Photon statistics of a ground-state-pumped laser

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A model of three-level laser, in which the pumping excites the atoms from the lower lasing level, is considered. The statistical properties of the field are calculated using full quantum treatment. It is shown that apart from threshold there is additional critical value of the pump rate, at which the laser field behaves in a phase-transition-like manner: the photon number drops down, the Fano factor has a strong peak, the relative photon number variance grows enormously, and the second-order coherence function jumps to 2, which is typical for thermal light.

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Lasers with incoherent pumping out of the lower lasing level can switch off when the pump rate is large enough [1–3]. This phenomenon was first predicted by Mu and Savage [1] who showed that it is possible in both one atom and multiatom lasers. We refer to such laser schemes as V-type lasers, as distinct from Λ schemes, in which the pumping excites the atoms from a nonlasing level. The comparison of the dynamical behavior of Λ- and V- type lasers has been done in Ref. [2]. In this paper we present the results of calculation of fluctuation properties of V-type lasers below, near, and above threshold. The calculations are based on the full quantum treatment approach developed in Refs. [4] and [5]. The relationship between three statistical parameters, the Fano factor, the second-order correlation function $G^{(2)}$, and the relative variance, as well as their relation to the minimization principle [5], are discussed.

We study the system of three-level atoms interacting with a single mode of electromagnetic field inside an optical cavity. In the V scheme shown in Fig. 1 the pumping excites the atom, with the rate γ_{02} from the lower lasing state $|0\rangle$, which is the ground state in this case, to the excited state $|2\rangle$, which is then depleted with the decay rate γ_{21} to the upper lasing state $|1\rangle$. The main feature of such a scheme is that the pumping is used to excite the lower lasing state $|0\rangle$ directly. It is this property of the V-type schemes that makes their dynamic behavior crucially different from that of the Λ-type schemes.

The interaction Hamiltonian, in the rotating-wave approximation, has the following standard form

$$H=\sum_{k=1}^{N} g(a^{\dagger}\sigma_{10}^{(k)}+\sigma_{01}^{(k)}a), \tag{1}$$

where g is the atom-cavity coupling constant, $a^{\dagger}$ and a are the photon creation and annihilation operators, $\sigma_{ij}^{(k)}=|j\rangle\langle i|$ are the atomic rising and lowering operators, and N is the total number of atoms. The atoms and the field are coupled to their reserviors. The Master equation for the atom-field density matrix ρ reads

$$\frac{\partial\rho}{\partial t}=(\Lambda_f+\Lambda_a-iL)\rho, \tag{2}$$

$$\Lambda_f\rho=\kappa([a\rho,a^{\dagger}]+[a,\rho a^{\dagger}]), \tag{3}$$

$$\Lambda_a\rho=\frac{1}{2}\sum_{k=1}^{N}\left(\sum_{i\neq j=0}^{2}\gamma_{ij}([\sigma_{ij}\rho,\sigma_{ji}]+[\sigma_{ij},\rho\sigma_{ji}]) + \sum_{i>j=0}^{2}\eta_{ij}([\sigma_{ij}^{3}\rho,\sigma_{ij}^{3}]+[\sigma_{ij}^{3},\rho\sigma_{ij}^{3}])\right), \tag{4}$$

Where $L\rho=[H,\rho]$, κ is the cavity decay rate, $\sigma_{ij}^{3}=\frac{1}{2}(\sigma_{ij}\sigma_{ji}-\sigma_{ji}\sigma_{ij})$, γ_{ij} is either the rate of spontaneous emission into nonlasing modes or the pump rate, and η_{ij} is the rate of collisional or/and reservior-induced dephasing at $i\rightarrow j$ transition.

From the Master equation (2) one can derive the Fokker-Planck equation for the field density matrix (see Refs. [4] and [5]), which allows one to calculate all statistical characteristics of the intracavity field. In this paper we present the results of an analytical solution of the Fokker-Planck equation in graphic format. The following statistical characteristics are studied: the Fano factor $F\equiv(\langle n^2\rangle-\langle n\rangle^2)/\langle n\rangle$, the photon number variance $V\equiv(\langle n^2\rangle-\langle n\rangle^2)/\langle n\rangle^2=F/\langle n\rangle$, and the second-order correlation function $G^{(2)}\equiv\langle a^{\dagger}a^{\dagger}aa\rangle/\langle a^{\dagger}a\rangle^2=1+(F-1)/\langle n\rangle$. The first two values, the Fano factor F and the variance V, are obtained from the same photon counting experiment and differ from each other by a factor of $\langle n\rangle$ only. However this differnce is essential for physical interpretation, as will be seen later. The correlation function $G^{(2)}$, though related to the Fano factor, is measured in a completely different experiment (see, for example, Ref. [6]) and is a measure of second-order coherence of optical fields.

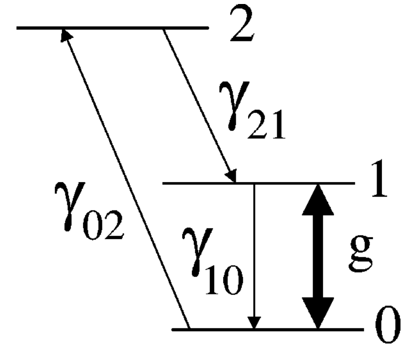

FIG. 1. The scheme of levels for V type laser.

Both the dynamics and the statistics of the laser field depend crucially on the fraction β of spontaneously emitted photons directed into the lasing mode, which can be defined as follows [3,7]

$$\beta = \frac{2g^2/(\gamma_\perp + \kappa)}{2g^2/(\gamma_\perp + \kappa) + \gamma_{10}}, \tag{5}$$

where

$$\gamma_\perp = \frac{1}{2}(\gamma_{10} + \gamma_{02} + \eta_{10}). \tag{6}$$

It is this spontaneous emission factor β that controls the value of the laser threshold [2,3,7]. It is convenient to express all the values in terms of few dimensionless parameters

$$\lambda = \frac{2\kappa}{\gamma_{21}}, s = \left(\frac{\gamma_{21}}{2g}\right)^2, p = \frac{\gamma_{02}}{\gamma_{21}}, \varepsilon = \frac{\gamma_{10}}{\gamma_{21}}, \mu = \frac{\eta_{10}}{\gamma_{21}}. \tag{7}$$

Then the spontaneous emission factor β becomes

$$\beta = [1 + \varepsilon s(p + \varepsilon + \mu + \lambda)]^{-1}. \tag{8}$$

Notice that the β factor depends upon the pump parameter p, namely, it decreases with pump. This dependence gives rise to the appearance of a second critical point at which lasing ceases [2].

In Fig. 2 we reproduce, for the comparison convenience, the results obtained earlier [2] for the dynamical values of the photon number $\langle n \rangle$ and the inversion $\langle d \rangle$. Both plots show clearly the existence of two critical points. If the rate of spontaneous emission out of lasing mode ε is neither too small nor too large, the photon number curves have a clear kink at threshold. The inversion becomes about zero above threshold, which is caused by the strong-field mixing of the atomic levels at lasing transition. If the spontaneous emission rate ε is small enough, so that the β factor is close to 1, the threshold becomes less clear and eventually disappears [see curves i and ii in Fig. 2(a)], i.e., the laser becomes thresholdless [1–3,7]. As the pump rate increases the photon number reaches its maximum value and then starts to decrease. Eventually, when the pump rate approaches the second critical point, the photon number drops down to the values less than 1 indicating that lasing ceases [1,2]. At that point the inversion grows to about 1 showing that most of the atoms occupy the upper lasing level and cannot emit photons into the lasing mode because of very small polarization. If the rate of spontaneous emission out of lasing mode ε is large enough, so that the β factor is less than some critical value [2], the laser does not operate as a laser.

Approximate expressions for the two critical values of the pump rate, threshold and turn-off points, can be calculated using semiclassical equations, as it has been done in Ref. [2]

$$p_{thr} = \varepsilon[1 + \delta(2+\varepsilon)(2\varepsilon + \mu)], \tag{9}$$

$$p_{off} = \frac{1}{\delta(1+\varepsilon)} - \frac{\mu + \varepsilon(\mu+3) + 2\varepsilon^2}{1+\varepsilon}. \tag{10}$$

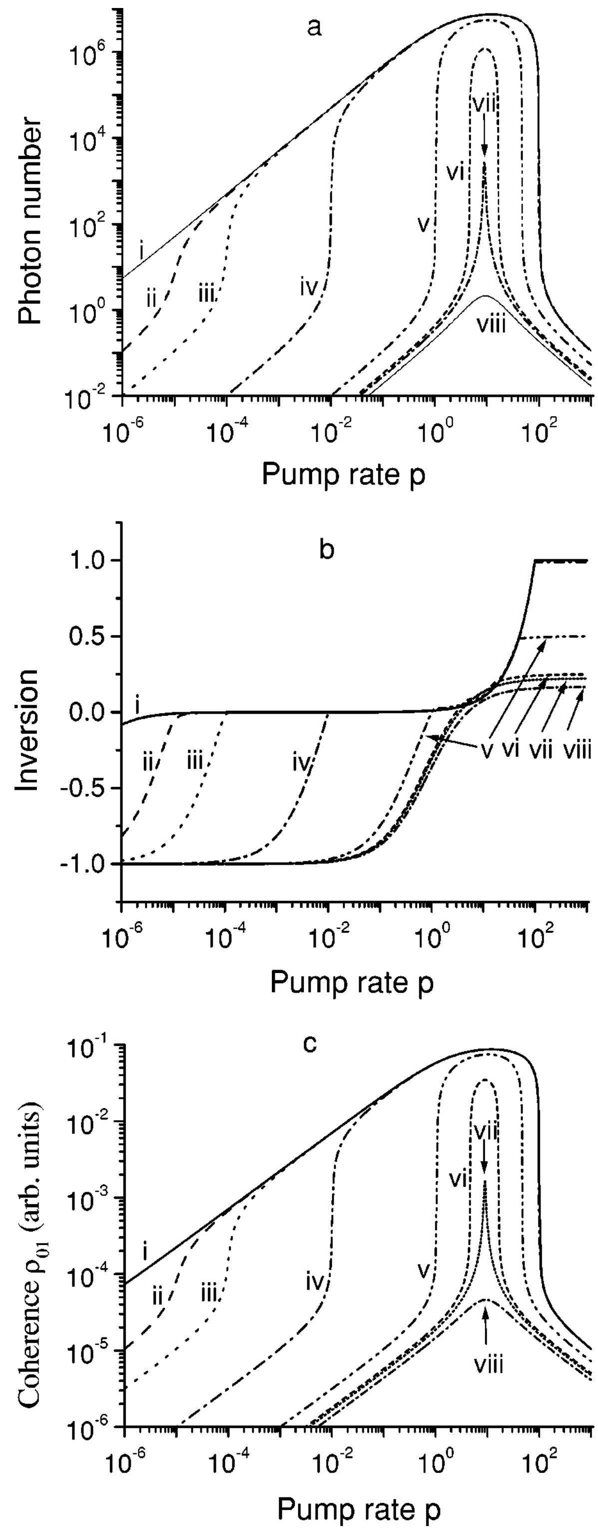

FIG. 2. Photon number (a) and population inversion (b) vs pumping rate p for various values of ε. $\lambda = 0.01$, $s = 10^5$, $N = 10^5$, $\mu = 0$, $\varepsilon = 0$ (i), 10^{-5} (ii), 10^{-4} (iii), 10^{-2} (iv), 1 (v), 3 (vi), 3.5 (vii), and 5 (viii).

These expressions are valid as long as $\delta \equiv \lambda s/N \ll 1$. One can see from Eqs. (9) and (10) that increasing the spontaneous emission rate ε gives rise to both increasing the threshold value and decreasing the pump rate at which the laser

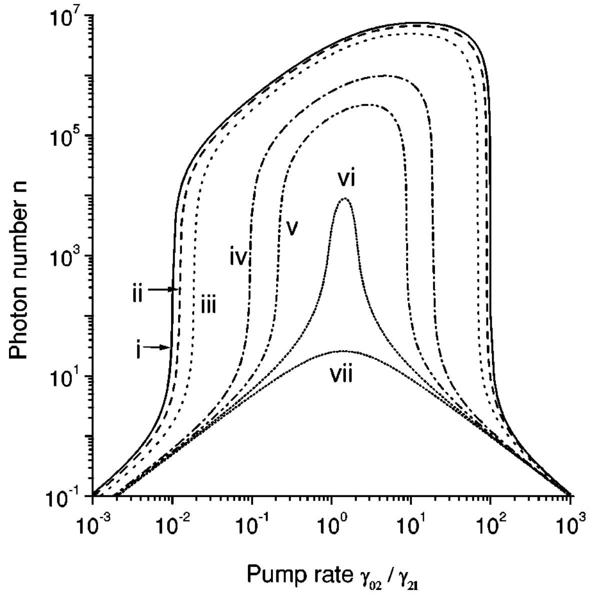

FIG. 3. Photon number vs pumping rate p for various values of scaled dephasing rate μ. $\lambda=0.01$, $s=10^5$, $N=10^5$, $\varepsilon=0.01$, $\mu=0$ (i), 10 (ii), 30 (iii), 80 (iv), 90 (v), 96 (vi), and 100 (vii).

turns off, thus resulting in narrowing the range of laser operation. Eventually the two critical points degenerate into a single point, and lasing cannot be obtained at larger ε.

It follows from Eqs. (9) and (10) that the effect of phase-destruction processes to the system behavior is similar to the effect of spontaneous emission. The difference in their effect is that the dephasing rate is allowed to be much larger than the spontaneous emission rate. To illustrate these points the photon number is plotted in Fig. 3 as a function of the pump rate p for different values of the dephasing rate μ.

Three statistical characteristics of the field, the Fano factor F, the second-order correlation function $G^{(2)}$, and the relative variance V, are plotted in Fig. 4 as functions of the pump rate. Again one can notice the same critical points as in Figs. 2 and 3. The Fano factor has two clear peaks unless the spontaneous emission rate ε is either too low or too high. These peaks indicate a phase-transition-like behavior at both threshold and the second critical point when lasing ceases. As ε increases, the two peaks become closer to each other [compare curves iv, v, and vi in Fig. 4(a)], then degenerate into a single peak (curve vii), and finally disappear in non-lasing regime (curve viii). The second-order correlation function is shown if Fig. 4(b). It demonstrates that the field turns from incoherent below threshold, where $G^{(2)}=2$, into coherent one above threshold, where $G^{(2)}=1$. The reverse transition happens at the second critical point, where the field becomes incoherent again. One can see from Fig. 4(b) that as the rate of spontaneous emission out of lasing mode ε increases, the range of pump rates at which the field is coherent becomes narrower. If all spontaneously emitted photons are directed into the lasing mode so that $\varepsilon=0$ and hence $\beta=1$, the laser becomes thresholdless, which is illustrated by curves i in Fig. 4.

The essential reason of existence of the second phase-transition point is dependence of the transverse relaxation

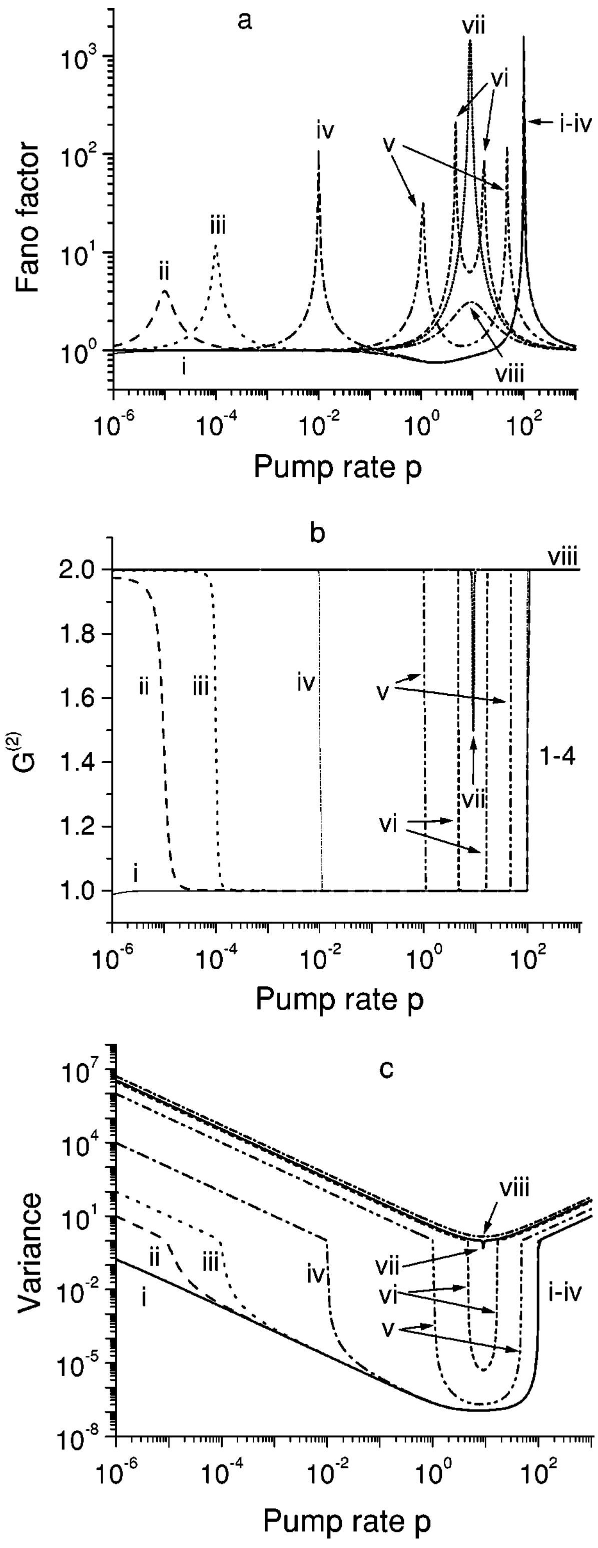

FIG. 4. Fano factor F (a), second-order correlation function $G^{(2)}$ (b), and relative variance V (c) vs pumping rate p for various values of ε. The parameters are the same as in Fig. 2.

rate $\gamma_\perp$, and hence of the spontaneous emission factor β, upon the pump rate, as discussed in Refs [1–3]. When pumping becomes strong enough it destroys atomic polarization, not allowing the efficient interaction between the atoms and the field.

It is interesting to compare the three statistical characteristics, the Fano factor F, the second-order coherence $G^{(2)}$, and relative variance V from the point of view of the information about the laser field they provide. Such a comparison can be done considering, for example, the curves iv in Fig. 4. All three graphs show clearly phase-transition-like behavior at the two critical points. The Fano factor has a well-known minimum [5,8] $F=0.75$ at $p=2$, or $\gamma_{02}=2\gamma_{21}$, which corresponds to maximal noise reduction described by the minimization principle [5]. At that point the photon statistics are sub-Poissonian and the laser light is squeezed. Neither the $G^{(2)}$ function nor the variance V are sensitive to this quantumness of the field. Moreover, calculating the coherence function for the number state $|n\rangle$ leads to $G^{(2)}=1-1/\langle n\rangle$, which means that at very large photon numbers, one cannot distinguish between the Fock state and the coherent state by means of the $G^{(2)}$ function. However both the Fano factor F and the variance V are equal to zero in the Fock state, which is thus a purely quantum state. On the other hand, both below and far above threshold we have $F\approx 1$, so the Fano factor does not allow one to distinguish between incoherent light below threshold and coherent one above threshold. In contrast, the coherence function, which is a measure of second-order coherence by its definition [6], demonstrates clearly the difference between these two regimes, indeed $G^{(2)}=2$ below threshold and $G^{(2)}=1$ above threshold.

In summary, quantum statistical properties of three-level lasers, with pumping out of the lower lasing level, have been considered. It has been shown that the statistical characteristics of laser field reveal phase-transition-like behavior at two critical points, one of which is threshold while another one is the switching off point. We have also demonstrated mutual complementarity of three statistical parameters, the Fano factor, the second-order coherence function, and the relative variance, which being measured in different experiments, provide us with different types of information about laser fields.

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