Adiabatic compression of a trapped Fermi gas

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We propose a method to reach conditions of high degeneracy in a trapped Fermi gas, based on the adiabatic transfer of a small fraction of atoms from a magnetic to a tighter optical trap. The transformation yields a large increase of the Fermi energy, without a significant change of the temperature. The large enhancement of the central density emphasizes the role of the interactions and makes the system much closer to the BCS transition. We show that, despite Pauli blocking, the thermalization processes ensuring equilibrium in the optical trap are not suppressed at low temperature.

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I. INTRODUCTION

The experimental realization of a highly degenerate atomic Fermi gas confined in traps is a task of primary importance, especially in view of the perspective of approaching the BCS transition to the superfluid phase. The regime of quantum degeneracy has been already reached in a sample of potassium atoms [1], where first signatures of Fermi statistics, such as the deviation of the velocity distribution from a Boltzmann profile and the increase of the kinetic energy with respect to the classical value, have been observed. The main difficulties in further lowering the temperature are due to the fact that the efficiency of the evaporative cooling process is strongly quenched [2,3]. In fact Fermi statistics inhibits collisional processes at low temperature, both directly by reducing the phase space available for collisions and indirectly by lowering the density of the sample because of Pauli repulsion. Procedures to optimize the evaporative process have recently permitted to reach lower temperatures, of the order of $0.2T_F-0.3T_F$ where T_F is the Fermi temperature [2].

In the present work we propose a method to reach conditions of high degeneracy, based on an adiabatic compression of the gas. The adiabatic increase of degeneracy has proven quite successful in producing Bose-Einstein condensation in a reversible way, starting from a trapped Bose gas above the critical temperature [4]. The main point is that, by changing the shape of the confinement from a harmonic to a nonharmonic trap, one can increase the degree of quantum degeneracy by keeping the entropy of the total system constant [5]. At the same time, despite the effects of Pauli blocking, the process of thermalization is not suppressed at low temperature and can take place over times much shorter than the lifetime of the cloud. In the following we consider a gas occupying two different spin states, initially confined by a harmonic trap. We then switch on adiabatically a second tighter trap (see Fig. 1). Experimentally this can be realized using a magnetic trap for the first confinement and an optical trap for the second one. As a consequence of the adiabatic process a fraction of atoms will move from the magnetic to the optical trap. This can provide several important advantages.

(i) The gas in the optical trap becomes much more degenerate than the original one. In particular, if the number of atoms transferred to the optical trap is a small fraction, the temperature will not change significantly with respect to the initial value but the Fermi energy will increase in a way proportional to the depth of the optical trap.

(ii) The gas in the optical trap is much denser due to the tighter confinement and the effects of interaction are thereby enhanced.

(iii) The Fermi energies of the two spin components, which initially were different because of the different magnetic trapping, become closer in the optical trap, thereby favoring the mechanism of Cooper pairing. In fact, Cooper pairing cannot take place if the difference between the two Fermi energies is larger than the BCS gap.

All the above effects favor the reachability of the BCS transition. Another important advantage of the proposed method is that the velocity distributions of the atomic clouds occupying the magnetic and optical traps can be measured separately. By releasing first the magnetic trap, one can measure the temperature of the sample. The effects of quantum degeneracy can then be investigated by measuring the velocity distribution of the gas confined in the optical trap. Finally, the use of optical traps opens up the possibility of tuning the scattering amplitude of the trapped atoms by exploiting the occurrence of Feshbach resonances.

The paper is organized as follows. In Sec. I we assume adiabaticity and we explore the properties of the new gas

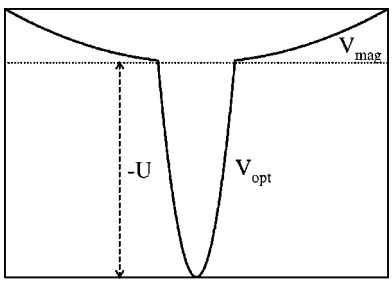

FIG. 1. Schematic representation of the confining potential $V_{ext}(\mathbf{r})$ in the presence of the magnetic and optical traps.

produced in the presence of the tight confinement by imposing entropy conservation. In Sec. II we discuss the relaxation mechanisms that bring about thermal equilibrium in the optical trap and provide estimates for the times required to ensure adiabaticity.

II. ADIABATIC TRANSFER

Let us consider a gas initially confined in a harmonic trap (hereafter called magnetic trap). We assume that the trapping frequencies and the number of atoms are the same for the two-spin species. The Fermi energy is given by $\epsilon_F^0 = \hbar\omega_{mag}(6N)^{1/3}$, where N is the number of atoms of each species and ω_{mag} is the geometrical average of the frequencies characterizing the magnetic trapping potential V_{mag}. We will consider systems lying initially in configurations of moderate degeneracy corresponding to $k_B T_{in} = 0.2\epsilon_F^0 - 0.5\epsilon_F^0$, where T_{in} is the initial temperature of the gas. For simplicity, we will assume that also the optical trap can be approximated by a harmonic potential V_{opt} having a tighter frequency $\omega_{opt} \gg \omega_{mag}$ and depth $V_{opt}(\mathbf{r}=0) = -U$. In our model the trapping potential, after switching on the optical trap, is defined as $V_{ext}(\mathbf{r}) = V_{opt}(\mathbf{r})$ inside the optical trap ($V_{opt} < V_{mag}$), and $V_{ext}(\mathbf{r}) = V_{mag}(\mathbf{r})$ outside (see Fig. 1). The maximum number of atoms that can be transferred in the optical trap is given by the value

$$N_{opt} = \frac{1}{6}\left(\frac{U}{\hbar\omega_{opt}}\right)^3. \tag{1}$$

As we will see, if we start from moderately low temperatures, Eq. (1) provides an accurate estimate of the number of atoms that are actually transferred by the adiabatic process, provided $N_{opt} \ll N$. The relative number of atoms transferred in the optical trap is then given by the useful expression

$$\frac{N_{opt}}{N} = \left(\frac{U\omega_{mag}}{\epsilon_F^0\omega_{opt}}\right)^3. \tag{2}$$

Typical values that will be considered are $\omega_{mag}/\omega_{opt} = 0.1$ and $U/\epsilon_F^0 = 5$, corresponding to $N_{opt}/N \sim 10\%$. Since the relative number of transferred atoms is small, one expects that the final temperature T_{fin} of the gas will not change significantly with respect to the initial value T_{in}. The final degree of degeneracy of the gas will be, however, significantly higher since the final Fermi energy is approximately given by $\epsilon_F \simeq U + \epsilon_F^0$. By increasing further the depth of the optical trap the relative number of transferred atoms increases significantly and so does the temperature. In the extreme situation, where all the atoms have been transferred into the optical trap, the temperature increase becomes equal to the increase of the Fermi energy and the degeneracy of the gas decreases back to its initial value. Due to the compression, the central density of the gas, that at small temperature $n(0) = (2m\epsilon_F^0/\hbar^2)^{3/2}/(6\pi^2)$ in the initial stage, will increase by the factor $(\epsilon_F/\epsilon_F^0)^{3/2}$ in the optical trap. The increase of the Fermi energy and of the central density has an important effect on the value of the BCS transition temperature that is expected to behave, for negative scattering lengths, as [8,9]

$$T_{\mathrm{BCS}} \sim \frac{\epsilon_F}{k_B}\exp\left[-\frac{\hbar\pi}{2p_F|a|}\right], \tag{3}$$

where $p_F = \hbar(6\pi^2)^{1/3}[n(0)]^{1/3}$ is the Fermi momentum calculated in the center of the trap. The value of the BCS critical temperature can increase significantly with respect to its value in the magnetic trap. For example, in a gas of ^{6}Li atoms with $a \sim -2000a_0$ (a_0 is the Bohr radius), by taking a ratio $U/\epsilon_F^0 = 5$ and an initial central density $n(0) \sim 10^{12}$ cm^{-3}, we obtain an increase of the density by a factor ~ 15 and the BCS transition temperature (3) is enhanced by the huge factor ~ 50, becoming comparable to the initial value of the Fermi temperature ϵ_F^0/k_B. The above discussion suggests that the proposed adiabatic mechanism might provide conditions of high degeneracy, not far from the transition to the BCS phase. With such a denser gas also the effects of the mean field on the density profile may be significant. An estimate is given by the ratio [7] $E_{int}/E_{ho} \simeq 0.3p_F a/\hbar$ between the interaction energy and the oscillator energy of a spherically symmetric trap. By using the values employed above one finds corrections to the ground-state properties (energy, density profile, etc.) of the order of 10%.

The high degeneracy realized in the gas is expected to show up in the velocity distribution and in the release energy. After completing the adiabatic transfer one can release the magnetic trap. Measuring the velocity distribution of these atoms then provides information on the temperature of the system, which is expected to be close to the initial value. The atoms of the optical trap can be imaged in a second step. For them one predicts that the ratio $E_{kin}/k_B T_{fin} \sim 3U/8k_B T_{fin}$ between the kinetic energy per particle and the thermal energy should be enhanced in a significant way if $U \gg k_B T_{fin}$ revealing the effects of quantum degeneracy.

To confirm the scenario emerging from the above discussion, we have carried out a numerical calculation of the thermodynamic functions before and after the adiabatic transformation. The calculation is obtained by imposing that the initial and the final configurations have the same entropy. This has been calculated using the semiclassical expression

$$\frac{S}{k_B} = \frac{1}{(2\pi\hbar)^3}\int d\mathbf{r}\, d\mathbf{p}\left[\frac{\epsilon(\mathbf{p},\mathbf{r})/k_B T - \ln z}{z^{-1}e^{\epsilon(\mathbf{p},\mathbf{r})/k_B T}+1} + \ln(1+ze^{-\epsilon(\mathbf{p},\mathbf{r})/k_B T})\right], \tag{4}$$

where $z = \exp(\mu/k_B T)$ is the gas fugacity and

$$\epsilon(\mathbf{p},\mathbf{r}) = \frac{p^2}{2m} + V_{ext}(\mathbf{r}) \tag{5}$$

are the semiclassical particle energies. The results are presented in Figs. 2–5. In Fig. 2 we show the relative number of atoms in the optical trap as a function of the depth of the optical trap U/ϵ_F^0 for two initial temperatures. For the chosen

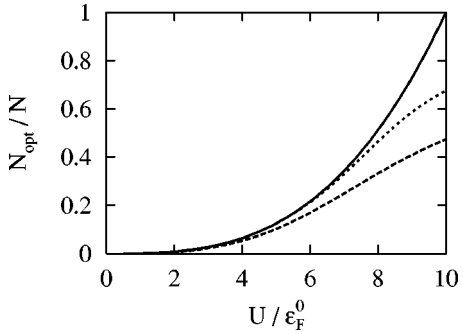

FIG. 2. Fraction of atoms in the optical trap as a function of U/ϵ_F^0 for two initial temperatures: $k_B T_{in}=0.5\epsilon_F^0$ (dashed line), and $k_B T_{in}=0.25\epsilon_F^0$ (dotted line). The solid line corresponds to the analytical estimate (2).

configuration with $\omega_{mag}/\omega_{opt}=0.1$ and final depth $U=5\ \epsilon_F^0$, the optical trap can host about 10% of atoms confirming the analytic prediction (2).

In Fig. 3 the final temperature of the gas is plotted as a function of U/ϵ_F^0. As already anticipated, the final temperature does not change significantly from the original value, except for values of U/ϵ_F^0 of the order of $\omega_{opt}/\omega_{mag}$. Notice however that, due to the tight confinement, the temperature of the gas can become comparable to the level splitting in the optical trap $\hbar\omega_{opt}/k_B$ if N is not large enough.

In Fig. 4 we show the kinetic energy per atom in units of the final temperature. The initial temperature is $k_B T_{in}=0.25\epsilon_F^0$. In the same figure, we also show the kinetic energy per particle of the gas occupying separately the magnetic and the optical trap. These values are obtained by averaging separately the kinetic energy over the particles occupying the optical trap $[\epsilon(\mathbf{p},\mathbf{r})<0]$, and the particles occupying the magnetic trap $[\epsilon(\mathbf{p},\mathbf{r})>0]$. The kinetic energy provides an important indicator of the quantum degeneracy of the gas (for a classical gas one would have $E_{kin}/k_B T_{fin}=3/2$). The effect is very spectacular for the atoms of the optical trap, where at $U=5\epsilon_F^0$ one finds $E_{kin}/k_B T_{fin}\simeq 7$. By choosing a higher initial temperature the effect is less pronounced, but still large ($E_{kin}/k_B T_{fin}\simeq 3$ for $k_B T_{in}=0.5\epsilon_F^0$ and $U=5\epsilon_F^0$). Because of the high degeneracy, the velocity distribution of the atoms in the optical trap deviates significantly from a Boltzmann distribution (Fig. 5). We have also

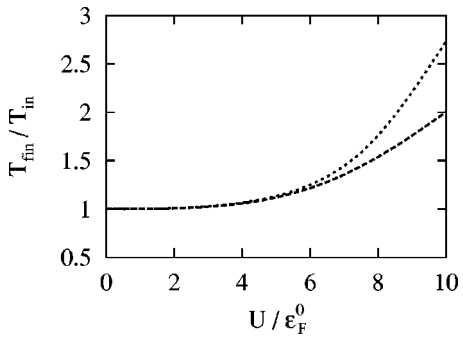

FIG. 3. Ratio between the final and initial temperature of the gas as a function of U/ϵ_F^0 for the same initial temperatures as in Fig. 2.

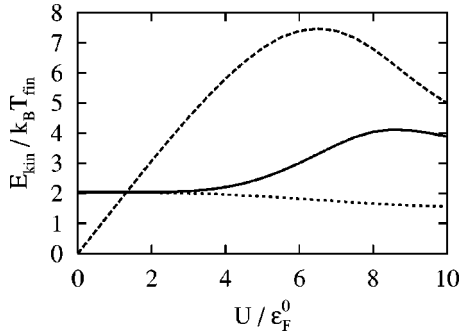

FIG. 4. Kinetic energy per particle E_{kin} in units of the final temperature $k_B T_{fin}$ as a function of U/ϵ_F^0. The initial temperature is $k_B T_{in}=0.25\epsilon_F^0$. The solid line corresponds to the kinetic energy per particle averaged over the entire system. Also shown are the kinetic energy per particle in the optical trap (dashed line) and in the magnetic trap (dotted line).

calculated the central density as a function of the depth U. For $U=5\ \epsilon_F^0$ and $k_B T_{in}=0.25\epsilon_F^0$, we find an increase by a factor 15 with respect to the initial value.

III. ROLE OF COLLISIONS

In this section we discuss how the adiabatic transfer of atoms can be realized in practice and what is the role of collisions. Let us recall that there are several time scales in the problem. A first scale is fixed by the inverse frequencies of the harmonic wells. These times (of order of 10^{-3}–10^{-1} s) are in general much shorter than the relaxation times due to collisions. The condition of reversibility would require that the relaxation time be much shorter than the time over which the optical trap is switched on. From a practical point of view we find that an efficient way to realize an almost adiabatic transformation is to switch on the optical trap on a time scale longer than the inverse periods of the harmonic wells but faster than the relaxation time, and then to let the system relax to equilibrium. The first step of this transformation corresponds to an adiabatic transfer of the lowest N_{opt} single-particle states from the magnetic to the optical trap. This transformation keeps the corresponding occupation numbers unchanged and hence conserves entropy. The sec-

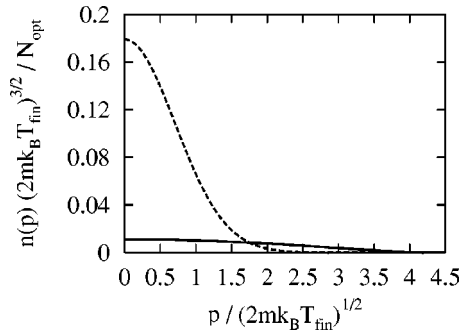

FIG. 5. Momentum distribution of the atoms occupying the optical trap with $U=5\epsilon_F^0$ (solid line). The initial value of the temperature is $k_B T_{in}=0.25\epsilon_F^0$. The dashed line corresponds to a Boltzmann gas of the same number of atoms at the temperature $T=T_{fin}$.

ond step (relaxation to equilibrium) produces an increase of entropy, which is however negligible if the final fraction of atoms transferred to the optical trap is small. By using the choice of parameters discussed in the first part of the work we have checked that in this two-step transformation all the relevant physical quantities differ very little from the ones obtained by imposing entropy conservation. For example, working at $k_B T_{in}=0.5\,\epsilon_F^0$ and $U=5\,\epsilon_F^0$, the final temperature $T_{fin}=1.16\,T_{in}$, is to be compared with the value $T_{fin}=1.12\,T_{in}$ obtained with the reversible transformation (Fig. 3); even smaller differences are found in the two cases for the final fraction of atoms in the optical trap.

It is important to discuss the time needed for the system to reach equilibrium in the second step of the transformation. We have simulated the process of thermalization by solving the quantum Boltzmann equation. We assume equal distribution functions for the two-spin components and that the phase-space distribution of particles is a function only of the single-particle energies $\epsilon(\mathbf{p},\mathbf{r})$ (ergodic assumption) [6,2]. This yields the following equation to solve:

$$\rho(\epsilon_1)\frac{\partial f(\epsilon_1)}{\partial t}=\frac{m\sigma}{\pi^2\hbar^3}\int d\epsilon_3 d\epsilon_4 \rho(\epsilon_{min}) \times\{f(\epsilon_3)f(\epsilon_4)[1-f(\epsilon_1)][1-f(\epsilon_2)] - f(\epsilon_1)f(\epsilon_2)[1-f(\epsilon_3)][1-f(\epsilon_4)]\}, \tag{6}$$

where $\rho(\epsilon)$ is the density of states in the potential V_{ext}, which we model by $\rho(\epsilon)=(\epsilon+U)^2/2(\hbar\omega_{opt})^3$ if $\epsilon<0$, and $\rho(\epsilon)=\epsilon^2/2(\hbar\omega_{mag})^3$ if $\epsilon>0$. Furthermore, $\epsilon_{min}=\min\{\epsilon_1,\epsilon_2,\epsilon_3,\epsilon_4\}$ is the minimum value of the four single-particle energies involved in the collisions and $\epsilon_2=\epsilon_3+\epsilon_4-\epsilon_1$ according to energy conservation. The cross section for collisions between the two distinguishable spin states $\sigma=4\pi a^2$ and is fixed by the *s*-wave scattering length *a*. In the numerical simulation we fix the ratio $\omega_{mag}/\omega_{opt}$ and the depth *U* of the optical trap. The initial energy distribution $f(\epsilon,t=0)$ is fixed by the adiabatic transfer of atoms realized in the first step of the transformation and its time evolution towards equilibrium is calculated by solving Eq. (6). From this equation, it turns out that the time scale of thermalization is fixed by $\tau^{-1}=\omega_{mag}(|a|/a_{mag})^2N^{2/3}$, where $a_{mag}=(\hbar/m\omega_{mag})^{1/2}$ is the magnetic oscillator length. If we choose $\omega_{mag}/\omega_{opt}=0.1$, $U=5\epsilon_F^0$, and $k_B T_{in}=0.5\epsilon_F^0$, we find that the time required for the system to reach equilibrium in the optical trap is $t_{rel}\simeq 10\tau$ (see Fig. 6). For a configuration of ^{40}K atoms with $N=10^6$, $\omega_{mag}=2\pi\times 100$ Hz, and $|a|/a_{mag}=5\times 10^{-3}$, this corresponds to $t_{rel}\sim 0.1$, a time easily accessible in experiments.

A remarkable feature emerging from our calculations is that the relaxation time needed to achieve equilibrium becomes shorter by lowering the initial temperature of the gas. This is clearly illustrated in Fig. 6 where we show how the relative number of atoms in the optical trap relaxes to the asymptotic value in the case of the two-step transformation for two different initial temperatures. The origin of this behavior can be understood if one considers the model case of

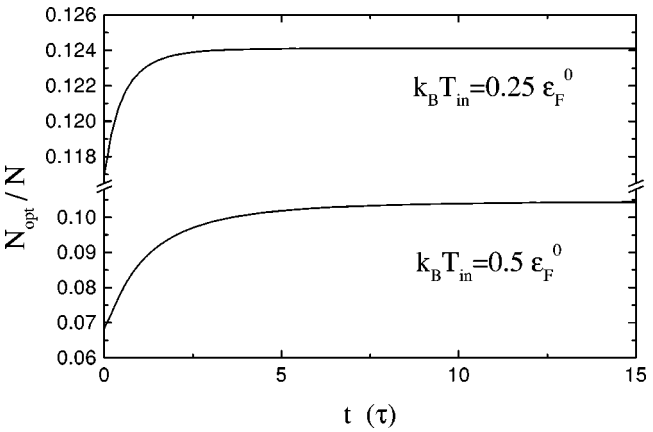

FIG. 6. Relaxation to equilibrium of the fraction of atoms in the optical trap with $U=5\epsilon_F^0$, for two initial temperatures: $k_B T_{in}=0.25\epsilon_F^0$ and $k_B T_{in}=0.5\epsilon_F^0$.

a very shallow optical trap ($U\ll\epsilon_F^0$). In this case, one can assume that all the single-particle states in the optical trap can be grouped into a single energy level $\epsilon_1\simeq -U$ and that the occupation of the energy levels in the magnetic trap is well approximated by the equilibrium value $f(\epsilon)=[\exp(\epsilon-\mu)/k_B T+1]^{-1}$ if $\epsilon\geqslant 0$. By using these assumptions one can solve analytically the quantum Boltzmann equation (6) and get the following result:

$$f(\epsilon_1,t)=[f^0-f^{eq}(\epsilon_1)]\ \ e^{-At/f^{eq}(\epsilon_1)}+f^{eq}(\epsilon_1). \tag{7}$$

In the above equation $f^0=[\exp(-\mu)/k_B T+1]^{-1}$ is the initial occupation of the level which, before the first step of the transformation, was at the bottom of the magnetic trap ($\epsilon=0$), and $f^{eq}(\epsilon_1)=[\exp(\epsilon_1-\mu)/k_B T+1]^{-1}$ is instead its final occupation at equilibrium (chemical potential and temperature do not change during the transformation if $N_{opt}\ll N$). The exponential relaxation to equilibrium is governed by the collision rate *A* which is fixed only by the atoms in the magnetic trap

$$A=\frac{m\sigma}{\pi^2\hbar^3}\int d\epsilon_3 d\epsilon_4\ \ [1-f(\epsilon_2)]f(\epsilon_3)f(\epsilon_4), \tag{8}$$

where ϵ_3, ϵ_4 run over the levels of the magnetic trap and $\epsilon_2=\epsilon_3+\epsilon_4-\epsilon_1$. By lowering the temperature, the rate *A* increases and the relaxation time is thereby decreased. For example, at $k_B T_{in}=0.5\epsilon_F^0$ one finds $A\simeq 0.78\,\tau^{-1}$, while at $k_B T_{in}=0.25\epsilon_F^0$ one has $A\simeq 1.9\,\tau^{-1}$. As $T_{in}\to 0$ the collision rate *A* approaches a constant value $A=6^{2/3}2/\pi\tau^{-1}\simeq 2.1\,\tau^{-1}$. This simple model only accounts for the collisional processes involving two atoms in the magnetic trap with one of the two particles scattering into the optical trap and is accurate if $U\ll\epsilon_F^0$. In the relevant situations discussed in this work, *U* is larger than ϵ_F^0 and additional collisional mechanisms should be considered. In particular, it is important to take into account also collisions where atoms in the highest levels of the optical trap scatter into the lowest levels by exchanging energy with the atoms in the magnetic trap. These collisions have the effect of slowing down the relax-

ation in the highest levels and accelerate the thermalization in the bottom part of the optical trap. Since the highest levels host the largest part of the particles in the optical trap, the relaxation time turns out to be considerably longer than the prediction of the simple model (7) (see Fig. 6). However, the temperature dependence of the relaxation time is qualitatively the same. This behavior should be compared with the corresponding mechanism taking place in evaporative cooling, where the rate of effective collisions is suppressed at low temperature by Pauli blocking [2].

We finally remark that, if one wants to improve the adiabaticity of the transformation and further reduce the entropy increase, one should ramp down the depth U of the optical trap on longer time scales. Of course, in this case, the total duration of the transformation will be longer. One should also take into account that the true shape of the optical trap differs from the harmonic potential employed above. Differences appear because a dipole trap can be safely approximated by a harmonic potential only at its center and, in general, can host more atoms than the corresponding harmonic trap with the same depth U and frequency ratio $\omega_{mag}/\omega_{opt}$. This results in more heating and, as a consequence, in a higher final temperature T_{fin}. We have modeled the optical trap potential using the shape discussed in Ref. [10]. For $k_B T_{in}=0.25\epsilon_F^0$, we find that the fraction of atoms in the optical trap at $U=5\epsilon_F^0$ is about 50% larger than the value obtained using the harmonic approximation, while the final temperature is only about 10% higher. The final degree of degeneracy of the gas in the optical trap is consequently slightly reduced, but the qualitative features discussed above remain unchanged.

IV. CONCLUSIONS

In this paper we have investigated the consequences of an adiabatic transfer of a gas of Fermions from a magnetic to a tighter optical trap. We have seen that it is possible to achieve configurations involving a significant number of atoms that, as a result of the transformation, will occupy a Fermi sea in conditions of high degeneracy. The large enhancement of the release energy should be easily observable through time of flight measurements. This method could make it possible to produce highly degenerate Fermi gases, which, in the case of gases interacting with negative scattering length, are close to the transition to the BCS phase.

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