Teleportation of atomic states within cavities in thermal states

Shi-Biao Zheng[1,2,*] and Guang-Can Guo[1,†]
[1]*Laboratory of Quantum Communication and Quantum Computation and Department of Physics, University of Science and Technology of China, Hefei 230026, People's Republic of China*
[2]*Department of Electronic Science and Applied Physics, Fuzhou University, Fuzhou 350002, People's Republic of China*
(Received 20 September 2000; published 15 March 2001)

A scheme is proposed for the teleportation of atomic states within cavities. The scheme is insensitive to the cavity field states and cavity decay. The teleportation may be achieved in a simple way. We show how the idea can be used to generate multiatom entangled states. In order to entangle n atoms, we require $n-1$ cavities and allow the first cavity to exchange energy with the environment. The other $n-2$ cavities are always in the vacuum state and thus the cavity decay is suppressed.

DOI: 10.1103/PhysRevA.63.044302 PACS number(s): 03.67.Lx, 03.65.Ta, 42.50.Dv

In recent years, much attention has been paid to quantum entanglement of two particles, which not only provides powerful tools for testing local hidden theory against quantum mechanics [1], but also resides in the heart of quantum information processing, e.g., teleportation of quantum states [2]. Recently, two-particle entangled states have been realized in both cavity QED [3] and ion traps [4]. Quantum teleportation has been demonstrated using optical systems [5] and NMR [6]. A scheme has been proposed to teleport the internal state of a trapped ion [7].

In the context of microwave cavity QED, several schemes have been proposed for the teleportation of an unknown atomic state [8]. In these schemes, the cavities act as memories, which store the information of an atom and then transfer to another atom after the conditional dynamics. Thus these schemes require that the cavities be initially cooled to the vacuum states and have a very high-quality factor, which is experimentally problematic. In the optical regime, Bose *et al.* [9] have proposed a novel scheme for teleportation of atomic states via cavity decay. The scheme requires the ability to trap a single three-level atom in a cavity. Again within the framewok of microwave cavity QED, we have proposed an alternative scheme for realizing two-atom entanglement and quantum computation and teleportation [10]. The distinct advantage of the proposed scheme is that during the operation, the cavity is only virtually excited and thus the efficient decoherence time of the cavity is greatly prolonged.

On the other hand, much interest has been paid to entangled states involving three or more particles, referred to as Greenberger-Horne-Zeilinger (GHZ) states [11]. With such states, a set of measurements is sufficient for demolishing local hidden theories, in contrast with the case using two-particle states where the contradictions with locality are of a statistical nature [1]. Apart from the fundamental tests of quantum mechanics, GHZ states are useful in quantum information processing, such as cryptographic conference, multiparticle generalization of superdense coding [12], reducing communication complexity [13], and quantum secret sharing [14]. Recently, three-photon GHZ states have been observed [15]. Entanglement of four trapped ions has also been demonstrated [16] using the technique proposed by Mølmer and Sørensen [17]. In the context of cavity QED, many schemes have been presented for the generation of multiatom GHZ states [18]. A three-particle entangled state has been successfully produced in cavity QED [19].

In this contribution, we propose an alternative scheme for the teleportation of atomic states within the framework of microwave cavity QED. A distinct advantage of the present scheme is that it allows the cavities to be initially in thermal states with a few photons and exchange energies with the environment. Furthermore, we do not require additional classical fields to perform Bell state measurement, which are necessary in previous schemes [8,10]. We show that the idea can also be used to generate entangled states of n atoms by using $n-1$ cavities. In this case only the first cavity is allowed to exchange energy with the environment. However, the other $n-2$ cavities are in vacuum states throughout the procedure and thus the effect of cavity decay is greatly suppressed.

We first consider two identical two-level atoms simultaneously interacting with a single-mode cavity field. The Hamiltonian for the system is given by

$$H=H_0+H_i, \tag{1}$$

where

$$H_0=\omega a^+a+\omega_0\sum_{j=1,2} S_{z,j}, \tag{2}$$

$$H_i=g\sum_{j=1,2}(a^+S_j^-+aS_j^+), \tag{3}$$

where $S_{z,j}=\frac{1}{2}(|e_j\rangle\langle e_j|-|g_j\rangle\langle g_j|), S_j^+=|e_j\rangle\langle g_j|$, and $S_j^-=|g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|g_j\rangle$ ($j=1,2$) being the excited and ground states of the jth atom, a^+ and a are the creation and annihilation operators for the cavity mode, ω_0 is the atomic transition frequency, ω is the cavity frequency, and g is the atom-cavity coupling strength. In the case $\delta=\omega_0-\omega \gg g\sqrt{\overline{n}}$, with $\overline{n}$ being the mean photon number of the cavity field, there is no energy exchange between the atomic system

*Email address: sbzheng@pub5.fz.fj.cn
†Email address: gcguo@ustc.edu.cn

and the cavity. The energy-conversing transitions are between $|e_1g_2n\rangle$ and $|g_1e_2n\rangle$. The Rabi frequency λ for the transitions between these states, mediated by $|g_1g_2n+1\rangle$ and $|e_1e_2n-1\rangle$, is given by [20]

$$\lambda=\frac{\langle e_1g_2n|H_i|g_1g_2n+1\rangle\langle g_1g_2n+1|H_i|g_1e_2n\rangle}{\delta}+\frac{\langle e_1g_2n|H_i|e_1e_2n-1\rangle\langle e_1e_2n-1|H_i|g_1e_2n\rangle}{-\delta}=\frac{g^2}{\delta}. \tag{4}$$

Since the two transition paths interfere destructively, the Rabi frequency is independent of the photon number of the cavity mode. Then the effective Hamiltonian is

$$H_e=\lambda\left[\sum_{j=1,2}(|e_j\rangle\langle e_j|aa^+-|g_j\rangle\langle g_j|a^+a)+(S_1^+S_2^-+S_1^-S_2^+)\right], \tag{5}$$

where $\lambda=g^2/\delta$. The first and second terms describe the photon-number-dependent Stark shifts, and the third and fourth terms describe the dipole coupling between the two atoms induced by the cavity mode.

The time evolution of this system is decided by Schrödinger's equation,

$$i\frac{d|\psi(t)\rangle}{dt}=H_e|\psi(t)\rangle. \tag{6}$$

Perform the unitary transformation

$$|\psi(t)\rangle=e^{-iH_0't}|\psi'(t)\rangle, \tag{7}$$

with

$$H_0'=\lambda\sum_{j=1,2}(|e_j\rangle\langle e_j|aa^+-|g_j\rangle\langle g_j|a^+a). \tag{8}$$

Then we obtain

$$i\frac{d|\psi'(t)\rangle}{dt}=H_i'|\psi'(t)\rangle, \tag{9}$$

where

$$H_i'=\lambda\sum_{j=1,2}(S_1^+S_2^-+S_1^-S_2^+). \tag{10}$$

Assume the atom 1, which is to be teleported, is initially in a superposition state,

$$|\phi_1\rangle=c_e|e_1\rangle+c_g|g_1\rangle, \tag{11}$$

where c_e and c_g are unknown coefficients. Atoms 2 and 3, initially in the state $|e_2\rangle|g_3\rangle$, are sent through a cavity simultaneously. After an interaction time t, we obtain

$$|\psi'(t)\rangle=\cos(\lambda t)|e_2\rangle|g_3\rangle-i\sin(gt)|g_2\rangle|e_3\rangle. \tag{12}$$

Using Eqs. (7) and (8), we obtain

$$|\psi(t)\rangle=e^{-i\lambda t}[\cos(\lambda t)|e_2\rangle|g_3\rangle-i\sin(gt)|g_2\rangle|e_3\rangle]. \tag{13}$$

With the choice of $\lambda t=\pi/4$, we obtain the maximally entangled two-atom state, i.e., the Einstein-Podolsky-Rosen pair (EPR pair) [21],

$$\frac{1}{\sqrt{2}}(|e_2\rangle|g_3\rangle-i|g_2\rangle|e_3\rangle), \tag{14}$$

where we have discarded the common phase factor $e^{-i\pi/4}$. Note that for the initial atomic state $|e_2\rangle|g_3\rangle$, the time evolution of the system is independent of the photon number of the cavity field and thus the scheme is insensitive to the cavity field state. This is due to the fact that the Rabi frequency does not depend on the photon number, and the photon-number-dependent Stark shifts of the two atoms have the same magnitudes but opposite signs. Thus the scheme allows the cavity field to be in any state with a few photons, e.g., a thermal state.

The state for the whole system can be expanded as

$$|\psi_{1,2,3}\rangle=\frac{1}{2}[|\Psi^+\rangle(c_e|e_3\rangle+c_g|g_3\rangle)+|\Psi^-\rangle(c_e|e_3\rangle-c_g|g_3\rangle)+|\Phi^+\rangle(c_e|g_3\rangle-c_g|e_3\rangle)+|\Phi^-\rangle(c_e|g_3\rangle+c_g|e_3\rangle)], \tag{15}$$

where $|\Psi^\pm\rangle$ and $|\Phi^\pm\rangle$ are the Bell states [22]

$$|\Psi^\pm\rangle=\frac{1}{\sqrt{2}}(-i|e_1\rangle|g_2\rangle\pm|g_1\rangle|e_2\rangle), \tag{16}$$

$$|\Phi^\pm\rangle=\frac{1}{\sqrt{2}}(|e_1\rangle|e_2\rangle\pm i|g_1\rangle|g_2\rangle). \tag{17}$$

Then atoms 1 and 2 are sent through another cavity. After an interaction time, we obtain the evolution of $|\Psi^\pm\rangle$,

$$|\Psi^\pm\rangle\to\frac{1}{\sqrt{2}}e^{-i\lambda\tau}\{(-i)[(\cos\lambda\tau)\mp\sin(\lambda\tau)]|e_1\rangle|g_2\rangle]\pm[(\cos\lambda\tau)\mp\sin(\lambda\tau)]|e_1\rangle|g_2\rangle\}. \tag{18}$$

Choosing $\lambda\tau=\pi/4$, we obtain

$$|\Psi^{\pm}\rangle \rightarrow \begin{cases} -i|e_1\rangle|g_2\rangle \\ -|g_1\rangle|e_2\rangle. \end{cases} \quad (19)$$

Again, the common phase factor $e^{-i\pi/4}$ is discarded. On the other hand, $|\Phi^{\pm}\rangle$ involve two terms $|e_1\rangle|e_2\rangle$ and $|g_1\rangle|g_2\rangle$, which do not undergo transitions but impart phase shifts during the interaction.

Then the sender (Alice) performs a joint measurement on atoms 1 and 2 separately. The outcomes $|e_1\rangle|g_2\rangle$ and $|g_1\rangle|e_2\rangle$ correspond to $|\Psi^{+}\rangle$ and $|\Psi^{-}\rangle$, respectively. If Alice obtains outcome $|e_1\rangle|g_2\rangle$, she can tell the receiver (Bob) that atom 3 has been prepared in the initial state of atom 1. While Alice obtains $|g_1\rangle|e_2\rangle$, she tells Bob to perform the following transformation on atom 3:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (20)$$

In this way Bob can exactly convert atom 3 into a replica of the original state of atom 1. On the other hand, Alice cannot distinguish $|\Phi^{+}\rangle$ from $|\Phi^{-}\rangle$. Thus, if she obtains outcome $|e_1\rangle|e_2\rangle$ or $|g_1\rangle|g_2\rangle$, the procedure fails. Therefore, the present scheme is a probabilistic one with the probability of success being 50%.

We now turn to the problem of generating multiatom entangled states. In order to do so, we use ladder-type three-level atoms. The highest, middle, and lowest levels are denoted by $|i\rangle$, $|e\rangle$, and $|g\rangle$, respectively. The transition frequency between the states $|e\rangle$ and $|i\rangle$ is highly detuned from the cavity frequency and thus the state $|i\rangle$ is not affected during the atom-cavity interaction. Atoms 1 and 2 are simultaneously sent through a cavity 1. The effective Hamiltonian is given by Eq. (5). Assume the atoms are initially in the state $|e_1\rangle|g_2\rangle$. After an interaction time $t=\pi/(4\lambda)$, we obtain the maximally entangled state for atoms 1 and 2,

$$\frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle - i|g_1\rangle|e_2\rangle). \quad (21)$$

Then atoms 2 and 3 pass through another cavity simultaneously. Assuming this cavity is initially in the vacuum state, the effective Hamiltonian reduces to

$$H_e=\lambda\left(\sum_{j=2,3}|e_j\rangle\langle e_j| + (S_2^{+}S_3^{-}+S_2^{-}S_3^{+})\right). \quad (22)$$

Assume that atom 3, before entering cavity 2, is prepared in the state

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}(|g_3\rangle + |i_3\rangle). \quad (23)$$

In order to obtain such a state, we first let atom 3, initially in the state $|g_3\rangle$, cross two classical fields tuned to the transitions $|g\rangle\rightarrow|e\rangle$ and $|e\rangle\rightarrow|i\rangle$, respectively. Choose the amplitudes and phases of the classical fields appropriately so that this atom undergoes the transition

$$|g_3\rangle \rightarrow \frac{1}{\sqrt{2}}(|g_3\rangle + |e_3\rangle) \rightarrow |\phi_3\rangle. \quad (24)$$

Since for the states $|g_2\rangle|g_3\rangle$, $|g_2\rangle|i_3\rangle$, and $|e_2\rangle|i_3\rangle$ the photon-number-dependent Stark shifts of atoms 2 and 3 do not compensate for each other, we require the second cavity to be in the vacuum state. The interaction time of the atoms 2 and 3 with the second cavity is set to be $t'=\pi/\lambda$. Then we obtain

$$\frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle|\phi_3\rangle - i|g_1\rangle|e_2\rangle|\phi_3'\rangle), \quad (25)$$

where

$$|\phi_3'\rangle = \frac{1}{\sqrt{2}}(|g_3\rangle - |i_3\rangle). \quad (26)$$

Since $|\phi_3\rangle$ is orthogonal to $|\phi_3'\rangle$, the state of Eq. (25) is a three-atom maximally entangled state.

In order for the scheme to be valid, the interaction time of atoms 1 and 2 with cavity 1 is one-quarter of that of atoms 2 and 3 with cavity 2. Assume that the atomic velocity $v=l\lambda/\pi$, with l being the length of the cavities. Then a static field should be applied to Stark shift atoms 1 and 2 far off-resonant with cavity 1 for a time $3\pi/(4\lambda)$ during their passage through this cavity so that the effective atom-cavity interaction time is $\pi/(4\lambda)$. This has been done in recent experiments [19,23].

We note that the scheme can be generalized to generate n-atom entangled states. We assume that atoms 1 and 2 are prepared in the EPR state of Eq. (21) after they cross cavity 1 simultaneously. Then atom 2 passes through $n-2$ cavities sequentially. When this atom enters the kth $(1<k<n-1)$ cavity, the $(k+1)$th atom initially prepared in the state

$$|\phi_{k+1}\rangle = \frac{1}{\sqrt{2}}(|g_{k+1}\rangle + |i_{k+1}\rangle) \quad (27)$$

enters this cavity simultaneously. Finally, the n atoms are prepared in the state

$$\frac{1}{\sqrt{2^{n-1}}}[|e_1\rangle|g_2\rangle(|g_3\rangle+|i_3\rangle)\cdots(|g_n\rangle+|i_n\rangle) \\ -i|g_1\rangle|e_2\rangle(|g_3\rangle-|i_3\rangle)\cdots(|g_n\rangle-|i_n\rangle)]. \quad (28)$$

In summary, we have proposed a very simple scheme to realize quantum teleportation of atomic states with dispersive cavity QED. In contrast to previous schemes [8,10], the present one is a probabilistic one with the probability of success being 50%. However, it has the following advantages. First, it allows the cavities to exchange energy with the environment. Thus the scheme is insensitive to both the cavity decay and the existence of thermal photons. Second, the required procedure is somewhat simplified. In order to perform joint measurement on atoms 1 and 2, previous schemes require additional classical fields, which are unnecessary in the present scheme. The reduction of the number of operations should decrease the experimental errors.

As an intermediate step, the scheme also provides a new way of generating atomic EPR pairs with a cavity, which may exchange energy with the environment. The idea can also be used to generate multiatom entangled states. In order to entangle n atoms, we should use $n-1$ cavities and only allow the first cavity to exchange energy with the environment. However, the other $n-2$ cavities are always in vacuum states and thus the cavity decay is suppressed in the procedure.

This work was supported by the National Natural Science Foundation of China under Grant No. 60008003, Science Research Foundation of Education Committee of Fujian Province under Grant No. K20004, and Funds from Fuzhou University.

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