Destruction of quantum coherence through emission of bremsstrahlung

Heinz-Peter Breuer[1] and Francesco Petruccione[1,2]
[1]*Albert-Ludwigs-Universität, Fakultät für Physik, Hermann-Herder Straße 3, D-79104 Freiburg im Breisgau, Federal Republic of Germany*
[2]*Istituto Italiano per gli Studi Filosofici, Palazzo Serra di Cassano, Via Monte di Dio 14, 80132 Napoli, Italy*
(Received 6 September 2000; published 5 February 2001)

The emergence of decoherence in quantum electrodynamics is investigated. On combining superoperator methods with functional techniques from field theory, the degrees of freedom of a thermal radiation field are eliminated and the influence phase functional is derived which governs the reduced dynamics of the matter variables. Employing a prototypical interference device, a decoherence functional is developed which provides a gauge invariant relativistic measure for the degree of decoherence. It is demonstrated that the decoherence functional describes the destruction of quantum coherence through the emission of bremsstrahlung which is caused by the relative motion of the interfering components of a superposition. Explicit analytical expressions for the vacuum and the thermal contribution to the decoherence functional and for the corresponding coherence lengths are determined. These expressions reveal that bremsstrahlung leads to a fundamental decoherence mechanism which dominates for short times and which is present even in the electromagnetic field vacuum at zero temperature. The influence of bremsstrahlung on the center of mass coordinate of a system of many identical charged particles is also studied and is shown to lead to a strong suppression of quantum coherence.

DOI: 10.1103/PhysRevA.63.032102 PACS number(s): 12.20.Ds, 03.65.Yz, 03.70.+k, 03.67.−a

I. INTRODUCTION

The interaction of a quantum mechanical system with its environment generally leads to a partial or total destruction of the interference between different components of the wave function. In the theory of open quantum systems [1] this destruction of quantum coherence is explained by an investigation of the reduced density matrix. The latter represents the quantum state of an open system as it is obtained after an average over the degrees of freedom of its environment and the resulting loss of information on the entangled state of the combined total system. Many theoretical studies performed on the basis of various system-plus-reservoir models (see, e.g., Refs. [2–6]) have shown that under quite general physical conditions the environment induces an extremely rapid transition of a coherent superposition to an incoherent statistical mixture. This transition is called decoherence and the associated decoherence time arises as the time scale of an exponential decay of the off-diagonal elements in the reduced density matrix. An important result of the theoretical investigations is that the decoherence time can differ substantially from the corresponding relaxation time of the system [2], which signifies the fundamental distinction between the notions of decoherence and of dissipation. Several interesting experimental studies of decoherence have been performed, e.g., in experiments on Schrödinger cat states of a cavity field mode [7] and on single trapped ions in a controllable environment [8].

To give a prominent example let us consider a quantum particle whose motion can be described by the Brownian motion master equation [9], a high-temperature Markovian quantum master equation for the reduced density matrix $\rho(t)$ given by

$$\frac{\partial}{\partial t}\rho(t) = -\frac{i}{\hbar}[H_S,\rho(t)] - \frac{i\gamma}{\hbar}[\vec{R},\{\vec{P},\rho(t)\}] - \frac{2M\gamma k_B T}{\hbar^2}[\vec{R},[\vec{R},\rho(t)]]. \tag{1}$$

Here, $\vec{R}$ denotes the particle position and $\vec{P}$ its canonically conjugated momentum while

$$H_S = \frac{1}{2M}\vec{P}^2 + V(\vec{R}) \tag{2}$$

is the system Hamiltonian with an external potential V. The quantity γ represents the relaxation rate, M the total mass, T the temperature of the environment, and k_B the Boltzmann constant. It is well known that this equation leads to a suppression of quantum coherence given through the factor

$$D = \exp\left[-\gamma\left(\frac{\Delta R}{\overline{\lambda}}\right)^2 t_f\right] \tag{3}$$

which multiplies the off-diagonal terms of the reduced density matrix $\rho(t_f,\vec{R},\vec{R}')$ in the position representation. The elapsed time is denoted by t_f, while $\Delta R = |\vec{R}-\vec{R}'|$ measures the distance to the diagonal of the density matrix. According to the master Eq. (1) the relevant length is given by the thermal wavelength

$$\overline{\lambda} = \overline{\lambda}_{\rm th} \equiv \hbar/\sqrt{2Mk_B T} \tag{4}$$

of the Brownian particle. Equation (3) implies an exponential loss of coherence on a time scale given by the decoherence time

$$\tau_D = \tau_R \left(\frac{\overline{\lambda}_{\rm th}}{\Delta R} \right)^2 . \tag{5}$$

Here, we have introduced the relaxation time $\tau_R = 1/\gamma$ associated with the damping process describing by the rate γ. Simple estimates on the basis of Eq. (5) then lead to the conclusion that for macroscopic objects and distances ΔR the decoherence time τ_D becomes smaller than the relaxation time τ_R by many orders of magnitude [5]. Alternatively, one can characterize decoherence by introducing a time-dependent coherence length $L(t_f)$ by writing the decoherence factor as

$$D = \exp\left[-\frac{(\Delta R)^2}{2L(t_f)^2} \right]. \tag{6}$$

For quantum Brownian motion this yields

$$L(t_f)_{\rm BM} = \frac{\overline{\lambda}_{\rm th}}{\sqrt{2\gamma t_f}} = \frac{1}{\sqrt{2\gamma t_f}} \frac{\hbar}{\sqrt{2Mk_{\rm B}T}} \propto t_f^{-1/2} \cdot T^{-1/2}, \tag{7}$$

which is seen to decrease with the inverse square root of the temperature and of the elapsed time.

If one considers a quantum state of a composite object which represents a spatial superposition of two different locations of its center of mass coordinate, decoherence results from spontaneous or thermally induced transitions involving internal degrees of freedom, or from the scattering of an incoming flux of real particles off the object. In these cases γ represents the transition or the scattering rate, whereas the relevant length $\overline{\lambda}$ is given by the wavelength of the radiation or by the de Broglie wavelength of the scattered particles [3].

In this paper we study the emergence of decoherence in quantum electrodynamics (QED) from an open system's viewpoint. More precisely, we consider the electromagnetic radiation field as environment and investigate its influence on the coherence of the matter variables. It will be demonstrated that the radiation degrees of freedom give rise to a further decoherence mechanism which cannot be modeled in any way as a Markovian process and which leads to a physical picture for the destruction of coherence which differs substantially from the one indicated above. To study this mechanism, we consider a prototypical interference device and ask for the reduction of the interference contrast induced by the presence of a thermal radiation field. It is shown that the radiation field leads to a loss of coherence which can be described by a gauge invariant relativistic decoherence functional, a certain functional of the matter current densities. This functional is Lorentz covariant at finite temperatures and invariant at zero temperature, that is for the electromagnetic field vacuum.

The obtained decoherence functional has been already derived in an interesting article by Ford [10] for the case of zero temperature, with the aim to determine the influence of conducting boundaries on electron coherence. In contrast, here we are interested in the vacuum and thermally induced decoherence itself, that is in the loss of coherence which is present even without boundaries. Other related approaches to this problem, which consider a single-electron coupled to the radiation field within the dipole approximation, have been proposed by Barone and Caldeira [11], by Dürr and Spohn [12], and by the authors [13]. In Ref. [11] it is emphasized that the electromagnetic field provides a super-Ohmic environment. In fact, if one invokes the dipole approximation for the matter-field coupling the spectral density of the radiation modes is seen to increase with the third power of the frequency. Such types of reservoirs are known to lead to important modifications of the simple physical mechanisms indicated above [14]. The result found in Ref. [11] differs from the one derived here, which is due to the use of a different initial condition for the coupled system. For a single electron moving in a harmonic potential the results obtained in Refs. [12] and [13] coincide in the high-temperature limit. However, the authors of Ref. [12] argue that there might be no decoherence effect in the vacuum case, whereas it has been shown in Ref. [13] that the electromagnetic field vacuum does lead to a loss of coherence.

It will be shown in this paper that the underlying physical mechanism for the loss of coherence described by the decoherence functional is the emission of bremsstrahlung through the matter currents. In fact, to observe an interference pattern between two spatially separated components of the state vector, these components must be moved to one location. It is thus the unavoidable creation of bremsstrahlung that causes a loss of coherence, that is, it is the relative motion of the interfering components of a quantum superposition which is responsible for this decoherence process. As a result, a superposition of two wave packets with zero velocity does not decohere and thus the conventional picture of decoherence as a decay of the off-diagonal peaks in the corresponding density matrix does not apply to this decoherence mechanism. Moreover, the time dependence for decoherence through bremsstrahlung is profoundly different from that given by the conventional theories and clearly exhibits the highly non-Markovian character of the process.

A crucial step in the analysis of the decoherence functional is to investigate its infrared and ultraviolet structure. It will be shown that a careful physical interpretation leads to the result that the decoherence functional is ultraviolet and infrared convergent for finite temperatures as well as in the vacuum case. In addition, the analysis reveals that decoherence through bremsstrahlung is in a certain sense the most fundamental process since it always dominates for short times and for large particle numbers and because it occurs even in the vacuum state of the electromagnetic field, that is, at zero temperature and without real photons in the initial state. An important conclusion is that the electromagnetic field vacuum leads to a drastic suppression of the capability of states of many identical charged particles to interfere.

The paper is organized as follows. In Sec. II we combine field theoretic methods with a superoperator approach to derive an exact, relativistic representation for the reduced density matrix pertaining to the matter degrees of freedom. This representation involves an influence phase functional that completely describes the influence of the electromagnetic radiation field on the matter dynamics.

Our central goal in Sec. III is the derivation of the relativistic decoherence functional which provides a quantitative measure for the degree of the coherence of a quantum superposition. In addition, we shall develop an appropriate technique which allows the explicit determination of the decoherence functional for simple interference devices. Thereby, special attention is paid to demonstrate that the decoherence measure is ultraviolet as well as infrared finite. The expressions obtained for the vacuum and thermal coherence lengths of QED will be compared with the corresponding ones of the conventional theories.

Finally, we investigate in Sec. IV the destruction of the coherence of many-particle states. It will be argued that, while the decoherence effect is small for single electrons at nonrelativistic speed, it is drastically amplified for certain superpositions of many-particle states. Our conclusions are drawn in Sec. V.

II. THE INFLUENCE PHASE FUNCTIONAL OF QED

Employing functional techniques, we shall derive in this section a superoperator representation for the influence phase functional of QED which completely describes the reduced matter dynamics under the influence of a thermal radiation field. The influence phase will be given in the form of a functional of the super-operators of the matter current density, involving certain Green functions of the radiation field.

A. Elimination of the radiation degrees of freedom

Our aim is to eliminate the variables of the electromagnetic radiation field to obtain an exact representation for the reduced density matrix ρ_m of the matter degrees of freedom. The starting point is the following formal equation which relates the density matrix $\rho_m(t_f)$ of the matter at some final time t_f to the density matrix $\rho(t_i)$ of the combined matter-field system at some initial time t_i,

$$\rho_m(t_f)=\mathrm{Tr}_f\left\{ T_{\leftarrow}\ \exp\left[\int_{t_i}^{t_f} d^4x \mathcal{L}(x) \right]\rho(t_i)\right\}. \tag{8}$$

The Liouville superoperator $\mathcal{L}(x)$ is defined by the relation

$$\mathcal{L}(x)\rho \equiv -i[\mathcal{H}(x),\rho],$$

where ρ is any density operator of the combined system and $\mathcal{H}(x)$ denotes the Hamiltonian density. Space-time coordinates are denoted by $x=x^\mu=(x^0,\vec{x})=(t,\vec{x})$ and the inner product of two four vectors is written as $xy=x^\mu y_\mu=x^0y^0-\vec{x}\cdot\vec{y}$. All fields are taken to be in the interaction picture. The dynamics of the interaction picture matter degrees of freedom may also contain an external classical field, for example, an external potential $V(x)$. The chronological time-ordering operator for the interaction picture fields is denoted by $T_{\leftarrow}$ and Tr_f stands for the trace over the variables of the radiation field. Throughout the paper we use Heaviside-Lorentz units, such that $\hbar=c=1$ and the fine structure constant is given by $\alpha\equiv e^2/4\pi\hbar c\approx 1/137$. Occasionally, we will reintroduce factors of c and $\hbar$.

To be specific we choose the Coulomb gauge in the following which means that the Hamiltonian density takes the form [15–17]

$$\mathcal{H}(x)=\mathcal{H}_{\mathrm{C}}(x)+\mathcal{H}_{\mathrm{tr}}(x). \tag{9}$$

Here,

$$\mathcal{H}_{\mathrm{tr}}(x)=j^\mu(x)A_\mu(x) \tag{10}$$

represents the density of the interaction of the matter current density $j^\mu(x)$ with the transversal radiation field,

$$A^\mu(x)=(0,\vec{A}(x)),\quad \vec{\nabla}\cdot\vec{A}(x)=0, \tag{11}$$

and

$$\begin{aligned}\mathcal{H}_{\mathrm{C}}(x)&=\tfrac{1}{2}j^0(x)A^0(x)\\ &=\tfrac{1}{2}\int d^3y \frac{j^0(x^0,\vec{x})j^0(x^0,\vec{y})}{4\pi|\vec{x}-\vec{y}|}\end{aligned} \tag{12}$$

is the Coulomb energy density such that

$$H_C(x^0)=\tfrac{1}{2}\int d^3x\int d^3y \frac{j^0(x^0,\vec{x})j^0(x^0,\vec{y})}{4\pi|\vec{x}-\vec{y}|} \tag{13}$$

is the instantaneous Coulomb energy.

Our first step consists in the decomposition of the chronological time-ordering operator $T_{\leftarrow}$ into a time-ordering operator $T^j_{\leftarrow}$ for the matter current and a time-ordering operator $T^A_{\leftarrow}$ for the electromagnetic field,

$$T_{\leftarrow}=T^j_{\leftarrow}T^A_{\leftarrow}. \tag{14}$$

This enables one to write Eq. (8) as

$$\begin{aligned}\rho_m(t_f)=T^j_{\leftarrow}\Bigg(\mathrm{tr}_f\Bigg\{ T^A_{\leftarrow}\ \exp\Bigg[\int_{t_i}^{t_f} d^4x\\ \times[\mathcal{L}_{\mathrm{C}}(x)+\mathcal{L}_{\mathrm{tr}}(x)]\Bigg]\rho(t_i)\Bigg\}\Bigg),\end{aligned} \tag{15}$$

where we have introduced the Liouville super-operators for the densities of the Coulomb field and of the transversal field,

$$\mathcal{L}_{\mathrm{C}}(x)\rho\equiv -i[\mathcal{H}_{\mathrm{C}}(x),\rho],\quad \mathcal{L}_{\mathrm{tr}}(x)\rho\equiv -i[j^\mu(x)A_\mu(x),\rho]. \tag{16}$$

The currents j^μ commute under the time ordering $T^j_{\leftarrow}$. We may therefore treat them formally as commuting c-number fields under the time-ordering symbol. Since the superoperator $\mathcal{L}_{\mathrm{C}}(x)$ only contains matter variables, the corresponding contribution can be pulled out of the trace. Hence we have

$$\rho_m(t_f)=T^j_{\leftarrow}\Bigg(\exp\Bigg[\int_{t_i}^{t_f}d^4x\mathcal{L}_{\rm C}(x)\Bigg]\times{\rm tr}_f\Bigg\{T^A_{\leftarrow}\exp\Bigg[\int_{t_i}^{t_f}d^4x\mathcal{L}_{\rm tr}(x)\Bigg]\rho(t_i)\Bigg\}\Bigg). \quad (17)$$

We now proceed by eliminating the time-ordering of the A fields. With the help of the Wick theorem [18] we get

$$T^A_{\leftarrow}\exp\Bigg[\int_{t_i}^{t_f}d^4x\mathcal{L}_{\rm tr}(x)\Bigg]=\exp\Bigg[\tfrac{1}{2}\int_{t_i}^{t_f}d^4x\int_{t_i}^{t_f}d^4x'[\mathcal{L}_{\rm tr}(x),\mathcal{L}_{\rm tr}(x')]\theta(t-t')\Bigg]\times\exp\Bigg[\int_{t_i}^{t_f}d^4x\mathcal{L}_{\rm tr}(x)\Bigg]. \quad (18)$$

In order to determine the commutator of the Liouville superoperators we invoke the Jacobi identity which yields for an arbitrary test density ρ,

$$\begin{aligned}[\mathcal{L}_{\rm tr}(x),\mathcal{L}_{\rm tr}(x')]\rho&=\mathcal{L}_{\rm tr}(x)\mathcal{L}_{\rm tr}(x')\rho-\mathcal{L}_{\rm tr}(x')\mathcal{L}_{\rm tr}(x)\rho\\&=-[\mathcal{H}_{\rm tr}(x),[\mathcal{H}_{\rm tr}(x'),\rho]]\\&\quad+[\mathcal{H}_{\rm tr}(x'),[\mathcal{H}_{\rm tr}(x),\rho]]\\&=-[[\mathcal{H}_{\rm tr}(x),\mathcal{H}_{\rm tr}(x')],\rho]. \quad (19)\end{aligned}$$

The commutator of the transversal energy densities may be simplified to read

$$[\mathcal{H}_{\rm tr}(x),\mathcal{H}_{\rm tr}(x')]=j^\mu(x)j^\nu(x')[A_\mu(x),A_\nu(x')], \quad (20)$$

since the contribution involving the commutator of the currents vanishes by virtue of the time-ordering operator $T^j_{\leftarrow}$. Thus it follows from Eqs. (19) and (20) that the commutator of the Liouville superoperators may be written as

$$[\mathcal{L}_{\rm tr}(x),\mathcal{L}_{\rm tr}(x')]\rho=-[A_\mu(x),A_\nu(x')][j^\mu(x)j^\nu(x'),\rho]. \quad (21)$$

It is useful to introduce current superoperators $J_+(x)$ and $J_-(x)$ by means of

$$J^\mu_+(x)\rho\equiv j^\mu(x)\rho,\quad J^\mu_-(x)\rho\equiv\rho j^\mu(x). \quad (22)$$

Thus $J_+(x)$ is defined to be the current density acting from the left, while $J_-(x)$ acts from the right on an arbitrary density. With the help of these definitions we may write the commutator of the Liouville superoperators as

$$\begin{aligned}[\mathcal{L}_{\rm tr}(x),\mathcal{L}_{\rm tr}(x')]&=-[A_\mu(x),A_\nu(x')]J^\mu_+(x)J^\nu_+(x')\\&\quad+[A_\mu(x),A_\nu(x')]J^\mu_-(x)J^\nu_-(x').\end{aligned}$$

Inserting this result into Eq. (18), we can write Eq. (17) as

$$\begin{aligned}\rho_m(t_f)=T^j_{\leftarrow}\Bigg(&\exp\Bigg[\int_{t_i}^{t_f}d^4x\mathcal{L}_{\rm C}(x)\\&-\tfrac{1}{2}\int_{t_i}^{t_f}d^4x\int_{t_i}^{t_f}d^4x'\,\theta(t-t')[A_\mu(x),A_\nu(x')]\\&\times J^\mu_+(x)J^\nu_+(x')+\frac{1}{2}\int_{t_i}^{t_f}d^4x\int_{t_i}^{t_f}d^4x'\,\theta(t-t')\\&\times[A_\mu(x),A_\nu(x')]J^\mu_-(x)J^\nu_-(x')\Bigg]\\&\times{\rm tr}_f\Bigg\{\exp\Bigg[\int_{t_i}^{t_f}d^4x\mathcal{L}_{\rm tr}(x)\Bigg]\rho(t_i)\Bigg\}\Bigg). \quad (23)\end{aligned}$$

This is an exact formal representation for the reduced density matrix of the matter variables. Note that the time ordering of the radiation degrees of freedom has been removed and that they enter Eq. (23) only through the functional

$$W[J_+,J_-]\equiv{\rm tr}_f\Bigg\{\exp\Bigg[\int_{t_i}^{t_f}d^4x\mathcal{L}_{\rm tr}(x)\rho(t_i)\Bigg]\Bigg\}, \quad (24)$$

since the commutator of the A fields is a c-number function.

B. The influence superoperator

The functional (24) involves an average over the field variables with respect to the initial state $\rho(t_i)$ of the combined matter-field system. It therefore contains all correlations in the initial state of the total system. Here, we are interested in the destruction of coherence. Our central goal is thus to investigate how correlations are built up through the interaction between matter and radiation field. We therefore consider now an initial state of low entropy which is given by a product state of the form

$$\rho(t_i)=\rho_m(t_i)\otimes\rho_f, \quad (25)$$

where $\rho_m(t_i)$ is the density matrix of the matter at the initial time and the density matrix ρ_f of the radiation field describes an equilibrium state at temperature T. Since we are using the Coulomb gauge here we may write the latter state as

$$\rho_f=\frac{1}{Z_f}\exp(-\beta H_f), \quad (26)$$

where H_f denotes the Hamiltonian of the free radiation field and the quantity $Z_f={\rm tr}_f[\exp(-\beta H_f)]$ is the partition function with $\beta=1/k_BT$. In the following we shall denote the average of some quantity $\mathcal{O}$ with respect to the thermal equilibrium state (26) by

$$\langle\mathcal{O}\rangle_f\equiv{\rm tr}_f\{\mathcal{O}\rho_f\}. \quad (27)$$

The influence of the special choice (25) for the initial condition can be eliminated by pushing $t_i\to-\infty$ and by switching on the interaction adiabatically. This is the usual procedure used in quantum field theory in order to define asymptotic states and the S matrix. The matter and the field

variables are then described as *in fields*, obeying free field equations with renormalized mass.

For an arbitrary initial condition $\rho(t_i)$ the functional $W[J_+,J_-]$ can be determined, for example, by means of a cumulant expansion. Since the initial state (25) is Gaussian with respect to the field variables and since the Liouville superoperator $\mathcal{L}_{\rm tr}(x)$ is linear in the radiation field, the cumulant expansion terminates after the second order term. In addition, a linear term does not appear in the expansion because of $\langle A_\mu(x)\rangle_f=0$. Thus we immediately obtain

$$W[J_+,J_-] = \exp\left[\frac{1}{2}\int_{t_i}^{t_f}d^4x\int_{t_i}^{t_f}d^4x'\langle\mathcal{L}_{\rm tr}(x)\mathcal{L}_{\rm tr}(x')\rangle_f\right]\rho_m(t_i). \tag{28}$$

Inserting the definition for the Liouville superoperator $\mathcal{L}_{\rm tr}(x)$ into the exponent of this expression one finds after some algebra

$$\begin{aligned}
&\tfrac{1}{2}\int_{t_i}^{t_f}d^4x\int_{t_i}^{t_f}d^4x'\langle\mathcal{L}_{\rm tr}(x)\mathcal{L}_{\rm tr}(x')\rangle_f\rho_m \\
&\quad\equiv-\tfrac{1}{2}\int_{t_i}^{t_f}d^4x\int_{t_i}^{t_f}d^4x'\ {\rm tr}_f\{[\mathcal{H}_{\rm tr}(x),[\mathcal{H}_{\rm tr}(x'),\rho_m\otimes\rho_f]]\}= \\
&\quad-\tfrac{1}{2}\int_{t_i}^{t_f}d^4x\int_{t_i}^{t_f}d^4x'[\langle A_\nu(x')A_\mu(x)\rangle_f J_+^\mu(x)J_+^\nu(x') \\
&\qquad+\langle A_\mu(x)A_\nu(x')\rangle_f J_-^\mu(x)J_-^\nu(x') \\
&\qquad-\langle A_\nu(x')A_\mu(x)\rangle_f J_+^\mu(x)J_-^\nu(x') \\
&\qquad-\langle A_\mu(x)A_\nu(x')\rangle_f J_-^\mu(x)J_+^\nu(x')]\rho_m .
\end{aligned}$$

On using this result together with Eq. (28), Eq. (23) can be cast into the form

$$\begin{aligned}
\rho_m(t_f)=T_{\leftarrow}^j\Bigg(\exp\Bigg[&\int_{t_i}^{t_f}d^4x\mathcal{L}_{\rm C}(x)+\frac{1}{2}\int_{t_i}^{t_f}d^4x\int_{t_i}^{t_f}d^4x' \\
&\times\{-(\theta(t-t')[A_\mu(x),A_\nu(x')] \\
&+\langle A_\nu(x')A_\mu(x)\rangle_f)J_+^\mu(x)J_+^\nu(x')+(\theta(t-t') \\
&\times[A_\mu(x),A_\nu(x')]-\langle A_\mu(x)A_\nu(x')\rangle_f)J_-^\mu(x) \\
&\times J_-^\nu(x')+\langle A_\nu(x')A_\mu(x)\rangle_f J_+^\mu(x)J_-^\nu(x') \\
&+\langle A_\mu(x)A_\nu(x')\rangle_f J_-^\mu(x)J_+^\nu(x')\}\Bigg]\Bigg)\rho_m(t_i).
\end{aligned} \tag{29}$$

At this point it is useful to introduce the commutator function

$$D(x-x')_{\mu\nu}\equiv i[A_\mu(x),A_\nu(x')], \tag{30}$$

and the anticommutator function

$$D_1(x-x')_{\mu\nu}\equiv\langle\{A_\mu(x),A_\nu(x')\}\rangle_f . \tag{31}$$

Correspondingly, we define a commutator superoperator $J_c(x)$ and an anticommutator superoperator $J_a(x)$,

$$J_c^\mu(x)\rho\equiv[j^\mu(x),\rho],\quad J_a^\mu(x)\rho\equiv\{j^\mu(x),\rho\}, \tag{32}$$

which are related to the previously introduced superoperators $J_\pm^\mu(x)$ by

$$J_c^\mu(x)=J_+^\mu(x)-J_-^\mu(x),\quad J_a^\mu(x)=J_+^\mu(x)+J_-^\mu(x). \tag{33}$$

In terms of these quantities we can now write Eq. (29) as

$$\rho_m(t_f)=T_{\leftarrow}^j\ \exp(i\Phi[J_c,J_a])\rho_m(t_i), \tag{34}$$

where the influence phase functional $\Phi[J_c,J_a]$ is given by

$$\begin{aligned}
i\Phi[J_c,J_a]=&\int_{t_i}^{t_f}d^4x\mathcal{L}_{\rm C}(x)+\int_{t_i}^{t_f}d^4x\int_{t_i}^{t}d^4x' \\
&\times\left\{\frac{i}{2}D(x-x')_{\mu\nu}J_c^\mu(x)J_a^\nu(x')\right. \\
&\left.-\frac{1}{2}D_1(x-x')_{\mu\nu}J_c^\mu(x)J_c^\nu(x')\right\}.
\end{aligned} \tag{35}$$

These equations provide an exact representation for the density matrix of the matter variables which takes on the desired form: It represents the influence of the radiation field on the matter dynamics in terms of the two fundamental two-point correlation functions $D(x-x')$ and $D_1(x-x')$. One observes that the motion of the matter is determined by a time-ordered exponential function whose exponent $i\Phi[J_c,J_a]$ is a bilinear functional of the current superoperators $J_c(x)$ and $J_a(x)$. Note that the double space-time integral in Eq. (35) is already a time-ordered integral since the integration over $t'=x_0'$ extends over the time interval from t_i to $t=x_0$. The first part of the influence phase involving the commutator function describes dissipative effects and $D(x-x')$ is usually called dissipation kernel in the context of nonrelativistic quantum Brownian motion [19]. The anticommutator function $D_1(x-x')$ is often referred to as noise kernel and it is the corresponding part of the influence phase which is responsible for the decoherence through bremsstrahlung, as will be discussed in the next section.

The representation (34) immediately yields the following second-order equation of motion for the density matrix of the matter degrees of freedom,

$$\begin{aligned}
\frac{d}{dt}\rho_m(t)=&\int d^3x\mathcal{L}_{\rm C}(x)\rho_m(t) \\
&+\int d^3x\int_{t_i}^{t}d^4x' \\
&\times\left[\frac{i}{2}D(x-x')_{\mu\nu}J_c^\mu(x)J_a^\nu(x')\rho_m(t')\right. \\
&\left.-\frac{1}{2}D_1(x-x')_{\mu\nu}J_c^\mu(x)J_c^\nu(x')\rho_m(t')\right].
\end{aligned} \tag{36}$$

It must be noted that this equation still provides a non-Markovian master equation since it involves the nonlocal dissipation and noise kernels. Various well-known master equations encountered in quantum optics and solid state physics can be derived from this equation. For example, the quantum optical master equation [1] is obtained from it by performing the Markovian and the rotating wave approximation.

Invoking the nonrelativistic (dipole) approximation of Eq. (36) one is led to the following equation of motion for a single electron with mass m:

$$\frac{d}{dt}\rho_m(t)=\frac{i}{2m^2}\int_{t_i}^{t}dt'D(t-t')[\vec{p}(t),\{\vec{p}(t'),\rho_m(t')\}] - \frac{1}{2m^2}\int_{t_i}^{t}dt'D_1(t-t')[\vec{p}(t),[\vec{p}(t'),\rho_m(t')]], \tag{37}$$

where $\vec{p}(t)$ represents the interaction picture momentum operator canonically conjugated to the electron's position coordinate $\vec{x}(t)$. The dissipation and the noise kernel of Eq. (37) can be expressed in terms of the spectral density

$$J(\omega)=\frac{e^2}{3\pi^2}\omega\Theta(\Omega-\omega) \tag{38}$$

as integrals over the photon frequencies ω,

$$D(t-t')=\int_0^\infty d\omega J(\omega)\sin\omega(t-t'), \tag{39}$$

$$D_1(t-t')=\int_0^\infty d\omega J(\omega)\coth(\beta\omega/2)\cos\omega(t-t'), \tag{40}$$

where we have introduced an ultraviolet frequency cutoff Ω. The role of this cutoff for decoherence phenomena will be discussed in detail in the following section. The equation of motion (37) should be contrasted to the quantum Brownian motion master Eq. (1). Note that Eq. (37) is formulated in the interaction picture, while Eq. (1) is written in the Schrödinger picture. One easily verifies that Eq. (37) yields the following equation for the expectation value of the particle position:

$$m\frac{d^2}{dt^2}\langle\vec{x}(t)\rangle+\frac{d}{dt}\int_{t_i}^{t}dt'D(t-t')\frac{d}{dt'}\langle\vec{x}(t')\rangle=-\langle\vec{\nabla}V(\vec{x})\rangle. \tag{41}$$

One observes that this is the Ehrenfest equation of motion corresponding to the classical Abraham-Lorentz equation for an electron [20]. The second term which contains the dissipation kernel $D(t-t')$ describes the radiative damping of the particle motion. It can be shown that this term leads to an electromagnetic mass renormalization and to a damping term involving the third derivative of the position coordinate [11,13].

We finally remark that several alternative strategies could be used to arrive at an expression of the form (35) as, for example, path integral techniques [21] or Schwinger's closed time-path method [22]. A similar expression for the influence phase functional has been given, for example, in Ref. [23] without the Coulomb term and for the special case of zero temperature. In our derivation we have combined superoperator techniques with methods from field theory, which seems to be the most direct way to obtain an operator representation of the reduced density matrix.

III. DECOHERENCE THROUGH EMISSION OF BREMSSTRAHLUNG

In this section we wish to develop a relativistic formulation for the loss of coherence induced by the emission of bremsstrahlung. The starting point of our discussion is Eq. (35) for the influence phase functional. The aim is to deduce from that equation a quantitative, relativistically covariant measure for the degree of coherence of a superposition of spatially separated states. This can be achieved with the help of a simple interference device which is used to quantify the degree of coherence of such a superposition through the resulting interference contrast. A similar approach has been employed in Ref. [2] to study decoherence in quantum Brownian motion.

A. Introducing the decoherence functional

The influence phase given in Eq. (35) provides an exact expression as long as the electromagnetic field variables are in a Gaussian state, as, for example, the equilibrium state (26), and as long as the initial state of the combined system is given by a low-entropy factorizing state (25). In Ref. [13] we have obtained exact analytical expressions for the single-electron propagator function within the non-relativistic dipole approximation. These expressions take into account the finite width as well as the spreading of the wave packets. In the general case, the formal Eq. (35) is, however, much too complicated to be evaluated exactly and one must restore to various approximation schemes.

For the purpose of defining a measure for the degree of decoherence we shall employ the prototypical interference device which is sketched in Fig. 1. A charged particle, say an electron, is emitted by the source Q and can move along two different world lines y_1 and y_2 to reach a screen at S, where an interference pattern is observed. These paths represent two quantum alternatives whose probability amplitudes may be described by two wave packets $|\Psi_1(t_i)\rangle$ and $|\Psi_2(t_i)\rangle$. With the help of the superposition principle we find that the wave function

$$|\Psi(t_i)\rangle=|\Psi_1(t_i)\rangle+|\Psi_2(t_i)\rangle \tag{42}$$

describes the physical situation depicted in the figure. Alternatively, the electron state can be represented in terms of the density matrix $\rho_m(t_i)=|\Psi(t_i)\rangle\langle\Psi(t_i)|$ which may be written as

$$\rho_m(t_i)=\rho_{11}(t_i)+\rho_{22}(t_i)+\rho_{12}(t_i)+\rho_{21}(t_i), \tag{43}$$

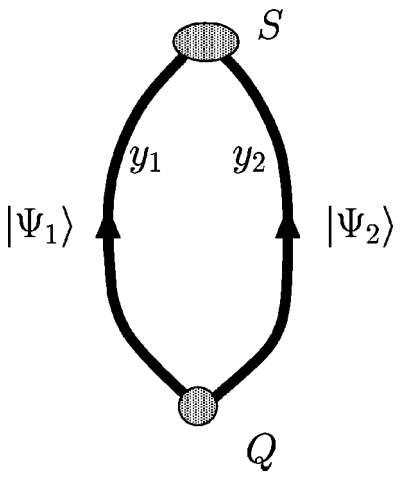

FIG. 1. Sketch of a prototypical interference device which is employed to introduce the decoherence functional. An electron emerges from the source Q and can follow two possible paths y_1 and y_2, which leads, in general, to an interference pattern observed on a screen at S. The two quantum alternatives may be described through the wave packets $|\Psi_1\rangle$ and $|\Psi_2\rangle$.

where $\rho_{ij}(t_i)=|\Psi_i(t_i)\rangle\langle\Psi_j(t_i)|$. One observes the emergence of the interference term $\rho_{12}(t_i)+\rho_{21}(t_i)$. Remember that we are working in the interaction picture and that we therefore have $\rho_m(t_i)=\rho_m(t)$ for all times in the case of a vanishing coupling between matter and electromagnetic field.

Our aim is to determine with the help of the influence superoperator the structure of the electron density matrix ρ_m in the presence of the electromagnetic radiation field. An essential simplification is achieved if the matter current density can be treated as a classical current. This approximation can be justified under the following conditions. First, we assume that the wavelength $\bar{\lambda}=c/\omega$ of the photons emitted by the currents is large compared to the Compton wavelength $\bar{\lambda}_C$ of the electron,

$$\bar{\lambda}\gg\bar{\lambda}_C=\frac{\hbar}{mc}, \tag{44}$$

and thus also large in comparison to the classical electron radius $r_e=\alpha\bar{\lambda}_C$. This requirement is equivalent to $\hbar\omega\ll mc^2$. In this low energy regime one may neglect pair creation and annihilation amplitudes and treat the matter current density as a given classical field [16,17]. The same procedure is used, for example, in the nonperturbative analysis of radiative corrections in the low frequency limit (see Sec. III B). In an experiment of the type sketched in Fig. 1 the paths involve an acceleration of the electron through a certain field of force. This force gives rise to a certain characteristic acceleration time τ_p. We define τ_p as the inverse of the highest frequency in the spectrum of the force acting on the electron. In the following we call τ_p the preparation time since it can be interpreted as the time required to set into motion the interfering wave packets. As a consequence of the existence of such a characteristic time we have a natural upper cutoff $\Omega_{\max}$ for the frequency spectrum of the emitted radiation which is of the order

$$\Omega_{\max}\sim\frac{1}{\tau_p}=\frac{c}{\sigma_0}, \tag{45}$$

where the length scale σ_0 represents the order of the minimal wavelength of the radiation. Our above requirement thus takes the form

$$\sigma_0\gg\bar{\lambda}_C. \tag{46}$$

This also implies that the characteristic acceleration time τ_p is large compared to r_e/c. It is known from classical electrodynamics that this condition ensures that the energy radiated is small compared to the kinetic energy of the particle and that therefore radiative damping effects are small [20].

The second condition is that the motion of the current can be reasonably described within a semiclassical approximation. This leads to the requirement $\Delta v/v\ll 1$, where v is a typical velocity and Δv the velocity uncertainty. Assuming that the wave packets represent states of minimal uncertainty with spatial width Δx one is led to the condition

$$\frac{\Delta v}{v}\sim\frac{\hbar}{mv\Delta x}\ll 1, \tag{47}$$

or, equivalently,

$$\frac{\bar{\lambda}_{\mathrm{dB}}}{\Delta x}\ll 1, \tag{48}$$

where $\bar{\lambda}_{\mathrm{dB}}=\hbar/mv$ is the de Broglie wavelength. This is the typical condition for a semiclassical treatment.

In view of these conditions we now assume that $\rho_m(t_i)$ represents a state which is an approximate eigenstate of the current density. Thus, if $\rho_m(t_i)$ is a pure state,

$$\rho_m(t_i)=|\Psi(t_i)\rangle\langle\Psi(t_i)|, \tag{49}$$

we suppose that

$$j^\mu(x)|\Psi(t_i)\rangle\approx s^\mu(x)|\Psi(t_i)\rangle, \tag{50}$$

where $s^\mu(x)$ is a classical current density. Hence we also have to the same degree of accuracy,

$$J_c^\mu(x)\rho_m(t_i)=[j^\mu(x),\rho_m(t_i)]\approx 0. \tag{51}$$

The initial state $\rho_m(t_i)$ does not necessarily have to be a pure state. It suffices to require Eq. (51), where

$$\langle j^\mu(x)\rangle=\mathrm{tr}_m\{j^\mu(x)\rho_m(t_i)\}=s^\mu(x) \tag{52}$$

is the expectation value of the current density. In any case we immediately obtain with the help of Eq. (51) and expression (35) for the influence phase

$$\rho_m(t_f)\approx\rho_m(t_i). \tag{53}$$

This equation states that the system is essentially unaffected by the radiation field, i.e., by virtue of our assumption that the initial state is a current eigenstate, the dynamics of the density matrix is that of a free system. The same conclusion has been obtained in Ref. [13], where the dynamics of Gaussian wave packets was investigated using the exact analytical expression for the propagator function in the dipole approximation.

Let us now return to the interference device and assume that the superposition (42) consists of two current eigenstates,

$$\begin{aligned} j^{\mu}(x)|\Psi_1(t_i)\rangle &\approx s_1^{\mu}(x)|\Psi_1(t_i)\rangle, \\ j^{\mu}(x)|\Psi_2(t_i)\rangle &\approx s_2^{\mu}(x)|\Psi_2(t_i)\rangle, \end{aligned} \tag{54}$$

where $s_1(x)$ and $s_2(x)$ are classical current densities. These currents are assumed to be concentrated within two world tubes around the paths y_1 and y_2 of the interference device, respectively. By virtue of Eq. (54) we have

$$J_c^{\mu}(x)\rho_1(t_i) \approx J_c^{\mu}(x)\rho_2(t_i) \approx 0,$$

$$J_c^{\mu}(x)\rho_{12}(t_i) \approx [s_1^{\mu}(x) - s_2^{\mu}(x)]\rho_{12}(t_i),$$

$$J_a^{\mu}(x)\rho_{12}(t_i) \approx [s_1^{\mu}(x) + s_2^{\mu}(x)]\rho_{12}(t_i),$$

and

$$\mathcal{L}_C(x)\rho_1(t_i) \approx \mathcal{L}_C(x)\rho_2(t_i) \approx 0,$$

$$\mathcal{L}_C(x)\rho_{12}(t_i) \approx -i[\mathcal{H}_{C1}(x) - \mathcal{H}_{C2}(x)]\rho_{12}(t_i),$$

where

$$\mathcal{H}_{C1,2}(x) = \tfrac{1}{2}\int d^3y \frac{s_{1,2}^0(x^0,\vec{x})s_{1,2}^0(x^0,\vec{y})}{4\pi|\vec{x}-\vec{y}|} \tag{55}$$

are the Coulomb energy densities associated with the current densities $s_1^{\mu}(x)$ and $s_2^{\mu}(x)$, respectively. We may suppose that the corresponding Coulomb energies for both possible paths are equal to each other. The expression (35) for the influence phase functional now immediately leads to

$$\begin{aligned} \rho_m(t_f) \approx \rho_1(t_i) + \rho_2(t_i) + \exp(i\Phi)\rho_{12}(t_i) \\ + \exp(-i\Phi^*)\rho_{21}(t_i), \end{aligned} \tag{56}$$

where

$$\begin{aligned} i\Phi = \int_{t_i}^{t_f} d^4x \int_{t_i}^{t} d^4x' \frac{i}{2} D(x-x')_{\mu\nu}[s_1^{\mu}(x) - s_2^{\mu}(x)] \\ \times [s_1^{\nu}(x') + s_2^{\nu}(x')] - \tfrac{1}{4}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' \\ \times D_1(x-x')_{\mu\nu}[s_1^{\mu}(x) - s_2^{\mu}(x)][s_1^{\nu}(x') - s_2^{\nu}(x')]. \end{aligned}$$

We see here that the electromagnetic field affects the interference terms through a complex phase $\Phi[s_1,s_2]$ which is a functional of the two possible classical paths y_1 and y_2, or, more precisely, of the associated current densities $s_1^{\mu}(x)$ and $s_2^{\mu}(x)$. The real part of Φ leads to a distortion of the interference pattern. What is interesting in the present context is the imaginary part of the phase functional which yields a suppression of the interference contrast. Thus we observe that it is the functional

$$\begin{aligned} \Gamma[s_1,s_2] = -\tfrac{1}{4}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' D_1(x-x')_{\mu\nu} \\ \times [s_1^{\mu}(x) - s_2^{\mu}(x)][s_1^{\nu}(x') - s_2^{\nu}(x')] \end{aligned} \tag{57}$$

which measures the degree of decoherence, and which will therefore be referred to as decoherence functional. Alternatively, we can write Γ in terms of the current difference,

$$c^{\mu}(x) = \frac{1}{\sqrt{2}}[s_1^{\mu}(x) - s_2^{\mu}(x)], \tag{58}$$

as follows:

$$\Gamma[c] = -\tfrac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' D_1(x-x')_{\mu\nu} c^{\mu}(x) c^{\nu}(x'). \tag{59}$$

The above expressions lead to several physically interesting interpretations which will be discussed in the next subsection. Explicit formulas for the decoherence functional will be obtained in Sec. III C.

B. Physical interpretation

Let us first discuss the transformation properties of the decoherence functional. Expression (59) describes the loss of coherence for physical situations like the one sketched in Fig. 1. We take t_0 to be the time corresponding to Q, that is as the time at which the wave packet is separated into two components, while t_f denotes the final time when both packets are recombined at S. It is obvious that the current difference $c^{\mu}(x)$ vanishes for times prior to t_0 and for times later than the final time t_f. In fact, the support of the current difference $c^{\mu}(x)$ lies in the interior of a closed world tube around the loop l which is formed by following y_1 in the positive and y_2 in the negative direction. Formally, we write this loop as

$$l = y_1 - y_2. \tag{60}$$

Current conservation therefore enables us to write the decoherence functional as follows:

$$\Gamma[c] = -\tfrac{1}{2}\int d^4x \int d^4x' D_1(x-x')_{\mu\nu} c^{\mu}(x) c^{\nu}(x'), \tag{61}$$

where the covariant form for the anti-commutator function can be used,

$$D_1(x-x')_{\mu\nu} = -g_{\mu\nu} D_1(x-x'), \tag{62}$$

with the scalar function

$$D_1(x-x')=\int \frac{d^3k}{2(2\pi)^3\omega}\{\exp[-ik(x-x')] + \exp[ik(x-x')]\}\coth(\beta\omega/2). \quad (63)$$

Equation (61) shows that the decoherence functional provides a decoherence measure which is both relativistically covariant and gauge invariant. In addition, it is Lorentz invariant in the vacuum case. To see this we recall that at zero temperature the anticommutator function is given by the invariant function

$$D_1(x-x')_{\rm vac}=\int \frac{d^3k}{2(2\pi)^3\omega}\{\exp[-ik(x-x')] + \exp[ik(x-x')]\} = -\frac{1}{2\pi}\,\mathrm{P}\,\frac{1}{(x-x')^2}, \quad (64)$$

where P denotes the principal value.

The form (61) suggests an interesting representation for the decoherence functional in terms of a double loop integral [10]. To this end, we consider a current density of the form

$$s^\mu(x)=e\int_{-\infty}^{\infty} d\tau u^\mu(\tau)\,\delta(x-y(\tau)), \quad (65)$$

where $y(\tau)$ is some timelike world line parameterized by the proper time τ, and

$$u^\mu(\tau)=\frac{dy^\mu(\tau)}{d\tau}, \quad u^\mu u_\mu=1,$$

is the four velocity. Apart from the assumption of an idealized line density this formula neglects the spin contribution to the current. In fact, one has from the Gordon decomposition of the Dirac current density [24],

$$s^\mu=e\bar{\psi}\gamma^\mu\psi=\frac{e}{2m}[\bar{\psi}(p^\mu\psi)-(p^\mu\bar{\psi})\psi]-\frac{ie}{2m}p_\nu(\bar{\psi}\sigma^{\mu\nu}\psi). \quad (66)$$

The first part on the right-hand side represents the convection current density which leads to Eq. (65), while the second part is known as the spin current density. The spin current density can be neglected as long as the length scales involved in the problem under consideration are large compared to the Compton wavelength [10]. Under this condition the decoherence functional (61) can be expressed as a double integral over the closed path l,

$$\Gamma[c]=-\frac{e^2}{4}\oint_l dx^\mu \oint_l dx'^\nu D_1(x-x')_{\mu\nu}. \quad (67)$$

Again, this representation exhibits the manifest Lorentz covariance and the gauge invariance of the decoherence functional.

An equation of the form (67) has been derived by Ford [10] using a different technique. We remark that there are, however, two substantial differences. First, we note that Eqs. (61) and (67) involve the temperature-dependent Green function $D_1(x-x')_{\mu\nu}$ of the electromagnetic field [see Eqs. (31) and (63)], whereas Ford's expression only involves the zero-temperature Green function defined as a vacuum expectation value. Equation (67) is therefore more general than the expression given by Ford. In particular, it allows to study the full temperature dependence of decoherence and enables one to compare the influence of the vacuum field with that of the thermal field (see the next subsection). Second, it must be emphasized that it is the influence of conducting boundaries on the loss of coherence what is investigated in Ref. [10]. To this end, the difference of the Green function satisfying the boundary conditions to the vacuum Green function is substituted into Eq. (67). This means that the difference between decoherence with and without boundaries is studied in Ref. [10], whereas here the vacuum decoherence itself will be examined.

For the physical interpretation of the decoherence functional it is useful to introduce the Feynman propagator and its complex conjugated ($T_\rightarrow$ denotes the antichronological time-ordering operator),

$$\begin{aligned} iD_F(x-x')_{\mu\nu} &\equiv \langle T_\leftarrow[A_\mu(x)A_\nu(x')]\rangle_f \\ &= \theta(t-t')[A_\mu(x),A_\nu(x')] + \langle A_\nu(x')A_\mu(x)\rangle_f, \\ iD_F^*(x-x')_{\mu\nu} &\equiv -\langle T_\rightarrow[A_\mu(x)A_\nu(x')]\rangle_f \\ &= \theta(t-t')[A_\mu(x),A_\nu(x')] - \langle A_\mu(x)A_\nu(x')\rangle_f, \end{aligned} \quad (68)$$

as well as the two-point correlation functions

$$\begin{aligned} D_+(x-x')_{\mu\nu} &\equiv \langle A_\mu(x)A_\nu(x')\rangle_f, \\ D_-(x-x')_{\mu\nu} &\equiv \langle A_\nu(x')A_\mu(x)\rangle_f. \end{aligned} \quad (69)$$

Expressing the anticommutator function in terms of the Feynman propagator,

$$D_1(x-x')_{\mu\nu}=iD_F(x-x')_{\mu\nu}-iD_F^*(x-x')_{\mu\nu}, \quad (70)$$

we obtain for the influence phase factor (57) the following expression:

$$\exp(i\Phi[s_1,s_2])=A[s_1]A[s_2]^* \times\exp\left[\tfrac{1}{2}\int d^4x\int d^4x' \times[D_-(x-x')_{\mu\nu}s_1^\mu(x)s_2^\nu(x') + D_+(x-x')_{\mu\nu}s_2^\mu(x)s_1^\nu(x')]\right], \quad (71)$$

where we have introduced the definition

$$A[j]=\exp\left[-\frac{i}{2}\int d^4x\int d^4x' D_F(x-x')_{\mu\nu}j^\mu(x)j^\nu(x')\right]. \tag{72}$$

At zero temperature $A[j]$ is the vacuum-to-vacuum amplitude [25] in the presence of a classical current density $j^\mu(x)$. The first term on the right-hand side in Eq. (71) is thus the product of the vacuum-to-vacuum amplitudes in the presence of the current densities s_1^μ and s_2^μ, respectively. This contribution describes virtual processes in which photons are emitted and reabsorbed by either the current s_1^μ or by s_2^μ. Correspondingly, the exponential on the right-hand side of Eq. (71) is the contribution of the emission of at least one photon. These processes also contribute to the decoherence functional since a photon can be emitted by both currents and carries away information on the path taken by the electron. Moreover, at finite temperatures thermally induced emission and absorption processes occur. It will be seen in the next subsection that the radiation field shows the typical spectrum of bremsstrahlung.

With the help of Eq. (70) we can write the decoherence functional as

$$\Gamma[c]=-\frac{i}{2}\int d^4x\int d^4x' D_F(x-x')_{\mu\nu}c^\mu(x)c^\nu(x')+\text{c.c.}, \tag{73}$$

where c.c. means complex conjugated. Thus we see that at zero temperature the decoherence factor can be expressed as

$$\exp(\Gamma[c])=A[c]A[c]^*. \tag{74}$$

Obviously, we have $\Gamma[c]\leqslant 0$, and $\Gamma[c]=0$ for $s_1^\mu=s_2^\mu$, that is for a vanishing current difference, $c^\mu=0$. Equation (74) gives rise to another interesting interpretation: The decoherence factor which multiplies the interference term is given by the no-photon emission probability in the presence of the current density c^μ. This current is the same as the current which would be created by two particles with opposite charges $\pm e/\sqrt{2}$, one moving along y_1 and the other along y_2. The smaller is the vacuum-to-vacuum amplitude for this current density the larger is the reduction of the interference contrast. This must have been expected since it is the difference between the currents s_1 and s_2 which determines the extent to which the two possible paths can be distinguished, and, thus, the degree of the loss of coherence.

These interpretations in terms of the emitted photons must be taken, however, with some care. The reason is that we consider here processes on a finite time scale and not transitions between asymptotic states. It is well known that certain matter currents emit an infinite number of long-wavelength (soft) photons whose frequencies approach zero, while their total energy adds up to a finite value. This is the so-called infrared catastrophe [15,16] which arises in the perturbative calculation of radiative corrections to any process involving charged matter. The complete removal of infrared divergences requires a nonperturbative treatment in which the amplitudes for the emission of real and virtual soft photons are summed to all orders, such that the processes involving real and virtual photons become indistinguishable in the low frequency limit. Infrared divergences can be shown to cancel provided a finite resolution $\Omega_{\min}$ for the photodetection is introduced: Insisting on the perturbative picture, one could say that there is always an infinite number of quanta, namely those whose frequency is lower than $\Omega_{\min}$, which escapes undetected and cannot be observed in principle.

Our analysis treats the matter current classically but it is nonperturbative [16]. In view of the above considerations it is obvious that the decoherence functional $\Gamma[c]$ does not lead to infrared divergences since it describes a process taking place in the finite time interval between the splitting of the wave packet at t_0 and the recombination at t_f. This gives rise to a natural frequency resolution of the order

$$\Omega_{\min}\sim\frac{1}{t_f-t_0}. \tag{75}$$

The emergence of this effective infrared cutoff will be seen explicitly in the calculations of the next subsection, where it will be demonstrated that the arising integrals over the photon frequencies converge at the lower limit $\omega\to 0$. In addition we also have an ultraviolet cutoff $\Omega_{\max}$ which has already been introduced in Eq. (45). This cutoff can accounted for by the introduction of a finite width σ_0 characterizing the current world tube, as will be seen in the next subsection.

C. Determination of the decoherence functional

We wish to derive here explicit formulas for the decoherence functional corresponding to the interference device depicted in Fig. 1. On using Eqs. (61), (62), and (63) we find

$$\Gamma[c]=-\int\frac{d^3k}{2(2\pi)^3\omega}\coth(\beta\omega/2)[-c^\mu(k)c_\mu(k)^*], \tag{76}$$

where we use the notation $k=(\omega,\vec{k})=(|\vec{k}|,\vec{k})$ for the wave vector and

$$c^\mu(k)\equiv\int d^4x\exp(-ikx)c^\mu(x) \tag{77}$$

is the Fourier transform of the current difference.

Let us first show explicitly how a finite width of the current world tubes gives rise to an ultraviolet cutoff scale. To this end, the currents $s_1^\mu(x)$ and $s_2^\mu(x)$ are taken to be concentrated within world tubes of spatial extent σ_0 around the world lines $y_1(\tau)$ and $y_2(\tau)$. To be specific we write

$$s_{1,2}^\mu(x)=e\int d\tau u_{1,2}^\mu(\tau)\,\delta_{\sigma_0}(x-y_{1,2}(\tau)), \tag{78}$$

where

$$\delta_{\sigma_0}(x-x')=\delta(x_0-x_0')\frac{1}{(2\pi\sigma_0^2)^{3/2}}\exp\left[-\frac{(\vec{x}-\vec{x}')^2}{2\sigma_0^2}\right] \tag{79}$$

is a smeared δ function described by a Gaussian with width σ_0. Using Eq. (78) in Eq. (77) we find for the Fourier transform of the current difference

$$c^\mu(k)=\frac{e}{\sqrt{2}}\left[\int d\tau u_1^\mu(\tau)\exp[-iky_1(\tau)] - \int d\tau u_2^\mu(\tau)\exp[-iky_2(\tau)]\right]\exp\left[-\frac{1}{2}\sigma_0^2\omega^2\right]. \tag{80}$$

We see that the finite width σ_0 of the current world tubes yields an effective ultraviolet cutoff $\Omega_{\max}\sim\sigma_0^{-1}$ as given in Eq. (45). Our main interest is an estimation of the decoherence functional for some specific situations. We therefore ignore in the following the specific form of the cutoff function in Eq. (80) and work with a sharp cutoff at the maximal frequency $\omega=\Omega_{\max}$. It will be seen below that the final expression for $\Gamma[c]$ depends on $\Omega_{\max}$ only through $\ln\Omega_{\max}$. This extremely weak logarithmic dependence shows that the precise value of σ_0 or of the preparation time τ_p is rather irrelevant. The important point to note here is that the emergence of an effective ultraviolet cutoff has a clear physical origin.

Thus we now write Eq. (76) as

$$\Gamma[c]=-\frac{e^2}{16\pi^3}\int_0^{\Omega_{\max}} d\omega\,\omega\coth(\beta\omega/2) \times\int d\Omega(\hat{k})[-c^\mu(k)c_\mu^*(k)], \tag{81}$$

where

$$c^\mu(k)=\frac{e}{\sqrt{2}}\oint_l dx^\mu\,\exp(-ikx), \tag{82}$$

and $d\Omega(\hat{k})$ denotes the element of the solid angle into the direction of the unit vector $\hat{k}\equiv\vec{k}/|\vec{k}|$. For simplicity let us consider the case that the loop l consists of four straight world line segments [see Fig. 2(a)]. The four vertices of the loop are denoted by a_0,a_1,a_2,a_3, whereas the corresponding four velocities are u_1,u_2,u_3,u_4. We further assume that the arrangement is symmetric, that is,

$$u_1=u_4,\quad u_2=u_3, \tag{83}$$

and

$$a_1-a_0=a_2-a_3,\quad a_2-a_1=a_3-a_0. \tag{84}$$

For a single line segment with initial point a, endpoint b, and four velocity u [see Fig. 2(b)] we obtain

$$\int_a^b dx\,\exp(-ikx)=i\frac{u}{ku}[e^{-ikb}-e^{-ika}]. \tag{85}$$

With the help of this formula the Fourier transform (82) of the current difference is found to be

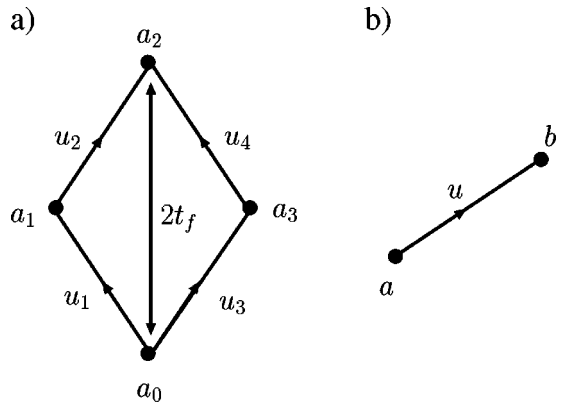

FIG. 2. (a) The closed loop l used for an explicit determination of the decoherence functional $\Gamma[c]$. The loop consists of four straight world line segments with four velocities u_1,u_2,u_3,u_4. The vertices are located at the space-time points a_0,a_1,a_2,a_3. As indicated, the loop corresponds to the total time $2t_f$. (b) A single line segment with initial point a, endpoint b, and four velocity u.

$$c(k)=\frac{ie}{\sqrt{2}}\left\{+\frac{u_1}{ku_1}[e^{-ika_1}-e^{-ika_0}]+\frac{u_2}{ku_2}[e^{-ika_2}-e^{-ika_1}] - \frac{u_3}{ku_3}[e^{-ika_3}-e^{-ika_0}]-\frac{u_4}{ku_4}[e^{-ika_2}-e^{-ika_3}]\right\}. \tag{86}$$

It should be noted that $k_\mu c^\mu(k)=0$ as required by current conservation. Note further that $c^\mu(k)$ shows the correct behavior under Lorentz transformations, in particular, one finds that $c^\mu(k)$ transforms into $c^\mu(k)\exp(-ikb)$ under a space-time translations by the four vector b. If we now use the symmetry properties of the loop we arrive at

$$c(k)=\frac{ie}{\sqrt{2}}\left[\frac{u_2}{ku_2}-\frac{u_4}{ku_4}\right]\mathcal{G}(k), \tag{87}$$

where we have introduced

$$\mathcal{G}(k)=e^{-ika_2}(1-e^{ik(a_2-a_3)})(1-e^{ik(a_2-a_1)}).$$

Using these results one is led to the following expression for the decoherence functional:

$$\Gamma[c]=\frac{\alpha}{8\pi^2}\int_0^{\Omega_{\max}}\frac{d\omega}{\omega}\coth(\beta\omega/2) \times\int d\Omega(\hat{k})\,\omega^2\left[\frac{u_2}{ku_2}-\frac{u_4}{ku_4}\right]^2|\mathcal{G}(k)|^2. \tag{88}$$

We denote the time interval associated with a single line element of the loop by t_f, that is we set $t_0=-t_f$ and $t_f-t_0=2t_f$ [see Fig. 2(a)]. Then we have

$$k(a_2-a_3)=\omega t_f(1-\hat{k}\cdot\vec{v}_4),\quad k(a_2-a_1)=\omega t_f(1-\hat{k}\cdot\vec{v}_2).$$

In order to estimate further the expression (88) we approximate

$$k(a_2-a_3)\approx k(a_2-a_1)\approx \omega t_f, \tag{89}$$

which leads to

$$|\mathcal{G}(k)|^2=8\left[(1-\cos\omega t_f)-\frac{1}{4}(1-\cos 2\omega t_f)\right].$$

This allows us to estimate the decoherence functional (88) as follows:

$$\begin{aligned}\Gamma[c]\approx&\frac{\alpha}{\pi^2}\int_0^{\Omega_{\max}}\frac{d\omega}{\omega}\coth(\beta\omega/2)\\&\times\left[(1-\cos\omega t_f)-\frac{1}{4}(1-\cos 2\omega t_f)\right]\\&\times\int d\Omega(\hat{k})\omega^2\left[\frac{u_2}{ku_2}-\frac{u_4}{ku_4}\right]^2.\end{aligned} \tag{90}$$

Let us first concentrate on the integral over the photon frequencies ω, that is on the first integral on the right-hand side of Eq. (90). The integrand is proportional to ω^{-1}, which is a typical signature for the spectrum of bremsstrahlung [20]. In addition to vacuum bremsstrahlung there may be thermally induced emission and absorption processes [18], which are embodied in the factor $\coth(\beta\omega/2)$. At zero temperature (vacuum field) this factor may be replaced by 1.

In order to calculate the frequency integral it turns out to be useful to decompose it into a vacuum contribution and a thermal contribution which vanishes for $T=0$. We therefore write

$$F\equiv\int_0^{\Omega_{\max}}\frac{d\omega}{\omega}\coth(\beta\omega/2)(1-\cos\omega t_f)\equiv F_{\rm vac}+F_{\rm th}, \tag{91}$$

where

$$F_{\rm vac}\equiv\int_0^{\Omega_{\max}}\frac{d\omega}{\omega}(1-\cos\omega t_f) \tag{92}$$

is the vacuum contribution, while

$$F_{\rm th}\equiv\int_0^{\Omega_{\max}}\frac{d\omega}{\omega}[\coth(\beta\omega/2)-1](1-\cos\omega t_f) \tag{93}$$

is the thermal contribution. The frequency integral $F_{\rm vac}$ can be evaluated as follows. Substituting $x=\omega t_f$ we get

$$F_{\rm vac}=\int_0^{\Omega_{\max}t_f}\frac{dx}{x}(1-\cos x)=\ln(g\Omega_{\max}t_f)+O\left(\frac{1}{\Omega_{\max}t_f}\right), \tag{94}$$

where $\ln g\approx 0.577$ is Euler's constant [26]. For $\Omega_{\max}t_f\gg 1$ we thus have asymptotically

$$F_{\rm vac}\approx\ln(g\Omega_{\max}t_f). \tag{95}$$

This relation demonstrates that the vacuum integral over the photon frequencies converges at the lower limit $\omega\to 0$ and that it gives rise to an effective infrared cutoff of the order $\Omega_{\min}\sim 1/t_f$, as discussed in the previous subsection [see Eq. (75)]. We also observe that the vacuum frequency integral increases weakly with the logarithm of $\Omega_{\max}t_f$. As indicated in Eq. (95) we keep for simplicity in the following only the leading contribution in our expressions. It should be kept in mind, however, that one can include without difficulties the terms of higher order which vanish in the limit $\Omega_{\max}t_f\to\infty$.

To determine the thermal contribution $F_{\rm th}$ we first write Eq. (93) as follows:

$$F_{\rm th}=\frac{1}{\beta}\int_0^{t_f}dt\int_0^{\beta\Omega_{\max}}dx[\coth(x/2)-1]\sin(tx/\beta), \tag{96}$$

where we have introduced the integration variable $x=\beta\omega=\omega/k_BT$. For temperatures T obeying

$$k_BT\ll\hbar\Omega_{\max} \tag{97}$$

the upper limit of the integral over x can be shifted to infinity. To give an example for this condition we take the ultraviolet cutoff $\Omega_{\max}\sim 10^{19}\ {\rm s}^{-1}$, corresponding to a length scale of the order $100\lambda\!\!\!^{-}_{\rm C}$. The requirement (97) then means that $T\ll 10^8$ K. Under this condition we find

$$\begin{aligned}F_{\rm th}\approx&\frac{1}{\beta}\int_0^{t_f}dt\int_0^{\infty}dx[\coth(x/2)-1]\sin(tx/\beta)\\=&\frac{1}{\beta}\int_0^{t_f}dt\left[\pi\coth\left(\frac{\pi t}{\beta}\right)-\frac{\beta}{t}\right]\\=&\ln\left(\frac{\sinh(t_f/\tau_B)}{t_f/\tau_B}\right).\end{aligned} \tag{98}$$

The quantity

$$\tau_B\equiv\frac{\beta}{\pi}\equiv\frac{\hbar}{\pi k_BT}\approx 2.4\times 10^{-12}\ {\rm s}/T[{\rm K}] \tag{99}$$

represents the correlation time of the thermal radiation field [1].

On using the results (95) and (98) we can now determine the frequency integral in Eq. (90),

$$\begin{aligned}&\int_0^{\Omega_{\max}}\frac{d\omega}{\omega}\coth(\beta\omega/2)\left[(1-\cos\omega t_f)-\frac{1}{4}(1-\cos 2\omega t_f)\right]\\&\approx\frac{3}{4}\ln(g\Omega_{\max}t_f)+\ln\left(\frac{\sinh(t_f/\tau_B)}{t_f/\tau_B}\right)\\&\quad-\frac{1}{4}\ln\left(\frac{\sinh(2t_f/\tau_B)}{2t_f/\tau_B}\right).\end{aligned} \tag{100}$$

It remains to calculate the angular integral in Eq. (90), that is, we have to evaluate integrals of the form

$$I(u_n,u_m)\equiv\int d\Omega(\hat{k})\omega^2\frac{u_nu_m}{(ku_n)(ku_m)}, \tag{101}$$

where $n,m=2,4$. To determine these integral we first note that the integrand does not depend on ω as is easily recognized with the help of the relations

$$u_n u_m = \gamma_n \gamma_m (1-\vec{v}_n \cdot \vec{v}_m),$$

$$k u_n = \omega \gamma_n (1-\hat{k}\cdot \vec{v}_n),$$

where

$$\gamma_n \equiv \sqrt{\frac{1}{1-|\vec{v}_n|^2}}.$$

Therefore we obtain

$$I(u_n,u_m)=\int d\Omega(\hat{k})\frac{1-\vec{v}_n\cdot\vec{v}_m}{(1-\hat{k}\cdot\vec{v}_n)(1-\hat{k}\cdot\vec{v}_m)}. \quad (102)$$

Next we observe that the combination $d\Omega(\hat{k})\omega^2$ is an invariant quantity, such that $I(u_n,u_m)$ is a Lorentz invariant integral. To determine this integral we may therefore transform to a coordinate system in which the second velocity is equal to zero, that is $\vec{v}_m=0$. In this system the magnitude $v_n=|\vec{v}_n|$ of the first velocity is equal to the relative velocity

$$v_{nm}\equiv\sqrt{1-\frac{1}{(u_n u_m)^2}}, \quad (103)$$

which is, by definition, a relativistic invariant quantity. Thus we now have by virtue of Eq. (102)

$$I(u_n,u_m)=\int d\Omega(\hat{k})\frac{1}{1-\hat{k}\cdot\vec{v}_n}=2\pi\int_{-1}^{+1}\frac{dz}{1-v_{nm}z}$$

$$=\frac{4\pi}{v_{nm}}\tanh^{-1}v_{nm}. \quad (104)$$

This formula is correct also for the case $u_n=u_m$, giving $I(u_n,u_n)=4\pi$, as may be seen directly from the expansion of $\tanh^{-1}(x)$ for small arguments x,

$$\tanh^{-1}x=x+\frac{1}{3}x^3+\frac{1}{5}x^5+\cdots. \quad (105)$$

Thus we find

$$\int d\Omega(\hat{k})\omega^2\left[\frac{u_2}{ku_2}-\frac{u_4}{ku_4}\right]^2$$

$$=I(u_2,u_2)+I(u_4,u_4)-2I(u_2,u_4)$$

$$=-8\pi\left(\frac{1}{v_{24}}\tanh^{-1}v_{24}-1\right). \quad (106)$$

Substituting Eqs. (106) and (100) into Eq. (90) we finally obtain

$$\Gamma[c]=\Gamma_{\rm vac}+\Gamma_{\rm th}, \quad (107)$$

where

$$\Gamma_{\rm vac}\approx-\frac{6\alpha}{\pi}\ln(g\Omega_{\max}t_f)\left(\frac{1}{v_{24}}\tanh^{-1}v_{24}-1\right) \quad (108)$$

is the vacuum decoherence functional and

$$\Gamma_{\rm th}\approx-\frac{8\alpha}{\pi}\left[\ln\left(\frac{\sinh(t_f/\tau_B)}{t_f/\tau_B}\right)-\frac{1}{4}\ln\left(\frac{\sinh(2t_f/\tau_B)}{2t_f/\tau_B}\right)\right]$$

$$\times\left(\frac{1}{v_{24}}\tanh^{-1}v_{24}-1\right) \quad (109)$$

is the thermal contribution to the decoherence functional. As expected, we see from these expressions that $\Gamma[c]$ strongly depends on the relative velocity v_{24} which is due to the fact that the decoherence is caused by the emission of bremsstrahlung. The larger v_{24} the larger is the involved acceleration of the charged particle which creates the radiation field.

An important result is that bremsstrahlung leads to a partial destruction of coherence even at zero temperature. The magnitude of the vacuum contribution $\Gamma_{\rm vac}$ is seen to increase as the logarithm of the time t_f if the relative velocity is held fixed. This weak dependence is connected to the effective infrared resolution $\Omega_{\min}\sim 1/t_f$ of the interference device: For increasing t_f photons of lower and lower frequencies could in principle be detected and thus more and more information is lost on tracing over the photon field. On the other hand, the term within the square brackets in Eq. (109) approaches $t_f/2\tau_B$ for $t_f\gg\tau_B$. Thus keeping fixed the relative velocity the magnitude of the thermal contribution $\Gamma_{\rm th}$ increases linearly with t_f for times $t_f\gg\tau_B$. This describes the decohering influence of absorption and emission processes induced by the thermal field. It follows that for short times the vacuum contribution dominates, while decoherence is mainly due to thermally induced processes for large times. The time t_f^* corresponding to the crossover between these two regimes is determined by the relation

$$\ln(g\Omega_{\max}t_f^*)=\frac{2}{3}\frac{t_f^*}{\tau_B}. \quad (110)$$

Taking $\Omega_{\max}\sim 10^{19}$ s^{-1} and $T=1$ K we find from this condition that the crossover time is of the order

$$t_f^*\approx 30\tau_B\approx 10^{-10}\ \text{s}. \quad (111)$$

This means that for the given example the vacuum decoherence dominates for times small compared to 10^{-10} s.

To facilitate the further discussion, let us investigate the case of opposite velocities with equal magnitude, that is $\vec{v}_1=\vec{v}_4=-\vec{v}_2=-\vec{v}_3$ [see Fig. 2(a)]. The relative velocity is then found to be

$$v_{24}=\frac{2v}{1+v^2}, \quad (112)$$

where

$$v=\frac{|\vec{a}_1-\vec{a}_3|}{2t_f}. \tag{113}$$

This situation corresponds to the case of two wave packets in a superposition which first move apart with opposite velocities $\vec{v}_1$ and $\vec{v}_3=-\vec{v}_1$, respectively, and, having reached their maximal distance $|\vec{a}_1-\vec{a}_3|$, approach each other again with velocities $\vec{v}_2$ and $\vec{v}_4=-\vec{v}_2$. For nonrelativistic velocities we have $v_{24}\approx 2v$ and we may use the expansion (105) to obtain

$$\Gamma_{\rm th}\approx -\frac{16\alpha}{3\pi}\frac{t_f}{\tau_B}v^2, \quad t_f\gg\tau_B. \tag{114}$$

One can then ask the following question: Given a fixed electron energy, that is a fixed velocity v, how far can we coherently separate the components of the electronic state without exceeding a given threshold $|\Gamma_0|$ for the decoherence? Provided the thermal contribution dominates, Eq. (114) leads to the condition

$$\frac{16\alpha}{3\pi}\frac{t_f}{\tau_B}v^2=|\Gamma_0|, \tag{115}$$

from which we obtain the maximal possible separation

$$d_{\rm max}=2vt_f=\frac{3\pi|\Gamma_0|}{8\alpha}\frac{c\tau_B}{v/c}. \tag{116}$$

Choosing $|\Gamma_0|=0.01$, which corresponds to a threshold of 1% decoherence, we find that the maximal distance at $T=300$ K is given by

$$d_{\rm max}\approx\frac{4}{v/c}\ \mu{\rm m}. \tag{117}$$

This shows that one can achieve rather large coherent separations for nonrelativistic electrons. For example, in the experiment performed by Hasselbach *et al.* [27] an electronic beam was coherently separated by a lateral distance of about $d=100\ \mu$m. To compare this experiment with our results we take an electron energy of 1 keV and use a fixed value of 10 cm for the distance corresponding to the time interval from t_0 to t_f. For $|\Gamma_0|=0.01$ condition (115) then yields $d_{\rm max}\approx 4.5$ cm at $T=1$ K and $d_{\rm max}\approx 0.26$ cm at $T=300$ K. Note that the quantity v in Eq. (115) represents the magnitude of the lateral component of the electron velocity in the experiment, which is due to the fact that it is the relative velocity that enters the formula for the decoherence functional. The values obtained for $d_{\rm max}$ are large compared with the lateral distance d reported in Ref. [27], demonstrating that our theory is in full agreement with experiment.

The result expressed through Eqs. (108) and (109) can also be discussed from another point of view. Namely, instead of keeping fixed the velocity v [Eq. (113)], we consider a fixed maximal spatial distance $|\vec{a}_1-\vec{a}_3|$ between the paths. Thus for increasing t_f the velocity v becomes smaller and smaller and, consequently, the decoherence effect through bremsstrahlung becomes smaller and smaller. For large enough times v is nonrelativistic such that the vacuum and the thermal contribution to the decoherence functional are given by

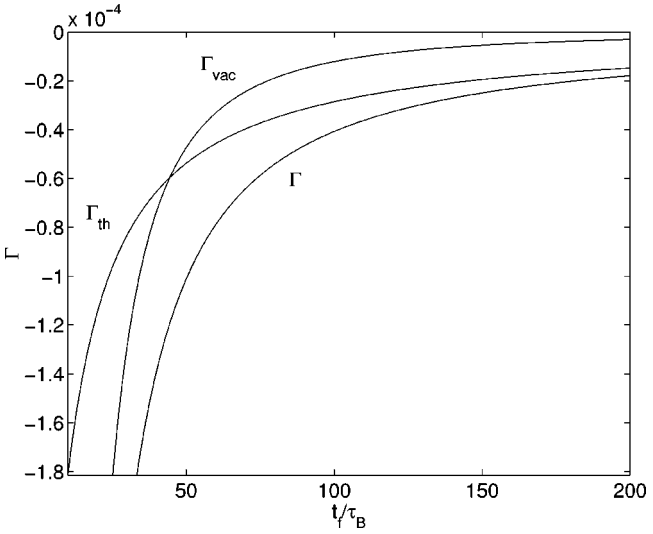

FIG. 3. The vacuum contribution $\Gamma_{\rm vac}$ and the thermal contribution $\Gamma_{\rm th}$ of the decoherence functional Γ. For a fixed maximal distance $|\vec{a}_1-\vec{a}_3|=c\tau_B$ between the paths, the two contributions are plotted against the time t_f which is measured in units of the thermal correlation time τ_B. The temperature was chosen to be $T=1$ K and $\Omega_{\rm max}=10^{19}\ {\rm s}^{-1}$. One observes the decrease of both contributions for increasing time, demonstrating the vanishing of decoherence effects for long times. The thermal contribution $\Gamma_{\rm th}$ vanishes as t_f^{-1}, while the vacuum contribution $\Gamma_{\rm vac}$ decays essentially as t_f^{-2}, leading to a crossover between two regimes dominated by the vacuum and by the thermal contribution, respectively.

$$\Gamma_{\rm vac}\approx -\frac{2\alpha}{\pi}\ln(g\Omega_{\rm max}t_f)\frac{|\vec{a}_1-\vec{a}_3|^2}{t_f^2}, \tag{118}$$

and

$$\Gamma_{\rm th}\approx -\frac{8\alpha}{3\pi}\left[\ln\left(\frac{\sinh(t_f/\tau_B)}{t_f/\tau_B}\right)-\frac{1}{4}\ln\left(\frac{\sinh(2t_f/\tau_B)}{2t_f/\tau_B}\right)\right]\frac{|\vec{a}_1-\vec{a}_3|^2}{t_f^2}. \tag{119}$$

According to Eq. (118) the magnitude of $\Gamma_{\rm vac}$ decreases essentially as t_f^{-2}, while Eq. (119) shows that the magnitude of the thermal contribution $\Gamma_{\rm th}$ decreases as t_f^{-1} for $t_f\gg\tau_B$,

$$\Gamma_{\rm th}\approx -\frac{4\alpha}{3\pi}\frac{|\vec{a}_1-\vec{a}_3|^2}{t_f\tau_B}. \tag{120}$$

We again observe the crossover between two regimes of times: For short times the vacuum decoherence dominates, whereas the thermally induced decoherence dominates for large times. This can be seen in Fig. 3 where we have plotted the expressions (108) and (109) as a function of t_f for a fixed value of $|\vec{a}_1-\vec{a}_3|$.

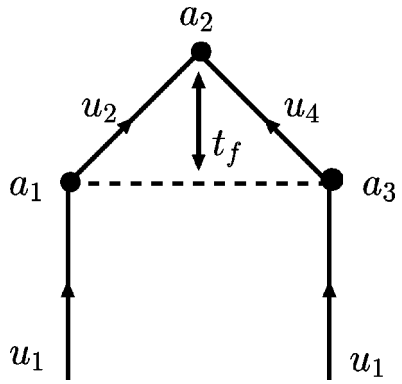

FIG. 4. A current loop involving two line segments which meet in the infinite past [compare with Fig. 2(a)]. This situation corresponds to an interference device in which the relative velocity v_{13} between the interfering wave packets vanishes initially.

The expressions (118) and (120) suggest to define a vacuum and a thermal coherence length by means of

$$\Gamma_{\rm vac} \equiv -\frac{|\vec{a}_1-\vec{a}_3|^2}{2L(t_f)^2_{\rm vac}}, \quad \Gamma_{\rm th} \equiv -\frac{|\vec{a}_1-\vec{a}_3|^2}{2L(t_f)^2_{\rm th}}, \tag{121}$$

in analogy to Eq. (6). This leads to (reintroducing factors of c)

$$L(t_f)_{\rm vac} = \sqrt{\frac{\pi}{4\alpha \ln(g\Omega_{\rm max} t_f)}}\, c t_f \approx \frac{10.4}{\sqrt{\ln(g\Omega_{\rm max} t_f)}}\, c t_f, \tag{122}$$

and

$$L(t_f)_{\rm th} = \sqrt{\frac{3\pi}{8\alpha}}\sqrt{c^2\tau_B t_f} \approx 12.7\sqrt{c^2\tau_B t_f} \propto t_f^{1/2} T^{-1/2}. \tag{123}$$

Equation (122) implies that the vacuum coherence length is roughly of the order

$$L(t_f)_{\rm vac} \sim c t_f. \tag{124}$$

This simple result means that for a given time t_f the radiation field does not destroy quantum coherence on length scales which are small compared to the distance that light travels during this time. This also explains why the radiation field is quite ineffective in destroying quantum coherence of single, localized electrons.

It is interesting to compare the thermal coherence length (123) with the corresponding result (7) obtained from the quantum Brownian motion master equation. We observe that both expressions are proportional to $T^{-1/2}$. The time dependence, however, is completely different: While $L(t_f)_{\rm BM}$ decreases as $t_f^{-1/2}$, the coherence length $L(t_f)_{\rm th}$ increases as $t_f^{1/2}$. This means that for $t_f \to \infty$ one has total destruction of coherence in the Brownian motion case, whereas coherence is completely maintained in the QED case.

We close this section by considering briefly another interference device which is depicted in Fig. 4: Here we suppose that the wave packets brought to interference are at rest initially. As indicated in the figure this case corresponds to a situation in which the line segments described by the four velocities u_1 and u_3 meet each other in the infinite past. Thus we set $u_1 = u_3$ in Eq. (86) to obtain

$$c(k) = \frac{ie}{\sqrt{2}}\left\{\frac{u_1}{ku_1}[e^{-ika_1} - e^{-ika_3}] + \frac{u_2}{ku_2}[e^{-ika_2} - e^{-ika_1}] - \frac{u_4}{ku_4}[e^{-ika_2} - e^{-ika_3}]\right\}. \tag{125}$$

Performing here the approximation (89) we are again led to expression (88), where now

$$|\mathcal{G}(k)|^2 = 2(1-\cos\omega t_f),$$

which immediately yields the expression

$$\Gamma[c] = -\frac{2\alpha}{\pi}\left[\ln\Omega_{\rm max} t_f + \ln\left(\frac{\sinh(t_f/\tau_B)}{t_f/\tau_B}\right)\right] \times \left(\frac{1}{v_{24}}\tanh^{-1} v_{24} - 1\right). \tag{126}$$

In the nonrelativistic limit we use the expansion (105) to obtain

$$\Gamma[c] \approx -\frac{8\alpha}{3\pi}\left[\ln(g\Omega_{\rm max} t_f) + \ln\left(\frac{\sinh(t_f/\tau_B)}{t_f/\tau_B}\right)\right] v^2, \tag{127}$$

where we have again assumed $\vec{v}_2 = -\vec{v}_4$. This result is identical to the one found in Ref. [13] using the dipole approximation for the matter-field coupling.

IV. DECOHERENCE OF MANY-PARTICLE STATES

For single electrons the vacuum decoherence through bremsstrahlung turns out to be small at nonrelativistic speeds. For example, taking $\Omega_{\rm max} \sim c/\bar{\lambda}_{\rm C}$ and t_f of the order of 1s, and using a velocity v which is already as large as 1/10 of the speed of light, one finds that $|\Gamma_{\rm vac}| \sim 10^{-2}$, which corresponds to a 1% suppression of interference. By virtue of the weak logarithmic dependence on the cutoff scale this estimate is true also for other particles carrying an elementary charge.

Matters change, however, drastically if one considers the superposition of two states of a system which is composed of N identical particles with mass m and charge e. Let us construct an effective master equation for the density $\rho_{\rm cm}(\vec{R},\vec{R}')$ of the center of mass coordinate

$$\vec{R} = \frac{1}{N}\sum_{i=1}^{N} \vec{x}_i \tag{128}$$

for such a system. Here, the $\vec{x}_i$ are the particle coordinates and we suppress, for simplicity, the spin degree of freedom. We introduce the relative coordinates $\vec{r}_i$ through

$$\vec{x}_i = \vec{R} + \vec{r}_i(q). \tag{129}$$

Since the $\vec{r}_i$ sum up to zero, they are functions of $3N-3$ internal variables which we denote collectively by q. Let us suppose that the state of the N-particle system is described in the position representation by a density matrix of the form

$$\rho_m = \rho_{\rm cm}(\vec{R},\vec{R}')\rho_{\rm int}(q,q'), \tag{130}$$

where $\rho_{\rm cm}$ and $\rho_{\rm int}$ are separately normalized to 1,

$$\int d^3R \rho_{\rm cm}(\vec{R},\vec{R}) = \int dq \rho_{\rm int}(q,q) = 1. \tag{131}$$

The density matrix $\rho_{\rm cm}$ describes the center of mass coordinate, while $\rho_{\rm int}$ represents the state of the internal degrees of freedom. For example, one finds that the quantity

$$w(\vec{x}-\vec{R}) = \frac{1}{N}\int dq \sum_{i=1}^{N} \delta(\vec{x}-\vec{R}-\vec{r}_i(q))\rho_{\rm int}(q,q) \tag{132}$$

is the density of finding a particle at $\vec{x}$ under the condition that the center of mass coordinate is at $\vec{R}$. This function is obviously normalized as

$$\int d^3x w(\vec{x}) = 1. \tag{133}$$

If the system described by the state (130) performs a translational motion it is reasonable to assume that its total current density can be approximated by an effective current density of the form

$$\vec{j}_{\rm cm}(\vec{x}) = \frac{Ne}{2M}\{\vec{P}w(\vec{x}-\vec{R}) + w(\vec{x}-\vec{R})\vec{P}\}, \tag{134}$$

where $\vec{P} = -i\partial/\partial\vec{R}$ is the total momentum canonically conjugated to the center of mass coordinate $\vec{R}$, Ne is the total charge, and $M = Nm$ the total mass. The expression (134) implies that the current density of the internal degrees of freedom vanishes. In particular, it excludes the possibility that the whole systems is in a rotational state which would require to introduce three further collective coordinates as, for example, the three Euler angles.

Equation (134) shows that the case of an N-particle system can be dealt with by using the replacements $e \to Ne$ and $m \to M = Nm$, and by interpreting the length scale σ_0, which appears in the UV cutoff scale $\Omega_{\rm max} \sim 1/\sigma_0$, as the linear extension of the one-particle density $w(\vec{x})$. A representation for the density matrix $\rho_{\rm cm}(\vec{R},\vec{R}')$ is then obtained from Eq. (34) by substituting the effective current (134) into the influence functional (35). The corresponding second order master equation is given by Eq. (36). Invoking the nonrelativistic (dipole) approximation one is led to the following representation for the center of mass density:

$$\rho_{\rm cm}(t_f) = \mathcal{T}_{\leftarrow}\Bigg(\exp\Bigg[\int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \times \Bigg\{ \frac{i}{2} D(t-t') \frac{\vec{P}_c(t)}{M} \frac{\vec{P}_a(t')}{M} - \frac{1}{2} D_1(t-t') \times \frac{\vec{P}_c(t)}{M} \frac{\vec{P}_c(t')}{M} \Bigg\} \Bigg] \Bigg) \rho_{\rm cm}(t_i), \tag{135}$$

where $\vec{P}_c(t)$ and $\vec{P}_a(t)$ denote the interaction picture commutator and anticommutator superoperators for the total momentum. The dissipation and the noise kernel are given by the expressions (39), where the spectral density now takes the form

$$J(\omega) = \frac{N^2 e^2}{3\pi^2} \omega \Theta(\Omega_{\rm max} - \omega). \tag{136}$$

The time-ordered exponential in Eq. (135) can be determined with the help of path-integral techniques. The details are given in Ref. [13] and we can immediately transfer the results given there to the present case with the help of the replacements given above. It follows that the vacuum decoherence functional for an N-particle state scales with the square N^2 of the particle number,

$$\Gamma_{\rm vac} \sim -N^2 \frac{8\alpha}{\pi} \ln(g\Omega_{\rm max} t_f) v^2. \tag{137}$$

This scaling with the particle number obviously leads to a large amplification of the decoherence effect. To give a very extreme example we take $N = 10^{22}$ which corresponds to $\sigma_0 \sim 1$ cm for typical free electron densities in metals. Let us ask for the maximal speed v leading to 1% decoherence. With the help of Eq. (137) we find $v \sim 3 \times 10^{-15}$ m s^{-1}. For a distance of 3 m this implies, for example, that t_f would be of the order of 10^{15} s, which is an extremely large time already comparable with the age of the universe!

V. CONCLUSIONS

The emission of bremsstrahlung leads to a loss of coherence whenever two spatially separated components of a superposition are set into relative motion in order to observe locally their capability to interfere. We have investigated a prototypical interference experiment which involves two possible paths of an interfering charged particle. The coherence loss in this type of experiment can be quantified with the help of a certain decoherence functional $\Gamma[c]$, which is a gauge invariant relativistic functional of the difference between the current densities corresponding to the two possible paths. As expected from the physical picture of decoherence through bremsstrahlung, the decoherence functional mainly depends on the invariant relative velocity between the interfering paths. It has been shown that decoherence induced by

the emission of radiation into the field vacuum dominates at short time scales, while thermally induced processes dominate at large times. We have estimated the crossover time which separates the vacuum from the thermal regime as well as the vacuum and the thermal coherence length.

Decoherence through bremsstrahlung exhibits a highly non-Markovian character since the decoherence functional depends on the whole paths of the interference device. This can be illustrated by a comparison of the results obtained for the two interference devices studied in Sec. III [compare Eq. (126) with Eqs. (108) and (109)]: After the time corresponding to the maximal distance between the wave packets we have in both cases two wave packets approaching each other with opposite velocities of the same magnitude v. The difference between the decoherence functionals obtained in the two cases shows that the suppression of quantum coherence through bremsstrahlung depends on the total history of the process and that the memory time is of the order of the total time t_f of the experiment.

An important conclusion is that the usual picture of decoherence as a decay of the off diagonals in the reduced density matrix does not apply to the destruction of coherence through bremsstrahlung. Namely, consider a superposition of two wave packets with zero mean velocity. The expression (118) for the decoherence function together with the estimate $L(t_f)_{\mathrm{vac}} \sim c t_f$ for the vacuum coherence length show that decoherence effects are negligible for times t_f which are large in comparison to the time it takes light to travel the distance between the wave packets. The off-diagonal terms of the reduced density matrix for the electron do therefore not decay for $t_f \to \infty$ which shows the profound difference between the decoherence mechanism through bremsstrahlung and other decoherence mechanisms. For the same reason a single localized wave packet moves essentially unaffected by the radiation field. If we separate, however, the wave packet into two components and recombine them a loss of coherence is observed.

One might ask the question of whether virtual vacuum processes or real photon emissions are responsible for the decoherence effect studied in this paper. The answer is contained in expression (71) for the influence phase factor $\exp(i\Phi)$. This equation tells us that the decoherence factor $\exp(\Gamma) = |\exp(i\Phi)|$ depends on virtual processes through the Feynman propagator $D_F(x-x')_{\mu\nu}$ (contained in the vacuum-to-vacuum amplitudes) as well as on real processes described by the Green functions $D_{\pm}(x-x')_{\mu\nu}$. Thus it is the combined effect of virtual and real processes which leads to decoherence. The physical reason is that the mere possibility of real photon emissions also reduces the vacuum-to-vacuum amplitudes which can already lead to a reduction of the interference contrast. For example, one can think of an interference device in which the path y_1 describes a uniform motion of the electron, whereas the other path y_2 is strongly bend, involving a large acceleration of the electron. Suppose we observe the photons emitted in the experiment. If we know that no photon has been emitted it is then very likely that the electron has taken the path y_1, which results in a suppression of the interference pattern formed by the electrons of the corresponding subensemble. This situation is similar to that of a two-level atom which is initially in a superposition of the excited state and the ground state. If we find that the atom did not emit a photon after a long time (long compared to the inverse of the emission rate) we have effectively measured the state of the atom to be the ground state. Thus the off-diagonal terms of the atomic density matrix approximately vanish without the emission of real photons.

We have also discussed the amplification of decoherence effects for systems of identical charged particles. The obtained scaling of $\Gamma[c]$ with the square N^2 of the particle number can be traced back to two facts. First, the radiative back action is proportional to the square of the total charge since the emitted radiation adds coherently in the limit of long wavelength. Second, the decoherence functional only depends on the logarithm of the cutoff $\Omega_{\max}$, which means that it depends only very weakly on the total mass or on the spatial extent of the N-particle state. In the cases discussed here one must expect, of course, a large radiative damping in addition to the decoherence effect. It is of great fundamental interest to investigate also the influence of these phenomena for composite neutral objects.

The phenomenon of decoherence is often linked to the emergence of dynamically induced superselection sectors of the underlying state space [6]. In this context one tries to show that the matrix elements of all observables between states belonging to different orthogonal subspaces, the superselection sectors, vanish in the limit $t_f \to \infty$. In this paper we have adopted a more practical viewpoint and defined the decoherence functional to describe the loss of coherence in an interference experiment which involves a process on a finite time t_f. It is this time which sets a natural infrared cutoff corresponding to the frequency resolution of the device and which ensures the infrared finiteness of the decoherence functional. Another point which might be important for further investigations is that the definition of the decoherence functional involves only the preparation and measurement of local quantities. Whether the resulting coherence loss is called true decoherence or not, is a matter of definition. It is, however, a fact that the influence of the radiation field leads to a reduction of the interference contrast which can be observed in an experiment that takes a finite time.

[1] C.W. Gardiner and P. Zoller, *Quantum Noise*, 2nd ed. (Springer-Verlag, Berlin, 2000).
[2] A.O. Caldeira and A.J. Leggett, Phys. Rev. A **31**, 1059 (1985).
[3] E. Joos and H.D. Zeh, Z. Phys. B: Condens. Matter **59**, 223 (1985).
[4] W.G. Unruh and W.H. Zurek, Phys. Rev. D **40**, 1071 (1989).
[5] W.H. Zurek, Phys. Today **44** (October), 36 (1991).
[6] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, and

H.D. Zeh, *Decoherence and the Appearence of a Classical World in Quantum Theory* (Springer-Verlag, Berlin, 1996).
[7] M. Brune, E. Hagley, J. Dreyer, X. Maître, A. Maali, C. Wunderlich, J.M. Raimond, and S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996).
[8] C.J. Myatt, B.E. King, Q.A. Turchette, C.A. Sackett, D. Kielpinski, W.M. Itano, C. Monroe, and D.J. Wineland, Nature (London) **403**, 269 (2000).
[9] A.O. Caldeira and A.J. Leggett, Physica A **121**, 587 (1983).
[10] L.H. Ford, Phys. Rev. D **47**, 5571 (1993).
[11] P.M.V.B. Barone and A.O. Caldeira, Phys. Rev. A **43**, 57 (1991).
[12] D. Dürr and H. Spohn, in *Decoherence: Theoretical, Experimental, and Conceptual Problems*, edited by Ph. Blanchard, D. Giulini, E. Joos, C. Kiefer, and I.-O. Stamatescu (Springer-Verlag, Berlin, 2000) p. 77–86.
[13] H.P. Breuer and F. Petruccione, in *Relativistic Quantum Measurement and Decoherence*, edited by H.P. Breuer and F. Petruccione (Springer-Verlag, Berlin, 2000).
[14] J.R. Anglin, J.P. Paz, and W.H. Zurek, Phys. Rev. A **55**, 4041 (1997).
[15] S. Weinberg, *The Quantum Theory of Fields*, Foundations Vol. I (Cambridge University Press, Cambridge, 1996).
[16] J.M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Springer-Verlag, New York, 1980).
[17] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (John Wiley, New York, 1998).
[18] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
[19] H. Grabert, P. Schramm, and G.-L. Ingold, Phys. Rep. **168**, 115 (1988).
[20] J.D. Jackson, *Classical Electrodynamics*, 3rd ed. (John Wiley, New York, 1999).
[21] R.P. Feynman and F.L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963).
[22] K.-c. Chou, Z.-b. Su, B.-l. Hao, and L. Yu, Phys. Rep. **118**, 1 (1985).
[23] L. Diósi, Found. Phys. **20**, 63 (1990).
[24] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
[25] R.P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).
[26] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, New York, 1980).
[27] F. Hasselbach, H. Kiesel, and P. Sonnentag, in *Decoherence: Theoretical, Experimental, and Conceptual Problems*, edited by Ph. Blanchard, D. Giulini, E. Joos, C. Kiefer, and I.-O. Stamatescu (Springer-Verlag, Berlin, 2000), pp. 201–212.