Elaboration of the Ahn-Weinacht-Bucksbaum scheme for information storage or retrieval through a quantum phase with a single operation

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We present a scheme for information storage or retrieval through a quantum phase with a single operation, which generalizes the Ahn-Weinacht-Bucksbaum (AWB) scheme proposed and demonstrated in a recent experiment on *N*-state Rydberg atoms by Ahn, Weinacht, and Bucksbaum [Science **287**, 463 (2000)]. Our scheme elaborates the AWB scheme through removing its original constraint on the smallness of the product of time and atom-light interactions by exactly evaluating the involved unitary transformations, and it is proved to have several pronounced advantages over the original one.

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In a recent spectacular experiment [1], Ahn, Weinacht, and Bucksbaum have demonstrated, based on their own scheme (we propose to call it AWB scheme), that information could be stored or retrieved by a single operation in agreement with Grover's proposal [2]. In that experiment, information was stored as quantum phase in an *N*-state Rydberg atom data register, and one or more flipped states stored in an eight-state atomic wave packet were retrieved in a single operation with the help of an ancillary state called the reservoir state and its interactions with the *N* states [1]. However, the AWB scheme is based on a lowest-order perturbation theory in evaluating its unitary transformations [1]. In this paper we shall show that we can derive exactly the unitary transformations induced by the laser pulses in the AWB scheme by including all the omitted higher-order perturbations. In this way, we are able to demonstrate that the AWB scheme is still an efficient scheme for information storage and retrieval without the constraint of the lowest-order approximation. As a matter of fact, the elaborated AWB scheme based on the exact transformation results may offer some additional advantages over the original one as will be seen later.

Consider a quantum system that has $N=2^L$ states $|1\rangle,|2\rangle,\ldots,|N\rangle$ satisfying orthonormal conditions $\langle j|k\rangle=\delta_{jk}$. One frequently occurred state for information processing is the equal-amplitude superposition state,

$$|\Psi_0\rangle=\xi\sum_{j=1}^{N}|j\rangle,\xi\equiv\frac{1}{\sqrt{N}}. \tag{1}$$

Information is stored in a state $|\Psi_s\rangle$,

$$\begin{aligned}|\Psi_s\rangle&=-\xi\sum_{k=1}^{s}|j_k\rangle+\xi\sum_{n\neq\{j_k\}}|n\rangle\\&\equiv-\sqrt{s}\xi|\mathrm{FL}\rangle+\sqrt{1-s\xi^2}|\mathrm{UF}\rangle,\end{aligned} \tag{2}$$

obtained by flipping *s* states in $|\Psi_0\rangle$ rewritten here as

$$|\Psi_0\rangle=\sqrt{s}\xi|\mathrm{FL}\rangle+\sqrt{1-s\xi^2}|\mathrm{UF}\rangle, \tag{3}$$

where $|\mathrm{FL}\rangle=(\sqrt{s})^{-1}\Sigma_{k=1}^{s}|j_k\rangle$ and $|\mathrm{UF}\rangle=(\sqrt{N-s})^{-1}\Sigma_{n\neq\{j_k\}}|n\rangle$ are two orthonormal kets called the flipped or marked state and the unflipped state, respectively, j_k's are integers (in the interval $[1,N]$) designating locations of stored information, and $\Sigma_{n\neq\{j_k\}}$ denotes summation over, except for j_k's, other integers in $[1,N]$ corresponding to the unflipped state.

In the AWB scheme [1], Ahn, Weinacht, and Bucksbaum have considered a $(N+1)$-state system by adding a normalized reservoir state $|0\rangle$ to the *N* states $|1\rangle,|2\rangle,\ldots,|N\rangle$ with $\langle 0|j\rangle=\delta_{0j}$ for $j=1,2,\ldots,N$. One possible physical realization for this kind of system is a Rydberg cesium atom where the reservoir state $|0\rangle$ corresponds to the atom's $7s$ state, other states $|j\rangle$ are high-lying Rydberg np states accessible from the $7s$ state by the absorption of a photon from an ultrafast Ti:sapphire chirped-pulse amplified laser pulse [1]. For a laser pulse containing *N*-frequency equal-amplitude components of the form $E\Sigma_{j=1}^{N}\eta_j\cos(\omega_j t)$ with $\eta_j=\pm1$, the system's interaction Hamiltonian in the interaction picture under the rotating-wave approximation reads $H_q=i(\Omega/2)\Sigma_{j=1}^{N}\eta_j(|j\rangle\langle 0|e^{i(\omega_{j0}-\omega_j)t}-|0\rangle\langle j|e^{-i(\omega_{j0}-\omega_j)t})$. Here $\omega_{j0}=(E_j-E_0)/\hbar$ denoting the atomic transition frequency between levels $|j\rangle$ and $|0\rangle$. Choosing the frequencies of the laser pulse's components such that $\omega_{j0}\approx\omega_j$, we can rewrite the interaction Hamiltonian as follows:

$$H_q=i\frac{\Omega}{2}\sum_{j=1}^{N}\eta_j(|j\rangle\langle 0|-|0\rangle\langle j|), \tag{4}$$

where $\eta_j=\pm1$, the subscript q denotes the number of negative η_j's, the Rabi frequency Ω describes the dipole coupling between $|0\rangle$ and $|j\rangle$ via the laser pulse for $j=1,2,\ldots,N$, and it is proportional to the product of $|\langle np|z|7s\rangle|$ and the square root of the laser intensity. The $\langle np|z|7s\rangle$ matrix elements are relatively real, and setting the phases (i.e., controlling the sign of η_j's) is a simple matter of adjusting the relative phase of the optical excitation radiation at each resonant frequency ω_{np-7s} [1]. Information processing in the AWB scheme is achieved through the system's unitary evolution characterized by the evolution operator $U_q(t)=\exp(-iH_q t)$ with the appropriate choice for q (i.e.,

choosing the number and the positions of the negative η_j's in Hamiltonian H_q) and the initial states. The AWB scheme is based on the assumption that $\Omega t/2=\epsilon$ is a small quantity, and hence $U_q(t)=\exp(-iH_qt)\approx 1-iH_qt$ by neglecting terms equal to and greater than the order ϵ^2. Here we do not assume the smallness of the quantity Ωt, and hence we need to evaluate the evolution operator exactly.

To calculate the evolution operator exactly, we rewrite Hamiltonian (4) as $H_q=-i\Omega\sqrt{N}F/2$. Here $F=(|0\rangle\langle\Psi_q|-|\Psi_q\rangle\langle 0|)$ and $|\Psi_q\rangle=(\sqrt{N})^{-1}\Sigma_{j=1}^{N}\eta_j|j\rangle$ with $\eta_{j_k}=-1$ for $k=1,2,\ldots,q$ and other η's equal to 1. Then one can readily show $F^{2k}=(-1)^{k+1}F^2$ for $k=1,2,\ldots$, and $F^{2j+1}=(-1)^jF$ for $j=0,1,2,\ldots$. It is now straightforward to obtain $U_q(t)=1-F\sin\tau-(\cos\tau-1)F^2$ or

$$U_q(t)=1-(|0\rangle\langle\Psi_q|-|\Psi_q\rangle\langle 0|)\sin\tau+(\cos\tau-1)(|0\rangle\langle 0|+|\Psi_q\rangle\langle\Psi_q|), \tag{5}$$

where $\tau=t\Omega\sqrt{N}/2\equiv\Omega t/(2\xi)$, and $|\Psi_q\rangle=\xi\Sigma_{j=1}^{N}\eta_j|j\rangle$ with $\eta_{j_k}=-1$ for $k=1,2,\ldots,q$ and other η's equal to 1. Note that the above expression for the evolution operator is valid for any possible arrangements of positive and negative η's.

Information storage and retrieval in our scheme correspond to U_s (obtained by setting $q=s$, i.e., $\eta_{j_k}=-1,k=1,2,\ldots,s$ and other η's equal to 1) and U_N (obtained by setting $\eta_j=-1$ for $j=1,2,\ldots,N$), respectively, or

$$U_s(t)=1-(|0\rangle\langle\Psi_s|-|\Psi_s\rangle\langle 0|)\sin\tau+(\cos\tau-1)(|0\rangle\langle 0|+|\Psi_s\rangle\langle\Psi_s|), \tag{6a}$$

$$U_N(t)=1+(|0\rangle\langle\Psi_0|-|\Psi_0\rangle\langle 0|)\sin\tau+(\cos\tau-1)(|0\rangle\langle 0|+|\Psi_0\rangle\langle\Psi_0|), \tag{6b}$$

where $\tau=\Omega t/(2\xi)$. It is worthwhile to mention that we know the values of j_k's in $|\Psi_s\rangle$ in the storage process (since they are just the locations where we want to produce flipped items) so that we can program the Hamiltonian corresponding to $U_s(t)$ in Eq. (6a) in this process, but we do not know them in the retrieval process and hence we program the Hamiltonian corresponding to $U_N(t)$ in Eq. (6b) in this retrieval process. Note that the above procedure to program the sign of η_j's in Hamiltonian (4) can be realized for a Rydberg atom radiated by an appropriate programmed ultrafast Ti-:sapphire chirped-pulse amplified laser pulse [1]. We are now ready to investigate the information storage or retrieval with a single operation based on the exact expression (6) of the evolution operator.

The information storage process is to produce the state $|\Psi_s\rangle$ in Eq. (2). To realize this process, the system is initially prepared in the reservoir state $|0\rangle$, and we switch on the unitary operation corresponding to the evolution operator $U_s(t)$ in Eq. (6a). Therefore, the system at time t will be in the state $U_s(t)|0\rangle$ or, by means of the expression (6a) and $\langle\Psi_s|0\rangle=0$,

$$U_s(t)|0\rangle=\sin\left(\frac{\Omega t}{2\xi}\right)|\Psi_s\rangle+\cos\left(\frac{\Omega t}{2\xi}\right)|0\rangle, \tag{7}$$

where $|\Psi_s\rangle$ is given in Eq. (2) and is the state to be produced. It can be clearly seen that the information storage state $|\Psi_s\rangle$ is obtained at time $t_s=\pi\xi/\Omega$ by a single unitary operation $U_s(t_s)$, i.e., $U_s(t_s)|0\rangle=|\Psi_s\rangle$. The system will remain in the state $|\Psi_s\rangle$ after time t_s if one switches off the laser and hence the interactions characterized by the interaction Hamiltonian immediately after time t_s. It is pointed out that the state $|\Psi_s\rangle$ can also be produced from the reservoir state $|0\rangle$ at other times $t_{s,n}\equiv(1+4n)t_s, n=1,2,\ldots$ as well. This fact carries implications for the information storage as we shall explain shortly.

The information retrieval process is to single out the flipped item $|\mathrm{FL}\rangle$ from the state $|\Psi_s\rangle$ in Eq. (2) without knowledge of the values of j_k's in the state $|\Psi_s\rangle$ now. But we can program the Hamiltonian (4) so that the evolution operator in this case is $U_N(t)$ in Eq. (6b), and the system initially in the state $|\Psi_s\rangle$ is in the state $U_N(t)|\Psi_s\rangle$ at time t. Utilizing Eq. (6b) and noting $\langle 0|\Psi_s\rangle=0$ and $\langle\Psi_0|\Psi_s\rangle=1-2s\xi^2$, we readily obtain

$$U_N(t)|\Psi_s\rangle=-2\sqrt{s}\xi|FL\rangle+\cos\theta\sin\tau|0\rangle+[1-(1-\cos\tau)\cos\theta]|\Psi_0\rangle, \tag{8}$$

where $\tau=\Omega t/(2\xi)$ and $\cos\theta=1-2s\xi^2$ or $\sin(\theta/2)=\sqrt{s}\xi$. Let t_r denote the smallest time satisfying the equation $\sin(\tau)=\sqrt{1-4s\xi^2}/(1-2s\xi^2)=1-2s^2\xi^4+O(\xi^6)$. Obviously, $t_r=\xi\pi/\Omega-4s\xi^3/\Omega+O(\xi^5)\approx\xi\pi/\Omega$ for sufficiently small $\xi=1/\sqrt{N}$. At time t_r, we have the result

$$U_N(t_r)|\Psi_s\rangle=-2\sqrt{s}\xi|FL\rangle+\sqrt{1-4s\xi^2}|0\rangle, \tag{9}$$

where $(1-\cos\tau_r)\cos\theta=1$ has been utilized. This equation demonstrates that the flipped item $|\mathrm{FL}\rangle$ has been sorted out from the state $|\Psi_s\rangle$ with all its unflipped states being completely suppressed by a single unitary operation $U_N(t_r)$. The system will stay in the state $-2\sqrt{s}\xi|\mathrm{FL}\rangle+\sqrt{1-4s\xi^2}|0\rangle$ if we switch off the interactions at time t_r. Once again, this switch-off action can be accomplished by shutting down the readout laser pulse. Once in the state given in Eq. (9), the system will have the probability $4s\xi^2$ in the state $|\mathrm{FL}\rangle$ and the probability $1-4s\xi^2$ in the reservoir state $|0\rangle$. Therefore one is able to identify the flipped item by the method utilized in the experiment of Bucksbaum's group [1]. Again, it is pointed out that besides time t_r, the state in Eq. (9) also emerges at other times such as $t_{r,n}=t_r+4n\xi\pi/\Omega$ for $n=1,2,\ldots$.

Before comparing our scheme with the AWB scheme [1], we need to establish the notation connection between them, and the approximated condition for the AWB scheme. Utilizing the connection that $|0\rangle$ and $|j\rangle$ for $j=1,2,\ldots,N$ are column vectors with $N+1$ elements, or more specific $|0\rangle=(1,0,\ldots,0)^T$, $|1\rangle=(0,1,0,\ldots,0)^T$, $\ldots$, and $|N\rangle=(0,\ldots,0,1)^T$, it is readily seen that matrices A and B in the AWB scheme as shown in Figs. 1A and 1B of Ref. [1], respectively, are, in our notation, $A=1+\epsilon(|0\rangle\langle 1|-|1\rangle\langle 0|)-\epsilon\Sigma_{j=2}^{N}(|0\rangle\langle j|-|j\rangle\langle 0|)\equiv 1-(\epsilon/\xi)(|0\rangle\langle\Psi_s|-|\Psi_s\rangle\langle 0|)$, and $B=1+\epsilon\Sigma_{j=1}^{N}(|0\rangle\langle j|-|j\rangle\langle 0|)\equiv 1+(\epsilon/\xi)(|0\rangle\langle\Psi_0|-|\Psi_0\rangle\langle 0|)$. Here $|\Psi_s\rangle=-\xi|1\rangle+\xi\Sigma_{j=2}^{N}|j\rangle$

is given in Eq. (2) with $s=1$ and $j_1=1$, and $|\Psi_0\rangle$ is given in Eq. (1). It is obvious, from Eq. (6) and taking $s=1$ and $j_1=1$, that $U_N \approx B$ and $U_s \approx A$ when $\Omega t/2 \equiv \epsilon \ll \xi^2$ [note $\tau \equiv \epsilon/\xi = (\epsilon/\xi^2)\xi \ll 1$ as $\epsilon \ll \xi^2 \equiv N^{-1}$]. Consequently, the original AWB scheme is based on an approximated theory under the condition $N\epsilon \ll 1$ or $N\Omega t/2 \ll 1$.

Now we are ready to compare our scheme with the AWB scheme [1]. First of all, ours is only a generalization of the original AWB scheme. They are very similar to each other in that both schemes deal with same $(N+1)$-state system (N information states plus an ancillary state $|0\rangle$ called as a reservoir state by Ahn *et al.*) and the same interaction model, i.e., the same form of the interaction Hamiltonian although Ahn *et al.* did not explicitly write it down [1]. This is why we only call our scheme an elaborated AWB scheme. However, the two schemes indeed have the following two crucial differences.

We deal with the information based on our exact results for the evolution operator, whereas the AWB scheme is based on the approximated expression of the evolution operator by using a lowest-order perturbation theory under the condition $N\Omega t/2 \ll 1$. Of course, the lowest-order approximation is a reasonable one (though it may not be the most convenient one) for small N situations just as the case with which the AWB experiment actually dealt [1]. However, the condition for justifying the lowest-order approximation may require too stringent constraints to be satisfied by experiments dealing with huge databases (i.e., sufficiently large N). As mentioned in the part after Eq. (7) [Eq. (9)], information storage (retrieval) in our scheme can be accomplished within either the time span t_s (t_r) or other time spans $t_{s,n}=(1+4n)t_s$ $(t_{r,n}=t_r+4n\pi\xi/\Omega)$ for $n=1,2,\ldots$. Consequently if the width T of the ultrafast laser pulse is too large to meet the requirement $T=t_s \equiv \pi/(\Omega\sqrt{N})$ $(T=t_r)$ for sufficiently large N, we can always choose the appropriate positive integer n such that $T \approx t_{s,n}$ $(T \approx t_{r,n})$ to accomplish the corresponding information storage (retrieval). This fact offers a practical advantage over the original AWB scheme for lessening the stringent constraint on the extreme narrowness of the laser pulse to perform information storage and retrieval, especially when the database is huge.

Based on the exact expression of the evolution operator, our scheme can accomplish more appropriate information storage and retrieval processes, some of which may be unaccessible to the original AWB scheme. For instance, in the information storage process, the final state in our scheme is exactly the information storage state $|\Psi_s\rangle$ [see the discussion after Eq. (7)]. It is worthwhile mentioning that to produce the exact information storage state $|\Psi_s\rangle$ is obviously desirable in information storage from a simple and practical viewpoint. However, the original AWB scheme is unable to achieve this goal since the final result of the information storage process in the AWB scheme is $|0\rangle+(\epsilon/\xi)|\Psi_s\rangle$ in our notation. Note that the ancillary state $|0\rangle$ is heavily dominating over the actual information storage state $|\Psi_s\rangle$ since the probability of the information storage state in the final result $|0\rangle+(\epsilon/\xi)|\Psi_s\rangle$ is extremely small due to the probability $(\epsilon/\xi)^2=\epsilon(\epsilon/\xi^2) \ll \epsilon \ll 1/N$. The final results of the two schemes in information retrieval also have an important difference, at least from a practical viewpoint. The flipped state $|\mathrm{FL}\rangle$ is much more heavily populated in our final state $-2\xi|\mathrm{FL}\rangle+\sqrt{1-4\xi^2}|0\rangle$ [Eq. (9) by taking $s=1$] than the final result $-2\epsilon|\mathrm{FL}\rangle+\sqrt{1-4\epsilon^2}|0\rangle$ (in our notation) in the AWB scheme [1] since the populated probability in our case is $4\xi^2$ much greater than the probability $4\epsilon^2$. As a matter of fact, due to $\epsilon \ll 1/N \equiv \xi^2$, the probability $4\xi^2 \gg 4\epsilon$ is at least N times bigger than the probability $4\epsilon^2$, implying that our scheme is able to accomplish information retrieval (or database searching) much easier than the AWB scheme does, especially for huge databases. The reason for this simplification is that one has to make a large number of identical subsystems to finally sort the flipped state out just as in the experiment of Bucksbaum's group [1], and the minimum number of the identical subsystems in the AWB scheme is $M_{min}^{AWB}=(4\epsilon^2)^{-1}$, while in our scheme it is only $M_{min}=(4\xi^2)^{-1} \ll M_{min}^{AWB}/N$, an extreme reduction when one deals with huge databases (i.e., sufficiently large N).

In summary, we have proposed a scheme for information storage and retrieval through a quantum phase with a single operation each, which generalizes the AWB scheme by removing its constraint on the smallness of the quantity Ωt. Based on the exact unitary transformations, we have shown that the elaborated AWB scheme, besides theoretical interests in its own right, has several pronounced practical advantages over the original one.

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