Damping of condensate collective modes due to equilibration with the noncondensate

J. E. Williams and A. Griffin
Department of Physics, University of Toronto, Toronto, Ontario, Canada M5S 1A7
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We consider the damping of condensate collective modes in the collisionless regime at finite temperatures arising from lack of equilibrium between the condensate and the noncondensate atoms, an effect that is ignored in the usual discussion of the collisionless region. As a first approximation, we ignore the dynamics of the thermal cloud. Our calculations should be applicable to collective modes of the condensate that are oscillating out-of-phase with the thermal cloud. We obtain a generalized Stringari equation of motion for the condensate at finite temperatures, which includes a damping term associated with the fact that the condensate is not in diffusive equilibrium with the static thermal cloud. This intercomponent collisional damping of the condensate modes is comparable in magnitude to the Landau damping considered in the recent literature.

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I. INTRODUCTION

The collective oscillations of a condensate at zero temperature $T=0$ are well described by the solutions of the linearized Gross-Pitaevskii (GP) time-dependent equation of motion for the condensate wave function $\Phi(\mathbf{r},t)$. At finite temperatures, the condensate dynamics is modified by interactions with the noncondensate atoms in the thermal cloud, which has the effect of renormalizing and damping the condensate oscillations. Recently the coupled dynamics of the condensate and the thermal cloud has been the subject of several theoretical studies [1–5]. Such calculations lead to a generalized GP equation for $\Phi(\mathbf{r},t)$ and some appropriate Boltzmann-like kinetic equation describing the dynamics of the noncondensate atoms. In the present paper, we make use of the recent formulation of Zaremba, Nikuni, and Griffin (ZNG) [3] to discuss a new kind of damping of condensate oscillations that arises from collisions between the condensate and noncondensate components. In contrast to Ref. [3], which discussed the collision-dominated hydrodynamic regime, here we discuss the collisionless regime. Within the well-known Thomas-Fermi (TF) approximation, we derive a generalized Stringari wave equation describing the condensate normal modes [6] that is valid at finite T and includes damping due to the fact that the condensate is not in equilibrium with the thermal cloud. This new source of damping is in addition to the usual Landau and Beliaev damping considered in the collisionless region at finite T [7–14].

Our theory can be used to generalize any discussion based on the usual GP equation at $T=0$. This simplicity is due to our neglect of any dynamics of the thermal cloud. Available studies of collective modes [3] at finite T suggest that, for any given mode symmetry, one mode mainly involves motion of the condensate (with a small out-of-phase motion of the noncondensate). This mode is a natural extension of the $T=0$ oscillation of a pure condensate and should be described by our theory. The other mode, of the same symmetry, mainly involves the motion of the thermal cloud (with a small in-phase motion of the condensate) and can be viewed as the natural extension of the oscillations above the critical temperature T_{BEC} [15,16]. Our present calculations do not apply to such ''normal-fluid'' oscillations, which include the Kohn mode at the trap frequency.

II. DERIVATION OF MODEL

Our starting point is the finite T generalized GP equation derived by ZNG (see also Refs. [1] and [2])

$$i\hbar\frac{\partial\Phi}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+U_{\mathrm{ext}}+gn_c+2g\tilde{n}-i\hbar R\right]\Phi, \tag{1}$$

where the interaction parameter $g=4\pi\hbar^2 a/m$, a is the s-wave scattering length, $n_c(\mathbf{r},t)=|\Phi(\mathbf{r},t)|^2$, and $\tilde{n}(\mathbf{r},t)$ is the noncondensate local density. The damping term in Eq. (1) is given by $R(\mathbf{r},t)\equiv\Gamma_{12}(\mathbf{r},t)/2n_c(\mathbf{r},t)$, with

$$\Gamma_{12}(\mathbf{r},t)=\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}C_{12}[f(\mathbf{p},\mathbf{r},t),\Phi(\mathbf{r},t)]. \tag{2}$$

This involves the collision integral $C_{12}[f,\Phi]$ describing collisions of condensate atoms with the thermal atoms, which also enters the approximate semiclassical kinetic equation for the single-particle distribution function $f(\mathbf{p},\mathbf{r},t)$ (valid for $k_{\mathrm{B}}T\gg gn_{c0}$ and $k_{\mathrm{B}}T\gg\hbar\omega_0$)

$$\frac{\partial f}{\partial t}+\frac{\mathbf{p}}{m}\cdot\nabla_{\mathbf{r}}f-\nabla U\cdot\nabla_{\mathbf{p}}f=C_{12}[f,\Phi]+C_{22}[f]. \tag{3}$$

Here the collision integral $C_{22}[f]$ describes binary collisions between noncondensate atoms. It does not change the number of condensate atoms and hence does *not* appear explicitly in Eq. (1). These coupled equations (1)–(3), along with Eqs. (23a) and (23b) of Ref. [3] defining the collision integrals C_{12} and C_{22}, were derived in the semiclassical approximation. However, they are expected to contain all the essential physics in trapped Bose-condensed gases at finite T, in both the collisionless and hydrodynamic domains. They assume that the atoms in the thermal cloud are well-described by the single-particle Hartree-Fock spectrum $\tilde{\varepsilon}_p(\mathbf{r},t)=p^2/2m+U(\mathbf{r},t)$, where $U(\mathbf{r},t)=U_{\mathrm{ext}}(\mathbf{r})+2g[n_c(\mathbf{r},t)+\tilde{n}(\mathbf{r},t)]$. We expect this semiclassical description to break down only for very low temperatures where the Bogoliubov excitation spectrum is more appropriate [17]. Our entire discussion is within what is called the Popov approximation in that we have ignored all effects associated with the anomalous pair correlations $\tilde{m}(\mathbf{r},t)=\langle\tilde{\psi}(\mathbf{r},t)\tilde{\psi}(\mathbf{r},t)\rangle$.

The coupled equations (1)–(3) have been used to derive the generalized two-fluid hydrodynamic equations in the collision-dominated region described by a local-equilibrium Bose distribution [3,18]. They also have been recently used to give a detailed analysis of condensate growth by quenching the thermal cloud distribution [19]. In these papers, Eqs. (1)–(3) are solved with both the condensate and noncondensate being treated dynamically and allowed to be out of equilibrium. The key limitation of the present paper is that we only consider the dynamics of the condensate, with the thermal cloud being in static equilibrium. This assumption allows a simple theoretical development and should be adequate for out-of-phase modes. The key role of the condensate and noncondensate being out of diffusive equilibrium was first stressed in a series of papers by Gardiner and co-workers [4,20]. These were based on a kinetic Master-equation formalism quite different from what we use, and no application was made to the damping of condensate collective modes.

It is important to understand what is meant by the *collisionless regime* and to clarify how this terminology relates to the present paper. Above $T_{\rm BEC}$ (where $C_{12}=0$), Eq. (3) reduces to the Boltzmann equation describing a normal gas [15,16]. In this case, the collisionless region is well defined and corresponds to having $\omega_i\tau_{\rm cl}\gg 1$, where ω_i is the collective-mode frequency of the gas, on the order of the trap frequency, and $\tau_{\rm cl}$ can be approximated by the mean time between collisions described by the classical Boltzmann collision integral C_{22}. In static equilibrium, this collision rate for a uniform gas is given by

$$\frac{1}{\tau_{\rm cl}}=\sqrt{2}n\sigma\overline{v}, \tag{4}$$

where n is the density of atoms, $\sigma=8\pi a^2$ is the quantum collision cross section, and $\overline{v}$ is the average speed of an atom in the gas. Both above and below $T_{\rm BEC}$, the analogous collision time corresponding to collision processes described by C_{22} in Eq. (3) will give an estimate of the lifetime of a *single-particle* excitation in the thermal cloud. This is distinct from the physics given by C_{12}, which describes the collisions of condensate atoms with atoms from the thermal cloud. In particular, one finds that if the thermal cloud is described by the equilibrium Bose distribution ($f=f^0$), then $C_{22}[f^0,\Phi]=0$ but $C_{12}[f^0,\Phi]\neq 0$. Thus C_{12} will give rise to damping of condensate oscillations even when the thermal cloud is treated statically. Of course, in the collisionless region, there is another source of damping arising from the dynamical mean-field coupling between the condensate and thermal cloud that is also included in Eq. (1) and (3); this is Landau damping [7–14], which will be discussed below.

A. Static Popov approximation

In the present paper, we use these equations to calculate the damped normal modes of the condensate given by the solutions of Eq. (1) assuming that the noncondensate atoms always remain in static thermal equilibrium. For our model, this means we take

$$f(\mathbf{p},\mathbf{r},t)\simeq f^0(\mathbf{p},\mathbf{r})=\frac{1}{\exp\beta[p^2/2m+U_0(\mathbf{r})-\tilde{\mu}_0]-1}, \tag{5}$$

where $\tilde{\mu}_0$ is the equilibrium chemical potential of the noncondensate and $U_0(\mathbf{r})=U_{\rm ext}(\mathbf{r})+2g[n_{c0}(\mathbf{r})+\tilde{n}_0(\mathbf{r})]$. The detailed analysis given by ZNG shows that the Bose-Einstein distribution in Eq. (5) is a stationary solution to Eq. (3) when the condensate and noncondensate are in diffusive equilibrium, which requires $\tilde{\mu}_0=\mu_{c0}$, where μ_{c0} is the equilibrium chemical potential of the condensate as described by Eq. (1).

Using our finite T ''static Popov'' approximation, Eq. (1) can be simplified to

$$i\hbar\frac{\partial\Phi}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+U_{\rm ext}+gn_c+2g\tilde{n}_0-i\hbar R_0\right]\Phi, \tag{6}$$

which describes the condensate motion within the static thermal cloud. Here $\tilde{n}_0$ is the equilibrium density of the noncondensate and the damping term R_0 is calculated using

$$\Gamma_{12}^0(\mathbf{r},t)\equiv\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}C_{12}[f^0(\mathbf{p},\mathbf{r}),\Phi(\mathbf{r},t)]. \tag{7}$$

Notice that $\Gamma_{12}^0(\mathbf{r},t)$ depends on time now only through $\Phi(\mathbf{r},t)$. Using the explicit general expression for C_{12} given in Eq. (23b) of ZGN, one finds (see also Ref. [4])

$$\Gamma_{12}^0(\mathbf{r},t)=\frac{n_c(\mathbf{r},t)}{\tau_{12}(\mathbf{r},t)}[e^{-\beta[\tilde{\mu}_0-\varepsilon_c(\mathbf{r},t)]}-1], \tag{8}$$

where we have defined the C_{12} collision time

$$\frac{1}{\tau_{12}(\mathbf{r},t)}\equiv\frac{2g^2}{(2\pi)^5\hbar^7}\int d\mathbf{p}_1\int d\mathbf{p}_2\int d\mathbf{p}_3\,\delta(\mathbf{p}_c+\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3)\times\delta(\varepsilon_c+\tilde{\varepsilon}_{p_1}-\tilde{\varepsilon}_{p_2}-\tilde{\varepsilon}_{p_3})(1+f_1^0)f_2^0f_3^0. \tag{9}$$

Here the condensate atom local energy is $\varepsilon_c(\mathbf{r},t)=\mu_c(\mathbf{r},t)+\frac{1}{2}mv_c^2(\mathbf{r},t)$ with the nonequilibrium condensate chemical potential

$$\mu_c(\mathbf{r},t)=-\frac{\hbar^2\boldsymbol{\nabla}^2\sqrt{n_c}}{2m\sqrt{n_c}}+U_{\rm ext}+gn_c+2g\tilde{n}_0. \tag{10}$$

The condensate atom momentum is $\mathbf{p}_c=m\mathbf{v}_c$, and $f_i^0=f^0(\mathbf{r},\mathbf{p}_i)$. We have introduced the usual condensate velocity defined in terms of the phase θ of the condensate $\Phi(\mathbf{r},t)=\sqrt{n_c(\mathbf{r},t)}\exp i\theta(\mathbf{r},t)$ as $\mathbf{v}_c=\hbar\boldsymbol{\nabla}\theta(\mathbf{r},t)/m$. A closed set of equations for $\Phi(\mathbf{r},t)$ is given by Eq. (6) and its complex conjugate combined with Eqs. (8)–(10).

We note that in terms of n_c and $\mathbf{v}_c$, Eq. (6) is completely equivalent to the coupled equations [3]

$$\frac{\partial n_c}{\partial t}+\nabla\cdot(n_c\mathbf{v}_c)=-\Gamma_{12}^0[f^0,\Phi],$$
$$m\left(\frac{\partial}{\partial t}+\mathbf{v}_c\cdot\nabla\right)\mathbf{v}_c=-\nabla\mu_c. \tag{11}$$

It is easy to see from Eq. (8) that when the condensate is in equilibrium with the thermal cloud according to $\mu_c\to\mu_{c0}=\tilde{\mu}_0$, $\Gamma_{12}^0(\mathbf{r},t)$ then vanishes. It is clear that the description of the system given by Eqs. (5) and (6), or equivalently Eq. (11), is valid only if the condensate is slightly perturbed from equilibrium and the condensate motion is essentially uncoupled from that of the thermal cloud, which we can then treat statically. In order to describe the condensate oscillations about equilibrium, we use the quantum hydrodynamic variables $n_c(\mathbf{r},t)=n_{c0}(\mathbf{r})+\delta n_c(\mathbf{r},t)$ and $\mathbf{v}_c(\mathbf{r},t)=\delta\mathbf{v}_c(\mathbf{r},t)$, where $n_{c0}(\mathbf{r})$ is the equilibrium density of the condensate with the associated equilibrium chemical potential μ_{c0}. Alternatively, one may work with the fluctuations of $\Phi(\mathbf{r},t)$ and derive coupled Bogoliubov equations [21,23,24] generalized to include the effect of the R_0 damping term. This generalization will be discussed elsewhere [22].

B. Finite-*T* Stringari wave equation

From Eq. (11), we can obtain linearized equations of motion for the condensate fluctuations δn_c and $\delta\mathbf{v}_c$. We use the fact that, to lowest order in the fluctuations from static equilibrium, Eq. (8) reduces to

$$\delta\Gamma_{12}^0=\frac{\beta n_{c0}(\mathbf{r})}{\tau_{12}^0(\mathbf{r})}\delta\mu_c(\mathbf{r},t), \tag{12}$$

where the ''equilibrium'' C_{12} collision rate is defined by

$$\frac{1}{\tau_{12}^0(\mathbf{r})}\equiv\frac{2g^2}{(2\pi)^5\hbar^7}\int d\mathbf{p}_1\int d\mathbf{p}_2\int d\mathbf{p}_3\delta(\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3)\times\delta\left(\frac{p_1^2-p_2^2-p_3^2}{2m}-gn_{c0}\right)(1+f_1^0)f_2^0f_3^0. \tag{13}$$

In the present paper, we restrict ourselves to the Thomas-Fermi limit, valid for large N_c,

$$\frac{\partial\delta n_c}{\partial t}+\nabla\cdot(n_{c0}\delta\mathbf{v}_c)=-\frac{1}{\tau'}\delta n_c, \tag{14}$$

$$m\frac{\partial\delta\mathbf{v}_c}{\partial t}=-g\nabla\delta n_c. \tag{15}$$

The collision time $\tau'(\mathbf{r})$ describes collisions between the condensate and noncondensate atoms when the condensate is perturbed away from equilibrium,

$$\frac{1}{\tau'(\mathbf{r})}=\frac{gn_{c0}(\mathbf{r})}{k_BT}\frac{1}{\tau_{12}^0(\mathbf{r})}. \tag{16}$$

In the TF approximation the equilibrium distribution reduces to $f_i^0=[\exp\beta(p_i^2/2m+gn_{c0})-1]^{-1}$. The new term on the right-hand side of Eq. (14) causes damping of the condensate fluctuations due to the lack of collisional detailed balance between the condensate and the static thermal cloud. We note that this collision time is only a function of $\mathbf{r}$ through its dependence on the static condensate density $n_{c0}(\mathbf{r})$. Plots of $1/\tau'(\mathbf{r})$ will be discussed below.

We can easily combine Eqs. (14) and (15) to obtain what we shall refer to as the finite T Stringari wave equation

$$\frac{\partial^2\delta n_c}{\partial t^2}-\frac{g}{m}\nabla\cdot(n_{c0}\nabla\delta n_c)=-\frac{1}{\tau'}\frac{\partial\delta n_c}{\partial t}. \tag{17}$$

Equation (17) is the main result of this paper. If we neglect the right-hand side, we obtain the undamped finite T Stringari normal modes $\delta n_c(\mathbf{r},t)=\delta n_i(\mathbf{r})\exp(-i\omega_i t)$ given by the solution of [6]

$$-\frac{g}{m}\nabla\cdot[n_{c0}(\mathbf{r})\nabla\delta n_i(\mathbf{r})]=\omega_i^2\delta n_i(\mathbf{r}). \tag{18}$$

As has been noted by several authors in recent papers [11,12,23], $n_{c0}(\mathbf{r})$ at finite T can be well approximated by the TF condensate profile at $T=0$ but with the number of atoms in the condensate $N_c(T)$ now being a function of temperature, since the static mean field of the noncondensate plays such a minor role. With this approximation for $n_{c0}(\mathbf{r})$, the solutions of the finite T Stringari equation (18) will be identical to those at $T=0$, since the $T=0$ Stringari frequencies do not depend on the magnitude of N_c. Of course, as shown by calculations solving the coupled Bogoliubov equations [24,23], the TF approximation breaks down when $N_c\lesssim10^4$. Thus the condensate collective mode frequencies will always become temperature dependent close to T_{BEC}, where the TF approximation is no longer valid. We generalize our present discussion to deal with this region in Ref. [22].

We can use the undamped Stringari modes as a basis set to solve Eq. (17) and find the damping of these modes. Writing $\delta n_c(\mathbf{r})=\Sigma_i c_i\delta n_i(\mathbf{r})$, and using the orthogonality condition $\int d\mathbf{r}\delta n_i(\mathbf{r})\delta n_j(\mathbf{r})=\delta_{ij}$, one obtains the following algebraic equations for the coefficients c_i

$$\omega^2c_i=\omega_i^2c_i-i\omega\sum_j\gamma_{ij}c_j, \tag{19}$$

where

$$\gamma_{ij}\equiv\int d\mathbf{r}\delta n_i(\mathbf{r})\delta n_j(\mathbf{r})/\tau'(\mathbf{r}). \tag{20}$$

Assuming the damping is small (we are in the collisionless region), Eq. (19) is easily solved using perturbation theory by setting $\gamma_{ij}=0$ for $i\neq j$. This gives the damped Stringari frequency (to lowest order) $\Omega_i=\omega_i-i\Gamma_i$, with

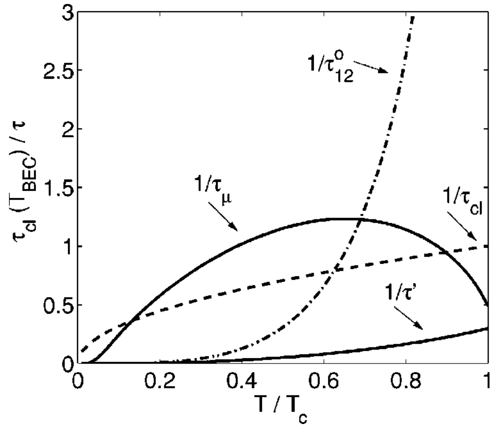

FIG. 1. Collision rates in a homogeneous Bose-condensed gas, normalized to the collision rate of a classical gas at the Bose-Einstein condensation transition temperature $\tau_{\rm cl}^{-1}(T_{\rm BEC})$. We have taken $gn=0.1k_{\rm B}T_{\rm BEC}$, where n is the total density. See also Refs. [25,26].

$$\Gamma_i \equiv \frac{\gamma_{ii}}{2} = \frac{1}{2}\int d\mathbf{r}\, \frac{\delta n_i(\mathbf{r})^2}{\tau'(\mathbf{r})}. \tag{21}$$

This result for Γ_i is reasonable, namely, it involves an average over $1/\tau'(\mathbf{r})$ weighted with respect to the undamped density fluctuations of the Stringari wave equation (18). We find that coupling to other modes ($\gamma_{ij}\neq 0$) is extremely small.

III. RESULTS

A. Homogeneous gas

Before treating the trapped gas, it is useful to first apply our theory to a homogeneous gas, which was considered previously in Ref. [25] in connection with the collision-dominated hydrodynamic region. For a homogeneous gas, τ' is independent of position and then Eq. (21) reduces to $\Gamma_i = 1/2\tau'$. Although our model in the present paper applies only to the collisionless region, it is useful to compare the intercomponent collision time in both the collisionless and hydrodynamic regimes. In ZNG, it was shown that the intercomponent collision time τ_μ in the hydrodynamic region is given by $\tau_\mu = \sigma\tau'$, where the temperature-dependent factor σ (not to be confused with the collision cross section) depends on various thermodynamic functions. In Fig. 1 we compare $1/\tau'$ and $1/\tau_\mu$ as functions of T. We see that σ dramatically alters the intercomponent relaxation rate $1/\tau_\mu$ appropriate to the hydrodynamic regime, as compared to $1/\tau'$ involved in the collisionless regime. For completeness, in Fig. 1 we also plot the often-used classical collision time given by Eq. (4) as well as τ^0_{12} defined in Eq. (13).

In a uniform Bose gas at finite temperatures, the Landau damping ($\omega = cq - i\Gamma_L$) of condensate modes has been evaluated in several recent papers [8–14]. Working within the full second-order Beliaev approximation, one finds

$$\Gamma_L = \left(\frac{3\pi}{8}\right)\frac{ak_{\rm B}Tq}{\hbar}. \tag{22}$$

This is clearly quite different from our intercomponent damping $\Gamma = 1/2\tau'$, as plotted in Fig. 1. Landau damping originates from the interaction of a condensate collective mode with the excitations of the thermal cloud but is not associated with C_{12} collisions, which give rise to τ'.

In the context of our generalized GP equation in Eq. (1), Landau damping comes from the fluctuations in the thermal cloud induced by the condensate mean field,

$$\delta\tilde{n} = \tilde{\chi}_0(2g\,\delta n_c). \tag{23}$$

In the finite temperature region of interest, $\tilde{\chi}_0$ can be approximated as the density response function of a noninteracting gas of atoms with a spectrum $\tilde{\varepsilon}_p$ and chemical potential μ_{c0}. For a uniform gas, one sees that using Eq. (23) in Eq. (1), with $R=0$, gives condensate modes satisfying $\omega^2 = c^2q^2(1+4g\tilde{\chi}_0)$ and thus $\omega = cq - i\Gamma_L$, where

$$\Gamma_L = 2gcq\,{\rm Im}\,\tilde{\chi}_0(q,\omega=cq). \tag{24}$$

Evaluating ${\rm Im}\,\tilde{\chi}_0$ in the limit of small q [7], one finds $\Gamma_L = \frac{4}{3}ak_{\rm B}Tq/\hbar$. Apart from the slightly larger numerical coefficient, this agrees with the exact result given in Eq. (22) [27].

B. Trapped gas

We now turn to explicit calculations of the intercomponent damping rate using our model for a trapped gas. In order to calculate Γ_i for a given mode, the equilibrium chemical potential $\mu_{c0} = \tilde{\mu}_0$ must be calculated self-consistently for a given total number N of atoms at a given temperature T. In the TF approximation, the procedure is straightforward (see, e.g., Ref. [28]). In the following, we consider a harmonic trap with axial symmetry $U_{\rm ext}(\mathbf{r}) = \frac{1}{2}m\omega_\rho^2(\rho^2+\lambda^2z^2)$, where $\lambda = \omega_z/\omega_\rho$ is the anisotropy parameter. In the TF approximation, the condensate density takes the explicit form $n_{c0} = [\mu_{c0} - m\omega_\rho^2(\rho^2+\lambda^2z^2)/2]/g$ within the TF radius, and the condensate chemical potential is $\mu_{c0} = \frac{1}{2}\hbar\omega_\rho[15\lambda N_c a/\rho_0]^{2/5}$, where $\rho_0 = \sqrt{\hbar/m\omega_\rho}$. The form of the Stringari normal modes $\delta n_i(\mathbf{r})$ is given explicitly in the literature [6,21]. We mainly consider the breathing mode ($n=1, l=0$) for which $\omega_{10} = \sqrt{5}\omega_\rho$, for $\lambda = 1$.

We choose experimentally accessible parameters in the following calculations for the collisionless region. However, we do not compare our results to the two available experiments on damping of normal modes at finite T, since the TF approximation is not valid for most of the data of Ref. [29], and the experiment described in Ref. [30] is approaching the collision-dominated hydrodynamic limit where the dynamics of the condensate and noncondensate become more strongly coupled. For ^{87}Rb the scattering length is $a \simeq 5.7$ nm [31]. We first consider a spherically symmetric trap $\lambda = 1$, with trap frequency $\nu_r = 10$ Hz, and we take $N = 2\times 10^6$. In the collisionless limit, we require $\omega_i\tau_{\rm cl} \gg 1$, taking $\tau_{\rm cl}$ as defined in Eq. (4). For a trapped gas, we obtain an upper limit on $1/\tau_{\rm cl}$ by taking the density in the center of the trap $n(r=0)$, which gives $1/\tau_{\rm cl} = 8a^2N\omega_\rho^3 m/(\pi k_{\rm B}T)$. For the parameters

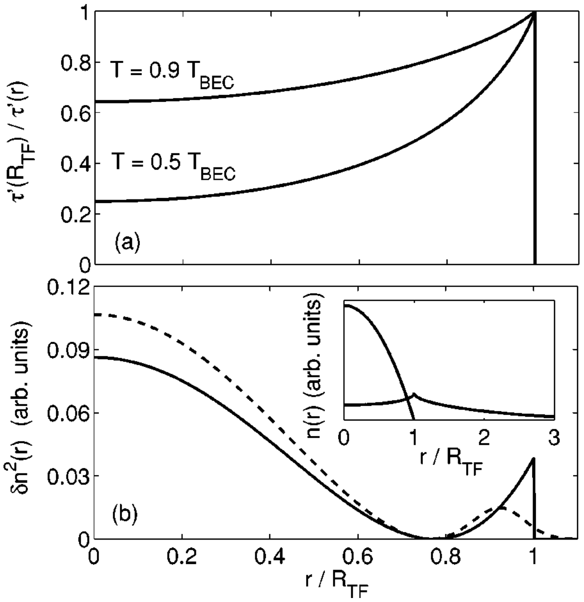

FIG. 2. Positional dependence of various quantities. In (a) we plot $1/\tau'(\mathbf{r})$ normalized by its value at the TF radius $R_{\rm TF}$. In (b) we show the density fluctuation δn_{10} of the Stringari breathing mode (solid) for a spherically symmetric trap. We also show the exact $T=0$ Bogoliubov mode (dashed) for $N_c(T=0.9T_{\rm BEC})=2.3\times10^5$. Both solutions are normalized to unity, $\int\delta n_{10}^2(\mathbf{r})d\mathbf{r}=1$. The densities of the condensate and thermal cloud are plotted in the inset for $T=0.9T_{\rm BEC}$.

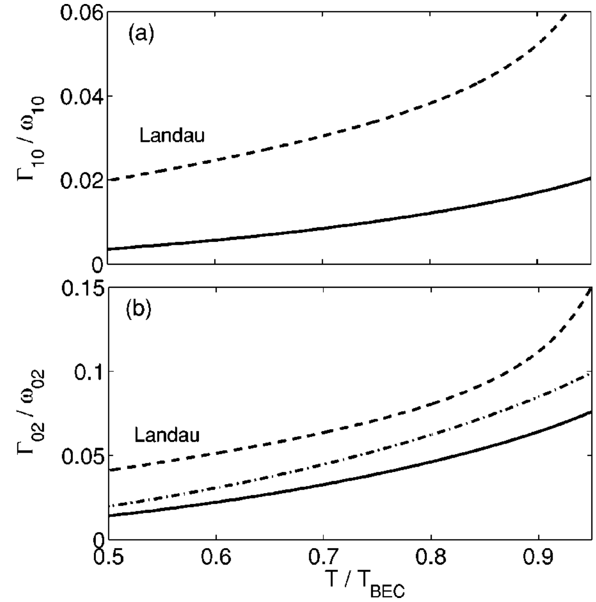

FIG. 3. Normal-mode damping rates vs temperature. In these plots, the damping rates are normalized by their corresponding mode frequencies and we only plot up to $T=0.95T_{\rm BEC}$, above which the Thomas-Fermi approximation will start to break down. In (a) we show the damping rate for the breathing mode ($n=1$, $l=0$) of a spherically symmetric trap, where the solid line corresponds to intercomponent collisional damping given in Eq. (21). In (b) we show damping rates for the quadrupole modes ($n=0$, $l=2$) in a cylindrical trap. The solid line is for the $m=0$ mode and the dot-dashed line is for $m=2$.

we use, $\omega_{10}\tau_{\rm cl}\approx19$ (compared to $\omega_{02}\tau_{\rm cl}\approx20$ for the data of Ref. [29], and $\omega_{02}\tau_{\rm cl}\approx2$ for the data of Ref. [30]).

In Fig. 2(a) we plot $1/\tau'(\mathbf{r})$ vs position for $T=0.9T_{\rm BEC}$ and $T=0.5T_{\rm BEC}$. We see that the collision rate increases steadily up to the condensate boundary, but as T increases, $1/\tau'(\mathbf{r})$ becomes relatively constant. The behavior of $1/\tau'(\mathbf{r})$ just seems to be mimicking the behavior of the noncondensate density $\tilde{n}(\mathbf{r})$, which we plot in the inset of Fig. 2(b) along with the condensate density. The condensate mean field pushes the noncondensate density out of the center of the trap, a well-known result [21]. We also show the breathing mode density fluctuation in Fig. 2(b). The sharp cusp of $\tilde{n}(\mathbf{r})$ and the sudden drop of $\delta n_i(\mathbf{r})$ and $1/\tau'(\mathbf{r})$ at the condensate boundary are all unphysical artifacts of the TF approximation. Inclusion of the kinetic-energy pressure in a more accurate calculation would have the effect of smoothing out this behavior at the boundary. To illustrate the effect of the kinetic-energy pressure, we also show in Fig. 2(b) the breathing mode obtained by solving the $T=0$ coupled Bogoliubov equations. We estimate that an improved treatment, which includes the kinetic-energy pressure, will modify our estimate of Γ_{10} by about 10–20 % [22].

In Fig. 3(a) we plot the damping rate Γ_{10} for the breathing mode ($n=1$, $l=0$) shown in Fig. 2(b) as a function of temperature up to $T=0.95T_{\rm BEC}$, where $N_c\approx7\times10^4$. At higher temperatures, the Thomas-Fermi approximation will start to break down and the mode frequencies become temperature dependent [24,23].

Landau damping of condensate modes in trapped gases has also been discussed in some detail in recent papers [10,12]. These papers give results that are in qualitative agreement with the expression for a uniform gas in Eq. (22), with $q=\omega_\rho/c$ and evaluating the Bogoliubov sound velocity c for the density at the center of the trap [9]. This simple estimate for Landau damping is plotted in Fig. 3 for comparison. We note that it is larger but comparable to the intercomponent collisional damping that we consider. Clearly, a fully satisfactory theory of finite T damping of normal modes must include *both* Landau damping as well as the damping we consider in this paper due to the condensate being out of diffusive equilibrium with the noncondensate.

Our theory is easily applied to anisotropic traps. In Fig. 3(b) we show the damping of $m=0$, 2 quadrupole modes for an axially symmetric trap with $\lambda=\sqrt{8}$. Here we choose a slightly tighter trap $\nu_r=23$ Hz, and we take $N=1\times10^6$ (in this case, $N_c\approx3\times10^4$ at $T=0.95T_{\rm BEC}$). For these parameters, we find $\omega_{20}\tau_{\rm cl}\approx6$. In Fig. 3(b) we see that Landau damping is about twice as large as our intercomponent collisional damping.

It is instructive to also consider the dependence of the mode damping Γ_i on the total population N. In Fig. 4, we show a shaded surface plot of Γ_{10} for the breathing mode in an isotropic trap as a function of T and N. The white line at $N=2\times10^6$ corresponds to the solid line plotted in Fig. 3(a). As one might expect, the intercomponent damping rate increases with increasing total population (since the density is increasing). It is also important to realize that in current experiments, the data taken is for a broad range of N due to

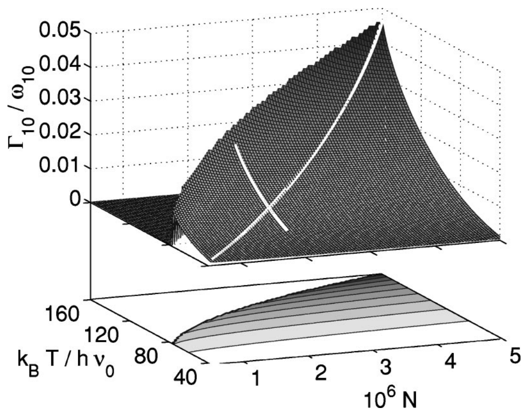

FIG. 4. Damping rate vs T and total N of the breathing mode. Here we plot the surface of Γ_{10} and its contours projected onto the plane below. The solid white line at $N=2\times10^6$ corresponds to the solid line plotted in Fig. 3(a). The longer line illustrates that in experiments, N decreases as the temperature is lowered due to evaporative cooling. The upper edge of the surface corresponds to the critical temperature $(k_B T_{BEC}/\hbar\omega_\rho)=0.94N^{1/3}$, above which Γ_{10} vanishes.

evaporative cooling losses [29,30]. The idealized fixed-N line can never be achieved in practice and one is instead dealing with a curve like the longer line on the surface in Fig. 4.

IV. CONCLUSION

In summary, we have calculated a new damping mechanism of condensate collective modes due to collisions with the thermal cloud, based on the finite-T equations derived in Ref. [3]. The essential mechanism involves the lack of diffusive equilibrium between the condensate and the thermal cloud [4], which also plays a key role in the theory of condensate growth [19,20]. Here we have carried out the first explicit calculation of this damping mechanism for a trapped gas in the collisionless regime (this intercomponent damping has recently also been evaluated in the collision-dominated hydrodynamic regime [32]). In recent discussions of the damping of condensate collective modes in the collisionless region, the mechanism we consider is omitted. One instead focuses on the dynamical mean-field coupling between the condensate and thermal cloud, which gives rise to Landau and Beliaev damping [7–14]. While we have not considered it in detail, we have indicated how we could include Landau damping by considering the noncondensate fluctuations in Eq. (1) induced by the condensate mean field [11]. Comparing Landau damping to the additional mechanism we have calculated, we find that the two are comparable in size. Further experimental studies of the collective modes at finite temperatures are needed to clarify the relative importance of these different sources of damping.

In this paper, we have argued that a good first estimate of the intercomponent damping of condensate collective modes can be obtained by coupling it to a static thermal cloud. A more systematic theory is clearly desirable in which the collisionless dynamics of the thermal cloud are allowed for. However, as noted in the introduction, we do not believe that this will lead to significant corrections to the intercomponent damping of out-of-phase condensate modes in which the motion of the thermal cloud is not significant. In future work, we hope to discuss the damping due to C_{12} collisions of collective modes that mainly involve the motion of the thermal cloud, with the condensate being treated statically.

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