Coupled Bose-Einstein condensate: Collapse for attractive interaction

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We study the collapse in a coupled Bose-Einstein condensate of two types of bosons 1 and 2 under the action of a trap using the time-dependent Gross-Pitaevskii equation. The system may undergo collapse when one, two, or three of the scattering lengths a_{ij} for scattering of boson i with j, $i,j=1,2$, are negative representing an attractive interaction. Depending on the parameters of the problem a single or both components of the condensate may experience collapse.

I. INTRODUCTION

The experimental detection [1,2] of Bose-Einstein condensation (BEC) at ultralow temperature in dilute trapped bosons (alkali metal and hydrogen atoms and the recent possibility in molecules) has spurred intense theoretical activities on various aspects of the condensate [3–7]. Many properties of the condensate are usually described by the mean-field time-dependent Gross-Pitaevskii (GP) equation [6,8]. One of the most interesting features of BEC has been observed in the case of attractive interatomic interaction [2,3]. In that case the condensate is stable for a maximum critical number of atoms, beyond which the condensate experiences a collapse. When the number of atoms increases beyond the critical number, due to interatomic attraction, the radius tends to zero and the central density of the condensate tends to infinity. Consequently, the condensate collapses emitting particles until the number of atoms is reduced below the critical number and a stable configuration is reached. The condensate may experience a series of collapses [2,3]. This phenomenon was observed in the BEC of ^{7}Li atoms with negative scattering length denoting attractive interaction where the critical number of atoms was about 1400 [2,3]. Theoretical analyses based on the GP equation in the case of ^{7}Li atoms also confirmed this collapse [2,3,6,7].

More recently, there has been experimental realization of BEC involving atoms in two different quantum states [9,10]. In one experiment ^{87}Rb atoms formed in the $F=1$, $m=-1$, and $F=2$, $m=1$ states by the use of a laser served as two different species, where F and m are the total angular momentum and its projection [9]. In another experiment a coupled BEC was formed with the ^{87}Rb atoms in the $F=1$, $m=-1$ and $F=2$, $m=2$ states [6,10]. It is possible to use the same magnetic trap to confine atoms in two magnetic states and this makes these experimental investigations technically simpler compared to a realization of BEC with two different types of atoms requiring two different trapping mechanisms. This is why so far it has not been possible to prepare a coupled BEC with two different types of atoms. In addition to coupled atomic condensates, there has been consideration of a hybrid BEC where one type of bosons are atoms and the other molecules [11]. These initiated theoretical activities in BEC with more than one type of bosons using the coupled GP equation [11,12].

In addition to just forming a coupled BEC with two quan-

tum states of the same atom, these studies also yielded crucial information about the interaction among component atoms and measured the percentage of each quantum state in the condensate [9,10]. It has been found that ^{87}Rb atoms have repulsive interaction in all three quantum states. Also, the strength of repulsive interactions in $F=1$, $m=-1$ and $F=2$, $m=2$ states are essentially identical. The interaction between an atom in the $F=1$, $m=-1$ state and another in the $F=2$, $m=2$ state is repulsive. As the change in the m value of a atomic quantum state does not correspond to a substantial structural change, it is likely that such change would not correspond to a large change in the atomic interaction.

Here we study theoretically the collapse in a coupled BEC composed of two quantum states 1 and 2 of a bosonic atom using the coupled time-dependent GP equation. We motivate this study by considering two possible atomic states of ^{7}Li whenever possible. An experiment of collapse in a coupled BEC has not yet been realized but could be possible in the future. In the case of ^{7}Li the interaction in state 1 is taken to be attractive, which is responsible for collapse. Here there are three types of interactions denoted by the scattering lengths a_{ij}, $i,j=1,2$, between states i and j. A negative (positive) scattering length denotes an attractive (repulsive) interaction. We study the collapse with different possibilities of attraction and repulsion between atoms in states 1 and 2. If one of the scattering lengths is negative, at least one component of the condensate may experience collapse. If two of the scattering lengths are negative one can have collapse in both components. Specifically, one can also have collapse of both components if a_{12} is negative and a_{ii}, $i=1,2$ are positive.

The usual GP equation conserves the number of atoms. The dynamics of the collapse (growth and decay of number of atoms) is best studied by introducing an absorptive contact interaction in the GP equation that allows for a growth in the particle number from an external source. One has also to introduce an imaginary quartic three-body interaction term responsible for recombination loss from the condensate [3]. If the strengths of these two terms are properly chosen, the solution of the time-dependent GP equation could produce a growth of the condensate with time when the number of atoms is less than the critical number. Once it increases past the critical number, the three-body interaction takes control and the number of atoms suddenly drops below the critical

level by recombination loss signaling a collapse [3]. Then the absorptive term takes over and the number of atoms starts to increase again. This continues indefinitely showing an infinite sequence of collapse.

II. COUPLED GROSS-PITAEVSKII EQUATION WITH ABSORPTION

We consider the following spherically symmetric coupled GP equation with two components at time τ for the condensate wave function $\psi_i(r,\tau)$ [12]:

$$\left[-\frac{\hbar^2}{2m} \frac{1}{r} \frac{\partial^2}{\partial r^2} r + \frac{1}{2} c_i m \omega^2 r^2 + \sum_{j=1}^{2} g_{ij} N_j |\psi_j(r,\tau)|^2 \right.$$

$$\left. - i\hbar \frac{\partial}{\partial \tau} \right] \psi_i(r,\tau) = 0, \qquad (2.1)$$

$i=1,2$, where m is the atomic mass. Here $g_{ij}=4\pi\hbar^2 a_{ij}/m$ is the coupling constant for atomic interaction, N_j the number of condensed atoms in state j, and ω the frequency of the harmonic-oscillator trap. The parameter c_i has been introduced to modify the frequency of the trap for the atoms in each quantum state.

As in Refs. [5] it is convenient to use dimensionless variables defined by $x=\sqrt{2}r/a_{\mathrm{ho}}$, and $t=\tau\omega$, where $a_{\mathrm{ho}} \equiv \sqrt{\hbar/(m\omega)}$, and $\phi_i(x,t)=x\psi_i(r,\tau)(\sqrt{2}\pi a_{\mathrm{ho}}^3)^{1/2}$. In terms of these variables Eq. (2.1) becomes [5]

$$\left[-\frac{\partial^2}{\partial x^2} + \frac{c_i x^2}{4} + \sum_{j=1}^{2} n_{ij} \frac{|\phi_j(x,t)|^2}{x^2} - i\xi_i \frac{|\phi_i(x,t)|^4}{x^4} \right.$$

$$\left. + i\gamma_i - i\frac{\partial}{\partial t} \right] \phi_i(x,t) = 0, \qquad (2.2)$$

where $n_{ij} \equiv 2\sqrt{2} N_j a_{ij}/a_{\mathrm{ho}}$ could be negative (positive) when the corresponding interaction is attractive (repulsive). In Eq. (2.2) we have introduced a diagonal absorptive $i\gamma_i$ and a quartic three-body term $-i\xi_i |\phi_i(x,t)|^4/x^4$ appropriate to study collapse [3]. For $\gamma_i=\xi_i=0, i=1,2$, the normalization condition of the wave function is

$$\int_0^{\infty} |\phi_i(x,t)|^2 dx = 1. \qquad (2.3)$$

The rms radius of the component i $x_{\mathrm{rms}}^{(i)}(t)$ at time t is defined by

$$x_{\mathrm{rms}}^{(i)}(t) = \left[\frac{\int_0^{\infty} x^2 |\phi_i(x,t)|^2 dx}{\int_0^{\infty} |\phi_i(x,t)|^2 dx} \right]^{1/2}. \qquad (2.4)$$

III. NUMERICAL RESULTS

To solve Eq. (2.2) we discretize it in both space (using step 0.0001) and time (using step 0.05) employing a Crank-

Nicholson-type rule and reduce it to a set of algebraic equations, which is then solved by iteration using the known boundary conditions, e.g., $|\phi_i(0,t)|=0$, and $\lim_{x\to\infty} |\phi_i(x,t)| \sim \exp(-x^2/4)$. The iteration is started with the known normalized (harmonic-oscillator) solution of Eq. (2.2) obtained with $n_{ij}=\gamma_i=\xi_i=0$. The nonlinear constants n_{ij} in this equation are increased by equal amounts over 500 to 1000 time iterations starting from zero until the desired final values are reached. This iterative method is similar to one in the uncoupled case [3,5]. A detailed account of the numerical procedure for the coupled case will appear elsewhere.

A. Stationary problem

First we consider the stationary solution of Eq. (2.2) with $\gamma_i=\xi_i=0$, which illustrates the collapse. As the three scattering lengths a_{ij} and two numbers N_i are all independent, the four parameters n_{ij} are also so with one restriction: the signs of n_{12} and n_{21} are identical.

Now we study the simplest case of collapse by taking only the interaction between the atoms in state 1 to be attractive corresponding to a negative a_{11}. All other scattering lengths $-a_{22}$ and $a_{12}(=a_{21})-$ are taken to be positive. Quite expectedly, here the first component of the condensate could experience collapse. Although the present formulation is generally valid, one has to choose numerical values of the parameters before an actual calculation.

The collapse of the first component is illustrated in Fig. 1(a) for $n_{11}=-3.814$, $n_{22}=4$, $n_{12}=n_{21}=1$, and $c_1=0.25$, $c_2=4$. These parameters are in dimensionless units and one can associate them with an actual physical problem of experimental interest. For this we consider the state 1 to be the states of ^{7}Li with attractive interaction as in the actual collapse experiment with $|a_{11}|/a_{\mathrm{ho}} \simeq 0.0005$ [2]. As $n_{11} = 2\sqrt{2} N_1 |a_{11}|/a_{\mathrm{ho}}$ this corresponds to a boson number $N_1 \simeq 2700$. This number is larger than the maximum number of atoms permitted in the BEC of a single-component of ^{7}Li, which is about 1400 [2]. The presence of the second component with repulsive interaction allows for a formation of a stable BEC with more ^{7}Li atoms in quantum state 1 than allowed in the single-component BEC. Similar conclusion was reached by Esry [13] in a study of a coupled BEC in a different context. We find from Fig. 1(a) that ϕ_1 is very much centrally peaked compared to ϕ_2. This corresponds to a small rms radius and large central density for ϕ_1 denoting an approximation to collapse. If the number N_1 is slightly increased beyond 2700 the first component of the condensate wave function becomes singular at the origin and no stable stationary solution to Eq. (2.2) could be obtained.

Next we discuss the collapse by taking only the interaction among atoms in two different states to be attractive corresponding to a negative $a_{12}(=a_{21})$. The atomic interaction in both quantum states 1 and 2 is taken to be repulsive corresponding to a positive a_{11} and a_{22}. Although it is a problem of theoretical interest for the study of collapse, it has no experimental analog in terms of ^{7}Li. We illustrate the approximation to collapse in this case in Fig. 1(b) for parameters $n_{11}=1$, $n_{22}=1.5$, $n_{12}=-5.95$, $n_{21}=-2$, $c_1=1$, and
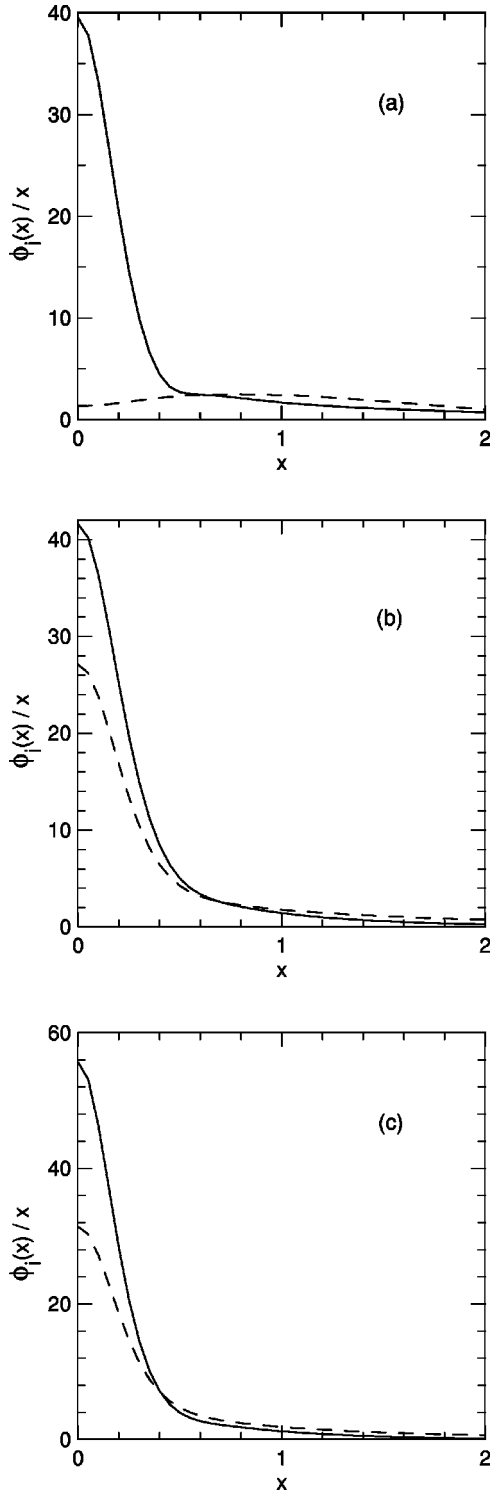
FIG. 1. Wave function components $\phi_1(x)$ (full line) and $\phi_2(x)$ (dashed line) vs x for two coupled GP equations with (a) $n_{11} = -3.814, n_{22} = 4$, $n_{12} = n_{21} = 1$, $c_1 = 0.25$, $c_2 = 4$; (b) $n_{11} = 1$, $n_{22} = 1.5$, $n_{12} = -5.95$, $n_{21} = -2$, $c_1 = 1$, $c_2 = 0.25$; and (c) $n_{11} = n_{22} = -1$, $n_{12} = n_{21} = -0.552$, $c_1 = 4$, and $c_2 = 0.25$.

$c_2 = 0.25$. Both wave function components are peaked near $x = 0$ and have small rms radii. The system would collapse with a small increase of $|n_{12}|$ and/or $|n_{21}|$. Here the interactions among atoms in states 1 and 2 are both repulsive. The

collapse is a consequence of the attraction between an atom in state 1 and one in state 2. This leads to a dominance of nonlinear off-diagonal coupling terms in the coupled GP equation.

Finally, in Fig. 1(c) we illustrate the approximation to collapse of both components when all scattering lengths are negative. This corresponds to taking all possible interactions attractive. The parameters in this case are $n_{11} = n_{22} = -1$, $n_{12} = n_{21} = -0.552$, $c_1 = 4$, and $c_2 = 0.25$. This has an experimental analog in terms of two states of ^{7}Li. We assume the atomic interaction in both states to be equally attractive corresponding to a negative scattering length: $a_{11} = a_{22}$. For $|a_{11}|/a_{\text{ho}} \simeq 0.0005$, as in the actual experiment [2], one has $N_1 = N_2 \simeq 700$. The total number of particles in this case is roughly 1400, which is equal to the critical number observed in the actual experiment of collapse in ^{7}Li. Both wave function components could become singular, in this case, as all possible interactions are attractive.

B. Time-dependent problem

Although the collapse of the coupled condensates could be inferred from the shape of the stationary wave functions of Fig. 1 (sharply peaked centrally with small rms radii), we also study the dynamics of collapse from a time evolution of the full GP equation (2.2) in the presence of an absorption and three-body recombination, e.g., for $\gamma_i \neq 0$ and $\xi_i \neq 0$ as in the uncoupled case [3]. For this purpose we consider the solution of Eq. (2.2) normalized according to Eq. (2.3) at $t = 0$ obtained with $\gamma_i = \xi_i = 0$ and allow this solution to evolve in time with $\gamma_i \neq 0$ and $\xi_i \neq 0$ by iterating the GP equation (2.2). The fractional change in the number of atoms due to the combined effect of absorption and three-body recombination is given by

$$\frac{N_i(t)}{N_i(0)} = \frac{\displaystyle\int_0^\infty |\phi_i(x,t)|^2 dx}{\displaystyle\int_0^\infty |\phi_i(x,0)|^2 dx}, \tag{3.1}$$

and the rms radii by Eq. (2.4). The continued growth and decay of the number of particles in the condensate would signal the possible collapse in a particular case. The oscillation of the rms radius would demonstrate the consequent radial vibration of the condensate.

Now we study the time evolution of the number of atoms of the two components and the corresponding rms radii. The general nature of time evolution is independent of the actual values of γ_i and ξ_i employed provided that a very small value for $\xi_i (\sim 0.001)$ and a relatively larger one for $\gamma_i (\sim 0.01$ to $0.1)$ are chosen [3]. The following parameters were chosen in case of models (a), (b), and (c) of Fig. 1: (a) and (b) $\gamma_1 = \gamma_2 = 0.03$ and $\xi_1 = \xi_2 = 0.001$, and (c) $\gamma_1 = 0.15$, $\gamma_2 = 0.03$, $\xi_1 = 0.002$, and $\xi_2 = 0.003$. The fractional change in the number of atoms for the two components are shown in Figs. 2(a)–2(c). The results for $0 < t < 100$ in Fig. 2 are calculated with 2000 iterations of the GP equation (2.2) using a time step of 0.05.
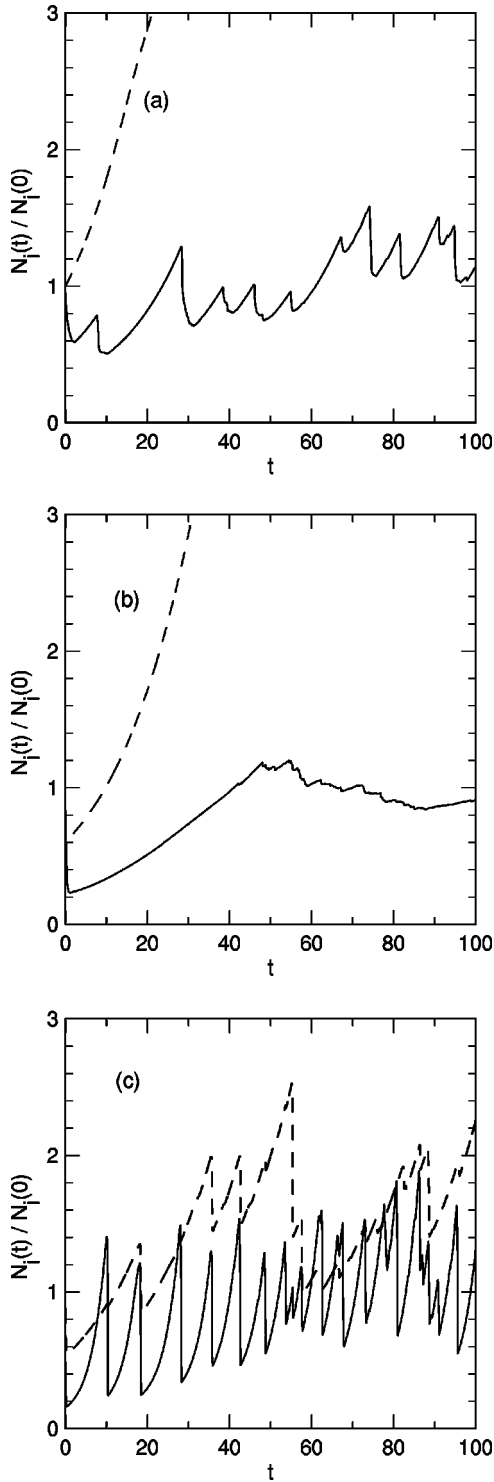
FIG. 2. The fractional change in the number of atoms $N_i(t)/N_i(0)$ vs t for component 1 (full line) and 2 (dashed line) for models (a) and (b) with $\gamma_1 = \gamma_2 = 0.03$ and $\xi_1 = \xi_2 = 0.001$, and for (c) with $\gamma_1 = 0.15$, $\gamma_2 = 0.03$, $\xi_1 = 0.002$, and $\xi_2 = 0.003$. The parameters are as in Fig. 1.

FIG. 3. The time dependence of rms radii $x_{\mathrm{rms}}^{(i)}(t)$ of models (a), (b), and (c) for components 1 (full line) and 2 (dashed line). The parameters are as in Figs. 1 and 2.

The quadratic nonlinear terms in model (a) are all repulsive in channel 2, the corresponding wave function (ϕ_2) of Fig. 1(a) does not show any sign of approximation to collapse as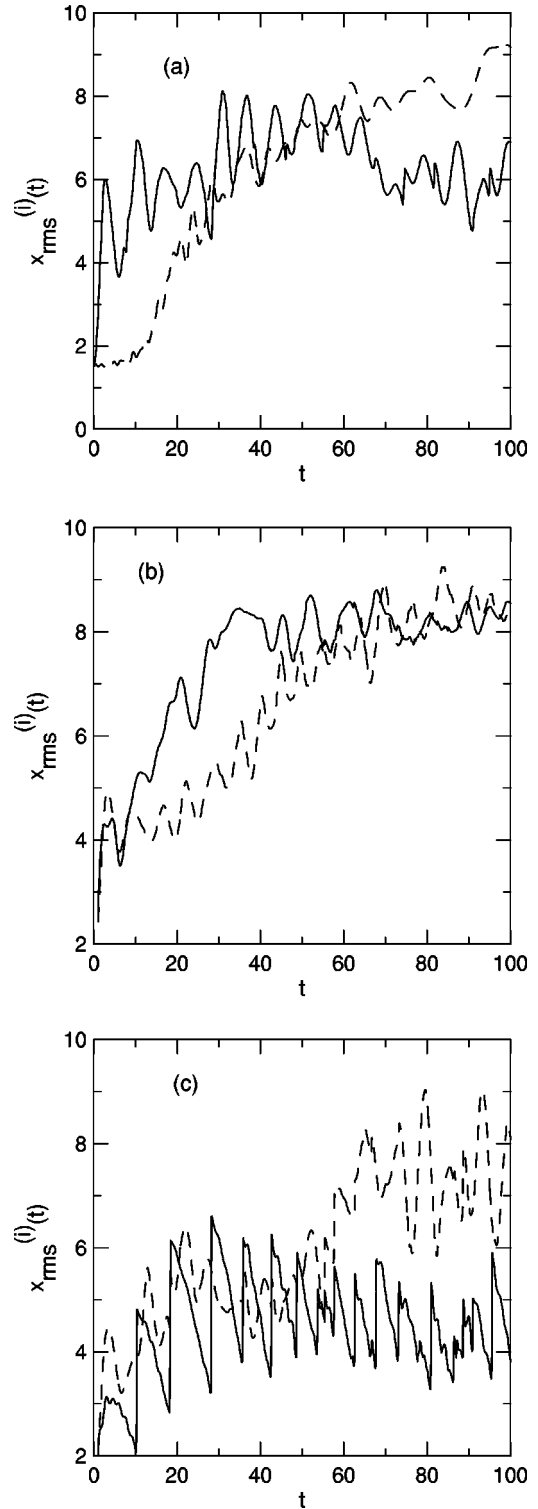 in channel 1 where the diagonal nonlinear term is attractive. The results reported in Fig. 2(a) are consistent with this. The number of particles N_1 of the first component undergoes successive growth and decay, whereas that of the second component keeps on growing indefinitely typical to a repulsive interaction.

For model (b) the effective nonlinear terms in channels 1 and 2 are both repulsive and it should be possible to have collapse in both channels by decreasing the dominating off-diagonal quadratic nonlinear terms n_{12} and n_{21} corresponding to an increase in attraction between an atom in state 1 and one in state 2. However, for the actual parameters of this model only the component 1 exhibits collapse. This is consistent with the more singular nature of ϕ_1 reported in Fig. 1(b), compared to ϕ_2. Consequently, in Fig. 2(b) only component 1 experiences collapse; the number of particles N_2 keeps on growing with time.

In model (c) all the quadratic nonlinear terms are attractive. Consequently, in Fig. 2(c) we find a series of collapse in both channels. The collapse is most favored in model (c) with attractive diagonal and nondiagonal nonlinear terms. This corresponds to attraction between two atoms in state 1, between two atoms in state 2, and between an atom in state 1 and another in state 2. The next favored case is of model (a) where the diagonal nonlinear term is negative in channel 1. Here only the atomic interaction in state 1 is attractive, all other atomic interactions are repulsive. The least favored case is of model (b) where only the off-diagonal nonlinear terms are negative. This corresponds to repulsion between two atoms in state 1, and between two atoms in state 2, and attraction between an atom in state 1 and another in state 2. In the last case, collapse takes place due to the dominance of the attractive nondiagonal nonlinear term over the repulsive diagonal one in channel 1. This is explicit in Fig. 2 where the frequency of collapse decreases from model (c) to (a) and then to (b).

Finally, in Figs. 3(a)–3(c) the rms radii for the two components are shown for models of Figs. 2(a)–2(c), respectively. In the case of models (a) and (b) we find from Figs. 2(a) and 2(b) that the number N_2 grows with time. This is reflected in the growth of the corresponding rms radii in Figs. 3(a) and 3(b). In the case of model (c) there is collapse in both channels and both the rms radii oscillate with time. This radial vibration of the collapsing condensate(s) also takes place in the uncoupled case [3]. However, from Figs. 3(a) and 3(b) we find that due to a collapse in one of the channels, both rms radii could execute oscillations. In one of the channels it is a direct consequence of collapse, in the other it is due to a coupling to the channel experiencing collapse.

IV. CONCLUSION

To conclude, we studied the collapse in a trapped BEC of atoms in states 1 and 2 using the GP equation when some of the atomic interactions are attractive. We motivate parts of this study with two atomic states of ^{7}Li. The component i of the condensate could experience collapse when the interaction among atoms in state i is attractive. Both components could experience collapse when at least the interaction between an atom in state 1 and one in state 2 is attractive. The collapse is predicted from a stationary solution of the GP equation. The time evolution of collapse is studied via the time-dependent GP equation with absorption and three-body recombination. The number of particles of the component(s) of BEC experiencing collapse alternately grows and decays with time. With the possibility of observation of coupled BEC, the results of this paper could be verified experimentally in the future.

ACKNOWLEDGMENTS

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