Phase control of group velocity: From subluminal to superluminal light propagation

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We show that the group velocity of a weak pulse can be manipulated by controlling the phases of two weak optical fields applied to a V-shaped three-level system. Such control can even cause the probe propagation to change from subluminal to superluminal. We consider two schemes: in the first, the excited states are coupled by decay-induced coherence, which is an inherent property of the medium, and in the second, quantum coherence is created by coupling the excited states to each other by a strong microwave field. We also discuss the group velocity reduction experienced by a single weak propagating probe due to decay-induced coherence.

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I. INTRODUCTION

Recently, the study of subluminal and superluminal light propagation has attracted a great deal of interest, especially due to the publication of a number of impressive experiments [1–6]. In [1], Hau *et al.* measured a group velocity v_g as low as 17 m/s in a gas of sodium atoms, cooled sufficiently to form a Bose-Einstein condensate. The atomic system was modeled as a Λ-type three-level system in an electromagnetically induced transparency (EIT) setup, in which a region of lossless normal dispersion is created between two absorption lines. The effect relies on the quantum coherence created by applying a strong pump pulse to one transition of the Λ system. The weak probe pulse, applied to the other transition, experiences transparency and very steep positive dispersion. The combination of low absorption and steep positive dispersion can lead to a dramatic slowing down of the group velocity of light and consequently large time delay. Most of the experimental demonstrations of subluminal group velocity [1–4] have been performed using the EIT setup in a Λ system. In another recent experiment Wang *et al.* [6] demonstrated superluminal light propagation using the region of lossless anomalous dispersion between two closely spaced gain lines. The gain doublet is created by applying two intense detuned cw pumps with slightly different frequencies to one transition of a Λ type three-level system in atomic caesium. A weak probe pulse is then applied to the other transition and the gain doublet is produced when the frequency of the probe is such that two-photon Raman resonance is achieved.

In this paper we exploit the fact that the properties of an atomic medium can be dramatically modified by controlling the *phases* of the applied fields, allowing us to manipulate the group velocity at which light propagates. The system we investigate here is a V-shaped closed three-level system, with lower level $|1\rangle$ and upper levels $|2\rangle$ and $|3\rangle$. Two weak fields couple the ground state to the two excited states. We apply *phase control* (PC) in two different schemes. First, when the excited states are coupled by their interaction with the vacuum, scheme A shown in Fig. 1(b), and second, when quantum coherence is created by coupling the two close-lying excited states by a strong microwave field, scheme B shown in Fig. 1(c). We show how *the group velocity of a weak pulse can be controlled by adjusting the relative phase of the two weak optical fields applied to the V-type three-level system and can even cause the probe propagation to change from subluminal to superluminal*.

Since decay-induced interference is an essential component of the former system [Fig. 1(b)], we first consider the effect of decay-induced interference on the propagation of a single weak pulse. This mechanism of creating quantum coherence by spontaneous emission, rather than by the application of a strong coupling field, can lead to similar effects to those observed in the EIT setup. These include ultranarrow resonances and transparency [7] and modification of the fluorescence spectrum [8] in a V-shaped three-level system

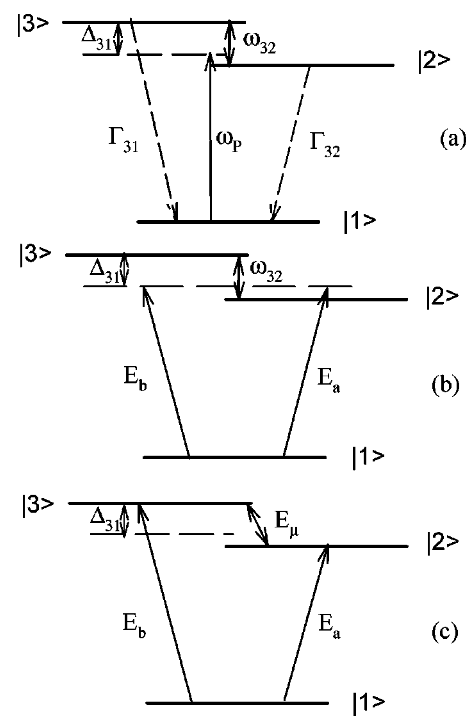

FIG. 1. (a) V-shaped three-level system illuminated by a single weak pulse with center frequency ω_P. (b) Scheme A: a V-shaped three-level system illuminated by two weak pulses with equal center frequencies $\omega_a = \omega_b = \omega$, but different phases Φ_a and Φ_b. The excited states are coupled by their interaction with the vacuum. (c) Scheme B: a V-shaped three-level system illuminated by two weak pulses with frequencies ω_a and ω_b, and phases Φ_a and Φ_b. The excited states are coupled by a strong microwave field.

[see Fig. 1(a)]. We will show that the transparency experienced by a weak pulse in such a system is accompanied by a steep linear positive dispersion, resulting in a reduction in the group velocity of the propagating pulse.

When a system forms a closed loop, its dynamics becomes phase dependent [9]. This fact has been exploited in controlling EIT in sodium atoms [10], where a double-Λ system was excited by laser radiation consisting of four optical frequencies. It was shown that the dark state necessary to establish EIT is created only for specific values of the relative phase $\delta\Phi$ of the applied fields. Paspalakis and Knight [11] showed that phase-dependent effects also arise in spontaneous emission spectra which can be efficiently controlled by varying the relative phase between two lasers, even under conditions that do not allow such control with a single laser. The atomic system used to demonstrate these effects was an open V system, where the ground state is coupled to two close-lying excited states by two lasers of equal frequencies. The excited states decay solely to a common state, thereby producing decay-induced interference effects. In another approach [12], quantum coherence was produced by a microwave field coupling the two excited states of the open V system, rather than by coupling via the vacuum modes. This scheme [13] for producing quantum coherence is appropriate when the transitions between the excited states and the ground state have orthogonal dipoles or, alternatively, when the difference in frequency between the excited states is large compared to their decay rates. The spontaneous emission is then controlled by the amplitudes and relative phases of the three applied fields.

The outline of the paper is as follows: in Sec. II we analyze the group velocity of a single weak pulse propagating in a closed V-shaped atomic system, in the presence of decay-induced quantum interference. In Sec. III, we investigate two schemes (schemes A and B) where the group velocity can be manipulated by controlling the relative phase of two weak fields interacting with a V-shaped system. Conclusions are presented in Sec. V.

II. GROUP VELOCITY OF A WEAK PULSE

We first analyze the propagation of a weak pulse in an atomic medium. The electric field $\tilde{E}(z,t)$ and the polarization $\tilde{P}(z,t)$ induced in the medium by this field are given by

$$\tilde{E}(z,t)=E(z,t)e^{-i(\omega_P\cdot t-k_P\cdot z)}+\text{c.c.}, \tag{1}$$

$$\tilde{P}(z,t)=P(z,t)e^{-i(\omega_P\cdot t-k_P\cdot z)}+\text{c.c.}, \tag{2}$$

where we have assumed that the weak pulse is sharply peaked around the center frequency $\omega=\omega_P$, with $k_P=\omega_P/c$. The Fourier transforms of the induced polarization and the weak electric field obey the linear relation $P(z,\omega)=\chi(z,\omega)E(z,\omega)$ where the real part of $\chi(z,\omega)$ is proportional to the dispersion and its imaginary part to the absorption. We assume that the spectral region of interest of the linear susceptibility $\chi(z,\omega)$ coincides with the optical bandwidth of the weak pulse and expand $\chi(z,\omega)$ around the center frequency ω_P. Inserting the expansion for $\chi(z,\omega)$ into Maxwell's equations leads to the following expression for the group velocity v_g of the propagating pulse [14]:

$$\frac{1}{v_g}=\frac{1}{c}\left[1+2\pi\text{Re}\chi(\omega_P)+2\pi\omega_P\text{Re}\left(\frac{\partial\chi}{\partial\omega}\right)_{\omega=\omega_P}\right]. \tag{3}$$

It is clear from this expression for v_g that when $\text{Re}\chi(\omega_P)$ is zero and the dispersion is very steep and positive, the group velocity is significantly reduced. On the other hand, strong negative dispersion can lead to an increase in the group velocity and even to its becoming negative [15–17] .

We now describe how decay-induced quantum interference in a closed V-shaped atomic system, with lower level $|1\rangle$ and upper levels $|2\rangle$ and $|3\rangle$, interacting with the same vacuum modes [see Fig. 1(a)] can lead to reduced group velocity of a single propagating weak pulse. First, we consider the coherence induced by spontaneous emission, in the absence of the probe. The equations of motion of the density matrix, in the interaction representation, are given by

$$\dot{\rho}_{22}=-\Gamma_{21}\rho_{22}-p\gamma_{23}(\rho_{23}e^{i\omega_{32}t}+\rho_{32}e^{-i\omega_{32}t}), \tag{4}$$

$$\dot{\rho}_{33}=-\Gamma_{31}\rho_{33}-p\gamma_{23}(\rho_{23}e^{i\omega_{32}t}+\rho_{32}e^{-i\omega_{32}t}), \tag{5}$$

$$\dot{\rho}_{21}=-\gamma_2\rho_{21}-p\gamma_{23}e^{-i\omega_{32}t}\rho_{31}, \tag{6}$$

$$\dot{\rho}_{31}=-\gamma_3\rho_{31}-p\gamma_{23}e^{i\omega_{32}t}\rho_{21}, \tag{7}$$

$$\dot{\rho}_{32}=-(\gamma_2+\gamma_3)\rho_{32}-p\gamma_{23}e^{i\omega_{32}t}(\rho_{22}+\rho_{33}), \tag{8}$$

with $\rho_{ij}^*=\rho_{ji}$ and $\rho_{11}+\rho_{22}+\rho_{33}=1$, where $\Gamma_{i1}(i=2,3)$ are the rates of spontaneous emission, and γ_i are the transverse decay rates. The term $\gamma_{23}=\frac{1}{2}\sqrt{\Gamma_{21}\Gamma_{31}}$ accounts for the effect of quantum interference due to spontaneous emission, and $p=\vec{\mu}_{21}\cdot\vec{\mu}_{31}/|\mu_{21}||\mu_{31}|=\cos\theta$ denotes the alignment of the dipole moments $\vec{\mu}_{21}$ and $\vec{\mu}_{31}$. When the dipoles are parallel, $p=1$ and the effect of quantum interference is maximized, whereas when the dipoles are orthogonal, $p=0$ and there is no interference effect due to spontaneous emission. It is clear from Eqs. (4)–(8) that this coherence is only significant when the two upper levels are close.

The absorption of a weak probe in this system has been analyzed and shown to exhibit ultranarrow resonances and even amplification without inversion [7]. We are mainly interested in the near-transparency dip, which is much narrower than the natural linewidth, since it is accompanied by steep dispersion with a positive slope [see Fig. 2(a)], which can lead to a very low group velocity for the probe on propagation.

The Bloch equations for this system, interacting with a weak probe, in the frame rotating with the probe frequency ω, are given by

$$\dot{\rho}_{11}=i(V_{13}\rho_{31}-\rho_{13}V_{31})+i(V_{12}\rho_{21}-\rho_{12}V_{21})+\Gamma_{31}\rho_{33}+\Gamma_{21}\rho_{22}+2p\gamma_{23}(\rho_{23}+\rho_{32}), \tag{9}$$

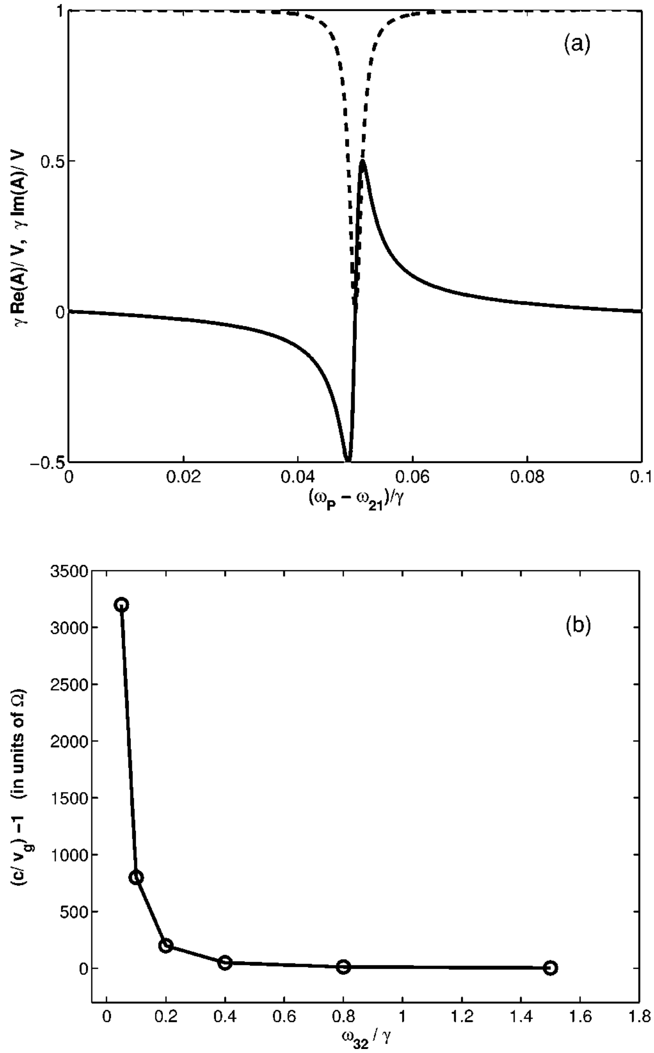

FIG. 2. (a) Real (solid line) and imaginary (dashed line) parts of the polarization induced by the probe (in dimensionless form), as a function of the detuning $(\omega-\omega_{21})/\gamma$, for $p=1$. The real part of $\gamma A/V$ is proportional to the dispersion and its imaginary part is proportional to the absorption. The relaxation rates satisfy $\gamma_2=\gamma_3=\gamma$ and $\Gamma_{21}=\Gamma_{31}=\Gamma=2\gamma$, and the upper level separation is $\omega_{32}=0.1\gamma$. (b) Reciprocal of the difference between the group velocity (dimensionless) in the medium and in vacuum, as a function of the separation between the two upper levels ω_{32} in units of γ. The atomic parameters are the same as in (a).

$$\dot{\rho}_{22}=-i(V_{12}\rho_{21}-\rho_{12}V_{21})-\Gamma_{21}\rho_{22}-p\gamma_{23}(\rho_{23}+\rho_{32}), \tag{10}$$

$$\dot{\rho}_{33}=-i(V_{13}\rho_{31}-\rho_{13}V_{31})-\Gamma_{31}\rho_{33}-p\gamma_{23}(\rho_{23}+\rho_{32}), \tag{11}$$

$$\dot{\rho}_{21}=-iV_{21}(\rho_{22}-\rho_{11})-iV_{31}\rho_{23}-(\gamma_2+i\Delta_{21})\rho_{21}-p\gamma_{23}\rho_{31}, \tag{12}$$

$$\dot{\rho}_{31}=-iV_{31}(\rho_{33}-\rho_{11})-iV_{21}\rho_{32}-(\gamma_3+i\Delta_{31})\rho_{31}-p\gamma_{23}\rho_{21}, \tag{13}$$

$$\dot{\rho}_{32}=iV_{31}\rho_{12}-iV_{12}\rho_{31}-(\gamma_2+\gamma_3-i\Delta)\rho_{32}-p\gamma_{23}(\rho_{22}+\rho_{33}), \tag{14}$$

where the Rabi frequencies are $2V_{21}=\mu_{21}E/\hbar$ and $2V_{31}=\mu_{31}E/\hbar$, and the detunings are $\Delta_{21}=\omega_{21}-\omega, \Delta_{31}=\omega_{31}-\omega$, and $\Delta=\Delta_{21}-\Delta_{31}=-\omega_{32}$. In general, the induced polarization is related to the density operator by

$$\tilde{P}=N\langle\tilde{\mu}\rangle=N\mathrm{Tr}(\hat{\rho}\hat{\mu}). \tag{15}$$

For the present case, the polarization is proportional to $A=\rho_{21}+\rho_{31}$. Solving Eqs. (9)–(14) in the steady-state weak field limit, we obtain

$$A(\omega)=\frac{iV_{21}(\gamma_3+i\Delta_{31}-\gamma_{23})+iV_{31}(\gamma_2+i\Delta_{21}-\gamma_{23})}{(\gamma_2+i\Delta_{21})(\gamma_3+i\Delta_{31})-p^2\gamma_{23}^2}. \tag{16}$$

The linear susceptibility χ, whose real and imaginary parts determine the dispersion and absorption of the weak probe, is proportional to $A(\omega)$. In Fig. 2(a) we plot the real and imaginary parts of $\gamma A(\omega)/V$ as a function of the dimensionless detuning $(\omega-\omega_{21})/\gamma$ for the case where $\gamma_2=\gamma_3=\gamma$, $V_{21}=V_{31}=V$, $\Gamma_{21}=\Gamma_{31}=\Gamma=2\gamma$ and the splitting frequency is taken to be $\omega_{32}=0.1\gamma$. We assume $p=1$ in order to maximize the quantum coherence. It can be seen that at $\omega_P-\omega_{21}=\omega_{32}/2$ the absorption line shape exhibits transparency and the slope of the dispersion becomes very steep and positive.

Now, consider a long, weak pulse with a spectral width similar to that of the transparency, induced by the decay interference. The pulse spectrum has a center frequency ω_P which we take equal to $(\omega_{21}+\omega_{31})/2$. When this pulse propagates in the medium, under the conditions leading to steep and positive dispersion shown in Fig. 2(a), its group velocity, given by $c/v_g=1+\Omega\,\partial\mathrm{Re}(\gamma A/V)/\partial(\omega\gamma^{-1})|_{(\omega_P)}$, is highly reduced. The parameter Ω depends on the characteristics of the medium and is given by $\Omega=\pi\omega_P N\mu^2/\gamma^2\hbar$, where N is the atomic density.

The group velocity is very sensitive to the upper level splitting ω_{32}. This is shown in Fig. 2(b), where (c/v_g-1) is plotted (in units of Ω) versus ω_{32} (in units of γ), for the same parameters as in Fig. 2(a). The slope of the dispersion becomes steeper as ω_{32} decreases, leading to an increasingly lower group velocity. For the parameters used in Fig. 2(b), the expression (c/v_g-1) reduces to the simple form $\Omega(8\gamma^2/\omega_{32}^2)$ [18]. This scheme can be contrasted with one involving EIT where the strength of the coupling field, rather than ω_{32}, determines the group velocity reduction. The plot shown in Fig. 2(b) can be alternatively interpreted in terms of the time delay, which is often the quantity measured in experiments. A weak pulse propagating through a distance L with a reduced group velocity is delayed compared to propagation in free space by $T_g=(L\Omega/c)\partial\mathrm{Re}(\gamma A/V)/\partial(\omega\gamma^{-1})|_{(\omega_P}$.

Although the condition of nonorthogonal dipole moments is rarely met in atomic systems, a number of ingenious proposals have been made to achieve decay-induced interference [19–26]. In a realistic experiment, one has to consider the effect of Doppler broadening, which tends to mask the interference effect [27]. Reduced group velocity as described in this work can be measured by propagating a weak pulse in a medium consisting of cold atoms and molecules, thereby eliminating Doppler broadening. Such cold atoms have already been used in an experiment to reduce the group velocity by Hau *et al.* [1] where they cooled a gas of sodium atoms by laser and evaporative cooling to temperatures below the transition temperature for Bose-Einstein condensation.

III. PHASE CONTROL OF GROUP VELOCITY

We now consider schemes A and B [Figs. 1(b) and 1(c)]. The relative phase of the two weak fields can be controlled, leading to phase-controlled changes in the group velocity of each of the fields. By changing the relative phase, we can demonstrate a wide range of behavior of the dispersion, from absorptionlike profiles where the group velocity cannot be defined by Eq. (3), to dispersionlike profiles with positive or negative slopes, leading to subluminal or superluminal group velocities. The subluminal group velocity can be accompanied by transparency or even gain, whereas the superluminal group velocity is accompanied by absorption.

A. Scheme A: V system with decay-induced interference

The electric fields interacting with the V system have the form

$$\tilde{E}_{a,b}(z,t)=E_{a,b}(z,t)e^{-i(\omega_{a,b}\cdot t-k_{a,b}\cdot z+\Phi_{a,b})}+\text{c.c.} \quad (17)$$

We assume the weak fields to have equal frequencies $\omega_a=\omega_b=\omega$, but different phases $\Phi_a$and Φ_b [11]. The equations of motion of the density matrix are the same as in Eqs. (9)–(14), but now the Rabi frequencies $2V_{j1}$, $j=2,3$ are defined by

$$2V_{21}=2V_{12}^{*}=\mu_{21}E_a/\hbar, \quad 2V_{31}=2V_{13}^{*}=\mu_{31}E_b/\hbar, \quad (18)$$

and the expressions for ρ_{ij} include phase terms.

The absorptive and dispersive response of the atomic medium to the interaction with two weak fields are related to the induced polarization $\tilde{P}$, given by Eq. (15). In the present case, the polarizations induced by E_a and E_b are proportional to ρ_{21} and ρ_{31}, respectively. Solving Eqs. (9)–(14) in the steady-state weak-field limit, in a frame rotating with the frequency ω, we obtain

$$\rho_{21}(\omega)=i\,\frac{V_{21}(i\Delta_{31}+\gamma_3)-V_{31}p\,\gamma_{23}\exp(i\,\delta\Phi)}{(i\Delta_{21}+\gamma_2)(i\Delta_{31}+\gamma_3)-p^2\gamma_{23}^2} \quad (19)$$

$$\rho_{31}(\omega)=i\,\frac{V_{31}(i\Delta_{21}+\gamma_2)-V_{21}p\,\gamma_{23}\exp(-i\,\delta\Phi)}{(i\Delta_{21}+\gamma_2)(i\Delta_{31}+\gamma_3)-p^2\gamma_{23}^2}, \quad (20)$$

where the detunings are $\Delta_{21}=\omega_{21}-\omega$ and $\Delta_{31}=\omega_{31}-\omega$, and the phase difference between the two probes is $\delta\Phi=\Phi_a-\Phi_b$. The linear polarization $\chi_{21}(\chi_{31})$, whose real and imaginary parts determine the dispersion and absorption of the weak field E_a (E_b), is proportional to $\rho_{21}(\omega)$ [$\rho_{31}(\omega)$] of Eq. (19) [Eq. (20)]. We now concentrate on the properties of $\rho_{21}(\omega)$ and note two main features. First, $\rho_{21}(\omega)$ is composed of two parts: the first proportional to the weak field E_a which couples levels $|1\rangle$ and $|2\rangle$, and the second proportional to E_b which couples levels $|1\rangle$ and $|3\rangle$. This second term is due to the coupling of the $|1\rangle$- $|2\rangle$ and $|1\rangle$- $|3\rangle$ transitions by decay-induced interference. Second, in this system phase control is only possible in the presence of decay-induced interference, which depends on the nonorthogonality of the transition dipole moments.

In the general case, the expression for $\rho_{21}(\omega)$ has extrema at the detunings

$$(\Delta_{21})_{1,2}=-\frac{\omega_{32}}{2}\pm\frac{1}{2}\sqrt{\omega_{32}^2+4\gamma_{23}^2(1-p^2)} \quad (21)$$

determined by the imaginary part of the denominator and at

$$(\Delta_{21})_3=-\omega_{32}\frac{\gamma_2}{\gamma_2+\gamma_3} \quad (22)$$

determined by the real part of the denominator. At the detuning $(\Delta_{21})_3$, we look for the conditions that give zero dispersion, that is, $\mathrm{Re}(\chi)=0$, since under these conditions, a weak pulse can propagate nearly unchanged. $\mathrm{Re}(\chi_{21})$ is zero when the parameters obey the relation

$$\sin\delta\Phi=\left(\frac{u\,\omega_{32}}{p\,\gamma_{23}}\right)\frac{\gamma_3}{\gamma_2+\gamma_3}, \quad (23)$$

where u is the ratio between the two weak probes $u=V_{21}/V_{31}$. In Figs. 3(a)–3(f) we plot the real and imaginary parts of $\gamma_2\rho_{21}/V_{21}$, as a function of the dimensionless detuning $(\omega-\omega_{21})/\gamma_2$, with $\omega_{32}/\gamma_2=0.2$ and $\gamma_3=0.75\gamma_2$, for different values of the relative phase $\delta\Phi$. We assume that the two weak fields have the same intensity, $V_{21}=V_{31}$. When the relative phase satisfies Eq. (23), the dispersion is zero at the detuning $(\Delta_{21})_3$, and its slope around this point is positive and very steep. This can lead to a subluminal group velocity, accompanied by a dip in the absorption line shape, which can lead to transparency [Fig. 3(a)] when the dipoles are nearly parallel with $p=0.87$ or even amplification [Fig. 3(b)] for maximal interference with $p=1$. Note that the positive dispersion is steeper when it is accompanied by amplification. For example, in Fig. 3(a) the slope of the dispersion is ~ 7 while in Fig. 3(b) it is ~ 2110. However, in recent experiments measuring subluminal and superluminal group velocity, transparency conditions were preferred since they minimize the reshaping of the propagating pulse. By varying $\delta\Phi$, the dispersion line shape can be modified dramatically. In Figs. 3(c)–3(e), we show the effect of phase control when $p=1$. At $\delta\Phi=\pi$, the slope of the dispersion becomes negative, leading to superluminal group velocity [Fig. 3(c)], which is accompanied by absorption. When we change $\delta\Phi$

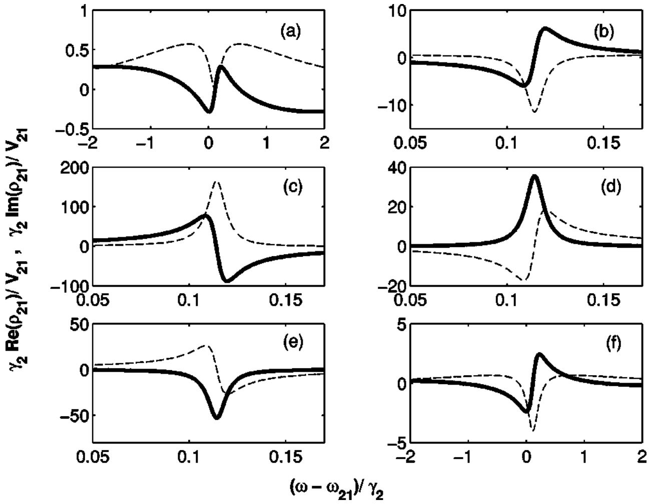

FIG. 3. Real (solid line) and imaginary (dashed line) parts of the polarization induced by E_a (in dimensionless form) in scheme A, as a function of the detuning $(\omega-\omega_{21})/\gamma_2$. The relaxation rates satisfy $\gamma_3=0.75\gamma_2$ and the upper level separation is $\omega_{32}=0.2\gamma_2$. For $V_{31}=V_{21}$ and the relative phase $\delta\Phi$ satisfying Eq. (23), (a) $p=0.87$, (b) $p=1$. Different values of $\delta\Phi$ with $p=1$ and $V_{31}=V_{21}$: (c) $\delta\Phi=\pi$, (d) $\delta\Phi=\pi/6$, and (e) $\delta\Phi=-\pi/6$. (f) Same as (a), with $V_{31}=2V_{21}$.

further, the dispersion becomes different from zero at the detuning $(\Delta_{21})_3$ and its line shape is no longer linear around this point. At $\delta\Phi=\pi/6$, the dispersion has an absorptive line shape while the absorption has a dispersive line shape [Fig. 3(d)], and at $\delta\Phi=-\pi/6$, the dispersion is gainlike [Fig. 3(e)]. These results clearly demonstrate the crucial role of phase control in determining pulse propagation. Moreover, they demonstrate that subluminal or superluminal propagation, without changing the shape of the pulse, can only be obtained when the phases of the interacting lasers satisfy a particular relation. Otherwise, the shape of the weak pulse becomes distorted during propagation by group velocity dispersion and other higher-order effects. Although we concentrate on phase control of the group velocity, it is worthwhile mentioning that varying the intensities of the weak probes can provide an additional means of control. For instance, when $V_{31}=2V_{21}$ and all the other parameters are kept as in Fig. 3(a), the slope of the dispersion is ~ 40 [Fig. 3(f)].

B. Scheme B: V system with a microwave field

In scheme B, the quantum coherence is achieved by coupling the two excited states of the V system by means of a strong microwave field; see Fig. 1(c). We assume the dipole moments of the two transitions to be orthogonal thereby *excluding* decay-induced interference effects. This scheme can thus be extended to large values of the separation between the excited states.

The electric field comprises the two weak beams of Eq. (17), with the addition of a microwave field with center frequency ω_μ which couples the two upper states. The frequencies of the three fields satisfy $\omega_a+\omega_\mu=\omega_b$. The equations of motion of the density matrix are given by

$$\dot{\rho}_{11}=i(V_{13}e^{i(\omega_b t+\Phi_b)}\rho_{31}-\text{c.c.})+i(V_{12}e^{i(\omega_a t+\Phi_a)}\rho_{21}-\text{c.c.})+\Gamma_{31}\rho_{33}+\Gamma_{21}\rho_{22}, \tag{24}$$

$$\dot{\rho}_{22}=-i(V_{12}e^{i(\omega_a t+\Phi_a)}\rho_{21}-\text{c.c.})-i(V_{32}e^{-i\omega_\mu t}\rho_{23}-\text{c.c.})+\Gamma_\mu\rho_{33}-\Gamma_{21}\rho_{22}, \tag{25}$$

$$\dot{\rho}_{33}=-i(V_{13}e^{i(\omega_b t+\Phi_b)}\rho_{31}-\text{c.c.})+i(V_{32}e^{-i\omega_\mu t}\rho_{23}-\text{c.c.})-\Gamma_{31}\rho_{33}-\Gamma_\mu\rho_{33}, \tag{26}$$

$$\dot{\rho}_{21}=-(\gamma_2+i\omega_{21})\rho_{21}-iV_{21}e^{-i(\omega_a t+\Phi_a)}(\rho_{22}-\rho_{11})+iV_{23}e^{i\omega_\mu t}\rho_{31}-iV_{31}e^{-i(\omega_b t+\Phi_b)}\rho_{23}, \tag{27}$$

$$\dot{\rho}_{31}=-(\gamma_3+i\omega_{31})\rho_{31}-iV_{31}e^{-i(\omega_b t+\Phi_b)}(\rho_{33}-\rho_{11})+iV_{32}e^{-i\omega_\mu t}\rho_{21}-iV_{21}e^{-i(\omega_a t+\Phi_a)}\rho_{32}, \tag{28}$$

$$\dot{\rho}_{32}=-(\gamma_\mu+i\omega_{32})\rho_{32}+iV_{32}e^{-i\omega_\mu t}(\rho_{22}-\rho_{33})+iV_{31}e^{-i(\omega_b t+\Phi_b)}\rho_{12}-iV_{12}e^{i(\omega_a t+\Phi_a)}\rho_{31}, \tag{29}$$

where $\gamma_2=\frac{1}{2}\Gamma_{21}+\gamma_2^{coll}$, $\gamma_3=\frac{1}{2}(\Gamma_{31}+\Gamma_\mu)+\gamma_3^{coll}$, and $\gamma_\mu=\frac{1}{2}(\Gamma_{21}+\Gamma_{31}+\Gamma_\mu)+\gamma_\mu^{coll}$. The Rabi frequencies V_{j1}, $j=2,3$, are given by Eq. (18), and $V_{32}=\Omega_\mu$ is the Rabi frequency of the microwave field. Solving Eqs. (24)–(29) in the steady-state weak-field limit and in the rotating-wave approximation, we obtain

$$\rho_{21}(\omega_a)=i\frac{V_{21}(i\Delta_{31}+\gamma_3)+iV_{31}V_{23}\exp(i\delta\Phi)}{(i\Delta_{21}+\gamma_2)(i\Delta_{31}+\gamma_3)+|V_{32}|^2} \tag{30}$$

$$\rho_{31}(\omega_b)=i\frac{V_{31}(i\Delta_{21}+\gamma_2)+iV_{21}V_{32}\exp(-i\delta\Phi)}{(i\Delta_{21}+\gamma_2)(i\Delta_{31}+\gamma_3)+|V_{32}|^2}, \tag{31}$$

where the detunings are $\Delta_{21}=\omega_{21}-\omega_a$ and $\Delta_{31}=\omega_{31}-\omega_b$, and the phase difference between the applied fields is $\delta\Phi=\Phi_a-\Phi_b$. As in scheme A, we concentrate on the properties of $\rho_{21}(\omega_a)$ which again comprises two parts: the first proportional to the weak field E_a which couples levels $|1\rangle$ and $|2\rangle$, and the second to E_b which couples levels $|1\rangle$ and $|3\rangle$. Here the absorption and refraction of E_a are influenced by E_b, due to the presence of the strong microwave which couples the two excited states, thus enabling the phase control of $\rho_{21}(\omega_a)$. It should be noted that Eq. (19) which refers to scheme A can be converted into Eq. (30) of scheme B by replacing $p\gamma_{23}$ by $-iV_{23}$.

The expression for $\rho_{21}(\omega_a)$ has extrema at the detunings

$$(\Delta_{21})_{1,2}=-\frac{\omega_{32}}{2}\pm\frac{1}{2}\sqrt{\omega_{32}^2+4(\gamma_2\gamma_3+|V_{32}|^2)} \tag{32}$$

determined by the imaginary part of the denominator and at $(\Delta_{21})_3$ determined by the real part of the denominator [see

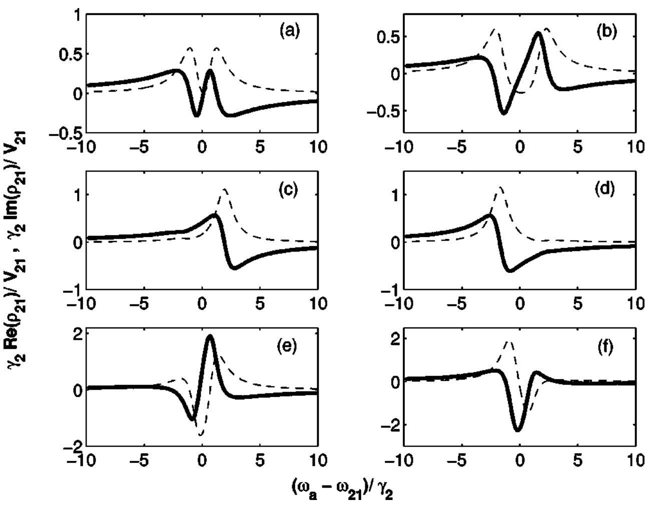

FIG. 4. Real (solid line) and imaginary (dashed line) parts of the polarization induced by E_a (in dimensionless form) in scheme B, as a function of the detuning $(\omega_a-\omega_{21})/\gamma_2$. The relaxation rates satisfy $\gamma_3=0.75\gamma_2$ and the upper level separation is $\omega_{32}=0.2\gamma_2$. For $V_{31}=V_{21}$ and relative phase $\delta\Phi$ satisfying Eq. (23), (a) $V_{32}=0.755\gamma_2$, (b) $V_{32}=1.8\gamma_2$. Different values of $\delta\Phi$ with $V_{32}=1.8\gamma_2$ and $V_{31}=V_{21}$: (c) $\delta\Phi=\pi$, (d) $\delta\Phi=2\pi$. For $V_{31}=4V_{21}$ with all other parameters as in (a), (e) $\delta\Phi$ satisfying Eq. (23), and (f) $\delta\Phi=\pi/6$.

Eq. (22)]. As before, we look for the conditions that give zero dispersion at the detuning $(\Delta_{21})_3$. This occurs when the parameters obey the relation

$$\cos\delta\Phi=-\left(\frac{\omega_{32}}{uV_{32}}\right)\frac{\gamma_3}{\gamma_2+\gamma_3}, \tag{33}$$

where u is the ratio between the two weak probes, $u=V_{21}/V_{31}$. In Figs. 4(a)–4(g) we plot the real and imaginary parts of $\gamma_2\rho_{21}/V_{21}$ as a function of the dimensionless detuning $(\omega_a-\omega_{21})/\gamma_2$, with $\omega_{32}=0.2\gamma_2$ and $\gamma_3=0.75\gamma_2$, for different values of the relative phase $\delta\Phi$. We assume that the two weak fields have the same intensity, $V_{21}=V_{31}$. In Fig. 4(a), a positive steep dispersion is accompanied by transparency at the detuning $(\Delta_{21})_3$ between two closely spaced absorption lines, when the relative phase satisfies Eq. (23) and $V_{32}=0.755\gamma_2$. This leads to a slowing down of the group velocity, which can be made even slower by increasing the microwave intensity to $V_{32}=1.8\gamma_2$, thus inducing amplification [Fig. 4(b)]. Note, however, that when the microwave intensity is increased further, the absorption lines drift apart, and the linear part of the dispersion around $(\Delta_{21})_3$ is reduced and becomes less steep. We now concentrate on the effect of phase control on the dispersion, and show that by varying $\delta\Phi$, the dispersion line shape can be modified considerably. At $\delta\Phi=\pi$ and 2π with $V_{32}=1.8\gamma_2$, the slope of the dispersion becomes negative, leading to superluminal group velocity [Figs. 4(c) and 4(d)]. Note, however, that while the subluminal group velocity is obtained at the detuning $(\Delta_{21})_3$ given by Eq. (22), superluminal behavior occurs within an absorption line, situated at one of the detunings of Eq. (21). This differs from the equivalent situation in scheme A, where both the subluminal and the superluminal dispersive features occur at the detuning $(\Delta_{21})_3$. By varying the Rabi frequencies of the weak pulses V_{21} and V_{31}, we can further modify the line shapes. In Figs. 4(e) and 4(f) we take $V_{31}=4V_{21}$, while keeping all the other parameters as in Fig. 4(a). The plots show that for this case the dispersion becomes steeper when the relative phase satisfies Eq. (23) [see Fig. 4(e)] and the dispersion exhibits an absorptive line shape while the absorption has a dispersive line shape at $\delta\Phi=\pi/6$ [Fig. 4(f)]. We have thus shown that by an appropriate choice of the phases and amplitudes of the applied fields, we can efficiently control the behavior of the dispersion and, hence, the propagation of a weak pulse.

IV. CONCLUSION

We have shown that decay-induced interference effect can significantly modify the propagation properties of a single weak pulse, leading to reduced group velocity. In addition, we have analyzed two schemes A and B where the group velocity can be manipulated by controlling the phases of the applied fields. In particular, we have shown that subluminal propagation occurs only when the relative phases satisfy a unique relation determined by the parameters of the specific system. Under these conditions a weak pulse propagates through the medium with a reduced group velocity and an undistorted shape. Deviation from this relation may lead to superluminal behavior or even to dispersion which is completely nonlinear leading to considerable distortion in the pulse shape. These results constitute a powerful demonstration of the role of phase control in influencing the propagation of a weak pulse.

The main difference between the two schemes is that in scheme A, phase control is determined by the inherent characteristics of the system, expressed by Eq. (23), whereas in scheme B, it is determined by the applied microwave field, expressed by Eq. (33). Thus, scheme B has an additional degree of freedom which can be adjusted in order to control the propagation of a weak pulse.

The effects of Doppler broadening on the dispersive properties of the two systems are quite different. In the case of scheme A, the dispersion line shape loses its linearity around the detuning $(\Delta_{21})_3$ when Doppler broadening effects are included, whereas in scheme B it remains linear but the steepness of the slope is reduced. Therefore, in an experimental demonstration of phase control of the group velocity in scheme A, cold atoms should be preferred, whereas this should be unnecessary in the case of scheme B.

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