PHYSICAL REVIEW A, VOLUME 63, 051601(R)

Dark-soliton creation in Bose-Einstein condensates

L. D. Carr,[1,*] J. Brand,[2] S. Burger,[3,†] and A. Sanpera[4]
[1]*Department of Physics, University of Washington, Seattle, Washington 98195-1560*
[2]*Department of Chemistry, University of Washington, Seattle, Washington 98195-1700*
[3]*Institut für Quantenoptik, Universität Hannover, 30167 Hannover, Germany*
[4]*Institut für Theoretische Physik, Universität Hannover, 30167 Hannover, Germany*
(Received 27 November 2000; published 9 April 2001)

It is demonstrated that stable, standing dark solitons can be created in current dilute-gas Bose-Einstein condensate experiments by the proper combination of phase and density engineering. Other combinations result in a widely controllable range of gray solitons. The phonon contribution is small and is calculated precisely. The ensuing dynamics should be observable *in situ*, i.e., without ballistic expansion of the condensate.

DOI: 10.1103/PhysRevA.63.051601 PACS number(s): 03.75.Fi, 05.30.Jp, 05.45.Yv

The observation of solitons in Bose-Einstein condensates (BEC's) brings together two heretofore disparate fields of study: quantum degenerate gases [1] and exactly integrable nonlinear systems [2]. Recently, three-dimensional, solitonic structures have been observed in weakly interacting atomic gas BEC's for the first time [3,4]. As fundamentally nonlinear, collective excitations of a macroscopic quantum wave they are intriguing in their own right. However, standing solitons in one dimension, which are expected to have a wealth of special properties [5], have not yet been created.

The Gross-Pitaevskii equation that describes the mean-field dynamics of a dilute BEC at low temperatures [1] reduces to a one-dimensional nonlinear Schrödinger equation (NLS) when the transverse dimensions of the condensate are on the order of its healing length and its longitudinal dimension is much longer than its transverse ones [6]. This is termed the *quasi-one-dimensional* (quasi-1D) regime of the Gross-Pitaevskii equation. If the transverse dimensions of the condensate are much less than the healing length, then the Gross-Pitaevskii equation no longer applies and other physical models must be applied [7]. The recent confinement of a BEC in a hollow blue–detuned laser beam demonstrates that the quasi-1D regime regime is experimentally realizable [8]. In three dimensions solitonic structures are unstable to transverse modulations, but in the quasi-1D regime they are stable, and are in fact the stationary states of the NLS [10].

The extent to which quantum fluctuations affect the BEC is an outstanding theoretical question [11]. As dark solitons are solutions to the mean-field theory, substantial instability of dark solitons at low temperature in the quasi-1D regime is an experimental measure of such higher-order effects. The present experimental technique for creating solitonic structures is based on phase engineering alone, makes additional transient density waves, and cannot be used to make a single standing dark soliton. Observations have been made in three-dimensional harmonic geometries or at temperatures for which about 10% of the atoms remained uncondensed. Time scales for observation are subsequently on the order of 10 ms, whereas the lifetime of the BEC is on the order of seconds, precluding the likelihood of observing the effect of quantum fluctuations. Finally, the length scale of these solitonic structures, i.e., the healing length, is so small that *in situ* observation of their dynamics is not possible and the condensate must first be expanded by turning off the confining potential. In the following, a method that remedies all of these difficulties is presented.

The NLS may be written [10] in the form

$$[-\xi^2\partial_{xx}+L|\psi(x,t)|^2+V(x)]\psi(x,t)=i\partial_t\psi(x,t), \quad (1)$$

where ψ is the macroscopic quantum wave function, L is a longitudinal confinement length, $V(x)=(2m\xi^2/\hbar^2)V_0(x)$ is a rescaling of a confining potential $V_0(x)$, $\xi\equiv(8\pi\bar{n}a)^{-1/2}$ is the healing length, a is the s-wave scattering length for binary atomic interactions, and $\bar{n}$ is the mean number density. ψ is normalized to 1 and has units of $L^{-1/2}$, x has units of length, and $t\equiv(2m\xi^2/\hbar)t_0$ is dimensionless, where t_0 has units of time.

In particular, we consider the case of ^{87}Rb confined in a boxlike potential, with $L=100$ μm and a transverse length of $L_t=10$ μm. For $N=1.1\times10^4$ atoms and $\bar{n}\equiv N/(LL_t^2)$, the effective healing length [12] is 2.5 μm. Thus the transverse length of the condensate is 4ξ, which satisfies the criteria of quasi-one-dimensionality as defined above and detailed in Refs. [6,10]. As the length scale of solitons is $\sim2\xi$ and the wavelength of the imaging radiation is ~0.5 μm, this experimentally realizable configuration ensures observability without needing to ballistically expand the condensate. It also ensures the possibility of phase and density engineering of structures on the scale of the healing length by means of the dipole potential of laser fields. A key point in this choice of parameters is the use of a boxlike, rather than harmonic, confining potential [13]. For harmonic confinement, the Thomas-Fermi radius [1] of the condensate scales as $N^{1/5}$, $\bar{n}\propto N/N^{3/5}$, and therefore $\xi\propto N^{-1/5}$, whereas for boxlike confinement $\xi\propto N^{-1/2}$.

Neglecting the effect of $V(x)$ and writing the wave function as $\psi(x,t)=\sqrt{\rho(x,t)}\exp[i\phi(x,t)-i\mu t]$, the single soliton

*Author to whom correspondence should be addressed.
†Present address: INFM–LENS, Largo E. Fermi, 2, 50125 Firenze, Italy.

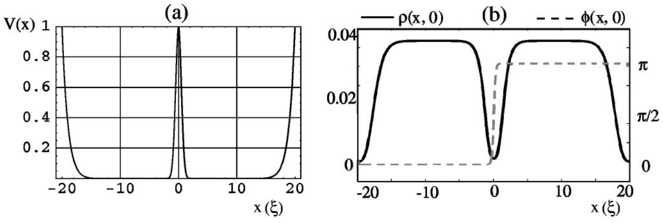

FIG. 1. (a) A combination of a boxlike potential and a tightly focused, blue-detuned laser beam is used to engineer the density. (b) The resulting wave function is phase engineered with a second, far-detuned laser beam, resulting in an initial state very close to a standing dark soliton.

solution to Eq. (1) takes the form

$$\rho(x,t)=\frac{1}{L}\{1-2\gamma^2\xi^2\,\mathrm{sech}^2[\gamma(x-ct)]\}, \tag{2a}$$

$$\phi(x,t)=\tan^{-1}\left\{\frac{2\xi^2\gamma}{c}\tanh[\gamma(x-ct)]\right\}+\frac{\pi}{2}, \tag{2b}$$

where

$$2\gamma^2\xi^2+c^2/(2\xi^2)=1. \tag{2c}$$

For $L\gg\xi$ Eqs. (2) are an excellent approximation away from the walls. The single-particle density $\rho(x,t)$ is constant except over a density notch of width $1/\gamma$, and the phase changes sharply and monotonically across the notch. The chemical potential $\mu=1$ in these units and c is the speed of the soliton. The constraint (2c) links the soliton velocity c to its depth $2\xi^2\gamma^2$ and thus leaves only a single free parameter for the soliton solution. In the case $c=0$, $1/\gamma=\sqrt{2}\xi$ and the soliton forms a node, is a stationary solution to Eq. (1), and is called a *dark soliton*. In the case $0<c<c_s$, where $c_s\equiv\sqrt{2}\xi$ is the Bogoliubov sound speed, the soliton is a moving density notch and is called a *gray soliton*. In the case $c\to c_s^-$, $1/\gamma$ diverges and the soliton depth approaches zero. Upon reinsertion of constants, it is found that $c_s=\sqrt{4\pi\bar{n}a}\hbar/m$. The total phase difference across the notch [14] is $\Delta\phi=2\tan^{-1}[\sqrt{1-(c/c_s)^2}/(c/c_s)]$.

As is apparent from Eq. (2), a soliton requires both a nontrivial phase and a nontrivial density profile. The technique of phase engineering used to make solitonic structures in previous experiments [3,4] imprinted a constant phase on one-half of a nearly uniform condensate. This resulted in a combination of transient density waves and one or more gray solitons. We propose instead the following scheme.

Atoms are condensed into a boxlike trap with a sharply focused, blue-detuned laser beam in the center of the long trap axis, closed in the longitudinal direction by two laser light sheets. Due to a large laser detuning from atomic resonances, spontaneous processes can be neglected on a time scale of seconds, which is longer than all relevant time scales. Thus the effect of the laser beam on the condensate atoms can be described by the optical dipole potential, which is proportional to the local light intensity. In Fig. 1(a) the potential used as a model is illustrated; a Gaussian with a

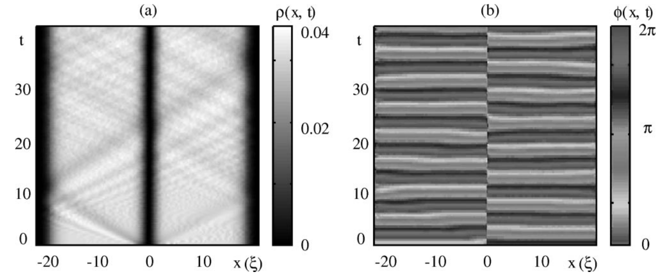

FIG. 2. A standing dark soliton is created with the proper combination of phase and density engineering. In addition to the central dark soliton, characterized by a π phase difference across a density notch that forms a node, a spray of phonons is emitted at the speed of sound ($c_s=\sqrt{2}$) to either side. A second, shallow soliton, emitted to the left at nearly c_s, offsets the effect of the finite slope of the initial phase due to the diffraction-limited fall-off of the phase-engineering laser beam.

width on the order of the healing length, i.e., $2\Delta x_v\sim 2.5\ \mu$m, steep walls at $\pm L/2$, and a height on the order of the chemical potential $\mu=1$:

$$V(x)=\exp\left(\frac{-x^2}{2(\Delta x_v)^2}\right)+\left(\frac{2x}{L}\right)^{20}. \tag{3}$$

In Fig. 1 and in all simulations the parameters $\xi=1$ and $L=40$ were used. It should also be possible to create a density notch by adiabatically ramping the intensity of a laser into an initially uniform BEC. The resulting deformation dips to $\sim$1% of the maximum density, or $\sim$10% of the maximum amplitude, as may be seen in Fig. 1(b). Because the scale of variation of $V(x)$ is $\lesssim\xi$, the kinetic-energy term in Eq. (1) cannot be neglected and the response of the condensate is not in the Thomas-Fermi regime. It is therefore obtained numerically by imaginary time relaxation of Eq. (1) with Eq. (3).

At $t=0$ the focused laser beam is switched off and a second, far-detuned laser pulse of uniform intensity distribution is shined on one half of the condensate. The pulse duration is chosen to be shorter than the correlation time of the condensate, $t_c=\xi/c_s$. This ensures that the light field changes only the phase distribution and not the density distribution of the BEC. In the simulation it therefore suffices to switch on a controllable phase $\phi(x)$ instantaneously [3]:

$$\phi(x)=\Delta\phi\tanh(2x/\Delta x_\phi) \tag{4}$$

was used as a model, which represents the diffraction-limited fall-off of the laser beam over the notch, with a width of Δx_ϕ and a total phase difference of $\Delta\phi$, as may be seen in Fig. 1(b), where the parameters $\Delta\phi=\pi$ and $\Delta x_\phi=\xi\sim 2.5\ \mu$m have been used.

The resulting notch is both density and phase engineered. In Fig. 2 its evolution is shown. For ^{87}Rb there are 2.27 ms per simulation time unit [10]. When $\Delta\phi=\pi$ was used it was found that the soliton drifted slowly to the left, i.e., was not completely dark. A true standing dark soliton requires a step function in the phase. Therefore, to counter the effect of the diffraction-limited fall-off of the second laser beam, $\Delta\phi$

$=1.05\times\pi$ was used. This creates a second, shallow gray soliton, which carries away the small momentum of the first without appreciably deforming the density. Thus a standing dark soliton is successfully created.

As is apparent in Fig. 2, some portion of the initial notch is radiated as phonons. The apportionment of the total energy into phonons and solitons may be calculated precisely. Because the NLS is nonlinear, it is not possible to build any particular solution by a summation over phonon modes. As phonons and solitons are the fundamental solution types of the 1D NLS it is therefore useful to single out their contribution to the dynamics. The NLS has a denumerably infinite number of integrals of motion [5]. In the context of the BEC, the first three correspond to normalization, energy, and momentum. For the simulations studied herein, it suffices to consider energy.

The second integral of motion of Eq. (1) is

$$I_2=\int_{-L/2}^{L/2}dx\left[\xi^2|\partial_x\psi|^2+\frac{L}{2}|\psi|^4+V(x)|\psi|^2\right]. \qquad (5)$$

Assuming $L\gg\xi$, the energy of a single soliton is

$$E_s\simeq N\int_{-\infty}^{\infty}dx\left[\xi^2|\partial_x\psi|^2+\frac{L}{2}\left(|\psi|^2-\frac{1}{L}\right)^2\right], \qquad (6)$$

where Eq. (5) has been renormalized to subtract out the constant background and the potential $V(x)$ has been assumed to be constant in the region of the soliton. The factor of N follows from the use of a single, rather than N, particle density for $|\psi(x,t)|^2$. Equations (2) may then be used to obtain the energy of a single soliton:

$$E_s\simeq\frac{N}{L}\frac{4}{3}c_s|\sin(\Delta\phi/2)|^3. \qquad (7)$$

E_s is dimensionless and depends on the line density N/L. The rescaling $[\hbar^2/(2m\xi^2)]E_s\propto(N/L)^{3/2}$ reinstates the units of energy. For our choice of parameters, the difference between the approximation and an exact expression for E_s is negligible and not observable in simulations.

Conservation of the total energy of the system may be expressed as $E_d+E_0=E_s+E_p+E_0$, where E_d, E_0, and E_p are the energies of the initial deformation, the undeformed background ground state, and the phonons, respectively. By use of Eq. (5) E_d and E_0 may be determined in the simulation. E_s may be calculated from Eq. (7). With this method it was determined that the total phonon contribution in Fig. 2 is 39% of the deformation energy E_d and the second soliton carries away only 1.5% of E_d. However, the phonons spread out over the whole box, while the 60% of the energy carried by the dark soliton is localized in a region of width $\sim 2\xi=2$. Therefore this method of dark soliton creation is highly efficient and phonons are essentially small fluctuations in the density.

It is important to explore the robustness of the proposed technique. To what extent does it depend on the parameters of Eqs. (3) and (4)? To this end many simulations were performed that varied Δx_v, Δx_ϕ, and $\Delta\phi$. It was found that

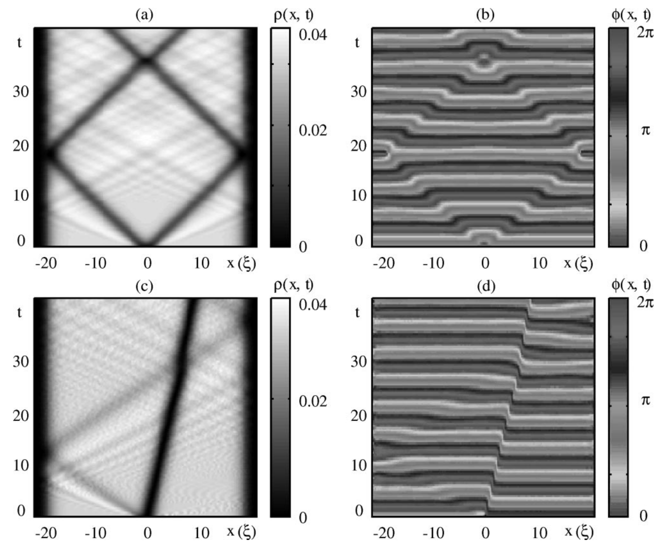

FIG. 3. A variety of physically intriguing possibilities arise from variations in the technique used to create a standing dark soliton. (a), (b) The use of density engineering alone creates a pair of equal and opposite gray solitons with a widely controllable range of velocities. (c),(d) Phase engineering with a phase different from π creates an asymmetric gray soliton pair. Note the spatial shift inherent in soliton-soliton interactions.

$2\Delta x_v\leqslant\xi$ was optimal. Note that the full width of the Gaussian is $2\Delta x_v$. $2\Delta x_v=2\xi$ produced an additional pair of very shallow gray solitons that would not be observable in experiments. $\Delta x_\phi\leqslant\xi$ was sufficient to produce a standing dark soliton, while $L\gg\Delta x_\phi>\xi$ set the soliton in motion. Finally, $\Delta\phi\sim\pi$ was required, as expected. It was also found that the amount of the deformation energy transformed into phonons was consistently about 40% to 50%. Outside of these parameter ranges, a number of intriguing possibilities arose. In the following a few such possibilities are illustrated.

If density engineering alone is used, a pair of equal and opposite gray solitons emerge. This effect is illustrated in Figs. 3(a) and 3(b). The velocities of the gray solitons may, by Eq. (7), be used to determine the total energy of the phonons. It was found in simulations that within the range of $2\Delta x=0.5$ to 2.5, from 40% to 70% of the deformation energy was converted into phonons and the remaining energy was converted into a symmetric gray soliton pair. In general, a lower energy notch resulted in slower solitons, with a lower limit on the soliton velocity of $\sim 0.45c_s$. In previous, solitonic-structure experiments, which used phase engineering alone [9], the velocities were $0.54c_s$ to $0.76c_s$ and $0.68c_s$ to c_s, respectively [3,4]. By the method of density engineering a wide range of velocities from $0.45c_s$ to c_s may be obtained by adjusting the width or depth and thereby the energy of the initial deformation.

Another variation on the technique, which uses both density and phase engineering and a phase difference of $\Delta\phi\neq\pi$, creates an asymmetric gray soliton pair. 60% to 70% of the deformation energy is radiated as phonons. For the phase profile of Eq. (4), $\pi<\Delta\phi<2\pi$ results in a faster soliton to the right. $0<\Delta\phi<\pi$ results in a faster soliton to the left, as is to be expected from the constant phase offset inherent in

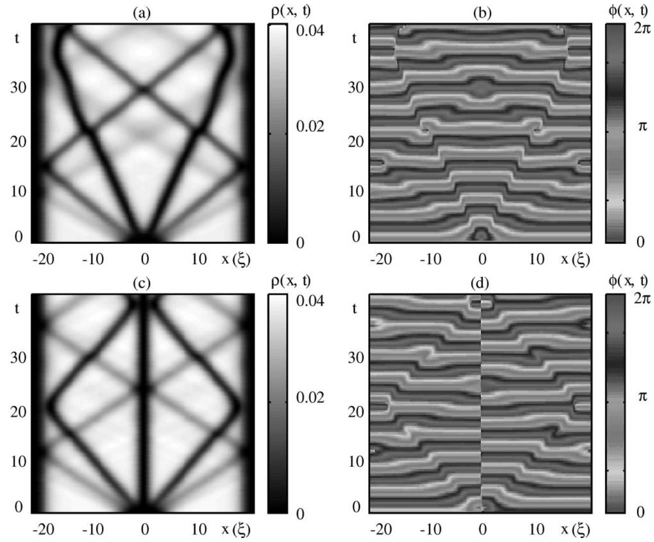

FIG. 4. (a),(b) Density engineering a notch wider than the healing length creates a symmetric fan of gray solitons. (c),(d) If phase engineering is added a dark soliton is created in the center of the fan.

the wave function. The total phase difference across the two solitons adds up to $\Delta\phi$. Such asymmetric interactions are useful in observing the delay inherent in soliton interactions [15].

If again density engineering alone is used, but with too large a notch, i.e., $\Delta x_v > \xi$, multiple pairs of equal and opposite gray solitons emerge, as has also been predicted elsewhere [14]. If phase engineering is added to this scenario, a central, dark soliton is created in the middle of the trap with equal and opposite pairs of gray solitons fanning outwards to either side. These cases are shown in Fig. 4.

In general, the consistent 40–70 % of the deformation energy radiated into phonons is representative of the difference in form of the density deformation, which results from the Gaussian function in Eq. (3) and the sech function in Eq. (2a). A different density notch could be expected to result in a different apportionment of energy. It should also be noted that solitons are robustly stable structures [5] and, though not shown here, the introduction of 10% stochastic noise into the simulations, which models experimental imperfections such as nonuniformity in the trapping potential, had no observable effect on the results.

In conclusion, a method of creating a single, standing dark soliton in a gaseous Bose-Einstein condensate has been presented. This experimentally realizable method combines phase and density engineering and requires one or two lasers and a quasi-one-dimensional trap. Variations result in the creation of gray solitons with a widely controllable range of velocities.

We thank Klaus Sengstock and William Reinhardt for helpful discussions. This work was supported by National Science Foundation Grant No. CHE97-32919 and by SFB407 of the *Deutsche Forschungsgemeinschaft* and by the Alexander von Humboldt Foundation.

[1] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999).
[2] V. E. Zakharov and A. B. Shabat, Zh. Éksp. Teor. Phys. **64**, 1627 (1973) [Sov. Phys. JETP **37**, 823 (1973)].
[3] S. Burger, K. Bongs, W. Ertmer, K. Sengstock, A. Sanpera, G. V. Shlyapnikov, and M. Lewenstein, Phys. Rev. Lett. **83**, 5198 (1999).
[4] J. Denschlag, J. E. Simsarian, D. L. Feder, C. W. Clark, W. P. Collins, J. Cubizolles, L. Deng, E. W. Hagley, K. Helmerson, W. P. Reinhardt, S. L. Rolston, B. I. Schneider, and W. D. Phillips, Science **287**, 97 (2000).
[5] Y. S. Kivshar, Phys. Rep. **298**, 81 (1998).
[6] L. D. Carr, C. W. Clark, and W. P. Reinhardt, Phys. Rev. A **62**, 063610 (2000).
[7] D. S. Petrov, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **85**, 3745 (2000).
[8] K. Bongs, S. Burger, S. Dettmer, D. Hellweg, J. Arlt, W. Ertmer, and K. Sengstock, e-print cond-mat/0007381.
[9] L. Dobrek, M. Gajda, M. Lewenstein, K. Sengstock, G. Birkl, and W. Ertmer, Phys. Rev. A **60**, R3381 (1999).
[10] L. D. Carr, M. A. Leung, and W. P. Reinhardt, J. Phys. B **33**, 3983 (2000).
[11] S. A. Morgan, J. Phys. B **33**, 3847 (2000).
[12] The effective healing length takes into account the transverse dimensions of the condensate by making an approximate separation of variables in the wave function and integrating over the transverse ground states [10].
[13] A hollow blue-detuned laser beam can be made to have a wide range of transverse confinement potentials by the use of higher modes. In particular, the quasi-1D experiment of Ref. [8] is capable of producing $V(\rho)=\rho^{20}$, where ρ is the radius.
[14] W. P. Reinhardt and C. W. Clark, J. Phys. B **30**, L785 (1997).
[15] S. Burger, L. D. Carr, P. Öhberg, A. Sanpera, and K. Sengstock (unpublished).