Teleportation using coupled oscillator states

P. T. Cochrane,* G. J. Milburn,† and W. J. Munro‡
Centre for Quantum Computer Technology, Department of Physics, The University of Queensland, St. Lucia, QLD 4072, Australia
(Received 10 April 2000; revised manuscript received 21 July 2000; published 10 November 2000)

We analyze the fidelity of teleportation protocols, as a function of resource entanglement, for three kinds of two-mode oscillator states: states with fixed total photon number, number states entangled at a beam splitter, and the two-mode squeezed vacuum state. We define corresponding teleportation protocols for each case including phase noise to model degraded entanglement of each resource.

PACS number(s): 03.67.−a

I. INTRODUCTION

Quantum entanglement plays a central role in the emerging fields of quantum computation [1–5], quantum cryptography [6,7], quantum teleportation [8–13], dense coding [14], and quantum communication [15–17]. The characterization of entanglement is a challenging problem [18–23] and considerable effort has been invested in characterizing entanglement in a variety of contexts [24–30].

One such context, quantum teleportation, has played a crucial role in understanding how entanglement can be used as a resource for communication. Recent experimental demonstrations [31,32] suggest that quantum teleportation could be viewed as an achievable experimental technique to quantitatively investigate quantum entanglement. Teleportation is a way of transmitting an unknown quantum state to a distant receiver with far better reliability than can be achieved classically. As the entanglement of the enabling resource is degraded, the fidelity of the teleportation protocol is diminished.

In this paper, we attempt to make this intuition more precise using specific examples from quantum optics. Three entangled resources are considered: states with fixed total photon number, number states entangled at a beam splitter, and the two-mode squeezed vacuum state [33]. The examples we discuss exhibit quantum correlations between the photon number in each mode and, simultaneously, between the phase of each mode.

In reality, the entanglement will not be perfect, but degraded to some extent by uncontrolled interactions with an environment during formation. To model this, we consider phase fluctuations on each mode independently. In the limit of completely random phase, we are left with only the classical intensity (photon number) correlations. The state is no longer entangled and the fidelity of the protocol depends only on the classical intensity correlations remaining in the resource.

II. ENTANGLEMENT AND TELEPORTATION

Intuitively entanglement refers to correlations between distinct subsystems that cannot be achieved in a classical statistical model. Of course correlations can exist in classical mechanics, but entanglement refers to a distinctly different kind of correlation at the level of quantum probability amplitudes. The essential difference between quantum and classical correlations can be described in terms of the separability of states [34–41]. A density operator of two subsystems is separable if it can be written as the convex sum [39]

$$\rho = \sum_A w_A \rho'_A \otimes \rho''_A , \tag{1}$$

where ρ'_A and ρ''_A are density matrices for the two subsystems and the w_A are positive weights satisfying $\Sigma_A w_A = 1$. For example, for two harmonic oscillators (a and b) the density operator which has correlated energy

$$\rho_{ab} = \sum_{n=0}^{\infty} p_n |n,n\rangle\langle n,n| \tag{2}$$

is separable (where we use the notation $|n,n\rangle = |n\rangle_a \otimes |n\rangle_b$) while the pure state

$$|\Psi\rangle_{ab} = \sum_{n=0}^{\infty} \sqrt{p_n} |n,n\rangle \tag{3}$$

has the same classical correlation but is not separable. In this form, we see that is possible for a separable and an entangled state to share similar classical correlations for some variables.

Consider a communication protocol in which the results of measurements made on a physical system are transmitted to a distant receiver. The goal of the receiver is to reconstruct the physical state of the source, using only local resources, conditioned on the received information. The communication that takes place is of course entirely classical. In a teleportation protocol there is one additional feature: quantum correlations (entanglement) are first shared between the sending and receiving station. The degree of entanglement shared by sender and receiver is called the teleportation *resource*. If there is no shared quantum correlation between the sender and receiver, the protocol is called classical.

The extent and nature of the quantum correlations in the resource determine the fidelity of the protocol. Under ideal conditions, the unknown state of some physical system at the transmitting end can be perfectly recreated in another physical system at the receiving end. There are many ways in

*Electronic address: cochrane@physics.uq.edu.au
†Electronic address: milburn@physics.uq.edu.au
‡Electronic address: billm@physics.uq.edu.au

which actual performance can differ from the ideal. In this paper, we analyze the change in the performance of teleportation protocols as the degree of entanglement in the resource is varied by decoherence. Our primary objective is to use the fidelity of a teleportation protocol to compare and contrast different kinds of entangled oscillator states.

A general teleportation protocol proceeds as follows. The sender, Alice, has a *target* state, $|\psi\rangle_T$, that she wishes to teleport to Bob, the receiver. Alice and Bob each have access to one part of an entangled bipartite physical system prepared in the state $|\psi\rangle_{AB}$. In this paper, the bipartite physical system is a two-mode electromagnetic field. In order to send the state of the target to Bob, Alice performs a joint measurement on the target and her mode. She then sends the information gained from these measurements to Bob via a classical channel. Bob performs local unitary transformations on the mode in his possession according to the information Alice sends to him, thereby attempting to recreate the initial target state. We quantify the quality of the protocol by the probability that Bob's received state is the same as the target state. This quantity is known as the *fidelity*.

The fidelity of quantum teleportation protocol is determined by the degree of shared entanglement, the quality of the measurements made by the sender, the quality of the classical communication channels used, and how well Bob can implement the desired unitary transformations. In this paper, we will discuss only the first of these; the amount of shared entanglement. In the original teleportation protocol [8], the bipartite system was made up of two systems each described by a two-dimensional Hilbert space, that is to say, two qubits, and the shared entangled state was a *maximally* entangled state [19]. In the case of two correlated harmonic oscillators, or two field modes, we cannot define maximal entanglement in quite the same way, as the entropy of each component system can be arbitrarily large. In this paper, we define extremal entangled pure states of two field modes in terms of the total mean photon number and the total maximum photon number.

In the case of a system with a finite-dimensional Hilbert space, a state of maximum (von Neumann) entropy is simply the identity operator in that Hilbert space. A natural generalization of this idea to infinite Hilbert spaces would define a maximum entropy state subject to some constraint, such as mean energy or total energy. These of course define the canonical ensemble and microcanonical ensemble of statistical mechanics. In the case of entangled pure states, the Araki-Lieb [42] inequality indicates that the entropy of each component system is equal. As the entropy of a harmonic oscillator scales with mean energy, this indicates that each component subsystem has the same mean energy. If we maximize the entropy of each subsystem subject to a constraint on the mean energy, the state must be a thermal state. The entangled pure two-mode state, for which the reduced density operator of each mode is thermal, is the squeezed vacuum state,

$$|\lambda\rangle=(1-\lambda^2)^{1/2}\sum_{n=0}^{\infty}\lambda^n|n,n\rangle. \tag{4}$$

The mean photon number in each mode is given by $\overline{n}=\lambda^2/(1-\lambda^2)$. If, however, we constrain the total photon number, N, of each mode, we get a very different expression for a maximally entangled state,

$$|N\rangle=\frac{1}{\sqrt{1+N}}\sum_{n=0}^{N}|N-n,n\rangle. \tag{5}$$

The entropy of the reduced state of each mode is $\ln(1+N)$ while the mean photon number is $N/2$. While squeezed vacuum states may be achieved in the laboratory, states with fixed total photon number have a not been produced, and will not be possible until we have a reliable N photon source. There are now a couple of proposals for such sources [43–45] and it may not be too long before they are used in teleportation schemes.

Teleportation fidelity for infinite-dimensional Hilbert spaces must necessarily vary from unity for an arbitrary target state, as the notion of a maximally entangled resource differs from the finite-dimensional case. The teleportation protocol can also be degraded by unknown incoherent processes that corrupt the purity of the shared entanglement. Of course, in some cases these incoherent processes may destroy the correlations entirely, for example by absorbing all the photons in each mode before Alice and Bob get to use them. In this paper, however, we will only consider those decoherence processes that change the purity of the states and leave unchanged the classical intensity correlations.

III. IDEAL RESOURCE

In a recent paper by Milburn and Braunstein [33], a teleportation protocol was presented using joint measurements of the photon number difference and phase sum on two field modes. This protocol is possible because the number difference and phase sum operators commute, thus allowing determination of these quantities simultaneously and to arbitrary accuracy.

Number *sum* and phase *difference* operators also commute, implying that if eigenstates of these operators can be found, then a teleportation protocol is possible. Such a protocol is discussed below. Recently, teleportation using number sum and phase difference measurements was described [46]. That work did not address how the degree of entanglement in the resource changes the teleportation fidelity as we do here.

Because the number sum and phase difference operators commute, we look for simultaneous eigenstates of these observables. Consider states of the form

$$|\psi\rangle_{AB}=\sum_{n=0}^{N}d_n|N-n\rangle_A|n\rangle_B \tag{6}$$

which are eigenstates of number sum with eigenvalue N. The labels A and B refer to the sender's and receiver's component of the entangled modes, respectively, and the d_n satisfy $\Sigma_n|d_n|^2$. This state will be maximally entangled when the d_n are all equal, giving the resource

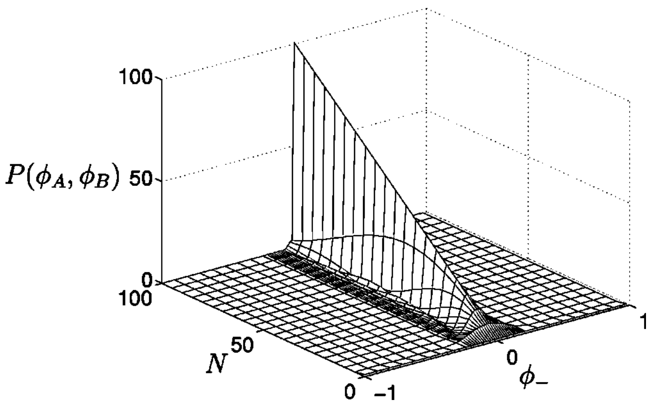

FIG. 1. Joint phase probability density. As N increases, the probability density becomes very narrowly peaked about $\phi_-=0$. The ϕ_- axis is in units of π.

$$|\psi\rangle_{AB}=\frac{1}{\sqrt{N+1}}\sum_{n=0}^{N}|N-n\rangle_A|n\rangle_B. \tag{7}$$

This state tends towards eigenstates of phase difference as $N\to\infty$. To see this, consider the joint phase probability density of Eq. (7), which is determined by the ideal joint phase operator projection operator $|\phi_A\rangle\langle\phi_A|\otimes|\phi_B\rangle\langle\phi_B|$ as $P(\phi_A,\phi_B)=\mathrm{tr}(\rho_{AB}|\phi_A\rangle\langle\phi_A|\otimes|\phi_B\rangle\langle\phi_B|)$, where [47–49]

$$|\phi\rangle=\sum_{n=0}^{\infty}e^{in\phi}|n\rangle. \tag{8}$$

Substituting the state in Eq. (6), we have

$$P(\phi_A,\phi_B)=\frac{1}{N+1}\left|\sum_{n=0}^{N}e^{in\phi_-}\right|^2, \tag{9}$$

where $\phi_-=\phi_A-\phi_B$. The probability density as a function of N and ϕ_- is shown in Fig. 1 and indicates that the density becomes sharply peaked about $\phi_-=0$ in the interval $[-\pi,\pi]$ as N gets larger. Hence the states of Eq. (7) tend to eigenstates of phase difference with increasing N.

The state to be teleported, the target state, can be written in the general form

$$|\psi\rangle_T=\sum_{m=0}^{\infty}c_m|m\rangle_T. \tag{10}$$

The input state to the protocol is then

$$|\psi\rangle=\frac{1}{\sqrt{N+1}}\sum_{m=0}^{\infty}\sum_{n=0}^{N}c_m|m\rangle_T|N-n\rangle_A|n\rangle_B. \tag{11}$$

If Alice measures the number sum of the target and her component of the entangled resource (i.e., $\hat{N}_A+\hat{N}_T$) with result q, the conditional state of the total system is

$$|\psi^{(q)}\rangle=[P_I(q)(N+1)]^{-1/2}\times\sum_n c_{q-N+n}|q-N+n\rangle_T|N-n\rangle_A|n\rangle_B, \tag{12}$$

where n runs from $\max(0,N-q)$ to N. The probability of obtaining the result q is

$$P_I(q)=\frac{1}{N+1}\sum_n|c_{q-N+n}|^2. \tag{13}$$

The subscript I emphasizes that this probability refers to the idealized resource. Alice now measures the phase difference with result ϕ_-. The conditional state of Bob's mode is then the pure state

$$|\psi^{(q,\phi_-)}\rangle_B=[P_I(q)(N+1)]^{-1/2}\sum_n e^{2in\phi_-}c_{q-N+n}|n\rangle_B. \tag{14}$$

Using the results q and ϕ_-, and knowledge of the number of Fock states in the resource (N), Bob has sufficient information to reproduce the target state. He does this by amplifying his mode so that $|n\rangle_B\to|q-N+n\rangle_B$ and phase shifting it by $e^{-i2n\phi_-}$. The unitary amplification operation is described in [50]. These operations complete the protocol, and the state Bob finally has in his possession is

$$|\psi^{(q)}\rangle_{\mathrm{out},B}=[P_I(q)(N+1)]^{-1/2}\sum_n c_{q-N+n}|q-N+n\rangle_B. \tag{15}$$

The fidelity of this protocol depends on the result q and is

$$F_I(q)=\sum_n|c_{q-N+n}|^2 \tag{16}$$

$$=(N+1)P_I(q). \tag{17}$$

As the fidelity depends on the result of the number sum measurement, it varies from one run to the next. To obtain an overall figure of merit for the protocol, we define the average fidelity,

$$\overline{F_I}=\sum_q F_I(q)P(q). \tag{18}$$

In this case, we find

$$\overline{F_I}=(N+1)\sum_{q=0}^{\infty}P_I(q)^2. \tag{19}$$

To see how well the teleportation protocol performs, we shall consider some examples.

Let the target state be a number state,

$$|\psi\rangle_T=|m\rangle_T, \tag{20}$$

so the only coefficient available is c_m, which is 1. We find that the teleportation fidelity is unity, independent of the

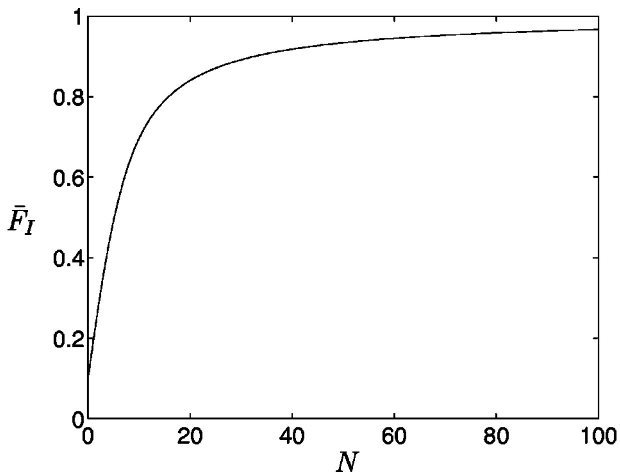

FIG. 2. Average fidelity as a function of the energy in the ideal resource, N, for a coherent state of amplitude $\alpha=3$.

measurement of q, because the only term appearing in the summations of both the fidelity and the probability is that corresponding to c_m. Hence, this protocol works perfectly if the target is a number state.

In Fig. 2, we show the average fidelity as a function of the total photon number in the resource for a coherent state target with amplitude $\alpha=3$. It is clear that increasing the number of photons in the entangled resource improves the teleportation protocol.

IV. BEAM-SPLITTER RESOURCE

The resource states discussed in Sec. III illustrate the protocol well, but are not produced by any known physical interaction. However, the beam-splitter interaction can be shown to give a resource with similar properties to Eq. (7). The beam-splitter interaction is described by [51]

$$|\psi\rangle_{AB}=e^{i(\pi/5)(a^\dagger b+ab^\dagger)}|N\rangle_A|N\rangle_B, \tag{21}$$

where the operators a, $a^\dagger$, b, and $b^\dagger$ are the usual boson annihilation and creation operators for modes A and B, and N is the number of photons at each input port of the beam splitter. Because the number sum of the two modes is a constant ($=2N$), we can rewrite the resource in terms of eigenstates of the number sum. The resource is now written as

$$|\psi\rangle_{AB}=\sum_{n=0}^{2N} d_{n-N}|n\rangle_A|2N-n\rangle_B. \tag{22}$$

The coefficients d_{n-N} are derived by first using Schwinger's boson representation of angular momentum, with total angular momentum quantum number $j=N$, and then identifying these coefficients as rotation matrix elements [51]. The d_{n-N} coefficients are defined by

$$d_{n-N}=e^{-i(\pi/2)(n-N)}D^N_{n-N,0}(\pi/2), \tag{23}$$

where

$$D^j_{m',m}(\beta)=[(j+m')!(j-m')!(j+m)!(j-m)!]^{1/2}\sum_s \frac{(-1)^{m'-m+s}\left(\cos\frac{\beta}{2}\right)^{2j+m-m'-2s}\left(\sin\frac{\beta}{2}\right)^{m'-m+2s}}{(j+m-s)!s!(m'-m+s)!(j-m'-s)!}. \tag{24}$$

The variable s ranges over all integer values where the factorials are non-negative [52]. It is easy to verify that all coefficients with n odd are zero.

This protocol proceeds identically to that discussed in Sec. III. We illustrate this variation of the protocol with the pure state form of the resource as given in Eq. (22). After a number sum and phase difference measurement on modes T and A, and then applying the amplification $|2N-n\rangle_B\rightarrow|q-n\rangle_B$ and phase shift $e^{-2in\phi_-}$, the output state becomes

$$|\psi^{(q)}\rangle_{\mathrm{out},B}=[P_{\mathrm{BS}}(q)]^{-1/2}\sum_{n=0}^{\min(q,2N)} c_{q-n}d_{n-N}|q-n\rangle_B, \tag{25}$$

where the probability for a number sum result q is

$$P_{\mathrm{BS}}(q)=\sum_{n=0}^{\min(q,2N)} |c_{q-n}|^2|d_{n-N}|^2. \tag{26}$$

The teleportation fidelity is found to be

$$F_{\mathrm{BS}}(q)=\frac{1}{P_{\mathrm{BS}}(q)}\left|\sum_{n=0}^{\min(q,2N)} |c_{q-n}|^2 d_{n-N}\right|^2. \tag{27}$$

If we again consider a coherent state target of amplitude $\alpha=3$, we can compare the beam-splitter-generated resource with the ideal resource in Sec. III. The average fidelity as a function of energy in the resource is shown in Fig. 3 and is almost identical to Fig. 2 except that its maximum is approximately one-half as opposed to unity. This is due to the fact that all terms in Eq. (22) with n odd are zero. Effectively only half of the perfect correlations in the ideal entangled resource are available, hence the maximum fidelity we would expect under such circumstances is 0.5. Even so, the state becomes a better teleportation resource with increasing N (see Fig. 3).

It is, however, possible to teleport those states which have no odd photon number components with near unit fidelity. An example is the even ''cat'' state, formed from the superposition of two coherent states of equal real amplitude but opposite sign [53],

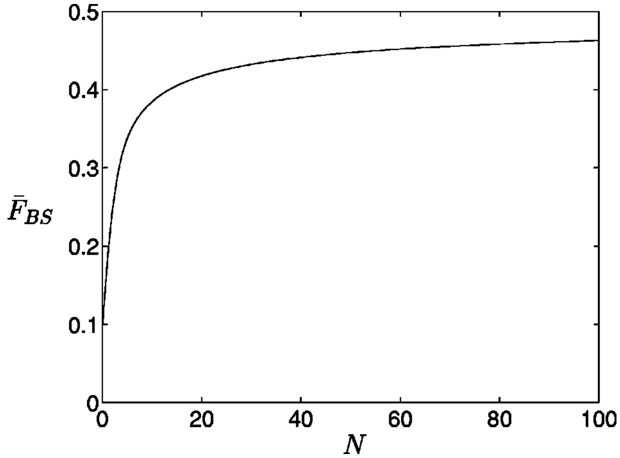

FIG. 3. Average fidelity as a function of energy in the beam-splitter resource, N, for a coherent state of amplitude $\alpha = 3$.

$$|\psi\rangle_T = \frac{|\alpha\rangle_T + |-\alpha\rangle_T}{\sqrt{2 + 2e^{-2|\alpha|^2}}}. \quad (28)$$

The average fidelity in this case is shown in Fig. 4. This result implies that it may be possible to tailor resources for given applications so that certain classes of states may be teleported well, without necessarily being able to teleport an arbitrary state.

V. DECOHERENCE

Teleportation requires quantum correlations, in the form of entanglement, to be shared by the sender and receiver. In this section we consider how teleportation fidelity changes if decoherence diminishes the extent of the correlation. We use a decoherence mechanism (phase diffusion), which does not change the intensity (photon number) correlations of the entanglement resource but does destroy the coherence in the number basis.

Phase diffusion is modeled by adding random phase fluctuations to each mode independently with the unitary operator

$$U(\theta) \equiv \exp[-i(\theta_a a^\dagger a + \theta_b b^\dagger b)], \quad (29)$$

where the phase sum and difference $\theta = \theta_a \pm \theta_b$ is taken to be Gaussian randomly distributed with a zero mean and variance σ,

$$P(\theta) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{\theta^2}{2\sigma}\right). \quad (30)$$

Even though a Gaussian distribution is not periodic, it can be taken to be an approximation of a true periodic distribution, such as $\cos^{2N}(\theta - \theta_0)$, which for sufficiently large N is approximately Gaussian near θ_0 with a variance of $1/2N$.

A. Squeezed-state resource

Reference [33] describes a teleportation protocol using two-mode squeezed vacuum states as an entanglement resource together with number difference and phase sum measurements. The resource for this protocol is written in the Fock basis as

$$|\psi\rangle_{AB} = \sqrt{1-\lambda^2} \sum_{n=0}^{\infty} \lambda^n |n\rangle_A |n\rangle_B. \quad (31)$$

The entanglement between resource modes may be altered by changing the squeezing parameter, λ, and by decohering the resource using phase diffusion. Applying the phase shift $U(\theta)$ and averaging over all realizations of the phase gives the resource as a density operator

$$\rho_{AB} = (1-\lambda^2) \sum_{n,n'=0}^{\infty} \lambda^n \lambda^{n'} e^{-\gamma(n-n')^2} |n\rangle_A \langle n'| \otimes |n\rangle_B \langle n'|, \quad (32)$$

where $\gamma = \sigma/2$ describes the degree of decoherence.

The number difference measurement can give a positive or negative result and we consider each case separately. If the state to be teleported is $\rho_T = \sum_{m,m'} c_m c^*_{m'} |m\rangle\langle m'|$, the output state at the receiver, conditioned on the positive number difference, q, is

$$\rho_{\text{out},B} = \frac{1-\lambda^2}{P_+(q)} \sum_{n,n'=0}^{\infty} c_{n+q} c^*_{n'+q} \lambda^n \lambda^{n'} \times e^{-\gamma(n-n')^2} |n+q\rangle_B \langle n'+q|, \quad (33)$$

with a corresponding fidelity given by

$$F_{+,\gamma}(q) = \frac{1-\lambda^2}{P_+(q)} \sum_{n,n'=0}^{\infty} |c_{n+q}|^2 |c_{n'+q}|^2 \lambda^n \lambda^{n'} e^{-\gamma(n-n')^2}, \quad (34)$$

where

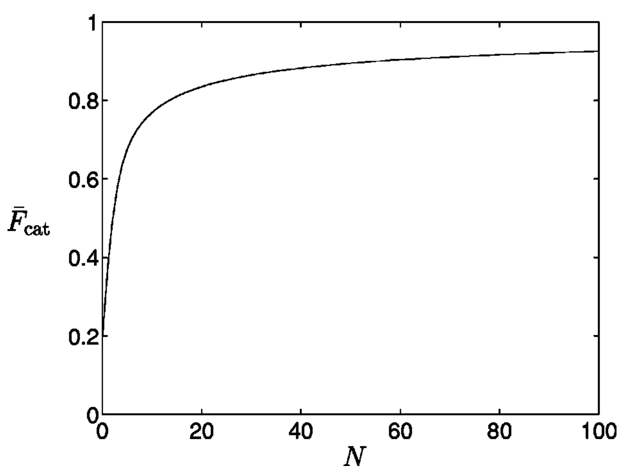

FIG. 4. Average fidelity as a function of the energy in the beam-splitter resource, N, for a ''cat'' state of amplitude $\alpha = 3$.

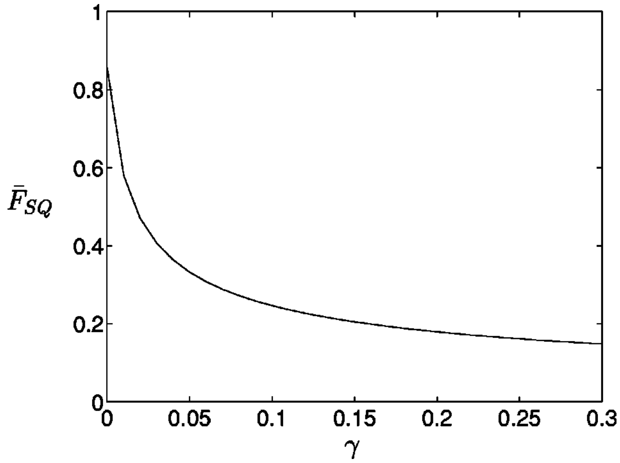

FIG. 5. Average fidelity as a function of degree of decoherence, γ, for a two-mode squeezed vacuum resource with a squeezing parameter value of $\lambda = 0.8$. Target is a coherent state of amplitude $\alpha = 3$.

$$P_+(q) = (1-\lambda^2)\sum_{n=0}^{\infty} |c_{n+q}|^2\lambda^{2n} \tag{35}$$

is the probability of obtaining a result q for photon number difference measurements at the sender, which does not depend on the decoherence.

For measurement of negative number difference, $q' = -q$, the fidelity after teleportation is

$$F_{-,\gamma}(q') = \frac{1-\lambda^2}{P_-(q')}\sum_{m,m'=0}^{\infty} |c_m|^2|c_{m'}|^2\lambda^{m+q'}\lambda^{m'+q'} \times e^{-\gamma(m-m')^2}, \tag{36}$$

where

$$P_-(q') = (1-\lambda^2)\sum_{m=0}^{\infty} |c_m|^2\lambda^{2(m+q')}. \tag{37}$$

The average fidelity as a function of degree of decoherence, γ, is shown in Fig. 5 and behaves as we would expect; decoherence in the resource reduces the output quality of the protocol implying that the entanglement available as a resource for teleportation has decreased.

B. Ideal resource

Applying our decoherence model to Eq. (7) and averaging over all realizations of the phase, we obtain the total state:

$$\rho_{TAB} = \frac{1}{N+1}\sum_{m,m'=0}^{\infty}\sum_{n,n'=0}^{N} c_m c_{m'}^* e^{-\gamma(n-n')^2} \times |m\rangle_T\langle m'| \otimes |N-n\rangle_A\langle N-n'| \otimes |n\rangle_B\langle n'|, \tag{38}$$

where γ is the degree of decoherence as before. After completion of the protocol, the fidelity is given by

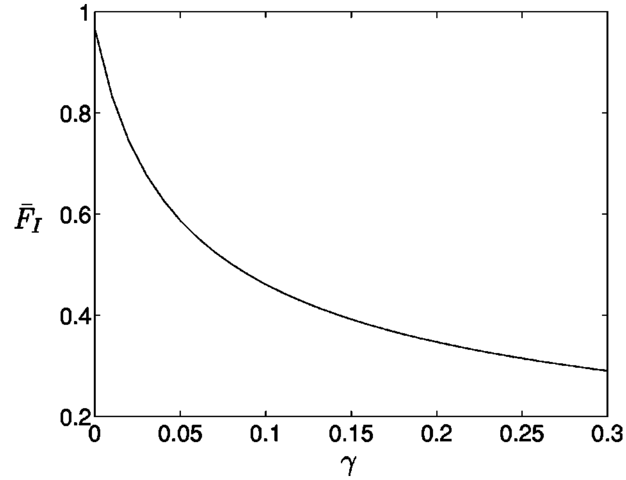

FIG. 6. Average fidelity as a function of degree of decoherence, γ, with an ideal resource energy corresponding to $N = 100$. Target is a coherent state of amplitude $\alpha = 3$.

$$F_{I,\gamma}(q) = \frac{1}{N+1}\frac{1}{P_I(q)}\sum_{n,m'} |c_{q-N+n}|^2|c_{p-N+n'}|^2 e^{-\gamma(n-n')^2}, \tag{39}$$

where n and n' run from $\max(0,N-q)$ to N and $P_I(q)$ is given by Eq. (13). It is not difficult to show that by setting $\gamma = 0$ we reproduce the result without noise, Eq. (17).

The average fidelity as a function of the degree of decoherence, γ, is shown in Fig. 6 for the example of a coherent state, $\alpha = 3$. As the degree of decoherence is increased, the fidelity drops away quickly. This is because the off-diagonal matrix elements of ρ_{AB} are being ''washed out'' by the $(n-n')^2$ term in the exponential. Physically, we are reducing the entanglement between the resource modes by making measurement of phase more random, and we would expect the ability of the technique to teleport a state to decrease; Fig. 6 shows this effect explicitly.

C. Beam-splitter resource

We add noise to the beam-splitter resource state in the same manner as described in Sec. V A, obtaining the total state,

$$\rho_{TAB} = \sum_{m,m'=0}^{\infty}\sum_{n,n'=0}^{2N} c_m c_{m'}^* d_{n-N} c_{n'-N}^* e^{-\gamma(n-n')^2} \times |m\rangle_T\langle m'| \otimes |n\rangle_A\langle n'| \otimes |2N-n\rangle_B\langle 2N-n'|. \tag{40}$$

After the teleportation protocol, we find that the fidelity with respect to the initial state is given by

$$F_{\mathrm{BS},\gamma}(q) = \frac{1}{P_{\mathrm{BS}}(q)}\sum_{n,n'=0}^{\min(q,2N)} |c_{q-n}|^2|c_{q-n'}|^2 \times d_{n-N} c_{n'-N}^* e^{-\gamma(n-n')^2}. \tag{41}$$

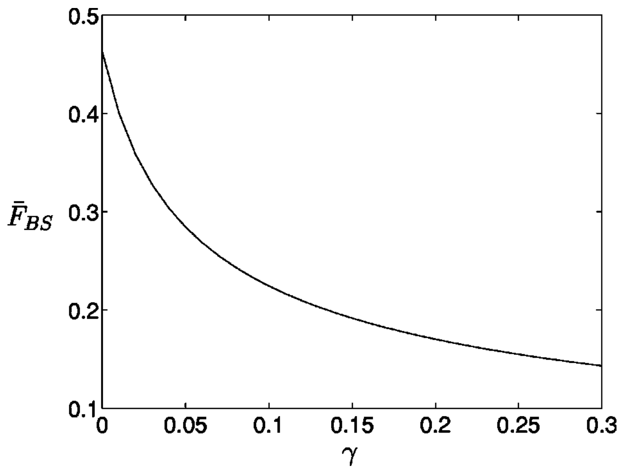

FIG. 7. Average fidelity as a function of degree of decoherence, γ, with a beam-splitter resource energy corresponding to $N=100$. Target is a coherent state of amplitude $\alpha=3$.

As we can see in Fig. 7, the fidelity decreases due to decoherence in the resource, except that the fidelity decreases from approximately $\frac{1}{2}$ instead of 1 as in Sec. III.

VI. FULL DECOHERENCE AND THE CLASSICAL LIMIT

Full decoherence corresponds to no entanglement between the resource modes and a completely flat phase probability distribution. A flat phase probability distribution is equivalent to taking the limit $\gamma\rightarrow\infty$ in the fidelities of Sec. V, thus making the off-diagonal terms in the density matrix representing the output state, $\rho_{\mathrm{out},B}$, zero. Physically, this limit corresponds to retaining the number correlations but making a measurement of phase completely arbitrary. We now suggest that this may be considered a *classical* limit of the teleportation protocol.

To motivate this point of view, we analyze a classical analog of the original qubit teleportation protocol [8]. Consider three classical bits, T,A,B, where A and B are correlated bits shared between the sender and receiver, respectively. The bit labeled T is the target bit and its state is specified by a distribution, $p_T(x)$, over the values of the binary variable. The bits A and B are correlated and have the state

$$p_{AB}(x,y)=\tfrac{1}{2}\delta_{x,y}, \tag{42}$$

where $\delta_{x,y}$ is the usual Kronecker delta. The total state of all three bits is $p_T(z)p_{AB}(x,y)$. We now suppose that the sender can measure the quantity $z\oplus x$ (addition mod2) on bits T and A. The result of this measurement is 0 if both T and A have the same value and 1 if they have different values. The sender A communicates this result to the receiver B.

The conditional state of the receiver—given the result of the measurement, w—is given by standard Bayesian conditioning as

$$p_B(y|w)=\frac{\Sigma_{x,z}p_T(z)p_{AB}(x,y)\,\delta_{w,z\oplus x}}{P_{TA}(w)}, \tag{43}$$

where $P_{TA}(w)$ is the probability that the joint measurement on A and T gives the result w. It can be shown that

$$p_B(x|0)=p_T(x), \tag{44}$$

$$p_B(x|1)=p_T(\neg x), \tag{45}$$

where $\neg$ is the logical NOT operation. The receiver B knows the result of the joint measurement and can implement a local NOT operation if the result of the measurement is 1. Given that local operation, we see that the state of the receiver, $p_B^{\mathrm{out}}(x)=p_B(x\oplus w|w)$, is now identical to the state of the target bit, that is to say it has exactly the same probability distribution. A little thought shows the protocol just described is exactly what would be implemented in the original qubit protocol if the shared resource between A and B were the completely decohered state $\rho_{AB}=(|00\rangle\langle 00| + |11\rangle\langle 11|)/2$. Note that in this case the only information that can be ''teleported'' is the probability distribution for the target bit in the basis in which ρ_{AB} is diagonal.

For all three entanglement resources considered, it can be shown that the average fidelity in the fully decohered limit $(\gamma\rightarrow\infty)$ reduces to

$$\overline{F}_\infty=\sum_{n=0}^{\infty}|c_n|^4. \tag{46}$$

For example, if the target is a coherent state, then this may be shown to be

$$\overline{F}(\alpha)=\frac{I_0(2|\alpha|^2)}{e^{2|\alpha|^2}}. \tag{47}$$

This is the fidelity between a pure state and a totally mixed state with the same photon number distribution. We conclude that if the resource contains only classical intensity correlations, it is only possible to teleport the number distribution of the target state: no phase information is teleported. In the sense of the qubit discussion in the preceding paragraph, we call this the classical limit of the protocol.

VII. CONCLUSIONS

We have shown that a teleportation scheme involving coupled oscillator states using number sum and phase difference measurements is possible, given sufficiently large numbers of Fock states in the resource. The ability of the scheme to reliably teleport a state was shown to improve as the number of Fock states in the resource increases. In the case of the beam-splitter-generated resource, this physically means more photons incident on the beam-splitter ports.

We have illustrated the effects of decoherence (in the form of phase diffusion) in three entanglement resources (ideal, beam-splitter-generated, and squeezed-state) on the fidelity of teleportation and have related this qualitatively to the change in entanglement of the resource. The decoherence maintains the classical intensity correlation inherent in the resource. In the limit of complete decoherence, the degraded state is only capable of teleporting the number distribution of the target state. As this result would have been obtained using standard Bayesian conditioning in a classical probabilistic protocol, we argue that it defines a classical limit for the quantum scheme.

ACKNOWLEDGMENTS

P.T.C. acknowledges the financial support of the Center for Laser Science and the University of Queensland.

[1] D. P. DiVincenzo, Science **270**, 255 (1995).
[2] L. K. Grover, Phys. Rev. Lett. **79**, 325 (1997).
[3] A. Ekert and R. Jozsa, Philos. Trans. R. Soc. London, Ser. A **356**, 1769 (1998).
[4] R. Jozsa, in *The Geometric Universe*, edited by S. Huggett, L. Mason, K. P. Tod, S. T. Tsou, and N. M. J. Woodhouse (Oxford University Press, Oxford, 1998), p. 369.
[5] V. Vedral and M. B. Plenio, Prog. Quantum Electron. **22**, 1 (1998).
[6] C. H. Bennett, G. Brassard, and N. D. Mermin, Phys. Rev. Lett. **68**, 557 (1992).
[7] J. Kempe, Phys. Rev. A **60**, 910 (1999).
[8] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[9] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett. **76**, 722 (1996).
[10] S. L. Braunstein, G. M. D'Ariano, G. J. Milburn, and M. F. Sacchi, Phys. Rev. Lett. **84**, 3486 (2000).
[11] M. B. Plenio and V. Vedral, Contemp. Phys. **39**, 431 (1998).
[12] S. L. Braunstein, C. A. Fuchs, and H. J. Kimble, J. Mod. Opt. **47**, 267 (2000).
[13] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
[14] S. L. Braunstein and H. J. Kimble, Phys. Rev. A **61**, 042302 (2000).
[15] C. H. Bennett, P. W. Shor, J. A. Smolin, and A. V. Thapliyal, Phys. Rev. Lett. **83**, 3081 (1999).
[16] B. Schumacher, Phys. Rev. A **54**, 2614 (1996).
[17] B. Schumacher and M. A. Nielsen, Phys. Rev. A **54**, 2629 (1996).
[18] S. Hill and W. K. Wootters, Phys. Rev. Lett. **78**, 5022 (1997).
[19] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).
[20] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[21] D. P. DiVincenzo, C. Fuchs, H. Mabuchi, J. A. Smolin, A. Thapliyal, and A. Uhlmann, in *Quantum Computing and Quantum Communications*, edited by C. P. Williams, Lecture Notes in Computer Science Vol. 1509 (Springer-Verlag, Berlin, 1999), p. 247 (NASA Conference on Quantum Computation, Palm Springs, 1998).
[22] V. Vedral, M. B. Plenio, M. A. Rippin, and P. L. Knight, Phys. Rev. Lett. **78**, 2275 (1997).
[23] V. Vedral and M. B. Plenio, Phys. Rev. A **57**, 1619 (1998).
[24] D. Jonathan and M. B. Plenio, Phys. Rev. Lett. **83**, 3566 (1999).
[25] D. Jonathan and M. B. Plenio, Phys. Rev. Lett. **83**, 1455 (1999).
[26] P. Horodecki, M. Horodecki, and R. Horodecki, Phys. Rev. Lett. **82**, 1056 (1999).
[27] V. Buzek, V. Vedral, M. B. Plenio, P. L. Knight, and M. Hillery, Phys. Rev. A **55**, 3327 (1997).
[28] W. Dür and J. I. Cirac, e-print quant-ph/0002028.
[29] M. Murao, M. B. Plenio, and V. Vedral, Phys. Rev. A **61**, 032311 (2000).
[30] V. Vedral, Phys. Lett. A **262**, 121 (1999).
[31] A. Furusawa, J. L. Sorensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, Science **282**, 706 (1998).
[32] D. Boschi, S. Branca, F. D. Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[33] G. J. Milburn and S. L. Braunstein, Phys. Rev. A **60**, 937 (1999).
[34] P. Rungta, W. J. Munro, K. Nemoto, P. Deuar, G. J. Milburn, and C. M. Caves, e-print quant-ph/0001075.
[35] A. O. Pittenger and M. H. Rubin, e-print quant-ph/0001110.
[36] W. Dür, J. I. Cirac, M. Lewenstein, and D. Bruß, e-print quant-ph/9910022.
[37] O. Rudolph, e-print quant-ph/0002026.
[38] P. Horodecki, Phys. Lett. A **232**, 333 (1997).
[39] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996).
[40] C. M. Caves and G. J. Milburn, e-print quant-ph/9910001.
[41] P. Deuar, W. J. Munro, and K. Nemoto, J. Opt. B: Quantum Semiclassical Opt. **2**, 225 (2000).
[42] A. Wehrl, Rev. Mod. Phys. **50**, 221 (1978).
[43] A. Imamoglu and Y. Yamamoto, Phys. Rev. Lett. **72**, 210 (1994).
[44] C. L. Foden, V. I. Talyanskii, G. J. Milburn, M. L. Leadbeater, and M. Pepper, e-print quant-ph/0004079.
[45] C. Brunel, B. Lounis, P. Tamarat, and M. Orrit, Phys. Rev. Lett. **83**, 2722 (1999).
[46] S. Yu and C.-P. Sun, Phys. Rev. A **61**, 022310 (2000).
[47] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1995).
[48] L. Susskind and J. Glogower, Physics (Long Island City, N.Y.) **1**, 49 (1964).
[49] S. L. Braunstein, C. M. Caves, and G. J. Milburn, Ann. Phys. (N.Y.) **247**, 135 (1996).
[50] H. M. Wiseman, Ph.D. thesis, The University of Queensland (1994).
[51] B. C. Sanders and G. J. Milburn, Phys. Rev. Lett. **75**, 2944 (1995).
[52] L. C. Biedenharn and J. D. Louck, *Angular Momentum in Quantum Physics—Theory and Application* (Addison-Wesley, Reading, MA, 1981).
[53] P. T. Cochrane, G. J. Milburn, and W. J. Munro, Phys. Rev. A **59**, 2631 (1999).