Balance of information in bipartite quantum-communication systems: Entanglement-energy analogy

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We adopt the view that information is the primary physical entity possessing objective meaning. Based on two postulates stating that (i) entanglement is a form of quantum information corresponding to internal energy and (ii) sending qubits corresponds to work, we show that in the closed bipartite quantum-communication systems, the information is conserved. We also discuss the entanglement-energy analogy in the context of the Gibbs-Helmholtz-like equation connecting the entanglement, of formation, distillable entanglement, and bound entanglement. Then we show that in the deterministic protocols of distillation, the information is conserved. We also discuss the objectivity of quantum information in the context of information interpretation of quantum states and algorithmic complexity.

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I. INTRODUCTION

It is astonishing that after over 60 years of study, the quantum formalism has only recently revealed us new possibilities due to entanglement processing being a root of such new quantum phenomena as quantum cryptography with the Bell theorem [1], quantum dense coding [2], quantum teleportation [3], quantum computation [4], etc. It shows how important it is to recognize not only the structure of the formalism itself, but also the potential possibilities encoded within.

In spite of many wonderful experimental and theoretical results on entanglement, there are still difficulties in understanding its many faces. This seems to be a reflection of the basic difficulties inherent in the interpretation of the quantum formalism as well as quantum-classical hybridism in our perception of Nature. To overcome the latter, it has been postulated that the existence of a unitary information field is a necessary condition of *any* communication (or correlation) [5–7]. In addition, the information interpretation of the quantum wave function has also been considered [6]. It rests on the generic information paradigm, according to which the notion of information represents a basic category, and it can be defined independently of probability itself [7–10]. It implies that Nature is an unbroken entity. However, according to the double, hylemorphic nature of the unitary information field, there are two mutually coupled levels of physical reality in Nature: *logical* (informational), due to the potential field of alternatives, and *energetic*, due the field of activities (events) [11]. From the point of view of the generic information paradigm, the quantum formalism is merely a set of extremely useful informational algorithms describing the above complementary aspects of the same, truly existing, unitary information field. It leads in a natural way to an analogy between information (entanglement) and energy being nothing but a reflection of unity of Nature.

Following this route, one attempts to find some useful analogies in the quantum-communication domain. Namely, physicists believe that there should exist laws governing entanglement processing in quantum-communication systems that are analogous to those in thermodynamics.

A short history of this view has its origin in the papers of Bennett *et al.*, who announced a possible irreversibility of the entanglement distillation process [12,13]. Popescu and Rohrlich [14] have pointed out an analogy between distillation formation of pure entangled states and the Carnot cycle, and they have shown that entanglement is an extensive quantity. The authors formulated the principle of entanglement processing analogous to the second principle of thermodynamics: *Entanglement cannot increase under local quantum operations and classical communication*. Vedral and Plenio [15] have considered the principle in detail and pointed out that there is some (although not complete) analogy between the efficiency of distillation and the efficiency of the Carnot cycle. In Refs. [16,17], the entanglement-energy analogy has been developed and conservation of information in closed quantum systems has been postulated in analogy with the first principle of thermodynamics: *Entanglement of compound systems does not change under unitary processes on one of the subsystems* [16]. An attempt to formulate the counterpart of the second principle in a way that is consistent with the above principle has been done (since in the original Popescu-Rohrlich formulation, entanglement was not conserved).

The main purpose of this paper is to support the entanglement-energy analogy by demonstrating that in the closed bipartite quantum-communication system, the information is conserved. The paper is organized as follows. In Sec. II, we describe a closed quantum-communication bipartite system. Section III contains a formal description of the balance of quantum information involving notions of physical and logical work. In Sec. IV, we introduce the concept of useful logical work in quantum communication. In Sec. V, we present the balance of information in teleportation. In

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Sec. VI, we discuss the entanglement analogy in the context of the Gibbs-Helmholtz-like equation connecting entanglement of formation, distillable entanglement, and bound entanglement. In Sec. VII, we present the balance of information in the process of distillation. In the final section, we discuss the objectivity of quantum information in the context of information interpretation of quantum states and algorithmic complexity.

II. CLOSED QUANTUM-COMMUNICATION SYSTEM: THE MODEL

Consider a closed quantum-communication (QC) system U composite of system S, measuring system M and environment R,

$$U = S + M + R, \tag{1}$$

where each system is split into Alice and Bob parts $S_X, M_X, R_X;\ X = A,B$.

It is assumed that Alice and Bob can control the system S_X, which does not interact with environment R_X. The M_X system consists of m_X qubits and continuously interacts with environment R_X. As a result, the system M_X, playing the role of ''ancilla,'' is measured on a distinguished basis $|x_1 x_2 \cdots\rangle$, $x_i = 0,1$ [18]. In this sense, the measurement is understood here as *the process of irreversible entanglement with some environment*, and the role of system R_X is to ensure this irreversibility. Note that in the above approach, the evolution of the system is unitary: we abandon the von Neumann projection postulate. Acting on one part of the entangled system, we have no way to *annihilate* entanglement. The latter can change only by means of the interaction of *both* entangled subsystems. It may be thought that we can destroy entanglement, e.g., by randomizing the relative phases on the subsystems of interest. However, if the reduction of the wave packet is *not* regarded as a real physical process, then the above operation must be considered as entangling the subsystem with some other system by means of a *unitary* transformation. Thus the entanglement will not vanish, rather it will *spread* over all three subsystems.

The operations Alice and Bob can perform in our QC system are as follows: (i) quantum communication: Alice and Bob can exchange particles from the system S_X; (ii) classical communication: Alice and Bob can exchange particles from the system M_X. Note that the number of qubits of the systems S_A and S_B can change but the total number of qubits of the system S is conserved (similarly for M). Besides, Alice and Bob can perform a unitary transformation over the system $M_X + S_X$, $X = A,B$.

We would like to stress once more that in our approach the measurement represents an irreversible entanglement rather than the ''projection'' of the state. To see this, consider the case in which Alice and Bob share a singlet state and Alice performs a measurement on it. The initial state of the system $M_A + S_A + S_B$ (M_A represents Alice's ancilla, while S_A and S_B correspond to the particles forming a singlet state) is

$$|\Psi\rangle_{M_A S_A S_B} = |0\rangle_{M_A} |\Psi_A^{\text{singlet}}\rangle = |0\rangle_{M_A} \frac{1}{\sqrt{2}} (|0\rangle_{S_A} |1\rangle_{S_B} - |1\rangle_{S_A} |0\rangle_{S_B}). \tag{2}$$

Then Alice performs the unitary operation U on subsystem $M_A + S_A$. This operation corresponds to the interaction between M_A and S_A and can be represented by a C-NOT gate. As a result, the whole system is in the state

$$|\Psi'\rangle_{M_A S_A S_B} = \frac{1}{\sqrt{2}} (|0\rangle_{M_A} |0\rangle_{S_A} |1\rangle_{S_B} - |1\rangle_{M_A} |1\rangle_{S_A} |0\rangle_{S_B}). \tag{3}$$

Furthermore, M_A can be irreversibly entangled with the environment system R_A (which models the irreversibility of the measurement). But R_A is still on Alice's side, hence we have entanglement between systems $(R_A + M_A + S_A)$ and S_B unchanged and equal to $E = 1$ *e*-bit (*e*-bit is a unit of entanglement: it is defined as entanglement of a two-qubit singlet state).

Of course, there are some interpretational problems if one imagines that Alice ''reads out'' the result of the measurement, as then we encounter problems coming from the possible extension of the model by the projection postulate. However, for practical reasons (i.e., as far as a quantum information qualitative description is concerned) informational processes such as, e.g., quantum teleportation do not require reading the data. Moreover, it must be noted that in the absence of the projection postulate, the above model can be viewed as being consistent with a ''many worlds'' interpretation [19].

III. CONSERVATION OF QUANTUM INFORMATION: FORMAL DESCRIPTION

To determine the balance of information in the closed system U, we adopt two basic postulates [16,17]: (i) entanglement is a form of quantum information corresponding to internal energy; (ii) sending qubits corresponds to work. In accordance with the postulate (i), the information is a physical quantity that, in particular, should be *conserved* in closed quantum systems, similar to energy. The second postulate allows us to deal with communication *processes* (in thermodynamics, work is a functional of process). To obtain the balance, we must define our ''energy'' and ''work'' quantitatively. To this end, consider system X described in the Hilbert space $\mathcal{H}$, $\dim \mathcal{H} = d$ being in a state ϱ_X. We define the *informational content* I_X of the state ϱ_X as follows (cf. [20]):

$$I_X = \log_2 \dim\ \mathcal{H} - S(\varrho_X), \tag{4}$$

where $\dim \mathcal{H} = d$ and $S(\varrho_X) \equiv S(X)$ are the dimension of the Hilbert space and the von Neumann entropy of the system state, respectively. Note that I_X satisfies the inequality $0 = I_X^{\min} \leqslant I_X \leqslant I_X^{\max} = \log_2 \dim\ \mathcal{H}$, where $I_X^{\min}$ and $I_X^{\max}$ are the information content of the maximal mixed state and pure state, respectively. Thus it is a well-defined quantity that measures the informational content of the system ϱ_X.

Formula (4) requires some explanation as usually one interprets the von Neumann entropy as a measure of information. In fact, there is no contradiction. Imagine for a moment that we admit the projection postulate, i.e., Alice knows the concrete result of the measurement. Then the von Neumann entropy measures the information gain *after* the measurement, while formula (4) corresponds to the information *prior to* the measurement, and this information, in particular, is maximal if the system is in a pure state. This is the reason that we use the term informational *content*, as it has actual rather than potential (i.e., related to future measurement) character. Below we shall see that, after we abandon the projection postulate, the above formula allows us to perform a balance of quantum information in a consistent way. Note that the Hilbert space dimension used in formula (4) is present also in the definitions of other notions (see below), in particular in the case of useful logical work (Sec. IV). It plays, to some extent, a role similar to that in channel capacities theory or error correction codes, in which the dimension of an ''error-free'' subspace is a central notion.

Consider now the QC system U, being in the initial pure state $\psi_{\rm in}$, described by the general Alice-Bob Hilbert space scheme as follows:

$$\left.\begin{array}{ccc} \mathcal{H}_A & & \\ \otimes & \otimes & \mathcal{H}_B \\ \mathcal{H}_{A'} & & \end{array}\right\} \psi_{\rm in}, \tag{5}$$

where $\mathcal{H}_A \otimes \mathcal{H}'_A$ and $\mathcal{H}_B$ are the Hilbert spaces of $S_A+M_A+R_A$ and $S_B+M_B+R_A$, respectively. Then in accordance with Eq. (4), the information contents of the Alice and Bob subsystems are defined as follows:

$$I_A = \log_2 \dim(\mathcal{H}_A \otimes \mathcal{H}_{A'}) - S(A+A'), \tag{6}$$

$$I_B = \log_2 \dim\, \mathcal{H}_B - S(B), \tag{7}$$

where $\dim(\mathcal{H}_A \otimes \mathcal{H}_{A'})$ and $\dim \mathcal{H}_B$ are the dimensions of the corresponding Hilbert spaces while $S(A+A')$ and $S(B)$ are the von Neumann entropies of the subsystems.

Now, after transmission of the system A' to the receiver (Bob), the Alice-Bob Hilbert space scheme is given by

$$\left.\begin{array}{ccc} & & \mathcal{H}_B \\ \mathcal{H}_A & \otimes & \otimes \\ & & \mathcal{H}_{A'} \end{array}\right\} \psi_{\rm out} \tag{8}$$

and the total system U is in the final state $\psi_{\rm out}$.

Now, in accordance with the above ''sending qubits – work'' postulate, we consider *physical work* performed over the system U being a *physical transmission* of particles. Consequently, we define W_p as the number of sent qubits of the system A',

$$W_p = \log_2 \dim\, \mathcal{H}_{A'}. \tag{9}$$

Note that after transmission of the system A' to Bob, there is an increase of the information content of his subsystem. Then we say that the system U performed the *logical work* W_l that is defined as an increase of the informational content of the Bob (in general the receiver) system,

$$W_l = I^B_{\rm out} - I^B_{\rm in}, \tag{10}$$

where $I^B_{\rm in} = I^B$, $I^B_{\rm out} = I^{B+A'}$. Then one can regard the physical work as sending ''matter'' and the logical work as sending ''form,'' which is consistent with the assumed hylemorphic nature of the information field. Subsequently, we can define the initial and final entanglement of the system U as

$$E_{\rm in} = S(B) = S(A+A'), \quad E_{\rm out} = S(A) = S(B+A'), \tag{11}$$

where obvious relations between the entropies of the subsystems hold. Now, in accordance with the first postulate, $E_{\rm in}$ and $E_{\rm out}$ are simply initial and final *potential* information contained in the total system. Having such defined quantities, it is not hard to obtain the following information balance equations:

$$E_{\rm in} + W_p = E_{\rm out} + W_l \tag{12}$$

or equivalently

$$I^A_{\rm in} + I^B_{\rm in} + 2E_{\rm in} = I^A_{\rm out} + I^B_{\rm out} + 2E_{\rm out} = \text{const.} \tag{13}$$

Note that the latter equation is compatible with the principle of information conservation expressed in the following form (equivalent to the one in the Introduction): For a compound quantum system, a sum of information contained in the subsystems and information contained in entanglement is conserved in unitary processes [16].

To see how the above formalism works, consider two simple examples with ideal quantum transmission. Suppose Alice sends an *unentangled* qubit of the system S to Bob. Then the physical work W_p is equal to one qubit. As a result, the informational content of Bob's system increases by 1, thus also the logical work W_l amounts to one qubit. Of course, in this case both ''in'' and ''out'' entanglement are 0.

Suppose now that Alice sends a maximally *entangled* qubit to Bob. Here, again, physical work is one qubit, and there is no initial entanglement. However, the final entanglement is one *e*-bit and logical work is 0, because the state of the Bob system is now completely mixed.

Now we see that, according to the balance equation (12), the difference $W_p - W_l$ between the physical and logical work is due to entanglement. Indeed, as in the above example, sending a particle may result in an increase of entanglement rather than performing nonzero logical work.

IV. USEFUL LOGICAL WORK: QUANTUM COMMUNICATION

The basic question arises in the context of quantum communication. Does the balance (12) distinguish between quantum and ''classical'' communication in our model? It follows from the definition that the physical work does not distinguish between these types of communication. But what about logical work? Suppose that Alice sent to Bob a particle of the system M_A in a pure state $|0\rangle$. But in our model such

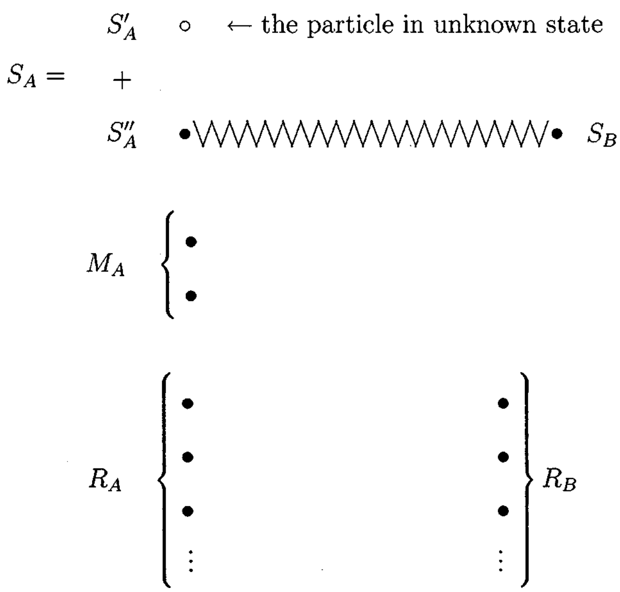

FIG. 1. Model of the quantum-communication system.

a state does not undergo decoherence. Then the logical work W_l is equal to one qubit [21]. Needless to say, it is not quantum communication. Hence the logical work is not ''useful'' in this case.

In quantum communication, we are usually interested in sending faithfully any superpositions without decoherence. Therefore, it is convenient to introduce the notion of *useful logical work* as follows.

Definition. Useful work is the amount of qubits of the system S transmitted without decoherence,

$$W_u = \log_2 \dim \mathcal{H}, \tag{14}$$

where $\mathcal{H}$ is the Hilbert space transmitted asymptotically faithfully. The latter means that *any* state of this space would be transmitted with asymptotically perfect fidelity. We see that the work performed in the previous example was not useful, since as a result of the process, only the states $|0\rangle$ or $|1\rangle$ can be transmitted faithfully.

V. BALANCE OF INFORMATION IN TELEPORTATION

To see how the above formalism works, consider the balance of quantum information in teleportation [3,22]. Now the system S_A consists of a particle in an unknown state and one particle from a maximally entangled pair, whereas the second particle from the pair represents the S_B system. The system M_A consists of two qubits that interact with environment R_A (Fig. 1).

The latter is only to ensure effective irreversibility of the measurement and it is evident that its action is irrelevant to the information balance in the case of teleportation. As one knows, the initial state can be written in the following form:

$$\psi_{\rm in} \equiv \psi_0 = \psi_{S'_A}^{\rm unknown} \otimes \psi_{S''_A S_B}^{\rm singlet} \otimes |00\rangle_{M_A}, \tag{15}$$

where $\psi_{S'_A}^{\rm unknown}$ is the state to be teleported, $\psi_{S''_A S_B}^{\rm singlet}$ is the singlet state of the entangled pair, and $|00\rangle_{M_A}$ is the initial state of the measuring system. It is easy to check that the initial entanglement $E_{\rm in}$ of the initial state is equal to one *e*-bit. Now Alice performs a ''measurement,'' which is the local unitary transformation on her joint system $S_{A'} + S_{A''} + M_A$. As a result, $\psi_{\rm in}$ transforms to

$$\psi_1 = \frac{1}{2} \sum_{i=0}^{3} \psi_{S'_A S''_A}^{i} \otimes \psi_B^{i({\rm unknown})} \otimes |i\rangle_{M_A}, \tag{16}$$

where $\psi^i_{S_A, S_{A''}}$ constitutes the Bell basis, $\psi_B^{i({\rm unknown})}$ is rotated $\psi_{S_A}^{\rm unknown}$, and $|i\rangle_{M_A}$ is the state of the system M_A indicating the result of the measurement (ith Bell state obtained). Since Alice's operation is unitary, it does not change the initial asymptotic entanglement. Subsequently, Alice sends the two particles of the system M_A to Bob. In accordance with definition (6), it corresponds to two qubits $W_p = 2$ of work performed over the system. At the same time, the state ψ_1 transforms to ψ_2 of the form

$$\psi_2 = \frac{1}{2} \sum_{i=0}^{3} \psi_{S'_A S''_A}^{i} \otimes \psi_B^{i({\rm unknown})} \otimes |i\rangle_{M_B}. \tag{17}$$

Finally, Bob decouples the system S_B from the other ones by unitary transformation, which of course does not change the asymptotic entanglement.

After classical communication from Alice, entanglement of the total system increased to the value $E_{\rm out} = 2$ *e*-bits. Indeed, Alice sends two particles of system M_A to Bob, which are entangled with particles S'_A, S''_A. On the other hand, the logical work performed by the system in the above process amounts to $W_l = 1$. One can see that the balance equation (12) is satisfied, and is of the following form:

$$(E_{\rm in} = 1) + (W_p = 2) = (E_{\rm out} = 2) + (W_l = 1). \tag{18}$$

One easily recognizes the result of the logical work in the transmission of the unknown state to Bob. Since it is faithfully transmitted independently of its particular form, we obtain also that useful logical work W_u is equal to one qubit. Hence in the process of teleportation, all the work performed by the system is useful, and represents quantum communication.

VI. THERMODYNAMIC ENTANGLEMENT-ENERGY ANALOGY: GIBBS-HELMHOLTZ-LIKE EQUATION

So far we have considered the balance of information in a closed QC system. For an open system (being, in general, in a mixed state), the situation is much more complicated, being a reflection of the fundamental irreversibility in the asymptotic mixed-state entanglement processing [12,13,23]. Namely, it has been shown [23] that there is a discontinuity in the structure of noisy entanglement. It appears that there are at least two quantitatively different types of entanglement: free, which means useful for quantum communication, and bound, which means a nondistillable, very weak, and

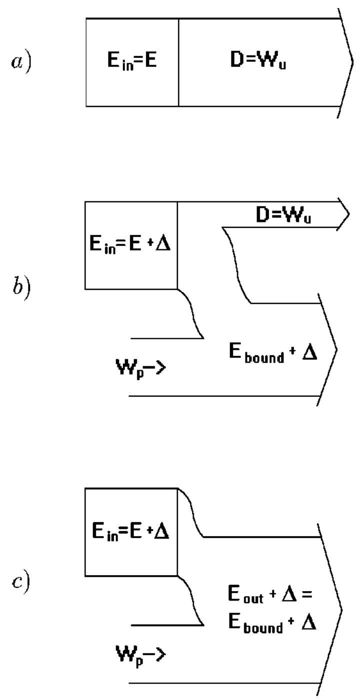

FIG. 2. This diagram illustrates the balance of quantum information in the entanglement distillation process for (a) the pure states case, (b) the general case, and (c) the bound entangled states case.

peculiar type of entanglement. In accordance with the entanglement-energy analogy, this new type of entanglement is defined by the equality

$$E_F = E_{\mathrm{bound}} + E_D, \tag{19}$$

where E_F and E_D are the asymptotic entanglement of formation [17,24] and distillable entanglement [12], respectively. Note that for pure entangled states $|\Psi\rangle\langle\Psi|$ we have always $E_F = E_D$, $E_{\mathrm{bound}} = 0$ [13]. Then in this case the whole entanglement can be converted into the useful quantum work [see Fig. 2(a)] with $E \equiv E_F(|\Psi\rangle\langle\Psi|)$. For bound entangled mixed states, we have $E_D = 0$, $E_F = E_{\mathrm{bound}}$. It is quite likely that $E_F > 0$ (so far we know only that $E_f > 0$ [25], where E_f is the entanglement of formation defined in Ref. [26]) and then all prior entanglement of formation would be completely lost. Thus in any process involving only separable or bound entangled states, useful logical work is just zero. In general, however, it can happen that the state contains two *different* types of entanglement:

$$E_{\mathrm{bound}} = E_F - E_D > 0. \tag{20}$$

States of such property have not been found so far, but are believed to exist [27] (cf. [29]). It can be viewed as an analog to irreversible thermodynamic processes where only the free energy (which is not equal to the total energy) can be converted to useful work. This supports the view [17] according to which Eq. (19) can be regarded as a quantum information counterpart of the thermodynamic Gibbs-Helmholtz equation $U = F + TS$, where the quantities E_F, E_D, and E_{bound} correspond to internal energy U, free energy F, and bound energy TS, respectively (T and S are the temperature and the entropy of the system).

The above entanglement-energy analogy has led to the extension [30] of the ''classical'' paradigm of local operations and classical communication (LOCC operations) by considering a new class of entanglement processing, called here entanglement enhanced LOCC operations (EELOCC). In particular, it suggested that entanglement can be pumped from one system to the other, producing different nonclassical chemical-like processes. In fact, it allowed us to find a new quantum effect, namely *activation* of bound entanglement that corresponds to the chemical activation process [31]. Similarly, a recently discovered *catalysis* of pure entanglement involves EELOCC operations [32]. As a result, the second principle of entanglement processing (see the Introduction) has been generalized [33] to cover the EELOCC paradigm: *By local action, classical communication, and N qubits of quantum communication, entanglement cannot increase more than N e-bits.*

Now, it is interesting in the above context to consider the problem of information balance in the cases in which systems are in mixed states.

VII. BALANCE OF INFORMATION IN THE DISTILLATION PROCESS

So far in our balance analysis the initial state of the QC system has been pure. Let us consider the more general case. Suppose that the initial state of the system S is mixed. We have not generalized the formalism to such a case. We can, however, perform the balance of information in the case of the distillation process [12] (see in this context [33]). This task would be, in general, very difficult, because almost all the known distillation protocols are *stochastic*. As one knows, the distillation protocol aims at obtaining singlet pairs from a large amount of noisy pairs (in the mixed state) by LOCC operations. A convenient form of such a process would be the following: Alice and Bob start with n pairs, and after distillation protocol, end up with m singlet pairs. Such a protocol we shall call *deterministic*. Unfortunately, in the stochastic protocols the situation is more complicated: Alice and Bob get with some probabilities a different number of output distilled pairs:

$$\varrho_{\mathrm{in}} = \underbrace{\varrho \otimes \varrho \otimes \cdots \otimes \varrho}_{n} \rightarrow \begin{cases} \rightarrow p_0, & \text{no output singlets} \\ \rightarrow p_1, & \text{one output singlet} \\ \rightarrow p_2, & \text{two output singlets} \\ \vdots & \end{cases}$$

Since we must describe the process in terms of a closed system, we will not see the above probabilities, but only their amplitudes. As a result, we will have *no* clear distinction between the part of the system containing distilled singlet pairs and the part containing the remaining states of no useful entanglement.

Consider, for example, the first stage of the Bennett *et al.* [12] recursive protocol. It involves the following steps: (i) take two spin-$\frac{1}{2}$ pairs, each in input state ϱ; (ii) perform operation XOR$\otimes$XOR; and (iii) measure locally the spins of the target pair [if the spins agree (probability p_a), keep the source pair, and if the spins disagree (probability p_d), discard both pairs]. After this operation, we have the following final ''ensemble:''

$$\{(p_a,\text{one pair in a new state } \tilde{\varrho}), \quad (p_d,\text{no pairs})\}.$$

If we include an environment in the description, the events ''no pair'' and ''one pair in state $\tilde{\varrho}$'' will be entangled with states of measuring apparatuses (and environment) indicating these events. Then we see that our total system becomes more and more entangled in various possible ways, so that it is impossible to perform the balance of information.

Fortunately, in a recent work Rains [33] showed that any distillation protocol can be replaced with a deterministic one, achieving the same distillation rate:

$$\varrho^{\otimes n} \rightarrow \varrho_{\text{out}} \simeq |\psi_{\text{distilled}}\rangle\langle\psi_{\text{distilled}}| \otimes \varrho_{\text{rejected}},$$

where $\psi_{\text{distilled}}$ is the state of m distilled singlet pairs while $\varrho_{\text{rejected}}$ is the state of the rejected pairs. In this case the system can be divided into two parts,

$$S = S_{\text{distilled}} + S_{\text{rejected}}, \tag{21}$$

where $S_{\text{distilled}}$ is disentangled with the rest of the universe and S_{rejected} is entangled with M, hence also with environment R.

This possibility of a clear partition between two systems is crucial for our purposes. Now the whole balance can be performed in this case as follows. As an input, we have the state ϱ with the value of asymptotic entanglement of formation $E = E_F(\varrho)$. Because it is mixed, we can take its purification (adding some ancilla) that would have entanglement E'. This is the initial entanglement in the process we examine. The operation of partial trace producing the state ϱ out of the purification can be composed of two local partial traces. Thus it cannot increase entanglement, so that E' is no less than E, and we have a non-negative deficit $\Delta \equiv E' - E \geqslant 0$. We can therefore split the total initial entanglement E' into E (carried by the state ϱ) and Δ, which is not accessible to Alice and Bob. Now one can perform the distillation process, having no access to the ancilla. After the process, the state of our whole system is still separated according to the formula (21), but now the state S_{rejected} involves the degrees of freedom of the ancilla. The balance of the information can now be easily performed taking into account, in particular, that distillable entanglement E_D can be interpreted as a useful work (14) W_u (Alice can always teleport through state $|\Psi_{\text{distilled}}\rangle\langle\Psi_{\text{distilled}}|$ if she wishes). To make the balance fully consistent, one should subtract from both input and output data the additional entanglement Δ coming from an extension of the system to the pure state. As the input physical work (connected with optimal distillation protocol) *is the same* regardless of the value Δ and the kind of ancilla, the whole balance is completely consistent. The input quantities of E, Δ, plus W_p as well as the output quantities $E_D = W_u$, Δ, and $E_{\text{out}} = E(\varrho_{\text{rejected}}) = E_{\text{bound}}$ are depicted in Fig. 2(b). In particular, if we deal with bound entangled states, then the corresponding diagram takes the form of Fig. 2(c).

VIII. OBJECTIVITY OF QUANTUM INFORMATION: INFORMATION INTERPRETATION OF QUANTUM STATES

As we have dealt with the balance of information in quantum composite systems, it is natural to ask about the objectivity of the entity that we qualify. In this section, we discuss that question and related ones in the context of quantum information theory and interpretational problems of quantum mechanics. As one knows, the latter holds up very well to commonly accepted interpretation. As a result, the number of different interpretations continues to grow while there are no operational criteria (except, maybe, the Ockham razor) to eliminate at least some of them.

It is characteristic that despite the dynamical development of interdisciplinary domain–quantum information, to our knowledge there is no impact of the latter on interpretational problems. In this context, a basic question arises: Do quantum information phenomena provide objective evidence for the existence of ''natural'' ontology inherent in quantum formalism?

It is interesting that out of the recently discovered quantum effects, only quantum cryptography [34] provides the answer ''yes.'' To see it clearly, consider quantum cryptographic protocol. A crucial observation is that the possibility of sharing a secret key is due to the fact that we send quantum states *themselves*, not merely the *classical information* about them [35]. Clearly, the latter could be duplicated, which is the reason why all classical cryptographic schemes are, in principle, not secure. Therefore, the use of qubits is *crucial* if we want like to take any advantage of the novel possibilities offered by quantum information theory.

Now, as there are experimental implementations of quantum information protocols [36], it follows that quantum information is objective and can provide a natural ontological basis for interpretation of quantum mechanics. Thus we arrive at the following important conclusion. Quantum states carry two complementary kinds of information, the ''classical'' information, involving quantum measurements, and ''quantum'' information, which cannot be cloned.

Note that this is consistent with an information interpretation proposed earlier of the wave function in terms of *objective* information content [6]. On the other hand, it contradicts the Copenhagen interpretation, according to which the wave functions have no objective meaning and only reality is the result of a measurement. It is remarkable that the above information interpretation of quantum states is compatible with the above-mentioned unitary information field concept, which rests on the assumption that information is physical [7,37,38] and can be defined independently of probability itself. The first axiomatic definition of classical information ''without probabilities'' was considered by Ingarden and Urbanik [9]. A quantum version of the definition was introduced by Ingarden and Kossakowski [10]. On the other hand, Kolmogorow [8], Solomonoff [39], and Chaitin [40] introduced the concept of classical algorithmic information or complexity. Recently, the classical algorithmic information was incorporated into the definition of the so-called physical entropy being a constant of ''motion'' under the ''demonic evolution'' [41,42].

Quite recently, algorithmic information theory was extended in different ways to quantum states by Vitanyi [43] and Berthiaume *et al.* [44]. In fact, one can convince oneself that the approaches [43] and [44] correspond to the above complementary kinds of information associated with the quantum state. Indeed, Vitanyi algorithmic complexity measures the amount of ''classical'' information in bits necessary to approximate the quantum state [45]. Needless to say, from the point of view of quantum cryptography such information is useless. On the other hand, the bounded fidelity version of quantum Kolmogorow complexity measures the amount of quantum information in a qubit string and it is closely related to quantum compression theory [46–48].

IX. SUMMARY

In conclusion, we have developed the entanglement-energy analogy based on some natural postulates: (i) entanglement is a form of quantum information being a counterpart of internal energy, (ii) the process of sending qubits is a counterpart of work. We also assume that the evolution of the quantum system is unitary.

Based on the above postulates, we have considered the balance of quantum information for bipartite quantum communication systems, i.e., the systems composed of two spatially separated laboratories endowed with a classical informational channel plus local quantum operations. We have introduced the notion of the informational content of the quantum state being the difference between the maximal possible von Neumann entropy and the actual one. Thus we have defined physical work as the number of qubits physically sent from Alice to Bob. We have also defined logical work as an increase in the informational content of the Bob state. To obtain a proper description of quantum-communication processes, we have also introduced the notion of useful logical work as the amount of qubits transmitted without decoherence.

Those tools have allowed us to perform the detailed balances of quantum information in two important processes of quantum communication: quantum teleportation and distillation of quantum noisy entanglement. In particular, we have discussed the question of balance of quantum information for open systems. In the context of the balance scheme and related notions, we conclude that the irreversibility connected with the existence of bound entanglement can be viewed as an analog to irreversible thermodynamic processes where only the free energy (which is not equal to the total energy) can be converted to useful work. This allows us to interpret the equation for entanglement of formation as the quantum information counterpart of the thermodynamic Gibbs-Helmholtz equation.

Finally, we have discussed the objectivity of quantum information in the general context of some recent achievements of quantum information theory including quantum cryptography and recent propositions of classical and quantum algorithmic information. This leads us to the conclusion that quantum states reflect properties of quantum information as an objective entity involving ''classical'' and ''quantum'' components that correspond to recently introduced ''classical'' and ''quantum'' algorithmic complexities. So the balance performed in the present paper concerns objective quantities rather than purely formal objects. We hope that the present informational approach to bipartite quantum communication systems, when suitably developed, may lead to a deeper understanding of the quantum information processing domain.

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