Stochastic nonrelativistic approach to gravity as originating from vacuum zero-point field van der Waals forces

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We analyze the proposal that gravity may originate from a van der Waals type of residual force between particles due to the vacuum electromagnetic zero-point field. Starting from the Casimir-Polder integral, we show that the proposed approach can be analyzed directly, without recourse to approximations previously made. We conclude that this approach to Newtonian gravity does not work, at least not with this particular starting point. Only by imposing different or additional physical constraints, or by treating the underlying dynamics differently than what are embodied in the inherently subrelativistic Casimir-Polder integral, can one expect to escape this conclusion.

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The present article analyzes in some detail a specific proposal on the physical origin of gravitation [1]. Most physicists regard gravitation as a very basic phenomenon, on par with the electromagnetic, weak, and strong interactions. However, trying to cast all four of these interactions under one unified theoretical description has proved to be enormously difficult. This difficulty contributed to Sakharov's proposal [2] that the gravitational interaction is not a fundamental interaction at all, but rather that it results from a ''change in the action of quantum fluctuations of the vacuum if space is curved.'' In turn, Sakharov's idea helped to motivate Puthoff's proposal in 1989 that '' . . . gravity is a form of long-range van der Waals force associated with particle *Zitterbewegung* response to the ZP (zero point) fluctuations of the electromagnetic field'' [3].

Several possible starting points were mentioned for the gravity related work in Ref. [1], including (i) Boyer's stochastic electrodynamics (SED) calculation of the van der Waals force between two classical, nonrelativistic, electric dipole harmonic oscillators [4], (ii) Renne's related nonrelativistic quantum electrodynamic (QED) calculation for a quantum harmonic-oscillator model [5], and (iii) fourth-order perturbation theory in QED leading to the (subrelativistic) Casimir-Polder integral [6]. All three of these approaches were discussed and related to each other in Ref. [4]. Since Puthoff explicitly referred to the first term in the Casimir-Polder integral [7], let us begin with this expression [6]:

$$U(R)=-\alpha^2\frac{\hbar c}{\pi}\int_0^\infty du\frac{u^4\omega_0^4}{(c^2u^2+\omega_0^2)^2}\frac{e^{-2uR}}{R^2}\left[1+\frac{2}{uR}+\frac{5}{(uR)^2}+\frac{6}{(uR)^3}+\frac{3}{(uR)^4}\right]. \tag{1}$$

Here, $U(R)$ is the Casimir-Polder potential between two neutral, polarizable particles, R is the distance between the particles, and ω_0 is the resonant frequency associated with the particles when they are treated as harmonic oscillators. The polarizability α is then given by $e^2/(m\omega_0^2)$.

A number of approximations were made to Eq. (1) in Ref. [1]. Only the first term in brackets in Eq. (1) was considered and $\omega_0=0$ was substituted into the integrand, based on the argument of a small effective resonant frequency. The upper limit of ∞ was replaced by an upper cutoff limit $u_c=\omega_c/c$. Some averaging arguments were then made that led to a $1/R$ effective potential between particles. Later, in response [8] to a criticism by Carlip [9] on the calculational procedure of the averaging steps, Puthoff gave some additional arguments and different reasoning to still yield this $1/R$ effective potential, now emphasizing that there should be physical reasons for imposing cutoffs in the integration that enable this $1/R$ form to be obtained.

We wish to make two key points here. First, one cannot simply extract the first term in Eq. (1), as all of the terms contribute on a roughly equal footing in the large distance regime. Second, Eq. (1) can be fully evaluated, as will be done here, and compared with any proposed approximations to the full integral. Unfortunately, as will be seen, the approximations in Refs. [1] and [8] do not hold, at least not without introducing additional assumptions that imply significantly different physical effects not embodied within the inherently subrelativistic full Casimir-Polder integral.

To begin, we make the substitution of $w=uR$ in Eq. (1) to obtain

$$U(R)=-\alpha^2\frac{\hbar c}{\pi}\frac{\omega_0^4}{c^4R^3}I\left(\frac{\omega_0R}{c}\right), \tag{2}$$

where

$$I(b) \equiv \int_0^\infty dw \frac{w^4 e^{-2w}}{(w^2+b^2)^2}\left[1+\frac{2}{w}+\frac{5}{w^2}+\frac{6}{w^3}+\frac{3}{w^4}\right]. \quad (3)$$

Thus, $U(R)$ has a functional form of $1/R^3$ times an integral that depends on $\omega_0 R/c$. A second argument to this integral could also be included [i.e., $I(b,w_c)$] if we replace the upper integration limit of infinity by a cutoff of $w_c = u_c R = \omega_c R/c$, such as might be imposed if the ZP spectrum was thought to be cutoff at sufficiently large frequencies [10]. Without imposing this cutoff, however, then it is easy to see from the above that if a $1/R$ potential is to emerge for the form of $U(R)$, under whatever limiting conditions one imposes (e.g., large R, small ω_0, etc.), then $I(b)$ must result in a b^{+2} dependence.

However, a full evaluation of Eq. (3) does not reveal any such dependency. As discussed in Ref. [11], each term in Eq. (3) can be analytically evaluated. Indeed, Fig. 1 in Ref. [11] shows a plot of $\ln[I(b)]$ versus $\ln(b)$, revealing that $I(b)$ is bounded from above by two curves that $I(b)$ asymptotically approaches at large and small values of b. For large $b = \omega_0 R/c$, the bounding curve is the retarded van der Waals expression of $I_r(b) \equiv 23/4b^{-4}$, yielding an overall $1/R^7$ dependence for $U(R)$ in this regime. At small b, $I(b)$ is bounded by the unretarded van der Waals expression of $I_{ur}(b) \equiv 3\pi/4b^{-3}$, yielding an overall $1/R^6$ dependence for $U(R)$ in this regime. At no point either between these extremes, or at these extremes, is there any behavior that remotely approaches a b^{+2} dependence that would be required to yield a net $1/R$ dependence for $U(R)$.

Reference [11] contains a detailed analysis on how $I_r(b)$ and $I_{ur}(b)$ can be extracted from Eq. (3). Moreover, the question is examined on what happens if an upper cutoff of $w_c = \omega_c R/c$ is imposed in the integration in Eq. (3). As shown there, if $\min(2\omega_0, 5c/R) \lesssim \omega_c$, where ω_0 is the resonant frequency of the oscillator system, then the integrations in Eqs. (1) or (3) will be barely affected. Since proposed upper frequency limits for the ZP spectrum are far, far larger than this restriction [10], then we must conclude that imposing a realistic upper frequency cutoff in the integration in Eq. (1) still yields that a Newtonian potential does not arise from the Casimir-Polder integral. An energy based argument discussed in Ref. [11] helps to support this point. It displays the remarkable implausibility of the low frequencies van der Waals force approach to Newtonian gravity formulated in Ref. [8] in response to the objections of Ref. [9]. In conclusion, barring the introduction of additional physical assumptions into the analysis in Refs. [1] and [8], the specific argument presented there involving an average force induced by ZP fields, will not yield a Newtonian gravitational force signature.

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[1] H. E. Puthoff, Phys. Rev. A **39**, 2333 (1989).
[2] A. D. Sakharov, Dokl. Akad. Nauk SSSR **177**, 70 (1967) [Sov. Phys. Dokl. **12**, 1040 (1968)].
[3] See Ref. [1], p. 2340.
[4] T. H. Boyer, Phys. Rev. A **7**, 1832 (1973); **11**, 1650 (1975); D. C. Cole, Phys. Rev. D **33**, 2903 (1986); Phys. Rev. A **42**, 1847 (1990).
[5] M. J. Renne, Physica (Amsterdam) **53**, 193 (1971).
[6] H. B. G. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).
[7] See footnote 29 in Ref. [1].
[8] H. E. Puthoff, Phys. Rev. A **47**, 3454 (1993).
[9] S. Carlip, Phys. Rev. A **47**, 3452 (1993).
[10] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965), p. 245.
[11] See EPAPS Document No. E-PLRAAN-63-089102 for supplementary material on analysis reported here. This document may be retrieved via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html) or from ftp.aip.org in the directory /epaps/. See the EPAPS homepage for more information.