PHYSICAL REVIEW A, VOLUME 63, 031802(R)

Synchronization of optical-feedback-induced chaos in semiconductor lasers by optical injection

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(Received 8 September 2000; published 7 February 2001)

We study the synchronization of feedback-induced chaos in semiconductor lasers by the optical injection scheme that consists of a transmitter laser with external optical feedback and a receiver laser with optical injection from the transmitter laser. The stable synchronization condition and the effects of the frequency detuning between the transmitter and the receiver lasers are investigated using numerical simulation. The results show that positive and negative frequency detuning exert different influences on the synchronization. A time lag between the waveforms of the transmitter and the receiver lasers in the case of a large frequency detuning is found and its experimental significance is commented on.

DOI: 10.1103/PhysRevA.63.031802 PACS number(s): 42.55.Px, 42.65.Sf, 05.45.Xt

Laser chaos has been of considerable interest in recent research on chaos synchronization and its applications to optical communications owing to its high dimension and large bandwidth [1]. In particular, semiconductor lasers with optical feedback or optical injection provide simple configurations for the generation of subnanosecond chaotic oscillations. Several algorithms have been demonstrated to achieve the synchronization of optical-feedback-induced chaos between two semiconductor lasers [2]. Recently, a direct injection scheme, i.e., an external cavity semiconductor laser on the transmitter side and a semiconductor laser with optical injection on the receiver side, was proposed to synchronize the feedback-induced optical chaos [3]. The direct injection scheme especially favors experimental implementation due to its simple configuration. Several experiments using this scheme have been conducted to show the synchronization between two lasers with oscillation frequencies ranging from a few kilohertz to a few gigahertz [4].

For the direct injection scheme, the existence of the synchronization state requires that the injection strength be identical to the feedback strength and that there be no detuning between the free-running frequencies of the transmitter and the receiver lasers. Although Ahlers *et al.* [5] have demonstrated that the direct injection scheme has quite a large tolerance to the mismatch in some laser parameters, the effect of frequency detuning and that of injection strength have not been clear. It is of both practical importance and theoretical interest to investigate the influence of frequency detuning and injection strength on the synchronization quality. This paper focuses on the issue of chaos synchronization through optical injection when a frequency detuning exists between the transmitter and the receiver lasers. First we show that perfect synchronization can be achieved with an appropriate injection strength only under the condition of no frequency detuning, i.e., $\omega_0^T = \omega_0^R$, where ω_0^T and ω_0^R are frequencies of free-running transmitter and receiver lasers, respectively. In the case of positive frequency detuning ($\omega_0^T > \omega_0^R$), the receiver laser is frequency-locked to the transmitter laser and good synchronization performance is achieved over a large detuning range. On the other hand, in the case of negative frequency detuning ($\omega_0^T < \omega_0^R$), the receiver fails to lock to the transmitter frequency even when the detuning is only a few hundred megahertz. In this situation, the quality of synchronization is poor. A time lag between the output waveforms of the receiver and the transmitter lasers is identified when the frequency detuning exceeds a certain positive value. This time-lag phenomenon implies that, when there exists a large frequency detuning, the experimentally observed time series may behave more like a direct response to the injection signal than a synchronized waveform.

We begin with a pair of single-mode semiconductor lasers coupled in a transmitter-receiver scheme as shown in Fig. 1. The output light from the transmitter laser diode (TLD) is divided into two beams, one is coupled back to the transmitter laser itself with the feedback strength $\eta_{\rm ext}$, and the other is injected into the receiver laser diode (RLD) with the injection strength $\eta_{\rm inj}$. The optical length of the external feedback path in TLD is $l_{\rm ext}$. An optical isolator is used to avoid both multiple reflection in the feedback loop of TLD and mutual coupling between TLD and RLD. For a single-mode laser diode with external feedback, we use the well known Lang-Kobayashi equation [6] to describe the complex electric field and the carrier density. Denoting the complex electric field of the transmitter laser diode as $E^T(t) = A^T(t)\exp(-i\omega_0^T t)$, one has the following equation for the

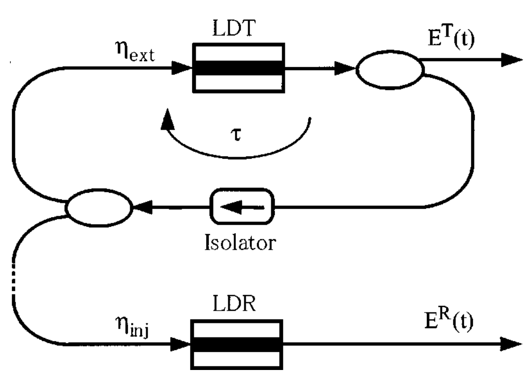

FIG. 1. Schematic of optical chaos synchronization via optical injection.

complex amplitude of the transmitter laser:

$$\frac{dA^T(t)}{dt} = -\frac{\gamma_c}{2}A^T(t) + i(\omega_0^T - \omega_c^T)A^T(t) + \frac{\Gamma}{2}(1 - ib)g^T A^T(t) + \eta_{\rm ext}A^T(t-\tau)\exp(i\omega_0^T\tau) + F_{\rm sp}^T, \quad (1)$$

where A is the total complex intracavity field amplitude at the oscillation frequency ω_0, γ_c is the cavity decay rate, ω_c is the longitudinal mode frequency of the cold laser cavity, b is the linewidth enhancement factor, Γ is the confinement factor, $\tau = l_{\rm ext}/c$ is the delay time of the external feedback, and g is the gain coefficient. $F_{\rm sp}$ is the complex Langevin noise term that is characterized by the spontaneous emission factor $R_{\rm sp}$ [7]. It is noted that the above equation also applies to a single-mode laser with weak to moderate optical feedback from an external mirror as shown in many previous works [2–4,6,8].

The receiver laser with the optical injection from the output of the transmitter laser can be described with following equation using $E^R(t) = A^R(t)\exp(-i\omega_0^R t)$ [9]:

$$\frac{dA^R}{dt} = -\frac{\gamma_c}{2}A^R(t) + i(\omega_0^R - \omega_c^R)A^R(t) + \frac{\Gamma}{2}(1-ib)g^R A^R(t) + \eta_{\rm inj}A^T(t-\tau_R)\exp(i\omega_0^T\tau_R - i\Omega t) + F_{\rm sp}^R, \quad (2)$$

where $\Omega = \omega_0^T - \omega_0^R$ is the detuning frequency between the transmitter and the receiver lasers and τ_R is the propagation time between TLD and RLD. In this paper we assume $\tau_R = \tau$ without loss of generality. The carrier density within the cavity is described by the following equation:

$$\frac{dN^{T,R}(t)}{dt} = \frac{J^{T,R}}{ed} - \gamma_S N^{T,R}(t) - \frac{2\varepsilon_0 n^2}{\hbar(\omega_0^{T,R})^2}g^{T,R}|A^{T,R}(t)|^2, \quad (3)$$

where the superscripts T and R denote transmitter and receiver, respectively, J is the injection current density, e is the electric charge, d is the active layer thickness, and γ_s is the carrier decay rate. The injection current density J can be measured with a normalized dimensionless parameter $\tilde{J} = (J/ed - \gamma_s N_0)/\gamma_s N_0$. Also, $\eta_{\rm ext}$ and $\eta_{\rm inj}$ are normalized with γ_c as $\eta_{\rm ext} = \gamma_c \xi_{\rm ext}$ and $\eta_{\rm inj} = \gamma_c \xi_{\rm inj}$, respectively. We assume identical values of b, Γ, γ_c, and γ_s for both TLD and RLD.

From the above equations, we can easily find that the perfect synchronization solution $A^R = A^T$ and $N^R = N^T$ exists when $\tilde{J}^R = \tilde{J}^T$, $\xi_{\rm inj} = \xi_{\rm ext}$, and $\Omega = 0$. One may think that perfect synchronization also exists even for a nonzero frequency detuning because the injection signal can lock RLD to the same frequency of TLD. Although injection locking does occur within a certain range of frequency detuning and injection strength [8], it also causes a phase shift in RLD and thus will destroy perfect synchronization.

The stability of perfect synchronization depends on the injection parameter. From Eqs. (1)–(3), we calculate the transverse Lyapunov exponents [10] for zero frequency detuning.

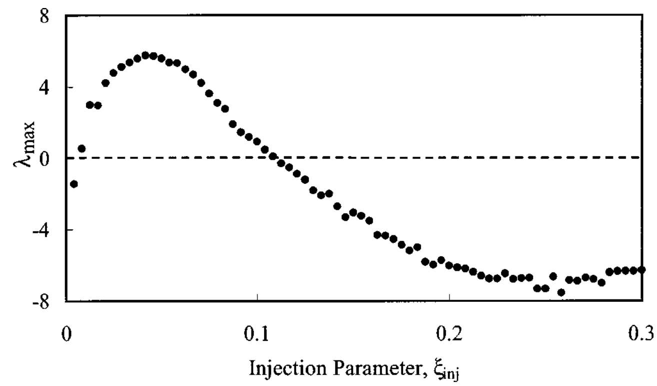

FIG. 2. Largest transverse Lyapunov exponent as a function of the injection parameter at $\tilde{J}^R = \tilde{J}^T = 0.67$, $l_{\rm ext} = 15$ cm ($\tau = 0.5$ ns), $\xi_{\rm inj} = \xi_{\rm ext}$, and $\Omega = 0$.

Figure 2 shows the largest transverse Lyapunov exponent $\lambda_{\rm max}$ as a function of the injection parameter $\xi_{\rm inj} = \xi_{\rm ext}$ varying from 0 to 0.3 for $\tilde{J}^R = \tilde{J}^T = 0.67$, $\tau = 0.5$ ns, and $\Omega = 0$. At this injection current density, the laser parameters are $b = 4$, $\Gamma = 0.4$, $\gamma_c = 2.4\times10^{11}$ s^{-1}, $\gamma_s = 1.458\times10^9$ s^{-1}, and $R_{\rm sp} = 4.7\times10^{18}$ V^2 m^{-2} s^{-1}, which are measured from a single-mode Fabry-Perot semiconductor laser [9]. We also simulated synchronization behavior from random initial conditions and found that $\lambda_{\rm max}$ gives a good test of synchronization. In general, we find that the synchronization of optical-feedback-induced chaos can only be achieved when the injection strength exceeds a certain level. For very small injection strengths ($\xi_{\rm inj} = \xi_{\rm ext} < 5\times10^{-3}$), we also find that the largest transverse Lyapunov exponent is negative, implying successful synchronization. Actually, with such a small feedback coefficient, the state of TLD is usually a stable fixed point and can be easily synchronized.

Next, we investigate the synchronization characteristics in the presence of a frequency detuning ($\Omega \neq 0$) by numerically calculating Eqs. (1)–(3). Figure 3 shows the output characteristics of TLD and RLD at $\Omega/2\pi = 1$ GHz, and $\xi_{\rm inj} = \xi_{\rm ext} = 0.25$. Other parameters are the same as those used for obtaining Fig. 2. In each plot of Fig. 3, the characteristics of the transmitter laser are presented in the upper trace, whereas those of the receiver laser are presented in the lower trace for direct comparison. Under this condition, TLD is in a completely chaotic state, as can be seen from the time series in Fig. 3(a) and the broadened peaks in the RF power spectrum in Fig. 3(c). The optical spectrum in Fig. 3(d) shows a large shift to lower frequencies due to the strong external feedback. The strong feedback also induced many peaks in the optical spectrum with the peak spacing corresponding to $1/\tau$. We summarize the phenomena in the presence of a positive frequency detuning (up to 10 GHz) as follows. (1) The optical frequency of RLD is locked to that of TLD. (2) The optical and RF spectra of RLD well coincide with those of TLD, indicating that a good correlation can be obtained between the output intensity waveforms of the two lasers. (3) Careful comparison of the time series of RLD and TLD shows that the high-frequency portions of two time series are well synchronized with each other, whereas the low-frequency portions with large-intensity fluctuations exhibit a

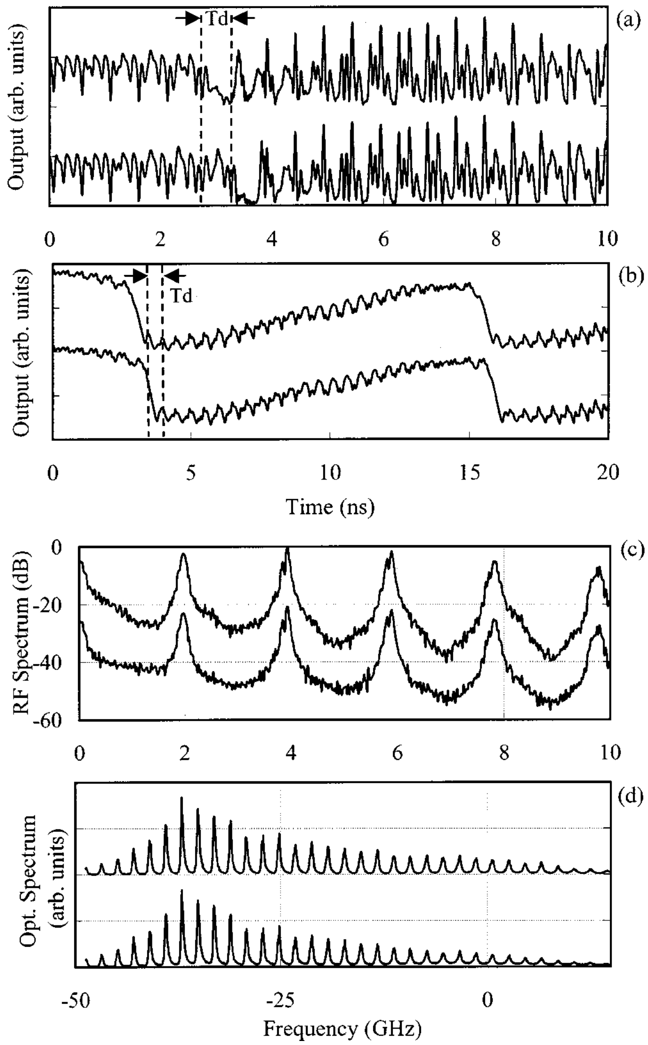

FIG. 3. Laser outputs at $\tilde{J}^R = \tilde{J}^T = 0.67$, $\tau = 0.5$ ns, $\xi_{\rm inj} = \xi_{\rm ext} = 0.25$, and $\Omega/2\pi = 1$ GHz. (a) Time series, (b) averaged time series of (a), (c) RF power spectrum, and (d) optical spectrum. Note that time scales in (a) and (b) are different.

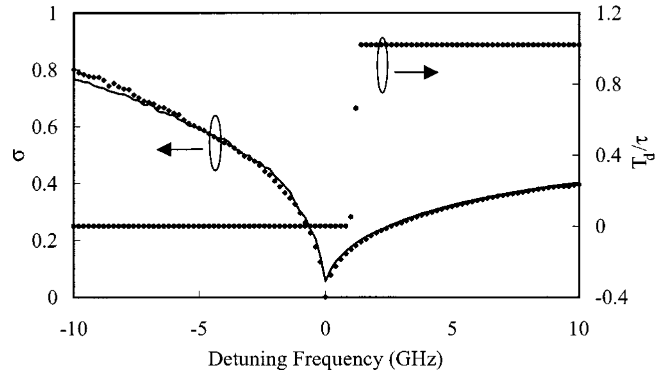

FIG. 4. Synchronization error (σ) and time lag (T_d) vs detuning frequency. The solid line corresponds to the case with Langevin noise. Parameters are the same as in Fig. 3.

discrepancy, a phenomenon also observed when there is a mismatch in other parameters as described in Ref. [5]. We further find that this discrepancy is characterized by a fine shift in time. (4) The amplitude and the rate of occurrence of this discrepancy increase with the increase in the frequency detuning. To further verify the above-mentioned temporal shift in the waveform caused by the frequency detuning, we average the time series in Fig. 3(a) with a 1-ns window. The result is shown in Fig. 3(b). This is the time series that would be observed in an experimental situation using a photodiode with a detection bandwidth of 1 GHz. In this case a time lag T_d between the receiver and the transmitter output intensity waveforms is clearly identified. The value of T_d is found to be equal to the time delay τ. Actually, the cross-correlation function $C_{T,R}(\Delta t) = \langle S^T(t) S^R(t+\Delta t)\rangle$ indicates that the maximum correlation occurs at the time delay $\Delta t = T_d = \tau$ when the frequency detuning exceeds a certain value.

The synchronization characteristics in the presence of negative frequency detuning are quite different from those in the presence of positive frequency detuning. We find that the receiver laser is not frequency-locked to the transmitter laser when the negative detuning frequency exceeds only a few hundred megahertz for the injection level up to $\xi_{\rm inj} = \xi_{\rm ext} = 0.3$. As a consequence, the synchronization performance is generally worse than that obtained in the case of positive frequency detuning. This can be qualitatively explained from the injection-locking properties of the receiver laser. As studied in previous works [9], the stable locking regime ranges roughly from -15 GHz to 0 GHz for $\xi_{\rm inj} = 0.25$. Meanwhile, with the external feedback, the oscillation frequency of the transmitter laser is shifted to the negative-frequency side [8]. With such a negative frequency shift in the transmitter laser caused by the feedback, a positive frequency detuning brings more frequency components into the stable locking region than a negative detuning does. Therefore, a positive frequency detuning results in a better locking performance than a negative frequency detuning. The detailed dynamics of a laser subject to optical injection from a chaotic laser with frequency detuning is still under investigation.

To quantitatively evaluate the synchronization performance, we define a synchronization error as $\sigma = \langle |S^R(t) - S^T(t)|\rangle / \langle S^T(t)\rangle$, where $\langle\,\rangle$ denotes the time average. σ is calculated to be about 16% in Fig. 3 for $\Omega/2\pi = 1$ GHz. Figure 4 shows the synchronization error σ and the time lag T_d as a function of the detuning frequency. The synchronization error has a minimum value at $\Omega = 0$ and increases as Ω deviates from 0. We find that σ in the region of $\Omega > 0$ is generally smaller than that in the region of $\Omega < 0$. The time lag, on the other hand, shows an abrupt jump from 0 to τ around a detuning frequency $\Omega/2\pi = 0.8$ GHz.

It is worthwhile to briefly discuss the significance of the time lag T_d. For perfect synchronization with no frequency detuning, since we assume $\tau_R = \tau$ in the simulation, the output of the RLD, $E^R(t)$, is synchronized and identical to the output of TLD, $E^T(t)$, while receiving the injection signal $E^T(t-\tau)$ from TLD. In the case of synchronization with a large frequency detuning, the existence of the time lag $T_d = \tau$ indicates that the RLD output $E^R(t)$ directly follows the injection signal $E^T(t-\tau)$ itself. Considering a practical experiment where a detector with a bandwidth of less than a few gigahertz is employed, we can predict the following results. For zero- or a small-frequency detuning, the output of

RLD reproduces the dynamics of TLD. In contrast, for a large positive frequency detuning beyond a few gigahertz, the RLD behaves more like a driven oscillator and its output directly responds to the injected signal although we may observe very similar optical and RF spectra between the two lasers in both cases.

The influence due to the mismatch between ξ_{inj} and ξ_{ext} is also investigated. In general, the synchronization error in the case where $\xi_{inj}>\xi_{ext}$ is smaller than that when $\xi_{inj}<\xi_{ext}$ because stronger injection results in better frequency locking according to the results shown in Fig. 2. For a large ξ_{inj}, we find that there also exists a lag time $T_d=\tau$ between the output waveforms of TLD and RLD, implying driven phenomenon rather than synchronization. The influence of the Langevin noise is investigated and the result is shown in Fig. 4 with solid curves. It is found that the laser noise causes a substantial increase in the synchronization error near $\Omega=0$, but causes little change at large detuning frequencies. Furthermore, the laser noise has little effect on the time lag and its dependency on Ω.

We briefly comment on the relationship between our calculations and several recent experiments. In experiments of fast oscillation synchronization by Fischer *et al.* and Fujino *et al.*, the time lag between waveforms of the transmitter and the receiver was calculated to be $T_d=\tau$ [4]. It is noted in both experiments the authors distinguished their observations from the synchronization predicted by the modeling [5]. Our numerical results suggest that the above observations are expected when a strong injection and/or a moderate positive detuning exist and therefore provide a reasonable interpretation for experimental observations.

In conclusion, we have investigated the synchronization of optical-feedback-induced chaos through optical injection by taking into account the frequency detuning, the injection strength, and the laser noise. In particular, we have the following conclusions. (1) Perfect synchronization of chaos can only be achieved at $\Omega=0$ with a strong injection strength. (2) In the presence of frequency detuning, reasonable synchronization performance can still be expected with strong injection. (3) In the case of positive frequency detuning ($\omega_0^T>\omega_0^R$), the optical frequency of the receiver laser is locked to that of the transmitter laser, whereas in the case of negative detuning ($\omega_0^T<\omega_0^R$), the frequency locking easily fails. As a consequence, positive frequency detuning results in better synchronization performance than negative detuning. (4) When measuring the output intensity with a slow detector, a time lag $T_d=\tau$ will be observed in the time series between the transmitter and the receiver lasers when the detuning frequency exceeds a few gigahertz. (5) A similar time lag phenomenon can be observed for $\xi_{inj}>\xi_{ext}$. Our results demonstrate the possibility of synchronizing the feedback-induced optical chaos through the optical injection scheme in a very practical sense. Furthermore, these results also provide experimental researchers with a means to distinguish the synchronization behavior from the driven oscillations in experiments.

We are grateful to Dr. B. Komiyama for encouragement and support. Y. Liu acknowledges the support of the Collaboration Project between ATR-ACR and UCLA. H. F. Chen and J. M. Liu were supported by the U.S. Army Research Office under Grant No. DAAG55-98-1-0269.

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