achieved independent of the crystal thickness and the spectral filter bandwidths, even with a femtosecond pulse pump.

In practice, however, one would not like to use a MZI in the experimental setup due to stability related issues. Therefore, we use a collinear scheme where two type-I BBO crystals are placed collinearly in the pump beam path; see Fig. 1(b). Two type-I BBO crystals with a thickness of 3.4 mm each (the first one is oriented horizontally and the second one is oriented vertically) are then pumped by a 45° polarized pump pulse. As described before, the quantum state resulting from the first BBO is $|V_{730}\rangle|V_{885}\rangle$ and that from the second BBO is $|H_{730}\rangle|H_{885}\rangle$. Since both crystals are pumped
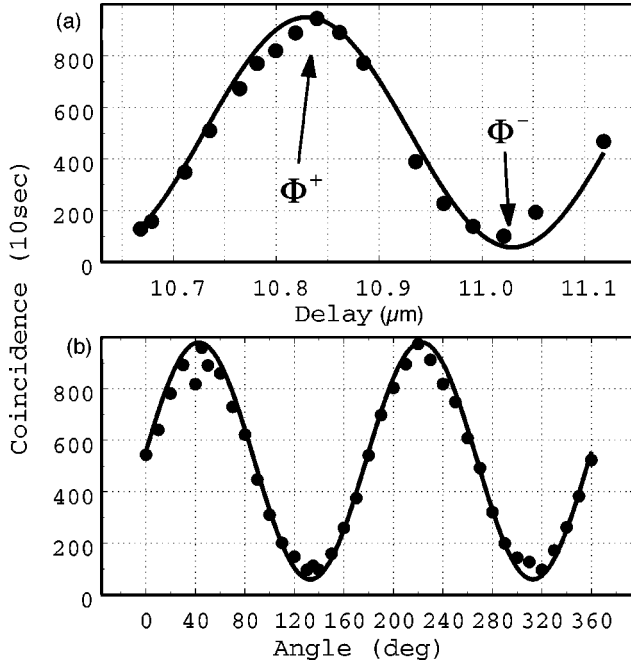
FIG. 2. Experimental data. (a) Space-time interference by varying the pump phase $\Delta\varphi_p$ when $\theta_1=\theta_2=45°$. The transition from Φ^+ to Φ^- is clearly demonstrated. (b) The polarization interference at Φ^+. A_1 is fixed at $\theta_1=45°$ and A_2 is rotated, i.e., θ_2 is varied. The solid lines are the theoretical curves.
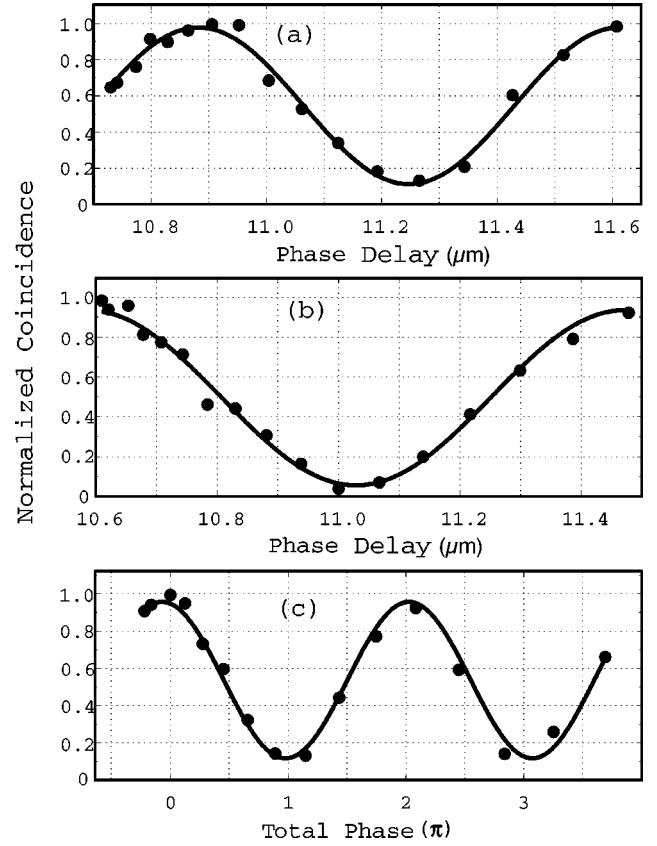

FIG. 3. Experimental data. (a) Space-time interference by varying $\Delta\varphi_s$ (730-nm modulation). (b) Space-time interference by varying $\Delta\varphi_i$ (885-nm modulation). (c) Two phases (730 nm and 885 nm) are varied at the same time. The solid line is the theory curve based on 400-nm modulation and agrees well with the data. This confirms $\Delta\varphi_s+\Delta\varphi_i=\Delta\varphi_p$. Note that $\theta_1=\theta_2=45°$ for all cases.

equally, the two amplitudes are equally probable. Due to the dispersion, however, $|V_{730}\rangle|V_{885}\rangle$ from the first BBO ($\odot$) and $|H_{730}\rangle|H_{885}\rangle$ from the second BBO ($\updownarrow$) are distinguishable in time. To make $|V_{730}\rangle|V_{885}\rangle$ and $|H_{730}\rangle|H_{885}\rangle$ indistinguishable in time, one needs to compensate the delay experienced by the SPDC photon pairs at each crystal. This compensation can be made by using a properly oriented quartz rod. If the compensation is made properly, either before or after the down-conversion nonlinear crystals, one will observe high-visibility quantum interference without any spectral postselection.

In the collinear scheme, having a perfect temporal compensation is difficult when the signal wavelength differs very much from the idler wavelength. This is because the signal-idler photon pairs created from the first BBO ($\odot$) experience different dispersion when they pass through the second BBO ($\updownarrow$). (The MZI scheme does not have this disadvantage.) In this experiment, for the wavelengths we are interested in, the temporal separation is rather small and does not affect the interference visibility. To prevent further dispersion effects, the compensation is made before the BBO crystals. The compensator consists of a quartz rod and two quartz plates whose optic axes are oriented vertically, see Ref. [11], and it imposes roughly a 1.5-psec required delay between the H- and V-polarized 400-nm pump pulse, which is mainly determined by the thickness of the BBO crystals. By tilting the two quartz plates in opposite directions, the phase delay $\Delta\varphi_p$ can be varied to prepare a Bell state. After the two BBO crystals, the remaining UV radiation is blocked by a UV reflecting mirror and the collinear SPDC is selected by a diaphragm. Then a dichroic beam splitter is used to reflect

the signal (730 nm) to D_1 and to transmit the idler (885 nm) to D_2. Two quartz plates are inserted in each beam path to vary the relative phase of the signal or the idler independently. The detector package consists of a single-photon counting module, an interference filter that is used to cut the pump noise [17], and a polarization analyzer.

To demonstrate the effectiveness of this scheme, we first study the space-time interference as a function of $\Delta\varphi_p$ by setting $\theta_1=\theta_2=45°$, where θ_1 and θ_2 are the angles of the analyzers A_1 and A_2 (measured from the vertical direction). According to Eq. (7), one should observe a pump wavelength modulation in the coincidence counting rate. Note that $\Delta\varphi_s$ and $\Delta\varphi_i$ are fixed. The observed modulation period is 400 nm, see Fig. 2(a), which agrees with the theory.

To prepare the Φ^+ (Φ^-) state, identified by constructive (destructive) interference, one just needs to set $\Delta\varphi_p-\Delta\varphi_s-\Delta\varphi_i=0,2\pi,4\pi,\ldots$ ($\Delta\varphi_p-\Delta\varphi_s-\Delta\varphi_i=\pi,3\pi,5\pi,\ldots$), which can be done by tilting the quartz plates so that the space-time interference fringe is at the maximum (minimum). Note that Ψ^+ and Ψ^- Bell states can also be easily prepared by introducing a $\lambda/2$ plate in one output port of the dichroic beam splitter.

We have also experimentally demonstrated the polarization interference for Φ^+ and Φ^-. For $\Phi^\pm$, the coincidence counting rate is calculated to be

$$R_c \propto |\langle \theta_2, \theta_1 | \Phi^{\pm} \rangle|^2 \propto \cos^2(\theta_1 \mp \theta_2). \qquad (8)$$

This means that one should observe high-visibility modulation in polarization correlation measurement for *arbitrary values* of θ_1 and θ_2. To confirm this experimentally, we first set $\theta_1 = 45°$ and varied θ_2. High-visibility polarization correlation is observed, see Fig. 2(b). We then repeated this measurement for many other values of θ_1 and observed that the visibility remained the same. This confirms Eq. (8). In other words, we have successfully prepared polarization Bell states from femtosecond pulse-pumped SPDC without amplitude and spectral postselection.

Unlike the usual degenerate two-photon sources, this source has one distinctive feature: one can vary three different phases independently, which is very useful for quantum communications. To demonstrate this interesting feature, we observe the space-time interference by varying the relative phases of the signal ($\Delta\varphi_s$) and the idler ($\Delta\varphi_i$) independently. In these measurements, θ_1 and θ_2 are set at 45°. The effect of $\Delta\varphi_p$ is already demonstrated in Fig. 2(a). When the signal phase $\Delta\varphi_s$ is varied, see Fig. 3(a), the signal wavelength (730 nm) modulation is observed in coincidence rate, while varying the idler phase $\Delta\varphi_i$, see Fig. 3(b), the idler wavelength (885nm) modulation is observed. Figures 2(a), 3(a), and 3(b) clearly demonstrate Eq. (7).

Finally, we tested the condition $\Delta\varphi_p = \Delta\varphi_s + \Delta\varphi_i$ by varying the signal phase $\Delta\varphi_s$ and the idler phase $\Delta\varphi_i$ at the same time. The quartz plates in both the signal and the idler paths are tilted at the same time with equal angles. To see whether the data agree with the theory, total phases accumulated in both beam path are calculated from the tilt angle, i.e., $\Delta\varphi_s + \Delta\varphi_i$. As evidenced from Fig. 3(c), the data agree well with Eq. (7).

In summary, we have demonstrated a scheme to prepare pulsed entangled photon pairs from which all four Bell states can easily be obtained. Amplitude and spectral postselection are not necessary. Note also that nonmaximally entangled states can be prepared by changing the relative intensities of the pump beams. The visibility and the photon flux are greatly enhanced by this method, although a femtosecond pulse laser is used as a pump. The signal, the idler, and the pump phases can be varied independently with different modulation frequency, which is very useful for quantum communications.

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[15] The visibility factor V is in general a function of $D_+ = 1/u_0(\Omega_p/2) - 1/u_0(\Omega_p)$, where u_0 is the group velocity of the ordinary ray, σ the pump pulse duration, and the delay T. Note that V is not a function of bandwidths of the spectral filters. One can also prepare a Bell state using two type-II SPDC. Equation (7) still remains valid, but the function V has a different width and shape. See Ref. [16].

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