Stochastic resonance in quantum trajectories for an anharmonic oscillator

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We demonstrate that the stochastic resonance phenomenon can occur in a pure quantum regime for a wider class of microscopic systems described by a dissipative anharmonic oscillator driven by two periodic forces. We apply a quantum-state diffusion method, and display the synchronization of quantum trajectories by the stochastic resonance process. The model proposed is accessible for experiments, and results in the formation of quantum states of the anharmonic oscillator, namely sub- and super-Poissonian statistics for time intervals exceeding the relaxation rate. We show that stochastic resonance phenomenon can be described in terms of the minimization of the quantum von Neuman entropy, and demonstrate the possibility of controlling both the dissipative dynamics and quantum statistics of anharmonic oscillator by two external periodic forces.

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I. INTRODUCTION

The interplay between noise and coherent driving in nonlinear dynamical systems gives rise to a variety of intriguing behaviors. The most extraordinary and counterintuitive example is the phenomenon of stochastic resonance (SR), whereby the response of the system to a coherent external signal can be enhanced by the assistance of noise. First proposed in 1981 [1], SR was first experimentally verified by Fauve and Heslot [2], and since then this phenomenon was demonstrated in sensory neurons, lasers, tunnel diodes, communication devices, etc. (See Refs. [3,4] for an extensive review and complete list of references.) The most known mechanism leading to SR concerns nonlinear bistable systems driven by a weak periodic signal, which in itself is insufficient to induce deterministic transitions between two metastable states. Then, in the presence of additional external noise, this system exhibits almost periodic transitions from one state to the other, for certain optimal noise intensities. It was shown that, as a consequence of the system's nonlinear intrinsic dynamics, SR can also occur in monostable systems [5] as well as in the absence of a periodic drive [6]. Recently, SR was proposed for a quantum regime where additional routes to quantum tunneling occur to overcome a potential barrier between two metastable states of a quantum double well [7]. Some developments and other applications and examples of quantum stochastic resonance (QSR) were given in Refs. [8,9].

The question was posed whether QSR can be realized in microscopic simple quantum systems where interactions are considered as quantum by quantum. One such scheme was proposed by Buchleitner and Mantegna [10], where QSR was demonstrated in a micromaser.

The goal of this paper is twofold: first, we show that QSR can occur in a wider class of microscopic systems described by an anharmonic oscillator driven by two periodic forces; and second, we demonstrate SR in pure quantum regime on quantum trajectories, as well as on ensemble-averaged results for the mean oscillatory number, variance of oscillatory number quantum fluctuations, and von Neuman entropy. The model of nonlinear oscillator we present here is accessible for experiments. It can be implemented, at least for the dynamics of strongly interacting photons, in optical cavity involved Kerr nonlinearity, and for a cyclotron oscillations of a single electron in a Penning trap.

A driven anharmonic oscillator (DAO) is a well-known and archetypal model for dealing with nonlinearities in quantum mechanics, and was widely used to describe various physical phenomena [11–15]. The DAO gives quantum interference even when coupled to a thermal reservoir [12], and describes a self-modulation of electromagnetic modes in a cavity filled by a $\chi^{(3)}$ nonlinear medium (see, for example, Ref. [13], and references therein). The DAO is also related to a description of Bose-Einstein condensates [14] and to hysteresis in atomic systems [15].

With dissipation included, the DAO model was exactly solved by Drummond and Walls [11] in the steady-state regime, and in terms of the Fokker-Plank equation in a complex P representation. An exact quantum theory for a parametrically driven anharmonic oscillator was given by Kryuchkyan and Kheruntsyan [16]. A characteristic feature of a dissipative DAO is the appearance of bistability and hysteresis effects. We exploit this feature by considering a model of a dissipative anharmonic oscillator (AO) driven by two periodic forces at different frequencies. As we show, this system exhibits remarkable nonlinear and quantum features, and displays both nonclassical, sub- and super-Poissonian statistics of elementary excitations. As to SR, it occurs in a regime where one of the two applied forces is weak compared to the other. This is an example of a quantum system in contact with its environment, where the controlling of dissipative dynamics as well as quantum statistics can be realized through an external time-dependent force. The model proposed cannot be described analytically. Our analysis is based on a quantum-state diffusion (QSD) approach [17], which naturally unravels a mixed state of open quantum system into component stochastic pure states. Thus it is expected that this approach will also be useful in obtaining insights into the problem of SR at the level of time-dependent quantum trajectories. As we show, it is possible to achieve SR in a deep quantum regime for strong anharmonicity, when a corresponding coupling constant is compa-

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rable with a dissipation decay rate. We suppose that such a model can, in particular, be realized in two experimental schemes. One of the candidates may be a scheme based on a model of anharmonic oscillator in a cavity, proposed in Ref. [18], which utilizes atomic dark resonances and allows one to achieve giant optical nonlinearities. The photon-photon interaction coefficient in such a nonlinear cavity, describing the anharmonicity coefficient, could easily be much larger than the cavity decay rate. We propose to use a modified version of such a DAO, whereby the high-finesse cavity is pumped by two coherent fields. Other proposal involves a single electron in a Penning trap under two low-frequency periodic drivings. Its anharmonicity comes from a nonlinear relativistic correction to an electron motion, while stochastic and dissipative effects arise from the spontaneous emission of synchrotron radiation, as predicted theoretically by Kaplan [19]. The trapped electron driven by a single coherent field was experimentally realized and studied by Gabrielse and co-workers [20].

The paper is organized as follows: In Sec. II, the model of a dissipative anharmonic oscillator driven by two periodic forces is presented, and the relevant QSD method is described. In Sec. III we demonstrate the SR phenomenon in a pure quantum regime of DAO from the point of view of QSD. The variance of the oscillatory excitation and the second-order correlation function are also calculated. Section IV is devoted to study of an information aspects of a DAO on the basis of quantum von-Neuman entropy. Finally, Sec. V presents our conclusion.

II. ANHARMONIC OSCILLATOR DRIVEN BY TWO FORCES

Let us discuss the proposed system in more detail. An anharmonic oscillator driven by two periodic forces at frequencies ω_1 and ω_2 is described by the Hamiltonian

$$H=\hbar\omega_0 a^\dagger a+\hbar\chi(a^\dagger a)^2 +\hbar\{[\Omega_1\exp(-i\omega_1 t)+\Omega_2\exp(-i\omega_2 t)]a^\dagger+\text{H.c.}\}. \tag{1}$$

Here a and $a^\dagger$ are boson annihilation and creation operators, ω_0 is an oscillatory frequency, and χ is the strength of the anharmonicity. The intensity of two driving forces is given by Rabi frequencies Ω_1 and Ω_2. We incorporate the dissipation by coupling the AO to a heat bath. The reduced density operator within the framework of the rotating-wave approximation and in a frame rotating with frequency ω_1 is governed by the master equation

$$\frac{\partial\rho}{\partial t}=-\frac{i}{\hbar}[H_0+H_{int},\rho] +\sum_{i=1,2}\left(L_i\rho L_i^+-\frac{1}{2}L_i^+L_i\rho-\frac{1}{2}\rho L_i^+L_i\right), \tag{2}$$

where

$$H_0=\hbar\Delta a^\dagger a,$$

$$H_{int}=\hbar\{[\Omega_1+\Omega_2\exp(-i\delta t)]a^\dagger +[\Omega_1^*+\Omega_2^*\exp(i\delta t)]a\}+\hbar\chi(a^\dagger a)^2. \tag{3}$$

Here $\Delta=\omega_0-\omega_1$ and $\delta=\omega_1-\omega_2$ are the detunings. L_i are the Lindblad operators,

$$L_1=\sqrt{(N+1)\gamma}a,\quad L_2=\sqrt{N\gamma}a^\dagger, \tag{4}$$

where γ is the spontaneous decay rate of the dissipation process, and N denotes the mean number of quanta of a heat bath.

In the case of single driving $\Omega_1\neq 0$ and $\Omega_2=0$, this model describes a dissipative DAO, which in the semiclassical approach and the steady-state regime exhibits bistability versus either the detuning Δ or the strength of driving Ω_1 [11]. However, the hysteresis of the mean oscillatory number $n(t)=\langle a^\dagger(t)a(t)\rangle$ is destroyed in the exact quantum-mechanical treatment as a consequence of ensemble averaging [16]. Nevertheless, hysteresis manifests itself on individual quantum trajectories as noise-induced transitions between two possible metastable states [21]. When two external forces are both present, we analyze the master equation numerically using the QSD method [17]. According to this method, the reduced density operator is calculated as the ensemble mean,

$$\rho(t)=M(|\Psi_\xi\rangle\langle\Psi_\xi|)=\lim_{N\to\infty}\frac{1}{N}\sum_\xi^N|\Psi_\xi(t)\rangle\langle\Psi_\xi(t)|, \tag{5}$$

over the stochastic pure states $|\Psi_\xi(t)\rangle$ describing the evolution along a quantum trajectory. The corresponding equation of motion is

$$|d\Psi_\xi\rangle=-\frac{i}{\hbar}(H_0+H_{int})|\Psi_\xi\rangle dt-\frac{1}{2}\sum_i(L_i^+L_i-2\langle L_i^+\rangle L_i +\langle L_i\rangle\langle L_i^+\rangle)|\Psi_\xi\rangle dt+\sum_i(L_i-\langle L_i\rangle)|\Psi_\xi\rangle d\xi_i. \tag{6}$$

Here ξ indicates the dependence on the stochastic process, the complex Wiener variables $d\xi_i$ satisfy the fundamental properties

$$M(d\xi_i)=0,\quad M(d\xi_i\,d\xi_j)=0,\quad M(d\xi_i\,d\xi_j^*)=\delta_{ij}\,dt, \tag{7}$$

and the expectation value $\langle L_i\rangle=\langle\Psi_\xi|L_i|\Psi_\xi\rangle$.

Below we shall give the results of numerical analysis of a dissipative anharmonic oscillator driven by two forces using an expansion of the state vector $|\Psi_\xi\rangle$ in Fock's number states of a harmonic oscillator:

$$|\Psi_\xi(t)\rangle=\sum_n a_n^{(\xi)}(t)|n\rangle. \tag{8}$$

In the following we consider the regime of strong anharmonicity $\chi/\gamma\sim 2$ and $\chi/\gamma\sim 0.7$, but not the strong coupling of AO to a reservoir, so that the Born-Markov approximation is valid. This regime is strongly quantum mechanical, as in the region of bistability, where SR occurs, the maximum number of oscillator states is less than 10.

Moreover, as we already noted, this regime is accessible on the practice. In the end of the section we briefly discuss this point, showing the conformity of the model proposed to the two above-mentioned physical schemes.

A. Anharmonic oscillator in a nonlinear cavity

It is well known that a single mode of the lossy cavity, involving third-order nonlinearity under coherent driving, presents an example of a dissipative DAO. In this model the anharmonicity of the mode dynamics comes from its self-phase modulation due to photon-photon interaction in the $\chi^{(3)}$ medium. Dissipative effects arise from the leakage of photons through the cavity mirrors, which damps the radiation field. It is easy to check that in the case of two classical driving fields this system is described by Hamiltonian (1), where operators a and $a^\dagger$ are annihilation and creation operators for the single-mode of the cavity at frequency ω_0, the photon-photon interaction term χ is proportional to the third-order nonlinear susceptibility $\chi^{(3)}$, and Ω_1 and Ω_2 are Rabi frequencies corresponding to two classical coherent fields, respectively. The damping of this oscillator with the cavity damping rate γ is described by the Lindblad operator [Eq. (4)], where N is the mean number of thermal reservoir photons.

We reiterate that the nonlinear cavity model described by the Hamiltonian of Eq. (1) in the case of one driving field was previously analyzed by several authors (see, for example, Refs. [11–16]). An important development in this area, including cavity QED, is the emerging capability for investigation both of atom-photon strong coupling and mode dynamics in a regime of strong photon-photon interaction (i.e., reduced γ and increased χ) [23]. In particular, Ref. [18] demonstrated a large resonant enhancement in nonlinearity for a low-density four-level atomic medium. A strong-coupling regime $\chi/\gamma=20$ was predicted for typical experimental parameters.

B. One-electron cyclotron oscillator in a Penning trap

As we noted, a single electron in a Penning trap was suggested as the realization of a quantum DAO interacting with a thermal reservoir [19–22]. Let us explain this statement in more detail. In fact, an electron stored in a Penning trap containing a magnetic field is a real quantum cyclotron oscillator, the anharmonicity of which comes from nonlinear effects that are caused by the relativistic motion of an electron in a trap. Furthermore, in such a system the dissipation effect arises from the spontaneous emission of the synchrotron radiation and thermal fluctuations of the cyclotron motion. The energy eigenstates of an AO are the number states, which are spaced in energy $E_{n+1}-E_n=\hbar[\omega_c-(n+1)\omega_e]$, where ω_c is the cyclotron frequency, ω_e is the relativistic anharmonic level shift, and $n=0,1,\ldots$. It was demonstrated that in the presence of an external periodic field, this system is described by a model of a dissipative DAO. The details of calculation can be found in Ref. [22]. The model we present here describes the interaction of a trapped electron moving in a magnetic field, with two coherent fields at different frequencies. The corresponding Hamiltonian is given by Eq. (1), where operators a and $a^\dagger$ describe cyclotron quantized motion at a modified cyclotron frequency ω_c. The parameter χ is the strength of the anharmonicity due to the relativistic effects, and Δ refers to the detuning between the eigenfrequency of the oscillator and the frequency of one of the driving fields. The parameters Ω_1 and Ω_2 characterize the amplitudes of the microwave driving fields. The interaction of the electron with its environment is the radiative coupling of the cyclotron motion with the thermal radiation field. Its radiative damping is described by the two Lindblad operators [Eqs. (4)] with the spontaneous decay rate of the cyclotron motion γ and the mean number of quanta N of the thermal radiation field. In principle, this scheme is easy to implement, as it generalizes a single-field case. It is important that a one-electron cyclotron oscillator allows one to achieve a relatively strong cubic nonlinearity $\chi/\gamma\gtrsim 1$. We use typical experimental values of the parameters to illustrate this property. It is easy to check that the nonlinear coupling χ in Hamiltonian (1) equals one-half of the relativistic anharmonic level shift, i.e., $\chi=\omega_e/2$. Although this shift is extremely small, $\omega_e/\omega_c=\hbar\omega_c/mc^2\simeq 10^{-9}$; however, since the spontaneous decay rates are also extremely slow, the transitions between lowest levels are well resolved with $\gamma/\omega_e \simeq 10^{-2}$. This gives an experimentally attainable value of $\chi/\gamma\simeq 50$.

III. CONTROLLING QUANTUM TRAJECTORIES AND QUANTUM STATISTICS

In this section we address the question of how SR is displayed in quantum trajectories in the pure quantum regime at a zero temperature of the heat bath [$N=0$, in the Lindblad operators (4)]. The difficulty in the realization of this process is obviously connected with the low level of quantum noise. To avoid this problem, we use a dissipative nonlinear system with a multiplicative noise, in which the noise level increases with the nonlinearity. In doing so, we first consider a model of DAO in the absence of periodic driving $\Omega_1\neq 0$ and $\Omega_2=0$, for the values of parameters Δ/γ and χ/γ, leading to bistable hysteresis depending on Ω_1/γ. The exact quantum analysis of this system is based on the well-known steady-state solution of the Fokker-Plank equation in the complex P representation [11]. Such a consideration leads, in particular, to the ensemble mean of the quantum-mechanical expectation number which is of interest here

$$\langle a^\dagger a\rangle=\frac{\Omega_1^2}{(\Delta+\chi)^2+(\gamma/2)^2}\,\frac{F(c+1,c^*+1,z)}{F(c,c^*,z)},\qquad(9)$$

where $F={}_0F_2$ is the generalized Gauss hypergeometric series:

$${}_0F_2(c,d,z)=\sum_{n=0}^{\infty}\frac{z^n}{n!}\,\frac{\Gamma(c)\Gamma(d)}{\Gamma(c+n)\Gamma(d+n)}.\qquad(10)$$

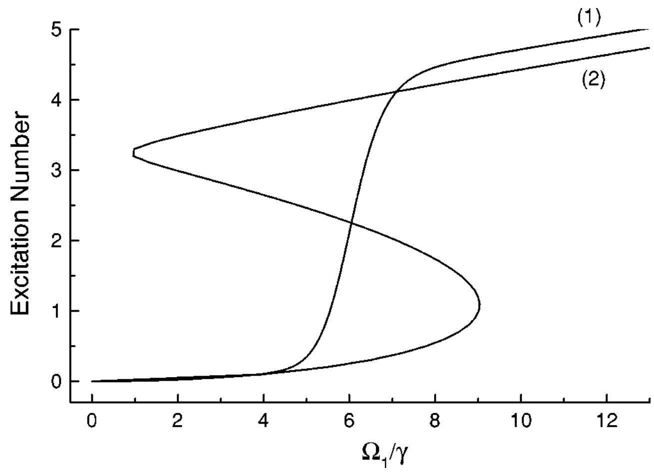

FIG. 1. Excitation numbers in quantum (1) and semiclassical (2) regimes for a single driven AO coupled with a vacuum reservoir vs the Rabi frequency, for the parameters $\chi/\gamma=2$ and $\Delta/\gamma=-15$.

The coefficients c and z depend on the physical parameters in the following ways: $c=(\Delta+\chi)/\chi-i\gamma/(2\chi)$ and $z=2(\Omega_1/\chi)^2$.

In the semiclassical limit and in the steady-state regime, the oscillatory excitation number $|\alpha|^2$, where α is the mean-field amplitude $\alpha=\langle a\rangle$, can be obtained by solving the equation

$$|\alpha|^2=\frac{\Omega_1^2}{(\gamma/2)^2+(\Delta+\chi+2\chi|\alpha|^2)^2}. \tag{11}$$

As shown in Ref. [21], the bistability and hysteresis behaviors take place for parameters satisfying the following conditions:

$$\chi(\Delta+\chi)<0,$$

$$\left|\frac{\Delta+\chi}{\gamma/2}\right|>\sqrt{3}, \tag{12}$$

$$\left[\frac{27\chi\Omega_1^2}{(\Delta+\chi)^3}+1+\left(\frac{3\gamma/2}{\Delta+\chi}\right)^2\right]^2<\left[1-3\left(\frac{\gamma/2}{\Delta+\chi}\right)^2\right]^3.$$

Outside of this range, the system has a monostable behavior.

We will use these results in order to choose parameters which are suitable for the SR process. Examples of Eqs. (9) and (11) for the discussed parameters are represented in Fig. 1. As we see, the quantum result does not show any hysteresis. The other peculiarity that can be concluded from results (9) and (11) is the increase of the quantum noise strength with the relative nonlinearity χ/γ. As a consequence, the characteristic threshold behavior, determined by a drastic increase of $\langle a^\dagger a\rangle$ in the transition region, disappears for large values of χ/γ. It is clear that this effect of the quantum noise increase will also be displayed in quantum trajectories.

Analyzing quantum trajectories, we set the system initially to the vacuum state of the corresponding harmonic oscillator, and integrate Eq. (6) for $\Omega_2=0$ and $N=0$ over a very long time, compared to the characteristic dissipative time $\sim 1/\gamma$. As expected, the analysis of the time-dependent stochastic trajectories for an expectation number $n_\xi(t)=\langle\Psi_\xi|a^\dagger a|\Psi_\xi\rangle$ shows that the system spends most of its time close to one of the semiclassical bistable solutions with quantum interstate transitions, occurring at random intervals. One of the possibilities for characterizing SR concerns statistical distributions of switching times of the stochastic process, including the residence-time distribution [3]. Here we concentrate on the escape-time distribution $P_e(t)$ of the time intervals it takes for the system to reach on upper state. The results are depicted on Fig. 2. In the absence of modulation forcing, $\Omega_2=0$, the distribution $P_e(t)$ has the expected exponential form [Fig. 2(a)], because quantum interstate transitions are statistically independent. The mean time of quantum noise-induced transitions follows from the formula

$$\overline{\tau}=\int_0^\infty \tau P_e(\tau)d\tau, \tag{13}$$

and equals $\overline{\tau}\simeq 314\gamma^{-1}$ for the parameters used. It is interesting to note that the mean time interval of the quantum transitions greatly exceeds the characteristic dissipation time $\sim\gamma^{-1}$ for $\chi/\gamma\lesssim 1$, and increases as this ratio decreases.

To observe SR phenomenon, we add a second driving force, which is kept weak enough ($\Omega_2\ll\Omega_1$) so that dynamical deterministic interstate transitions never occur, when the anharmonic oscillator is isolated from the bath. For convenience, we choose a Rabi frequency Ω_1 in the center of the bistability range shown on Fig. 1, i.e., $\Omega_1=5.8\gamma$. In this case the transition rates from one state to the other become approximately equal.

Returning to the full QSD equations with modulation terms [Eq. (6)], we assume that the period $T=2\pi/|\delta|$ is close to twice the mean time of the quantum transitions $2\overline{\tau}$ deduced from Fig. 2(a). This means that the frequency difference should be close to the characteristic frequency $\delta_{SR}=\pi/\overline{\tau}$, i.e., $\delta\simeq\delta_{SR}$. The numerical results for the $P_e(t)$ distribution obtained from an analysis of quantum trajectories are presented in Figs. 2(b) and 2(c). This simulation shows that in the presence of a modulation forcing $\Omega_2\neq 0$, distribution $P_e(t)$ exhibits a peak structure. The resonance condition is achieved by varying the modulation frequency δ, and a resonantlike process is identified with a separate peak of the distribution. This is the case when a maximal strength of the first peak is reached [Fig. 2(b)].

In Fig. 3 we show the effect of varying the modulation frequency δ in quantum trajectories for oscillatory excitation numbers $n_\xi(t)$. Here QSR is displayed as a strict synchronization of quantum trajectories, when the period of modulation of the driving force is close to $2\overline{\tau}$, i.e., $\delta\simeq\delta_{SR}=\pi/\overline{\tau}$. In this case the ensemble-averaged oscillatory number $\langle a^\dagger a\rangle$ exhibits a periodic modulation. The modulation is approximately sinusoidal with a period $2\pi/\delta$ of the driving force, as depicted in Fig. 4 (curve 1). At large and small values of δ compared with the characteristic frequency δ_{SR}, the synchronization of quantum trajectories is violated. This effect is well known for various nonlinear systems in the presence

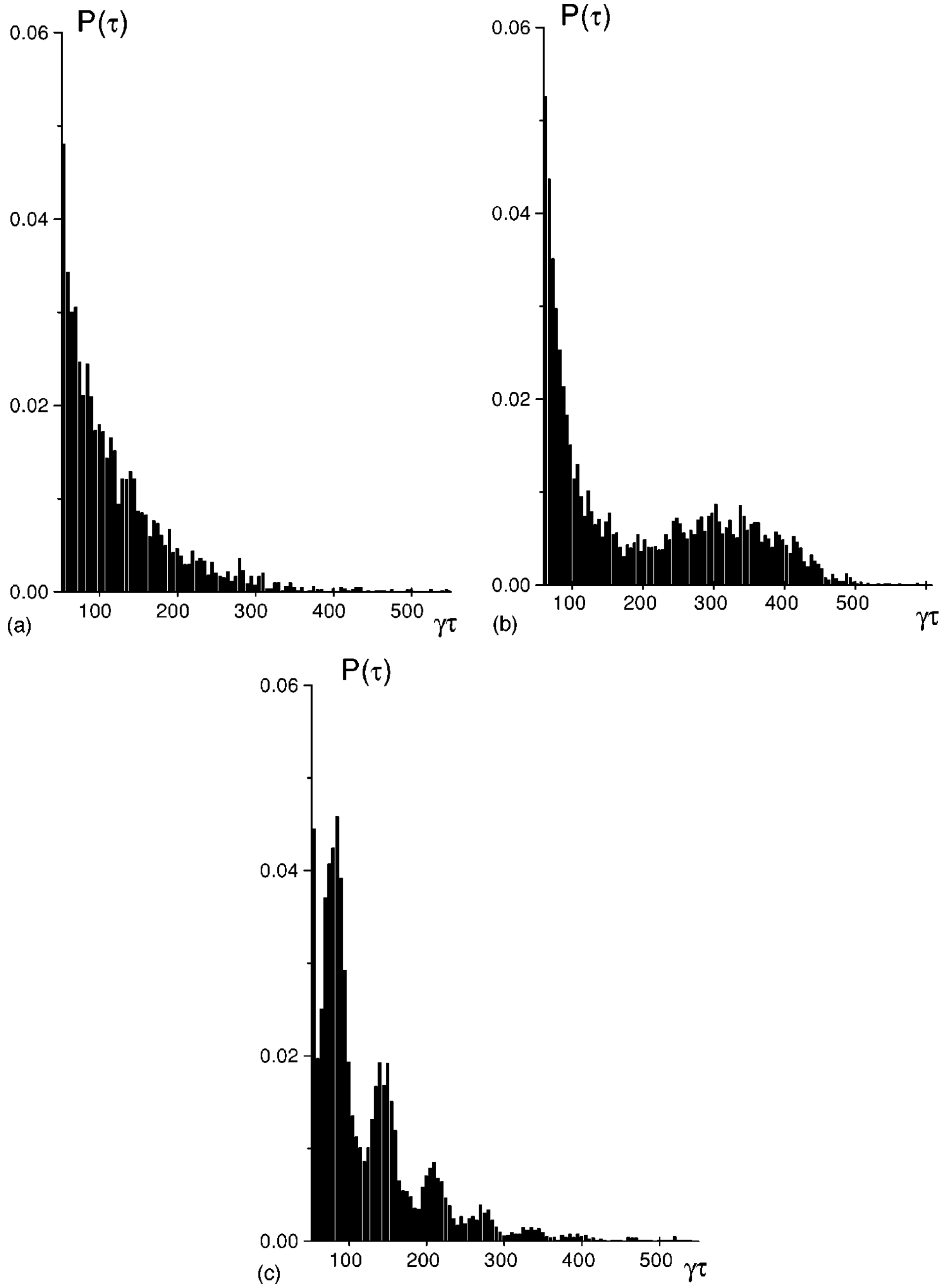

FIG. 2. Histograms for the residence times of the quantum jumps, (a) without time modulation; (b) with two drivings, provided that SR occurs, for $\delta=\delta_{SR}$; (c) with two drivings for $\delta=5\delta_{SR}$. The parameters are $\chi/\gamma=2$, $\Delta/\gamma=-15$, $\Omega_1/\gamma=5.8$, $\Omega_2/\gamma=1.2$, and $\delta_{SR}/\gamma=0.01$.

of thermal noise. The novelty here is that we demonstrate a manifestation of SR due only to quantum fluctuations. The obtained result can also be interpreted as controlling the stochastic dynamics of the quantum system by an external time-dependent force.

Other information about controlled stochastic dynamics can be obtained from an analysis of the quantum statistics of elementary oscillatory excitations. This decision can be made with the help of the Fano factor, which describes the excitation number uncertainty and is equal to the variance $\langle(\Delta n)^2\rangle=\langle(a^\dagger a)^2\rangle-\langle a^\dagger a\rangle^2$, normalized to the level of fluctuations for coherent states of the harmonic oscillator $\langle n\rangle=\langle a^\dagger a\rangle$, i.e.,

$$F=\frac{\langle(\Delta n)^2\rangle}{\langle n\rangle}. \tag{14}$$

The Fano factor is larger than unity if the statistics of elementary excitations are super-Poissonian, equal to unity if the statistics are Poissonian, and smaller than unity if the statistics are sub-Poissonian.

The means over an ensemble of QSD trajectories is calculated in formulas (5)–(8). In particular, for the variance, this method gives

$$\langle(\Delta n)^2\rangle=\lim_{N\to\infty}\frac{1}{N}\sum_{\xi}^{N}\left[\langle\Psi_\xi|(a^\dagger a)^2|\Psi_\xi\rangle - |\langle\Psi_\xi|(a^\dagger a)|\Psi_\xi\rangle|^2\right]. \tag{15}$$

The result for the time evolution of the Fano factor averaged over quantum trajectories is depicted in Fig. 4 (curve 2) for parameters leading to the SR-like process. In this case the

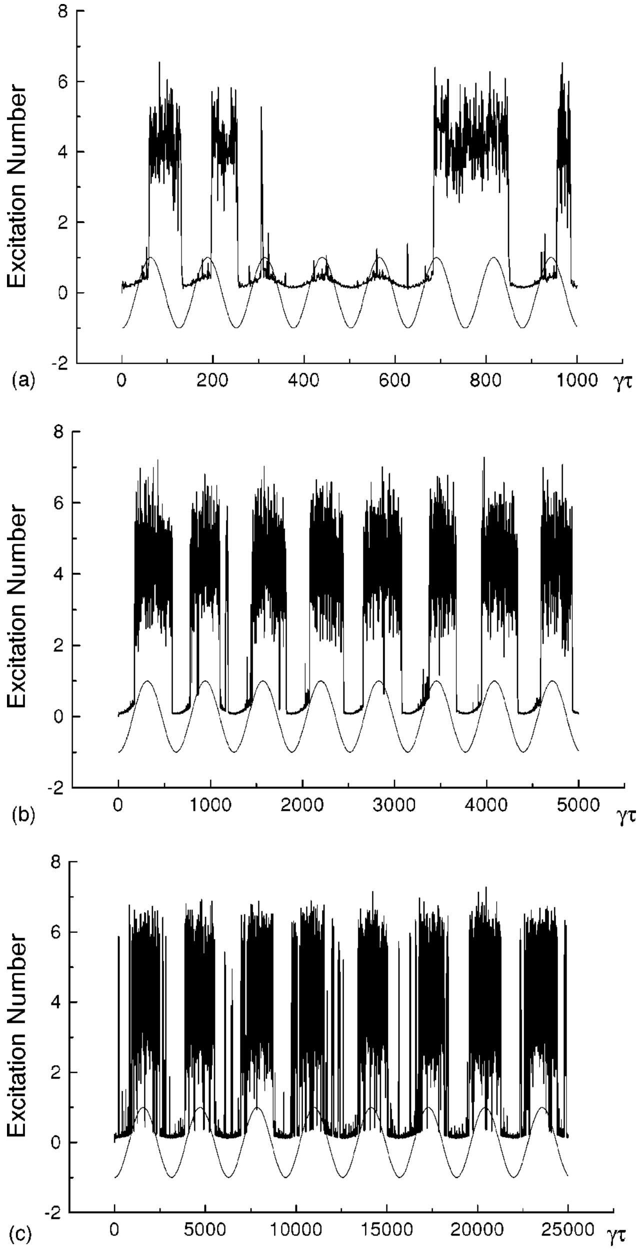

FIG. 3. Single quantum trajectories for a doubly driven dissipative AO for decreasing δ: (a) $\delta=5\delta_{SR}$, (b) $\delta=\delta_{SR}$, and (c) $\delta=0.2\delta_{SR}$. The parameters are $\chi/\gamma=2$, $\Delta/\gamma=-15$, $\Omega_1/\gamma=5.8$, $\Omega_2/\gamma=1.2$, and $\delta_{SR}/\gamma=0.01$. The time-dependent driving on the modulation frequency δ, shown below trajectories, is in arbitrary units.

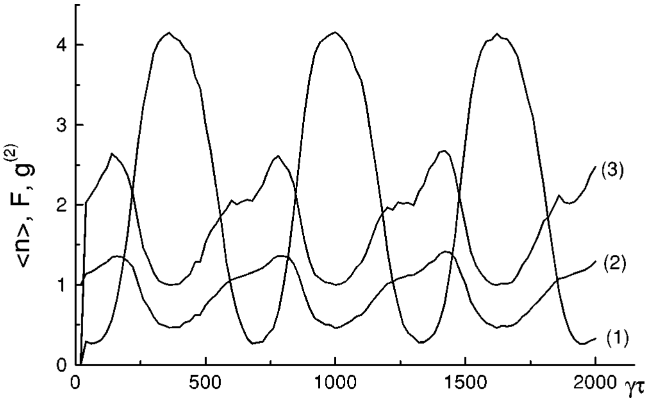

FIG. 4. Time evolution of the ensemble-averaged photon number (curve 1) and Fano factor (curve 2) for a doubly driven AO, coupled with a vacuum reservoir. The parameters are $\chi/\gamma=2$, $\Delta/\gamma=-15$, $\Omega_1/\gamma=5.8$, $\Omega_2/\gamma=1.2$, and $\delta_{SR}/\gamma=0.01$. The averaging is over 1500 trajectories.

Fano factor also shows a time-dependent modulation, with the phase shifted approximately on π relative to the excitation number curve [Fig. 4 (curve 1)]. Surprisingly, the oscillatory excitation number fluctuations of our DAO model are squeezed below the coherent level $\langle(\Delta n)^2\rangle<\langle n\rangle$. This nonclassical effect of reduction of quantum fluctuations, i.e., the formation of sub-Poissonian statistics, occurs for strongly definite time intervals $t=360\gamma^{-1},1000\gamma^{-1},\ldots,$ when the mean oscillatory number reaches its maximal values. The minimum of the Fano factor [Eq. (14)] is $F\simeq0.46$.

It is quite obvious to explain super-Poissonian statistics with a physical mechanism leading to elementary excitations of the DAO. In analogy with quantum-electromagnetic processes, where super-Poissonian photon statistics is stipulated by two-photon emissions into the resonance fluorescence [24], we connect the super-Poissonian statistics here with the contribution of the two-boson excitation process in our model. The contribution of these processes will be dominant in the second-order correlation function $g^{(2)}(\tau)$. In order to elucidate this situation, analogous calculations are performed for the normalized second-order correlation function $g^{(2)}=\langle a^\dagger a^\dagger aa\rangle/\langle a^\dagger a\rangle^2$ for zero delay time, i.e., $\tau=0$. The result is shown in Fig. 4 (curve 3) for the same parameters as for the photon number and Fano factor in Fig. 4 (curves 1,2). As we see, the correlation function of the states of anharmonic oscillator exhibits both antibunching ($g^{(2)}<1$) and superbunching ($g^{(2)}>2$) effects, which alternate with one another for definite time intervals. It is shown that the anticorrelation of excitations ($g^{(2)}<1$) arises for time intervals where $F<1$. For other times, the correlation function reaches its maximal value $g^{(2)}\simeq2.67$, if the Fano factor is equal to $F\simeq1.42$. The large superbunching effect is the result of super-Poissonian statistics of the DAO, and can be understood only by the quantum nature of oscillatory excitations. Note that these conclusions are in conformity with the formula $F=\langle n\rangle(g^{(2)}-1)$, connecting the Fano factor and the correlation function.

Interestingly, there is a possibility to change the statistics of oscillatory excitation from super- to sub-Poissonian periodically depending on the parameters of the driving forces. Thus one of the interesting conclusions which can be made from these studies is the possibility of controlling the quantum dynamics, as well as the quantum statistics, of a dissipative system by an external time-dependent force.

For the purpose of illustration we studied the QSR phenomenon for very slow oscillations at a frequency difference δ in comparison with the relaxation rate, i.e., $\delta\ll\gamma$, and for strong anharmonicity $\chi/\gamma=2$. However, analogous results can be obtained for other ranges of the parameters: χ/γ, Δ/γ, Ω_1/γ, Ω_2/γ, and δ/γ. As mentioned above, the applicability of this model for a realization of SR in a pure quantum regime is restricted only to the case of strong anharmonicity.

IV. ENTROPIC DESCRIPTION OF QSR

The challenge we want to address next concerns the quantum information aspects of the problem. Information-theory concepts, such as entropy and mutual information, were previously used in the study of SR [25]. We expand these studies, and present quantitative analysis of SR in the quantum range on the basis of von Neuman entropy. This quantity is a measure of dissipation, quantum entanglement, and the purity of quantum states [26], and is defined through the reduced density operator as

$$S=-\mathrm{Tr}(\rho\ln\rho). \tag{16}$$

In doing this, we consider the case of $T\neq 0$ temperature of the reservoir, but in a deep quantum regime, when the mean photon number of the reservoir is smaller than the mean oscillatory number $N\ll 1$, $N\ll\langle n\rangle$. The stochastic resonance condition in this case can be achieved by varying either the frequency δ or the temperature noise intensity N. We expect to manifest QSR as an optimal ordering degree of the system, due to its quantum evolution and quantum diffusion processes. We calculate the time evolution of the entropy by formula (16) for different levels of noise, using the results for the reduced density matrix, expressed by the ensemble of the trajectories. The calculations are performed by a diagonalization of the matrix $\rho_{nm}=\langle n|\rho(t)|m\rangle$ in the Fock-state basis. This simulation shows that for times t, larger than the time scale of the transient dynamics, the entropy $S(N,\Omega_1,\Omega_2)$ of the full system acquires the periodicity of the external driving Ω_2. As expected, the entropy exhibits maximum deviations from its time-independent value $S(N,\Omega_1,0)$ at an optimal noise pump rate. The entropy $S(N,\Omega_1,0)$ corresponds to the usual anharmonic oscillator driven by the single force $\Omega_1\neq 0$, $\Omega_2=0$. Both entropies $S(N,\Omega_1,\Omega_2)$ and $S(N,\Omega_1,0)$ depend on the system's parameters χ/γ and Δ/γ, and increase with intensity of external noise N. To clearly identify QSR, we demonstrate the numerical results for the quantum conditional entropy, which is defined as the difference

$$S(N,\Omega_1|\Omega_2)=S(N,\Omega_1,\Omega_2)-S(N,\Omega_1,0), \tag{17}$$

and we express the residual information of the system. The time evolution of the conditional entropy, starting from time intervals exceeding the transient regime, i.e., $t\gamma\gtrsim 50$, is presented in Fig. 5 for three values of N.

We observe that the quantum entropy of a subsystem [a dissipative anharmonic oscillator with no modulation force ($\Omega_1\neq 0$, $\Omega_2=0$)], dominates over the entropy of a full system, and therefore, the quantum conditional entropy is negative. Such a behavior indicates the effective time ordering of the dissipative AO by an external time-dependent force. The small irregular positive regions of $S(N,\Omega_1|\Omega_2)$ appear to be due to an insufficient number of averaged stochastic trajectories. For simplicity, we show only two periods of entropy

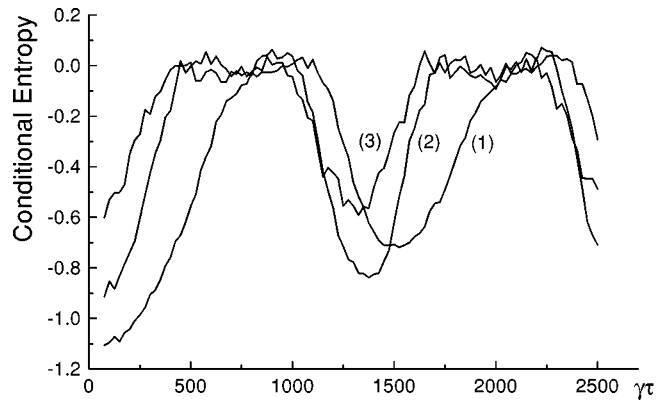

FIG. 5. Conditional von Neumann entropy in the presence of external noise for a doubly driven AO, coupled with thermal reservoir: (curve 1) $N=0.1$, (curve 2) $N=0.3$, and (curve 3) $N=0.6$. SR is realized for $N=0.3$. The parameters are $\chi/\gamma=0.7$, $\Delta/\gamma=-15$, $\Omega_1/\gamma=6.8$, $\Omega_2/\gamma=1.2$, and $\delta_{SR}/\gamma=0.005$. The averaging is over 1500 trajectories.

oscillations. Here QSR is observed at $N=0.3$ as the conditional entropy with minimum value [Fig. 5 (curve 2)].

It should be noted that the time ordering of the dissipative, stochastic dynamics by the external periodic force is displayed as a synchronization of quantum trajectories in the framework of the QSD approach. We have above demonstrated this effect in the pure quantum regime of a DAO. However, it is easy to check that such synchronization takes place much more often in the presence of thermal noise. Below, we show the other interesting consequence of the synchronization of quantum trajectories, which concerns the excitation number uncertainty. For this goal the mean values $\langle(\Delta n)^2\rangle$ and $\langle n\rangle$ are calculated by averaging over an ensemble of 4000 trajectories. In Fig. 6 we report the time evolution of the Fano factor for three values of N, which is increased from low values up to large values, crossing the resonance value $N=0.3$. These results, when plotted in dependence on time, display the periodic behavior of the Fano

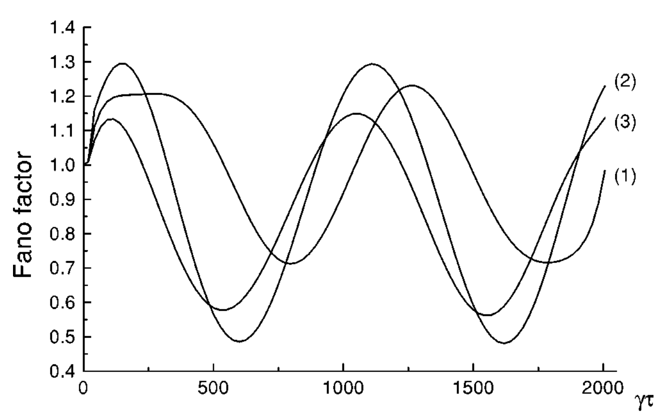

FIG. 6. Time evolution of an ensemble-averaged Fano factor of a doubly driven AO coupled with a thermal reservoir ($T\neq 0$). The effect of varying the external noise strength are (curve 1) $N=0.1$, (curve 2) $N=0.3$, and (curve 3) $N=0.6$. The parameters are $\chi/\gamma=0.7$, $\Delta/\gamma=-15$, $\Omega_1/\gamma=6.8$, $\Omega_2/\gamma=1.2$, and $\delta_{SR}/\gamma=0.005$.

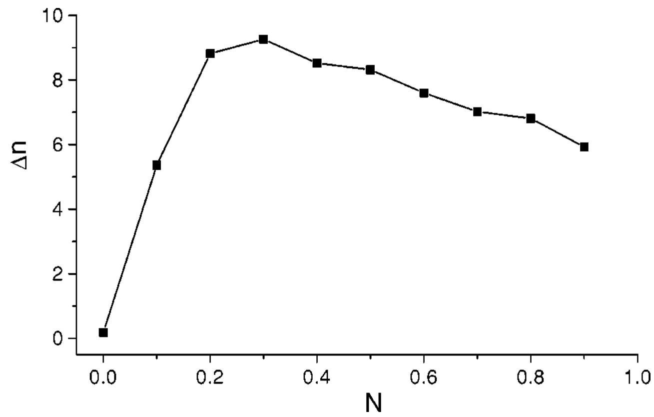

FIG. 7. The stochastic resonant behavior of $\Delta n = \langle n \rangle_{\max} - \langle n \rangle_{\min}$ vs the noise strength for the parameters in Fig. 6.

factor. The modulation with maximum and minimum values of F, which are equal to $F_{\max}=1.29$ and $F_{\min}=0.48$, is reached when the resonance condition is achieved. The periodic modulation also exhibits the ensemble-averaged oscillatory number $\langle n \rangle$. We have not shown this result; however, in Fig. 7 we plot, for an illustration, the difference between maximal and minimal values of $\langle n \rangle$ during one period of modulation, i.e., the quantity $\Delta n = \langle n \rangle_{\max} - \langle n \rangle_{\min}$, versus the external noise strength N. This result shows the stochastic resonance behavior.

The analysis above also indicates that it is possible to control the behavior of a quantum system by an external time-dependent force even when the system is coupled to a thermal reservoir. In this spirit, we emphasize that the idea of controlling the dynamics of a quantum system in the presence of dissipation and decoherence by an external periodic driving was exploited by many authors (see, for example, Refs. [27–33]). We note the effect of coherent destruction of tunneling [27], the control of decoherence and relaxation of frequency modulation of heat bath [28], examples of long-lived Schrödinger cat states in the context of superradiance [29], the suppression of decoherence by the specific sequences of radio-frequency pulses [30,31] and dynamical localization for two-level atom interacting with a standing wave [32,33]. In addition to these results, here we demonstrated the possibility of controlling the dynamics as well as the statistics and entropy of a nonlinear dissipative oscillator driven by two forces.

V. CONCLUSION

Our work demonstrates that the SR phenomenon can be realized in a pure quantum regime for a model of an anharmonic oscillator driven by two periodic forces. We are sure that the possibility of QSR, as a result of correlation between quantum noise and nonlinear evolution, contains a potential for applications. As an illustration of the possible potential of this model, we have demonstrated the synchronization of quantum trajectories by SR which leads to the controlling evolution of the quantum open system. From the perspective of quantum optics, the investigations suggest the nonclassical effect of sub- and super-Poissonian statistics of the oscillatory excitation number. These effects become maximal when SR occurs. We have also described the SR phenomenon from the point of view of quantum information on the basis of von Neuman entropy. The QSR phenomenon was illustrated in the regime of strong anharmonicity when the anharmonicity parameter is comparable to a dissipative decay rate. In addition to the fundamental interest in the exploration of the QSD method for an estimation of QSR, the investigation of a double driven anharmonic oscillator seems interesting for various applications. Although the primary motivation for this study was theoretical, results may be observable experimentally. As candidates we suggest two experimental schemes, which operate with a nonlinear cavity or a single electron in a Penning trap.

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