Accelerating decay by multiple 2π pulses

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We show how a control of the dynamics of a decay process can be achieved by the application of a series of 2π pulses on an auxiliary transition. The 2π pulse changes the phase of the ground state by π while leaving the phase of the excited state unaltered. This produces quantum interferences between the transition amplitudes for evolution in the short interval, just before and after the 2π pulse. Such an interference under suitable tailoring of the density-of-states of the bath and the time τ leads to *accelerated* decay.

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In a recent letter Kofman and Kurizki [1] have demonstrated the opposite of the quantum Zeno (suppression of decay) effect [2–8]. They show, by a number of calculations, that the accelerated decay is much more ubiquitous. They derive the following expression for the net decay rate in terms of the coupling constant $g(\omega)$; and the final density-of-states $\rho(\omega)$

$$R=2\pi\int_{o}^{\infty}d\omega|g(\omega)|^2\rho(\omega)F(\omega), \tag{1}$$

where the function $F(\omega)$ is related to the measurements at the intervals of τ

$$F(\omega)=\frac{\tau}{2\pi}\mathrm{sinc}^2\left(\frac{(\omega-\omega_a)\tau}{2}\right) \tag{2}$$

and ω_a is the frequency of the excited state. They show that both the quantum Zeno effect as well as the quantum anti-Zeno effect follow from Eq. (1) depending on the relation between the width Γ_R of $\rho(\omega)$ to the measurement rate τ^{-1} and the relative separation between the peaks ω_m of $\rho(\omega)$ and ω_a of $F(\omega)$. In particular if $|\omega_m-\omega_a|\tau\gg 1$, i.e., if ω_a is detuned from the nearest maximum of $\rho(\omega)$, then one finds accelerated decay by frequent measurements i.e., the quantum anti-Zeno effect [1,9]. We further note that continuous measurements [10] should be distinguished from a series of a large number of discrete set of measurements. Explicit calculations [11] have shown that in such measurements, in contrast to quantum Zeno effect, the state of the system invariably evolves. The classic work of Mishra and Sudarshan [2] on quantum Zeno effect and the recent work of Kofman and Kurizki [1] on quantum anti-Zeno effect invoke in an important way the idea of frequent measurements and consequently these predictions are dependent on the projection hypothesis. In the present report we examine an *alternate scenario* where we do not make use of frequent measurements, however we use a kind of coherent control [12]. We report how coherent control can lead to both acceleration and inhibition of decay depending on the structure of the density-of-states of the bath. In our scheme the system evolves *unitarily* under the influence of a series of ultrashort 2π pulses applied at intervals of τ on an auxiliary transition.

We consider the following scheme, which is similar to the one proposed in Ref. [13] in connection with the inhibition of decay. In the present paper we operate in a different region of parameters so as to produce acceleration of the decay. Initially the atom is prepared in the excited state and we consider its decay to the ground state $|g\rangle$. We consider systematic application of ultrashort 2π pulses on a long-lived transition $|g\rangle\leftrightarrow|l\rangle$. We also assume that during the 2π pulse duration the evolution of the states $|e\rangle,|g\rangle$ due to the vacuum field is negligible. The pulses are applied at $t=\tau,2\tau,\ldots,2N\tau$. In between the pulses the system evolves due to interaction H_1 with the vacuum field. We will show how this scheme can lead to the acceleration of the decay process if the transition frequency and the density-of-states of the bath are appropriately chosen.

Let us assume that τ is small enough so that we can apply first-order perturbation theory as far as the interaction with the vacuum field is concerned. The interaction Hamiltonian in the interaction picture is

$$H_1(t)=\hbar\sum_k |e\rangle\langle g|g_k a_k e^{i\delta_k t}+\mathrm{H.c},$$

$$\delta_k=\omega_{eg}-\omega_k. \tag{3}$$

Here a_k is the annihilation operator for the vacuum field and g_k is the coupling constant. Depending on the model of the vacuum field (free space, photonic band gap, etc.), the index k can also include the polarization index. The first-order perturbation theory gives the evolution of the state $|e\rangle$ as

$$|\Psi(t)\rangle\equiv|e\rangle-\sum_k ig_k^*|g,1_k\rangle\frac{(e^{-i\delta_k t}-1)}{(-i\delta_k)}, \tag{4}$$

which on application of the 2π pulse at $t=\tau$ transforms into

$$|\Psi(\tau^+)\rangle=|e\rangle+\sum_k ig_k^*|g,1_k\rangle\frac{(e^{-i\delta_k\tau}-1)}{(-i\delta_k)}. \tag{5}$$

Here we use the notation τ^+ to indicate the state just after the application of the 2π pulse. Note that the effect of the 2π

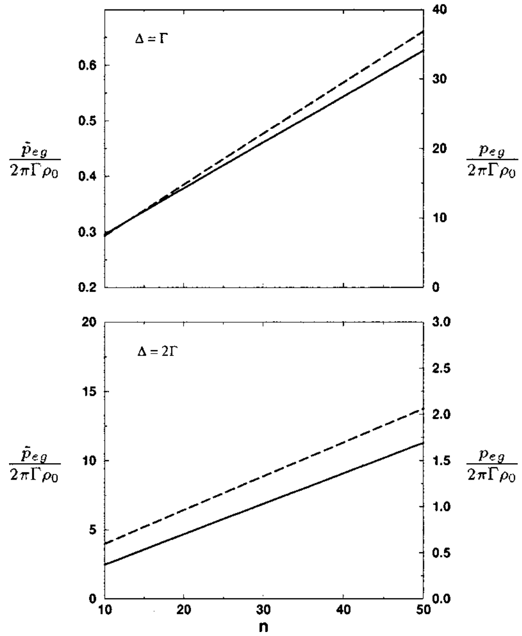

FIG. 1. The two frames show the probability of occupation as a function of n (time) for a continuum with Gaussian density-of-states. The solid curve is for $\tilde{p}_{eg}/2\pi\Gamma\rho_0$ and the dashed curve is for $p_{eg}/2\pi\Gamma\rho_0$. The value of Δ is given in the frames and $\tau\Gamma=1$.

pulse is to change the state $|g\rangle$ to $-|g\rangle$. This π phase change is crucial for our argument. The system evolves under H_1 from τ^+ to 2τ leading to

$$|\Psi(2\tau)\rangle=|e\rangle-i\sum_k g_k^*|g,1_k\rangle\frac{(e^{-i\delta_k\tau}-1)}{(-i\delta_k)}e^{-i\delta_k\tau}+i\sum_k g_k^*|g,1_k\rangle\frac{(e^{-i\delta_k\tau}-1)}{(-i\delta_k)}, \tag{6}$$

which on the application of the second 2π pulse at 2τ becomes

$$|\Psi(2\tau^+)\rangle\equiv|e\rangle+i\sum_k g_k^*|g,1_k\rangle\frac{(e^{-i\delta_k\tau}-1)^2}{(-i\delta_k)}+0(g^2). \tag{7}$$

We note in passing that the standard result (i.e., without 2π pulses) will be obtained from Eq. (7) by replacing $(e^{-i\delta_k\tau}-1)^2$ by $(1-e^{-2i\delta_k\tau})$. We can now continue this evolution till time $2N\tau^+$ and ask what is the probability $\tilde{p}_{ge}$ of finding the atom in the state $|g\rangle$ with the emission of one photon. Clearly $\tilde{p}_{ge}$ is given by

$$\tilde{p}_{ge}=\sum_k |\langle g,1_k|\Psi(2N\tau^+)\rangle|^2. \tag{8}$$

Our calculation leads to

$$\tilde{p}_{ge}=\sum_k |g_k|^2\tan^2\left(\frac{\delta_k\tau}{2}\right)\frac{\sin^2(\delta_k\tau N)}{(\delta_k/2)^2}. \tag{9}$$

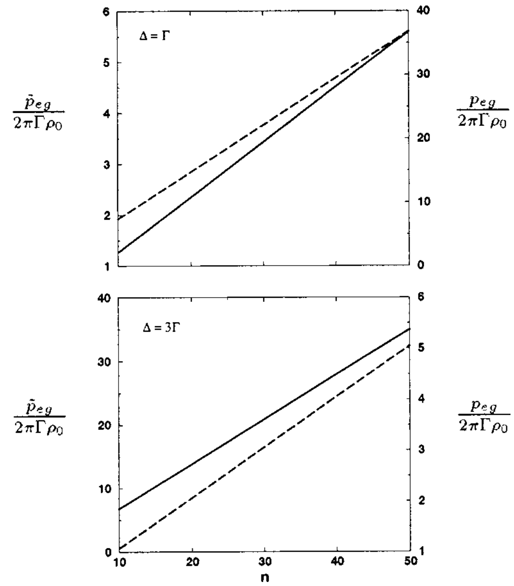

FIG. 2. The two frames show the probability of occupation as a function of n (time) for a continuum with exponential density-of-states. The solid curve is for $\tilde{p}_{eg}/2\pi\Gamma\rho_0$ and the dashed curve is for $p_{eg}/2\pi\Gamma\rho_0$. The value of Δ is given in the frames and $\tau\Gamma=1$.

We note that if we had not applied 2π pulses on the auxiliary transition $|g\rangle\leftrightarrow|l\rangle$ then the result would be given by Eq. (9) *without* the factor $\tan^2(\delta_k\tau/2)$. Evidently, now we have a handle for *manipulating decay characteristics* by suitably choosing the function $\tan^2(\delta_k\tau/2)$. We will call such a function the interference function, which arises from the interference between the transition amplitudes in the intervals $0<t<\tau$ and $\tau^+<t<2\tau$. This is clearly seen from Eqs. (6) and (7). We note in passing that several proposals exist in literature [13–15] for the inhibition of decay using pulses. The actual decay characteristics are quite sensitive to the parameters of the pulses. For example Viola and Lloyd [15] in their work on the interaction of a spin with an Ohmic bath also discover accelerated decay when pulses are not applied very frequently compared to the correlation time of the bath.

In order to understand the effect of the interference function, i.e., application of 2π pulses, we consider a one-dimensional continuum limit of (9):

$$\tilde{p}_{ge}=\int_{-\omega_{eg}}^{\infty}dx\tan^2\left(\frac{x\tau}{2}\right)\frac{\sin^2(x\tau N)}{(x/2)^2}\rho(x), \tag{10}$$

where $\rho(x)$ contains the effect of both the density of final states as well as the transition matrix element. The probability of finding the atom in the ground state has a form similar to the rate R appearing in the work of Kofman and Kurizki. We now evaluate Eq. (10) for a structured continuum defined by both Gaussian and exponential density-of-states

$$\rho(x)=\rho_o\exp\{-|x-\Delta|^2/\Gamma^2\},$$

$$\rho(x)=\rho_o\exp\{-|x-\Delta|/\Gamma\}, \tag{11}$$

where Δ is the difference between the transition frequency ω_{eg} and the central frequency of the structured continuum.

We assume that $\Gamma \ll \omega_{eg}$. We examine the behavior of $\tilde{p}_{ge}$ for $\Delta/\Gamma \gtrsim 1$ and for $\Delta\tau \gtrsim 1$. We show in Figs. 1 and 2 the behavior of $\tilde{p}_{ge}$ for a continuum with Gaussian (exponential) density-of-states as a function of N. We also show the behavior of the standard result p_{ge}, which is obtained by dropping the $\tan^2(x\tau/2)$ from the integrand in Eq. (10). Figures 1(b) and 2(b) clearly demonstrate that $\tilde{p}_{ge} > p_{ge}$, i.e., the decay process has been accelerated by the interference function $\tan^2(x\tau/2)$. At the same time it is also clear that for smaller Δ values (cf. Ref. [13] for $\Delta = 0$) one can obtain inhibition of decay [Figs. 1(a), 2(a)]. Using the ultrashort 2π pulses we have thus achieved a control of the decay process. We again emphasize that this control is achieved without performing either continuous measurements or a series of discrete measurements and thus our control does not involve the postulate of collapse of the wave function.

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