Breit correction to the parity-nonconservation amplitude in cesium

V. A. Dzuba,[1] C. Harabati,[1] W. R. Johnson,[2] and M. S. Safronova[2]
[1]*School of Physics, University of New South Wales, Sydney 2052, Australia*
[2]*Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556*
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Including the Breit interaction leads to a 0.6% reduction in the magnitude of the $6s$-$7s$ parity-nonconservation (PNC) amplitude in ^{133}Cs, confirming a result recently obtained by A. Derevianko [Phys. Rev. Lett. **85**, 1618 (2000)]. A revised value of the theoretical PNC amplitude for ^{133}Cs is given; the corresponding value of the weak charge shows no noticeable deviation from the standard model.

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I. INTRODUCTION

Parity nonconservation (PNC) in atoms, described in the standard model of the electroweak interaction by exchange of Z bosons between bound electrons and nuclear quarks, leads to nonvanishing electric-dipole matrix elements between atomic states with the same parity. The dominant part of PNC matrix elements (arising from the vector nucleon current) is proportional to a conserved weak charge Q_W, which is sensitive to physics beyond the standard model such as the existence of additional neutral Z' bosons.

Measurements of the $6s$-$7s$ PNC amplitude in ^{133}Cs, following the procedure described by Bouchiat and Bouchiat [1], were carried out at the 2% level of accuracy by Gilbert and Wieman [2]. When combined with calculations of the PNC amplitudes, which were estimated to be accurate to 1% [3–5], the measurements led to an experimental value for the weak charge,

$$Q_W(^{133}\mathrm{Cs}) = -71.04 \pm 1.81, \tag{1.1}$$

where the uncertainties in the calculations are added in quadrature with the experimental errors. This experimental value differed from the weak charge predicted by the standard model [6,7] (which includes radiative corrections)

$$Q_W(^{133}\mathrm{Cs}) = -73.09 \pm 0.03 \tag{1.2}$$

by 1.1 standard deviation.

In recent years, there has been a substantial improvement in the experimental determination of Q_W in cesium, due primarily to the precise measurement of the ratio of the $6s$-$7s$ PNC amplitude to β, the vector part of the Stark polarizability, by Wood *et al.* [8]. Another factor responsible for the improvement is a measurement of β, by Bennett and Wieman [9], in terms of M_{hf}, the off-diagonal hyperfine matrix element between the $6s$ and $7s$ states [10–12]. A reevaluation of the accuracy of the atomic structure calculations used in the determination of Q_W was also carried out in Ref. [9] based on comparisons of theoretical polarizabilities, hyperfine constants, and transition matrix elements with recent measurements, suggesting that the uncertainty in the theoretical PNC amplitude should be reduced from 1% to 0.4%. Combining the recent experiments with the revised estimate of the theoretical uncertainty led to the value

$$Q_W(^{133}\mathrm{Cs}) = -72.06 \pm 0.44, \tag{1.3}$$

where the theoretical uncertainty ± 0.34 is added in quadrature with the experimental error ± 0.28. This value differs by 2.3 standard deviations from the theoretical value in Eq. (1.2). Although the 0.6% accuracy of the experimental value of the weak charge is comparable to the accuracy of other experimental parameters used in tests of the standard model cited in Ref. [6], 2.3σ is one of the two largest differences with standard model predictions. Implications of this relatively large difference for physics beyond the standard model have been the subject of several recent investigations [13–15].

The contribution of the Breit interaction to the $6s$-$7s$ PNC amplitude in ^{133}Cs was recently investigated by Derevianko [16] and found to be substantially larger than previously estimated. The increased contribution accounted for a substantial part of the 2.3σ discrepancy discussed above. In the present paper, we evaluate the Breit correction both in the lowest-order Dirac-Hartree-Fock (DHF) approximation and including higher-order correlation effects. The present result for the Breit correction in the DHF approximation, 0.3% of the PNC-DHF amplitude[1] or $+0.002$, agrees precisely with the earlier estimate in [4,5]. Furthermore, our result for the Breit correction to the correlated PNC amplitude agrees well with the value found in Ref. [16], 0.6% of the correlated amplitude or $+0.0054$, confirming the principal conclusion of [16] and practically removing the deviation from the standard model claimed in [9].

II. CALCULATION

In the "frozen-core" DHF approximation, the perturbation $\tilde{\psi}_v^{\mathrm{HF}}$ to a valence electron state ψ_v^{HF} induced by the weak interaction h_{PNC} satisfies the inhomogeneous DHF equation

$$(h_0 + V^{\mathrm{HF}} - \epsilon_v^{\mathrm{HF}})\tilde{\psi}_v^{\mathrm{HF}} = -h_{\mathrm{PNC}}\psi_v^{\mathrm{HF}}. \tag{2.1}$$

In this equation, V_{HF} is the HF potential of the closed xenon-like core and ϵ_v^{HF} is the eigenvalue of the unperturbed DHF

[1]We use units $iea_0 \times 10^{-11}(-Q_W/N)$ throughout for the PNC amplitude.

TABLE I. Contributions, in the PNC-DHF approximation, to the $6s$-$7s$ PNC amplitude in ^{133}Cs. The two terms in Eq. (2.2) and their sum $E_{\rm PNC}^{\rm HF}$ are evaluated using the Coulomb DHF potential and the Coulomb + Breit DHF potential.

Type	$\langle \psi_{7s}^{\rm HF}\vert D\vert \tilde{\psi}_{6s}^{\rm HF}\rangle$	$\langle \tilde{\psi}_{7s}^{\rm HF}\vert D\vert \psi_{6s}^{\rm HF}\rangle$	$E^{\rm HF}$PNC
Coul. only	0.27492	−1.01439	−0.73947
Coul. + Breit	0.27411	−1.01134	−0.73722
Δ%	−0.29%	−0.30%	−0.30%

equation. The perturbed DHF equations are solved to give $\tilde{\psi}_{6s}^{\rm HF}$ and $\tilde{\psi}_{7s}^{\rm HF}$ and the PNC amplitude is given by the sum of two terms

$$E_{\rm PNC}^{\rm HF}=\langle \psi_{7s}^{\rm HF}|D|\tilde{\psi}_{6s}^{\rm HF}\rangle+\langle \tilde{\psi}_{7s}^{\rm HF}|D|\psi_{6s}^{\rm HF}\rangle, \qquad (2.2)$$

where D is the dipole operator. Values of the two terms and the resulting sum for the $6s$-$7s$ PNC transition in ^{133}Cs are given in Table I. In the first row, we list values obtained using the Coulomb HF potential to calculate unperturbed and perturbed orbitals and eigenvalues. In the second row, we list values obtained after adding the Breit interaction to the Coulomb interaction. The Coulomb + Breit DHF potential is used to obtain the unperturbed orbitals $\psi_v^{\rm HF}$, eigenvalues $\epsilon_v^{\rm HF}$, and the perturbed orbitals $\tilde{\psi}_v^{\rm HF}$ used to evaluate the PNC amplitude. We see that each of the two terms in Eq. (2.2) and their sum are reduced by 0.3%.

The dominant correlation corrections to the PNC amplitude can be included by replacing the DHF orbitals $\psi_v^{\rm HF}$ in Eq. (2.2) by Brueckner orbitals $\psi_v^{\rm Br}$ and by including polarization corrections to the weak-interaction operator $h_{\rm PNC}$ and the dipole operator, as described in [3]. Brueckner orbitals are obtained by solving Hartree-Fock-like equations for states of external electrons with an additional operator $\hat{\Sigma}$,

$$(h_0+V_{\rm HF}+\hat{\Sigma}-\epsilon_v^{\rm Br})\psi_v^{\rm Br}=0. \qquad (2.3)$$

$\hat{\Sigma}$ is a self-energy operator that is also often called the ''correlation potential.'' It accounts for the correlation interaction between valence and core electrons. Calculation of $\hat{\Sigma}$ is discussed in detail elsewhere [17]. The DHF approximation corresponds to $\hat{\Sigma}=0$.

Core polarization is taken into account by replacing the operator of external field h (where h is either $h_{\rm PNC}$ or D) by $h+\delta V_h$, where δV_h is a correction to the HF potential induced by external field h.

With correlations and core polarizations taken into account, Eq. (2.1) becomes

$$(h_0+V^{\rm HF}+\hat{\Sigma}-\epsilon_v^{\rm Br})\tilde{\psi}_v^{\rm Br}=-h_{\rm PNC}\psi_v^{\rm Br}-\delta V_{\rm PNC}^{\rm HF}\psi_v^{\rm Br}, \qquad (2.4)$$

and the PNC amplitude is given by

$$E_{\rm PNC}=\langle \psi_{7s}^{\rm Br}|D+\delta V_{\rm D}^{\rm HF}|\tilde{\psi}_{6s}^{\rm Br}\rangle+\langle \tilde{\psi}_{7s}^{\rm Br}|D+\delta V_{\rm D}^{\rm HF}|\psi_{6s}^{\rm Br}\rangle. \qquad (2.5)$$

TABLE II. Contributions, in the Brueckner approximation, $6s$-$7s$ PNC amplitude in ^{133}Cs. The two terms in Eq. (2.5) and their sum, the correlated PNC amplitude $E_{\rm PNC}$, are evaluated using the Coulomb interaction only and using the sum of the Coulomb and Breit interactions.

Type	$\langle \psi_{7s}^{\rm Br}\vert D\vert \tilde{\psi}_{6s}^{\rm Br}\rangle$	$\langle \tilde{\psi}_{7s}^{\rm Br}\vert D\vert \psi_{6s}^{\rm Br}\rangle$	$E^{\rm PNC}$
Coul. only	0.43942	−1.33397	−0.89456
Coul. + Breit	0.43680	−1.32609	−0.88929
Δ%	−0.60%	−0.59%	−0.59%

The Breit interaction is included in $V^{\rm HF}$, $\delta V_{\rm PNC}^{\rm HF}$, and $\delta V_{\rm D}^{\rm HF}$. Results for the correlated PNC amplitude calculated without and with the Breit interaction are shown in Table II. We see that the Breit corrections to each term in Eq. (2.5) and to the sum are 0.6%. The Coulomb $E_{\rm PNC}$ amplitude is very close to the results of [3–5]. The difference is caused by some additional small corrections that are not considered in this work, including structural radiation, normalization, and double core polarization by simultaneous action of weak interaction and photon electric field.

It is interesting to examine the origin of the 0.6% Breit correction. To this end, we decompose each term in Eq. (2.5) into a sum over intermediate states,

$$E_{\rm PNC}=\sum_n \frac{\langle 7s|\tilde{D}|np\rangle\langle np|\tilde{h}_{\rm PNC}|6s\rangle}{E_{6s}-E_{np}} + \frac{\langle 6s|\tilde{D}|np\rangle\langle np|\tilde{h}_{\rm PNC}|7s\rangle}{E_{7s}-E_{np}}. \qquad (2.6)$$

Here $\tilde{h}_{\rm PNC}=h_{\rm PNC}+\delta V_{\rm PNC}^{\rm HF}$, $\tilde{D}=D+\delta V_{\rm D}^{\rm HF}$, and $6s$, $7s$, and np designate Brueckner orbitals. Each term in the sum over states has three factors subject to Breit corrections, the matrix element of $\tilde{h}_{\rm PNC}$, the matrix element of $\tilde{D}$, and the energy denominator. The contributions of the Breit interaction from these three factors are −0.6%, −0.4%, and 0.4%, respectively. The corresponding contributions in the PNC-DHF approximation (2.2) are −0.3%, −0.3%, and 0.3%, respectively. Thus, the Breit corrections to the sum arising from corrections to dipole matrix elements or to energy denominators remain very close in correlated and uncorrelated calculations, whereas Breit corrections arising from the weak matrix elements approximately double in the correlated calculation.

The corrections to the correlated PNC amplitude from $h_{\rm PNC}$, D, and energies found in [16] were −0.5%, −0.4%, and 0.3%, respectively, giving a total of −0.6% in agreement with the present result. We disagree, however, with the assertion in [16] that one should ignore the Breit correction to the energies and consider only the −0.5%–0.4% =−0.9% correction to PNC amplitudes. That assertion was based on the incorrect assumption that experimental energies, which implicitly include Breit corrections, were used in Refs. [3–5]. In both [3] and [4,5], theoretical Coulomb energies were used to evaluate the theoretical PNC amplitude.

Although the Breit interaction accounts for a substantial part of the difference between theory and experiment, it should be emphasized that, at the fraction of a percent level, there are various other small theoretical corrections that must be considered. Among these are higher-order many-body Coulomb corrections, corrections due to differences in neutron and proton distributions discussed in Ref. [18], and those from higher order in $Z\alpha$ radiative corrections discussed in Ref. [19]. Since these small corrections may add coherently, it appears premature to assign an uncertainty smaller than 1% to the theoretical PNC amplitude.

The experimental value of the weak charge Q_W is found by dividing the experimental PNC amplitude by the theoretical amplitude expressed in terms of Q_W. We take the value of the theoretical Coulomb amplitude to be -0.9075, which is the average of -0.908 from [3] and -0.907 from [4,5]. (The underestimated Breit correction 0.002 has been removed from the value -0.905 given in [4,5] to obtain the value -0.907 for the Coulomb amplitude.) Adding the Breit correction to this average leads to a revised theoretical value for the PNC amplitude:

$$E_{\mathrm{PNC}}=-0.902. \tag{2.7}$$

The corresponding theoretical amplitude -0.9065 used in Ref. [9] differs by 0.5% from the present value. Combining the present theoretical amplitude with the experimental amplitude from [8,9] leads to a value of Q_W that differs from the standard model by 1.5σ if 0.4% theoretical accuracy is still assumed. However, if a more realistic 1% theoretical uncertainty is assumed, the value of the weak charge becomes

$$Q_W(^{133}\mathrm{Cs})=-72.42\pm(0.28)_{\mathrm{expt}}\pm(0.74)_{\mathrm{theor}}, \tag{2.8}$$

which is larger than the value given in [9] by 0.5% and shows no significant deviation from the standard model. Corrections arising from the difference between neutron and proton distributions, discussed in Ref. [18], have *not* been included in Eq. (2.8). The value of Q_W given in Eq. (2.8) differs from the value -72.65 from [16]. The difference is explained by the fact that nucleon distribution corrections from [18] were included in [16], and a 0.9% Breit correction, in which Breit corrections to energy denominators were improperly omitted, was assumed.

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