Quantum teleportation and dense coding by means of bright amplitude-squeezed light and direct measurement of a Bell state

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Simple schemes to realize quantum teleportation and dense coding for continuous variables are proposed, in that entangled EPR beams are produced by combining two bright amplitude squeezed beams. The measurement of the "Bell state" is accomplished by means of direct detection for photocurrents and two rf power splitters. As a local oscillator and balanced homodyne detector are not needed, the proposed system is easy to realize experimentally.

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Recent theoretical and experimental studies on quantum communication through the use of nonlocal quantum correlations of entangled states show great promise for establishing new kinds of information processing with no classical counterparts. One example of this is the concept of quantum teleportation which is the disembodied transport of an unknown quantum state from one place (Alice) to another (Bob). Teleportation was originally proposed for discrete variables in finite-dimensional Hilbert spaces [1], and later also for continuous variables in infinite-dimensional Hilbert spaces [2]. Discrete and continuous variable teleportations have been performed experimentally for single-photon polarization states [3] and coherent state of electromagnetic field modes [4], respectively. In the experiment of Refs. [4,5], an entanglement source was built from two single-mode phase squeezed vacuum states combined at a beamsplitter. The "Bell-state" measurement at Alice needs two sets of balanced homodyne detectors and local oscillators (LO's). Another scheme proposed in Ref. [6] is a similar arrangement in which two bright squeezed sources were used to produce the EPR beams, and a LO was needed at Alice for the Bell-state measurement. Another example of the application of entanglement to communication is dense coding, in that the single bit sent from Alice to Bob can successfully carry two bits of classical information if an entanglement pair of qubits was already shared previously by Alice and Bob [7]. The dense coding doubles the information capacity, and it is especially useful in some instances, such as when two bits must be simultaneously sent but only one quantum channel is available.

It is a common pursuit in scientific and technical fields to find a more easily realizable scheme for quantum teleportation and dense coding. In this paper, we consider a teleportation scheme similar to that in Ref. [6] but using a simple Bell-state measurement at Alice instead of the typical two balanced homodyne detectors. One of the EPR beams is phase shifted $\pi/2$, and then combined with an input beam of the same intensity on a 50% beamsplitter. The amplitude and phase quadrature are obtained from a direct detection implemented with two photodetectors and two rf splitters; therefore, a local oscillator is not needed. A realizable experimental system to achieve continuous-variable teleportation by means of bright EPR beams is described. Then a dense coding for a continuous variable based on using bright EPR beams and a direct detection of the Bell state is discussed. The transmitted phase and amplitude signals are modulated on the phase and amplitude quadrature of one of the EPR beams, respectively. Bob combines it with the other half of the EPR beam, and retrieves the original classical signal with a precision beyond the standard quantum-limit (SQL) by a direct detection of the Bell state. This scheme is actually equal to dense coding for discrete variables [7]. The dense coding capacity approaches twice that of coherent- and squeezed-state communications.

The schematic diagram for the direct measurement of the Bell state is shown in Fig. 1. Two bright coherently related amplitude squeezed beams with almost identical intensities are expressed by the annihilation operators a and b. A phase shift of $\pi/2$ is imposed on beam b, and then the beams are mixed on a 50% beamsplitter. The resulting output beams c and d are given by

$$c=\frac{\sqrt{2}}{2}(a+ib), \quad d=\frac{\sqrt{2}}{2}(a-ib). \tag{1}$$

We define upper-case operators in the rotating frame about the center frequency ω_0,

$$O(t)=o(t)e^{i\omega_0 t}, \tag{2}$$

with $O=[a,b,c,d]$ and $o=[A,B,C,D]$. By the Fourier transformation we have

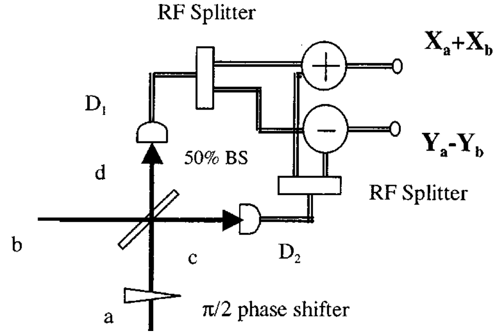

FIG. 1. Direct measurement of the Bell state. BS: 50% beam splitter. D_1 and D_2: detectors.

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$$O(\Omega)=\frac{1}{\sqrt{2\pi}}\int dt\,O(t)e^{-i\Omega t}. \quad (3)$$

The fields are now described as functions of the modulation frequency Ω. Thus we can consider any field as a carrier $O(0)$ oscillating at frequency ω_0, with an average value equal to the steady-state field, surrounded by ''noise sidebands'' $O(\Omega)$ oscillating at a frequency $\omega_0+\Omega$ with zero average values. The amplitude and phase quadrature can be written as

$$X_O(\Omega)=O(\Omega)+O^{+}(-\Omega),$$

$$Y_O(\Omega)=\frac{1}{i}[O(\Omega)-O^{+}(-\Omega)]. \quad (4)$$

The two bright output beams can be directly detected by D_1 and D_2. The discussed photocurrents are normalized with the average value of the field. The normalized output photocurrents spectra are given by

$$\hat{i}_c(\Omega)=\frac{1}{\langle c\rangle}\int dt\,e^{-i\Omega t}c^{+}c=\frac{1}{2}[X_a(\Omega)+Y_a(\Omega)-Y_b(\Omega)+X_b(\Omega)], \quad (5)$$

$$\hat{i}_d(\Omega)=\tfrac{1}{2}[X_a(\Omega)-Y_a(\Omega)+Y_b(\Omega)+X_b(\Omega)].$$

Equation (5) shows that the photocurrent of each arm of the 50% beamsplitter consists of two parts. Part 1 (terms 1 and 4) consists of self-terms of two input fields $X_a(\Omega)$ and $X_b(\Omega)$ at the beamsplitter, and part 2 comprises interference terms (2 and 3) including phase quadratures $Y_a(\Omega)$ and $Y_b(\Omega)$ derived from the $\pi/2$ phase shifter. Equation (5) shows that the self-terms of two arms are correlated (or in phase), and that interference terms are anticorrelated (or out of phase). Each of the photocurrents is divided into two parts through the rf power splitter. The sum and difference of the divided photocurrents are

$$\hat{i}_{+}(\Omega)=\frac{1}{\sqrt{2}}[\hat{i}_c(\Omega)+\hat{i}_d(\Omega)]=\frac{1}{\sqrt{2}}[X_a(\Omega)+X_b(\Omega)], \quad (6)$$

$$\hat{i}_{-}(\Omega)=\frac{1}{\sqrt{2}}[Y_a(\Omega)-Y_b(\Omega)].$$

We can see that the sum of the photocurrents of two arms c and d leaves only the self-term, which is the amplitude quadrature of the signal field combined with one of the EPR beams; the difference photocurrent leaves the interference terms, which the EPR beams phase quadrature. Thus a Bell-state measurement of two beams is achieved with this simple direct detection.

In recent years, compact, reliable, bright squeezed sources such as parametric amplifier have attracted attention [8]. We propose an experimental setup to generate bright EPR beams using two coherent amplitude squeezed beams that can be produced from two OPA's pumped by a laser or a ring OPA with two outputs; the relative phase between these can be locked very securely using mature phase-locking technology [4,5]. Two coherent amplitude squeezed beams were mixed on a 50% beamsplitter to form a pair of EPR beams. Because both coherently related amplitude squeezed beams a and b are intense, the amplitude quadratures of output beams c and d are given by

$$X_c(\Omega)=\frac{1}{2\bar{i}}[\langle a\rangle X_a(\Omega)-\langle a\rangle Y_b(\Omega)+\langle b\rangle Y_a(\Omega)+\langle b\rangle X_b(\Omega)], \quad (7)$$

$$X_d(\Omega)=\frac{1}{2\bar{i}}[\langle a\rangle X_a(\Omega)+\langle a\rangle Y_b(\Omega)-\langle b\rangle Y_a(\Omega)+\langle b\rangle X_b(\Omega)].$$

Here $\bar{i}=\langle c\rangle=\langle d\rangle=\sqrt{(\langle a\rangle^2+\langle b\rangle^2)/2}$. In a similar way, the phase quadratures are equal to

$$Y_c(\Omega)=\frac{1}{2\bar{i}}[\langle a\rangle Y_a(\Omega)-\langle a\rangle X_b(\Omega)+\langle b\rangle X_a(\Omega)+\langle b\rangle Y_b(\Omega)], \quad (8)$$

$$Y_d(\Omega)=\frac{1}{2\bar{i}}[\langle a\rangle Y_a(\Omega)+\langle a\rangle X_b(\Omega)-\langle b\rangle X_a(\Omega)+\langle b\rangle Y_b(\Omega)].$$

From Eqs. (7) and (8), we can readily write the cross-correlation between the variances of two outgoing fields as

$$\langle\delta(X_c+X_d)^2\rangle=\frac{1}{2\bar{i}^2}(\langle a\rangle^2 V_{X_a}+\langle b\rangle^2 V_{X_b})\to 0, \quad (9)$$

$$\langle\delta(Y_c-Y_d)^2\rangle=\frac{1}{2\bar{i}^2}(\langle b\rangle^2 V_{X_a}+\langle a\rangle^2 V_{X_b})\to 0,$$

where $\langle\delta(X_a)^2\rangle=V_{X_a}$ and $\langle\delta(X_b)^2\rangle=V_{X_b}$. From Eq. (9) it is obvious that if the amplitude squeezing of two input beams are perfect, i.e., $V_{X_a}\to 0$ and $V_{X_b}\to 0$, the two output beams are in a perfectly entangled state which will exhibit EPR-type correlations [4,6,9]. Correlations of the sum of the amplitude quadrature and the difference of the phase quadrature relate to the squeezed degree of the two amplitude-squeezed beams and the intensity of steady-state field, results little different from the those of Ref. [6], in which one of the amplitude-squeezed beams is much lower than the other one ($\langle a\rangle\gg\langle b\rangle\approx 0$); therefore, correlations of the sum of the amplitude quadrature and the difference of the phase quadrature in Ref. [6] relate to the squeezed degree of one of the two amplitude-squeezed beams, respectively. Usually a small unbalance of the 50/50 splitter can be corrected by aligning the incident angle, and the residual unbalance, that only slightly

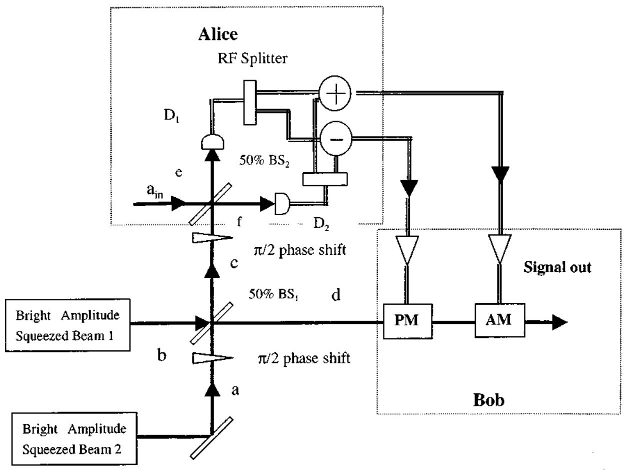

FIG. 2. Schematic of bright amplitude squeezed beams for teleportation. (The bright amplitude squeezed beams 1 and 2 are coherent.)

influences the precision of measurement, cannot destroy the experiment like the situation in Refs. [4] and [5].

The scheme for teleportation of continuous variables using direct detection of the Bell state and the bright EPR beams is shown in Fig. 2. One of the EPR beams is sent to Alice to mix with an input signal beam of the same intensity and the $\pi/2$ phase shift on the second 50% beamsplitter (BS2). The bright output beams *e* and *f* are directly detected by D_1 and D_2. Considering the detection efficiency η, the propagation efficiency ξ_c of the EPR beam *c*, and $\langle a_{in}\rangle = \xi_c\langle c\rangle$, according to Eq. (1) we have

$$e=\frac{\sqrt{2}}{2}\,\eta[a_{in}+i(\xi_c c+\sqrt{1-\xi_c^2}\nu_c)]+\sqrt{1-\eta^2}\,\nu_e\,,$$
$$f=\frac{\sqrt{2}}{2}\,\eta[a_{in}-i(\xi_c c+\sqrt{1-\xi_c^2}\nu_c)]+\sqrt{1-\eta^2}\,\nu_e\,, \tag{10}$$

where ν_c and ν_e are the coupled vacuum noises due to the losses. Each of the photocurrents at D_1 and D_2 is divided into two parts through the rf power splitter. The sum and difference of the divided photocurrents are expressed as

$$\hat{i}_+(\Omega)=\frac{1}{\sqrt{2}}[\eta X_{in}(\Omega)+\eta\xi_c X_c(\Omega)+\eta\sqrt{1-\xi_c^2}X_{\nu_c}(\Omega)]+\frac{1}{2}[\sqrt{1-\eta^2}X_{\nu_e}(\Omega)+\sqrt{1-\eta^2}Y_{\nu_e}(\Omega)+\sqrt{1-\eta^2}X_{\nu_f}(\Omega)-\sqrt{1-\eta^2}Y_{\nu_f}(\Omega)], \tag{11}$$

$$\hat{i}_-(\Omega)=\frac{1}{\sqrt{2}}[\eta Y_{in}(\Omega)-\eta\xi_c Y_c(\Omega)-\eta\sqrt{1-\xi_c^2}Y_{\nu_c}(\Omega)]+\frac{1}{2}[\sqrt{1-\eta^2}X_{\nu_e}(\Omega)+\sqrt{1-\eta^2}Y_{\nu_e}(\Omega)-\sqrt{1-\eta^2}X_{\nu_f}(\Omega)+\sqrt{1-\eta^2}Y_{\nu_f}(\Omega)].$$

Then the photocurrents are sent to amplitude and phase modulators in Bob, respectively. The amplitude and phase modulators transform the photocurrents into the other half *d* of the EPR beams. The output beam from modulators is found to be

$$a_{out}(\Omega)=\frac{1}{2}[\xi_d d+\sqrt{1-\xi_d^2}\nu_d+g_+\hat{i}_+(\Omega)+ig_-\hat{i}_-(\Omega)], \tag{12}$$

where g_+ and g_- describes suitably normalized amplitude and phase gain at the receiving station of Bob for the transformation from the photocurrent to output beam, and ξ_d is the propagation efficiency of the EPR beam *d*. The amplitude and phase variances of output beam are given by

$$V_{X_{out}}=g'^2V_{X_{in}}+(g'\xi_c+\xi_d)^2\frac{V_{X_{squee}}}{2}+(g'\xi_c-\xi_d)^2\frac{V_{Y_{squee}}}{2}+[g'^2(1-\xi_c^2)+(1-\xi_d^2)+2g'^2(1/\eta^2-1)], \tag{13}$$

$$V_{Y_{out}}=g'^2V_{Y_{in}}+(g'\xi_c+\xi_d)^2\frac{V_{X_{squee}}}{2}+(g'\xi_c-\xi_d)^2\frac{V_{Y_{squee}}}{2}+[g'^2(1-\xi_c^2)+(1-\xi_d^2)+2g'^2(1/\eta^2-1)],$$

where $g_+=g_-=g$, $g'=g\eta/\sqrt{2}$, $\langle\delta[X_a(\Omega)]^2\rangle=\langle\delta[X_b(\Omega)]^2\rangle=V_{X_{squee}}$, $\langle\delta[Y_a(\Omega)]^2\rangle=\langle\delta[Y_a(\Omega)]^2\rangle=V_{Y_{squee}}$, $\langle\delta[X_{out}(\Omega)]^2\rangle=V_{X_{out}}$, $\langle\delta[Y_{out}(\Omega)]^2\rangle=V_{Y_{out}}$, $\langle\delta[X_{in}(\Omega)]^2\rangle=V_{X_{in}}$, and $\langle\delta[Y_{in}(\Omega)]^2\rangle=V_{Y_{in}}$. We can calculate the fidelity $F=|\langle a_{in}^+a_{out}\rangle|^2$ characterizing the ''quality'' of teleportation with the variances of Eq. (13) [4]. If the EPR beams are in a coherent state, the variance $V_{X_{out}}=V_{Y_{out}}=3$ and the amplitude $\langle a_{out}\rangle=\langle a_{in}\rangle$. Since the fidelity $F_{Class}^{Theor}=0.5$ is the boundary between quantum and classical teleportation for coherent-state inputs, to meet the requirement of the quantum teleportation $F>0.5$ Alice and Bob must share a quantum source of a nonlocal correlation EPR pair.

In the past decade, a variety of squeezed state light has been applied in optical measurements with a precision beyond that of the SQL [10]. However, in these measurements only one quadrature component was examined. Here we propose a scheme of dense coding in which simultaneous measurements of phase and amplitude signals, with a sensitivity beyond that of the SQL, is achieved by means of bright amplitude squeezed light and direct measurement of the Bell state. As shown in Fig. 3, the classical amplitude and phase signals are modulated on one of the entangled pair of EPR beams, which leads to a displacement of a_s:

$$c'=c+a_s\,. \tag{14}$$

Here a_s is the signal sent via the quantum channel. From Eqs. (7) and (8) we know that the amplitude and phase quadrature of EPR beams have a large noise $\langle\delta(X_c)^2\rangle\rightarrow\infty$, $\langle\delta(Y_c)^2\rangle\rightarrow\infty$. The signal to noise ratio R is given by

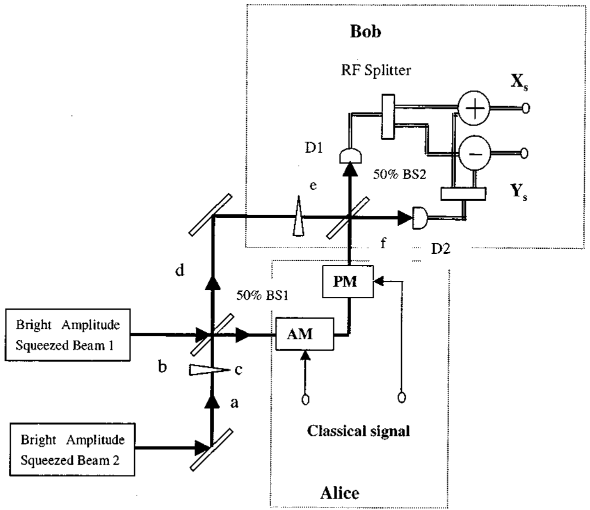

FIG. 3. Schematic of bright amplitude squeezed beams for dense coding.

$$\mathcal{R}_X = \frac{\langle \delta(X_s)^2\rangle}{\langle \delta(X_c)^2\rangle} \to 0, \quad \mathcal{R}_Y = \frac{\langle \delta(Y_s)^2\rangle}{\langle \delta(Y_c)^2\rangle} \to 0. \tag{15}$$

No one other than Bob can gain any signal information from the modulated EPR beam under ideal conditions, because the signal is submerged in large noises. The signal is demodulated with the aid of the other half of the EPR beams, which is combined with the modulated first half of the EPR beams on another 50% beamsplitter (BS2), and before combination a $\pi/2$ phase shift is imposed between them. The two bright output beams are directly detected by D_1 and D_2. Each photocurrent of D_1 and D_2 is divided into two parts through the power splitter. The sum and difference of the divided photocurrents are expressed by

$$\hat{i}_+(\Omega) = \frac{1}{\sqrt{2}}[X_c(\Omega) + X_d](\Omega) + X_s(\Omega)] = \frac{1}{\sqrt{2}} X_s(\Omega), \tag{16}$$

$$\hat{i}_-(\Omega) = \frac{1}{\sqrt{2}}[Y_c(\Omega) - Y_d(\Omega) + Y_s(\Omega)] = \frac{1}{\sqrt{2}} Y_s(\Omega),$$

where $\langle \delta[X_c(\Omega) + X_d(\Omega)]^2\rangle \to 0$ and $\langle \delta[Y_c(\Omega) - Y_d(\Omega)]^2\rangle \to 0$ under ideal conditions. Thus we obtain the amplitude and phase quadrature for the signal beam with a sensitivity below that of the SQL by simple direct detection. In fact, the sum of the amplitude quadrature of the EPR beams commutes with difference of its phase quadrature; thus the variances in them can be below that of the SQL at the same time, and not violate the uncertainty principle [11]. The signal-to-noise ratio of the detected signal is given by

$$\mathcal{R}_X = \frac{\langle \delta(X_s)^2\rangle}{\langle \delta(X_c + X_d)^2\rangle} \to \infty, \quad \mathcal{R}_Y = \frac{\langle \delta(Y_s)^2\rangle}{\langle \delta(Y_c - Y_d)^2\rangle} \to \infty. \tag{17}$$

We can see that the original signal is retrieved with a high fidelity with the help of the other EPR beam. Compared with Ref. [10], the simultaneous measurement of the small phase and an amplitude signal beyond the shot-noise limit can be achieved by using EPR beams. Hence it can be said that the coding capacity of this scheme is twice that of the measurements completed in Ref. [10].

In summary, we proposed a simple direct measurement scheme for the Bell state, and a system producing an EPR pair, by combining two bright amplitude squeezed lights. We theoretically demonstrated that the proposed protocol can be used to accomplish quantum teleportation of continuous variables, and dense coding. Mature techniques for producing coherent amplitude squeezed beams from the parametric down converter and the electronic phase locking, as well the simplicity of direct measurement, make the scheme valuable for performing experiments.

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