PHYSICAL REVIEW A, VOLUME 62, 052311

Bell-state analyzer with channeled atomic particles

Erika Andersson[1] and Stephen M. Barnett[2]
[1]*Department of Physics, Royal Institute of Technology, Lindstedtsvägen 24, S-10044 Stockholm, Sweden*
[2]*Department of Physics and Applied Physics, University of Strathclyde, Glasgow G4 ONG, Scotland*
(Received 15 May 2000; published 17 October 2000)

Recent advances in cooling and trapping of atomic particles has opened up the possibility of building microscopic networks of tubelike traps. As possible applications, we describe how a quantum controlled-NOT gate and a Bell-state analyzer could be implemented using atomic particles, which propagate through the system of channels and interact with the device potential and each other.

PACS number(s): 03.67.Hk, 03.75.−b

I. INTRODUCTION

Recently there has been considerable experimental interest in cooling and controlling atomic motion very accurately. Our motivation for this work is the possibility to make these atom traps very small [1], in the form of channels which guide the atoms. By arranging a network of miniaturized particle traps on a surface, one might construct quantum apparata for processing information and performing computations. An equivalent point of view has been expressed by Schmiedmayer in [2], who has discussed the possibility of fabricating quantum dots and quantum wires for atoms. Neutral atoms trapped in optical lattices might be used to achieve entanglement, conditional logic, and to perform computations [3–6]. In our previous work [7–9], we have investigated some aspects of how networks of matter wave guides might be used, and employed wave packet calculations to verify the operation of these devices.

Another reason why microscopic traps are of interest is the possibility of performing fundamental tests of quantum mechanics. Most tests have so far been performed using photons, but it is of interest to conduct experiments also with material particles, which display particle interactions and energy dispersion. Moreover, both bosons and fermions are readily available. Similar experiments are in principle possible with electrons. Here, it is straightforward to fabricate the devices using semiconductor heterostructures, but it is far less trivial to launch single conduction electrons in well controlled states. The advantage with an atomic environment is that the input states are easier to control. Yamamoto's group, however, has been able to show quantum correlations for electrons in an experiment which is the analog of a beam splitter for photons [10]. Buks *et al.* studied dephasing of interference by a ''which-path'' detector in an interferometer for electrons [11].

One way of trapping and guiding neutral atoms is to combine evanescent wave mirrors [12,13] or magnetic mirrors [14] with charged structures [2]. These structures can readily be fabricated on top of the mirrors using standard nanofabrication technology, which implies precision in the design. Another possibility is to use purely magnetic guidance [15–19]. The guides may be fabricated on a surface to form atom optical elements, e.g., beam splitters [20–23]. The ultimate goal is to reach truly microscopic dimensions; to confine the particles to a region of the size of a few nanometers or even less. An alternative way of achieving guided motion and possibly controlled interactions between atoms is to utilize hollow optical fibers with evanescent waves trapping the atoms to narrow channels at the center of the fiber [24–27]. These can eventually be fused to provide couplers similar to those used for optical signal transmission in fibers. Also the pure atomic wave guide achievable by the use of hollow laser modes may be used [28]. Atoms may be trapped in optical lattices [29]; these might also be used to store and manipulate atomic qubits [3–6].

In the following section, we consider the different mechanisms that contribute to coupling in the devices; tunneling from one channel to another and particle-particle interaction. In Sec. III, we briefly review some formalism concerning beam splitters and interferometers that will be needed to describe the quantum apparata we consider in Sec. IV. As a first example of a possible application for quantum information processing with particle networks, in Sec. IV A, we describe a controlled-NOT gate, an essential building block of a quantum computer. The second example, in Sec. IV B, considers the construction of a Bell-state analyzer. The ability to perform Bell measurements is of great importance, since Bell states, i.e., orthogonal, maximally entangled states of two two-level systems, appear in many applications of quantum information theory and quantum communication. It was recently shown that a Bell measurement may not be effected using linear elements only [30]. In our scheme, the nonlinear ingredient is provided by the particle-particle interaction.

II. COUPLING MECHANISMS

The devices we envisage consist of potential grooves which steer the particles and let them interact with the device potential and each other, thus effecting, e.g., logical operations and information processing. If two wave guides are brought close to each other, they are coupled by tunneling, which allows us to implement a beam splitter in a fashion completely analogous with an optical coupler. Moreover, if two particles are close to each other, they experience a coupling due to their interaction. This interaction enables us to effect conditional logical operations.

A. Tunneling between channels

Two channels close to each other can be thought of as forming a double-well potential, where particles will tunnel back and forth from one channel to the other. A wave packet in a one-dimensional double-well potential $U(x)$ will tunnel across the barrier with a rate

$$T \sim \exp\left\{ -\int \sqrt{2m[U(x)-E]}\,dx \right\}, \tag{1}$$

where we have put $\hbar \equiv 1$, and the integral should be evaluated over the potential barrier separating the two wells. By making this potential barrier higher or lower, we can control the tunneling between the two channels.

The coupling can also be understood in terms of the eigenfunctions of the double well potential. The ground state is symmetric, ψ_S, with energy E_S, and the first excited state antisymmetric, ψ_A, with energy E_A. We define the tunneling frequency 2Ω according to

$$\begin{aligned} E_A &= \bar{E} + \hbar\Omega, \\ E_S &= \bar{E} - \hbar\Omega. \end{aligned} \tag{2}$$

The time evolution of any initial state can easily be found using the eigenstates. We are interested in where the particle is localized; thus we form the states

$$\begin{aligned} \varphi_L &= \frac{1}{\sqrt{2}}(\psi_S + \psi_A), \\ \varphi_R &= \frac{1}{\sqrt{2}}(\psi_S - \psi_A), \end{aligned} \tag{3}$$

where the subscripts L (R) denote left (right) localization. Starting from φ_L at time $t=0$, we obtain

$$\begin{aligned} \Psi(t) &= \exp(-iHt/\hbar)\varphi_L \\ &= \frac{1}{\sqrt{2}}\exp(-i\bar{E}t/\hbar)(e^{i\Omega t}\psi_S + e^{-i\Omega t}\psi_A) \\ &= \exp(-i\bar{E}t/\hbar)(\cos\Omega t\;\varphi_L + i\sin\Omega t\;\varphi_R), \end{aligned} \tag{4}$$

which shows that the particle is oscillating back and forth between the two wells. For example, if

$$\Omega t = \frac{\pi}{4}, \tag{5}$$

the two coupled wells act as a 50/50 beam splitter.

The idea we have in mind is particles trapped in any type of ''neutral atom quantum dots,'' and moved to interact with each other by allowing for a time dependence in the potential. It also is easy to imagine that two coupled channels, where the particles move forward at a steady pace, will behave in the same fashion. A wave packet tunnels when the channels are close to each other and is split between the guides. By varying the distance between the channels, the length of the coupling region (i.e., the coupling time) and the height of the potential barrier between the guides, one is able to tune the device to perform, e.g., 50/50 beam splitting. In this case, the tunneling frequency Ω will obviously depend on the position along the channels, since the energy levels depend on the potential. It turns out, however, that one can describe the behavior of the two-channel system quite accurately using the simplified treatment above by defining a constant effective Ω and a constant effective coupling time τ [7].

B. Phase shift due to interaction

Two particles which are brought close to each other by the channels of the device will perturb each other's energy levels due to their mutual interaction. If we assume the interaction to be weak, i.e., the interaction energy to be much less than the level spacing of the modes in the guide, then the shape of the particle wave functions will essentially remain unchanged due to the interaction, and the only result will be a phase shift for the two-particle state. This approximation is reasonable since neutral atoms interact very weakly, with scattering lengths usually of the order of a few nm. If the energy shift due to the interaction is denoted by ΔE_{int}, the perturbative expression for this additional phase shift is

$$\Delta\phi_{\mathrm{int}} \approx \frac{1}{\hbar}\int dt\,\Delta E_{\mathrm{int}}, \tag{6}$$

where ΔE_{int} obviously depends on how close the two particles are at each time instant.

The fact that $\Delta\phi_{\mathrm{int}}$ is large if two particles are found close to each other gives us a way of implementing conditional logic. This has been used by Calarco *et al.* [5] to suggest a realization of a phase gate for two atoms trapped in time-dependent optical lattices. In their case, the cold atoms are allowed to collide and undergo interaction conditioned on their internal states. In principle, the calculations in our paper are valid both for fermions and bosons. The dominant collisional interaction, however, is the *s*-wave scattering, which for identical atoms in the same internal state occurs only for bosons. The interaction may also be provided by laser-induced dipole-dipole interactions, as discussed by Brennen *et al.* [3].

In our previous publication [7], we verified that the effect of particle interactions on a beam splitter such as the one considered in Sec. II A, is to change the effective coupling constant of the device. A device which acts as a 50/50 beam splitter for a single particle, will no longer perform this action when two particles are incident simultaneously, one through each input. This can easily be understood in the language of Sec. II A, since the effect of the interaction is to change the energy levels of the two-particle system, so that the effective coupling Ω is changed. This effect occurs when ΔE_{int} is of the same order of magnitude as the splitting between the symmetric and antisymmetric eigenfunctions of the two wave guides ΔE. One can, however, adjust the potential so that the splitting is 50/50 for two incident particles (in which case the action will obviously not be 50/50 splitting for a single particle).

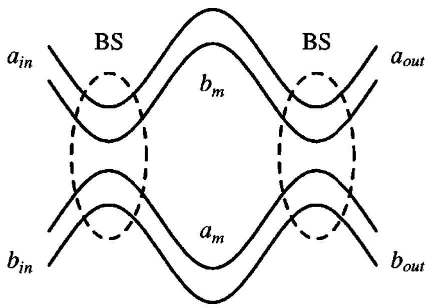

FIG. 1. Schematic view of an interferometer built of coupled wave guides. The regions marked ''BS'' encircled with dashed lines act as beam splitters as described in Sec. II.

III. BEAM SPLITTERS AND INTERFEROMETERS

When particles are directed into the incoming modes of a symmetric beam splitter, they are piloted into the outgoing modes according to the relations

$$\begin{bmatrix} a_{\rm out}^\dagger \\ b_{\rm out}^\dagger \end{bmatrix} = \begin{bmatrix} t^* & r^* \\ r^* & t^* \end{bmatrix} \begin{bmatrix} a_{\rm in}^\dagger \\ b_{\rm in}^\dagger \end{bmatrix}, \tag{7}$$

where t and r are the beam splitter transmission and reflection coefficients, obeying

$$|r|^2+|t|^2=1, \quad rt^*+tr^*=0; \tag{8}$$

see, for example, Ref. [31]. In particular, for a 50/50 beam splitter, we are allowed to choose $t=1/\sqrt{2}$ and $r=i/\sqrt{2}$.

An optical interferometer can be obtained using two 50/50 beam splitters and two mirrors. Using microtrap wave guides, an interferometer might be implemented as schematically depicted in Fig. 1. It is easily seen that a particle incident in one of the input channels can be directed into either of the output channels by varying the phase difference along the two paths of the interferometer. We take the incident state to be $|\Psi\rangle = a_{\rm in}^\dagger|0\rangle$, which implies that the state after the first beam splitter is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(a_m^\dagger + i b_m^\dagger)|0\rangle. \tag{9}$$

Suppose that particles propagating along the two paths acquire a phase difference $\Delta\phi$ (in the applications below, this phase difference will arise from the particle-particle interaction). As a result, the state incident on the second beam splitter is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(a_m^\dagger + i e^{i\Delta\phi} b_m^\dagger)|0\rangle. \tag{10}$$

Thus, the output state of the interferometer is

$$\begin{aligned} |\Psi\rangle &= \frac{1}{2}[a_{\rm out}^\dagger + i b_{\rm out}^\dagger + i e^{i\Delta\phi}(b_{\rm out}^\dagger + i a_{\rm out}^\dagger)]|0\rangle \\ &= \frac{1}{2}[(1-e^{i\Delta\phi})a_{\rm out}^\dagger + i(1+e^{i\Delta\phi})b_{\rm out}^\dagger]|0\rangle. \end{aligned} \tag{11}$$

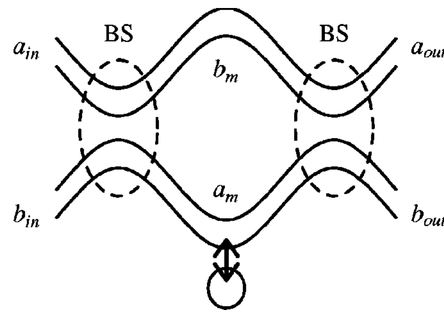

FIG. 2. Particle detector consisting of an interferometer and a dotlike trap. Conditioned on the presence of a particle in the trap, a particle incident on the interferometer will exit in different output modes, enabling us to detect the trapped particle. The double arrow symbolizes the particle-particle interaction, which induces a phase shift $\Delta\phi_{\rm int}=\pi$ if particles are present both in the dot and the guide denoted by a_m.

Depending on the phase shift introduced, the particle can be made to exit in either output channel, or, for that matter, coherently be split between the two outputs.

IV. DEVICES WITH CHANNELED PARTICLES

A. Particle detector or CNOT gate

We now turn to consider possible devices built of microscopic wave guides for material particles. To start with, we will outline the construction of a particle detector, which alternatively could serve as a controlled-NOT (CNOT) gate. In the previous section, we were able to obtain a 50/50 beam splitter by coupling two guiding channels. By coupling them twice, one obtains an interferometer as in Fig. 1. Now, add a dotlike particle trap close to one of the arms of the interferometer as depicted in Fig. 2. The action of the coupling due to interaction should be to perform a phase shift if particles are present both in the dot and in the lower interferometer arm, i.e., a phase gate. This phase shift may be obtained via collisional interaction, or alternatively via laser-induced dipole-dipole interaction as stated in Sec. II B. For a discussion of the working mechanisms of such a phase gate in the former case, we refer to Ref. [5]. If necessary, the particles may here perform a number of complete oscillations between the dot and the arm. An important point is then to make sure that the trapped particle does not leak out of the dot into the interferometer. This will require the dot potential to be temporarily lowered, or the dot to be moved closer to the interferometer arm, only when the interaction is to take place.

Suppose that a particle is incident in one of the interferometer input arms, say in input mode $a_{\rm in}$. If no particle is trapped in the dot, it will emerge in output mode $b_{\rm out}$, since the phase shift $\Delta\phi$ in Eq. (11) is zero. Alternatively, if the particle is incident in input mode $b_{\rm in}$, it will emerge in output mode $a_{\rm out}$. The presence of a particle in the dot will induce an extra phase shift for the part of the wave function in mode b_m. If this phase shift is tuned to be π, we conclude from Eq. (11) that the particle incident in the interferometer input $a_{\rm in}$ would exit in output mode $a_{\rm out}$ instead of $b_{\rm out}$; correspondingly, a particle incident in mode $b_{\rm in}$ would exit in $a_{\rm out}$. Thus, where the interferometer particle emerges is con-

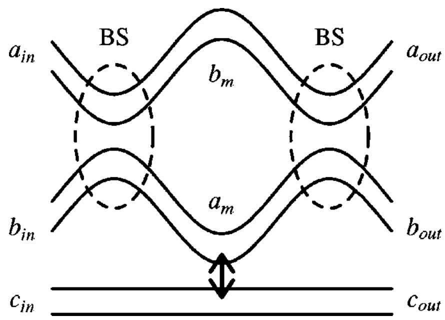

FIG. 3. Controlled-NOT gate realized with an interferometer plus an additional wave guide for the control bit. The data bit incident in one of the interferometer inputs can be redirected conditioned on the presence of a particle in the control wave guide, due to the additional phase shift $\Delta\phi_{\rm int}$ introduced by the particle-particle interaction.

ditioned on whether there is a particle trapped in the dot or not. This enables us to detect the trapped particle; alternatively, this particle serves as the control bit which steers the interferometer (data) particle. If the dotlike trap is replaced by a third wave guide, as in Fig. 3, then the control particle should be injected in this guide at a suitable time to make it reach the coupling point simultaneously with the data particle. This would correspond to the control bit being equal to one, flipping the data bit. The control bit being equal to zero simply means that this particle should be injected in some other wave guide, or delayed with respect to the data bit.

B. Bell-state analyzer

A Bell-state analyzer is a component needed to take full advantage of applications such as quantum teleportation [32–34] and quantum dense coding [35,36]. In principle, if we are able to perform a CNOT operation, and, in addition to this, linear one-qubit operations, we are also able to conduct Bell measurements.

Combining two interferometers I1 and I2 as in Fig. 4, we are able to implement a Bell-state analyzer in a simple and straightforward fashion. The two interferometers should be imagined to lie on top of each other. Again, the arrows indicate phase gates. The four possible Bell states we will want to distinguish with this device are

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(a_1^\dagger b_2^\dagger \pm b_1^\dagger a_2^\dagger)|0\rangle,$$

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(a_1^\dagger a_2^\dagger \pm b_1^\dagger b_2^\dagger)|0\rangle, \tag{12}$$

where $a_1^\dagger$ and $b_1^\dagger$ are creation operators referring to the two modes of interferometer No. 1 in Fig. 4, and $a_2^\dagger$ and $b_2^\dagger$ are creation operators for the modes of interferometer No. 2, and $|0\rangle$ denotes the vacuum state.

To demonstrate the operation of the device, we note that the reverse operation of the analyzer would be entangling incident nonentangled states. Let us assume the state $a_{1,\rm in}^\dagger a_{2,\rm in}^\dagger|0\rangle$ to be incident on the analyzer. After passing the first pair of beam splitters, this will have evolved into

$$\frac{1}{2}(a_{1,m}^\dagger + i b_{1,m}^\dagger)(a_{2,m}^\dagger + i b_{2,m}^\dagger)|0\rangle. \tag{13}$$

The particle-particle interaction will induce a phase shift in the two-particle wave function for the combinations $a_{1,m}^\dagger a_{2,m}^\dagger$ and $b_{1,m}^\dagger b_{2,m}^\dagger$, so that the two-particle state will evolve into

$$\frac{1}{2}(e^{i\Delta\phi_{\rm int}} a_{1,m}^\dagger a_{2,m}^\dagger + i a_{1,m}^\dagger b_{2,m}^\dagger + i b_{1,m}^\dagger a_{2,m}^\dagger - e^{i\Delta\phi_{\rm int}} b_{1,m}^\dagger b_{2,m}^\dagger)|0\rangle. \tag{14}$$

After the second pair of beam splitters, the out-going state is easily seen to be

$$\frac{1}{2}[(e^{i\Delta\phi_{\rm int}}-1)a_{1,\rm out}^\dagger a_{2,\rm out}^\dagger - (e^{i\Delta\phi_{\rm int}}+1)b_{1,\rm out}^\dagger b_{2,\rm out}^\dagger]|0\rangle. \tag{15}$$

For $\Delta\phi_{\rm int}=\pi/2$ this is a Bell state

$$\frac{1}{2}(1+i)(i a_{1,\rm out}^\dagger a_{2,\rm out}^\dagger - b_{1,\rm out}^\dagger b_{2,\rm out}^\dagger)|0\rangle = \frac{1}{\sqrt{2}} e^{i\pi/4}(a_{1,\rm out}^\dagger \tilde{a}_{2,\rm out}^\dagger - b_{1,\rm out}^\dagger b_{2,\rm out}^\dagger)|0\rangle = e^{i\pi/4}|\Phi^-\rangle, \tag{16}$$

where $\tilde{a}_{2,\rm out}^\dagger = i a_{2,\rm out}^\dagger$. Similarly, we can check how the input states $a_{1,\rm in}^\dagger b_{2,\rm in}^\dagger|0\rangle$, $b_{1,\rm in}^\dagger a_{2,\rm in}^\dagger|0\rangle$, and $b_{1,\rm in}^\dagger b_{2,\rm in}^\dagger|0\rangle$ are transformed. We conclude that Bell states fed into the analyzer will result in outputs according to

$$|\Psi^-\rangle \to e^{i\pi/4} a_1^\dagger b_2^\dagger|0\rangle,$$

$$|\Psi^+\rangle \to e^{i3\pi/4} b_1^\dagger a_2^\dagger|0\rangle,$$

$$|\Phi^-\rangle \to e^{-i\pi/4} a_1^\dagger a_2^\dagger|0\rangle,$$

$$|\Phi^+\rangle \to e^{-i3\pi/4} b_1^\dagger b_2^\dagger|0\rangle. \tag{17}$$

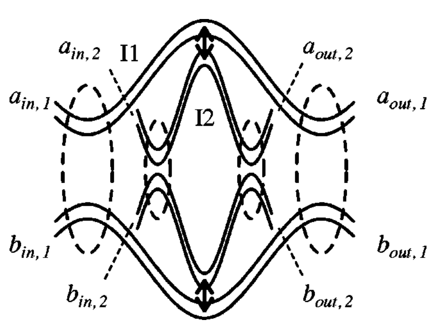

FIG. 4. Schematic depiction of a Bell-state analyzer consisting of two interferometers I1 and I2. The encircled regions act as beam splitters. The double arrows indicate phase gates where $\Delta\phi_{\rm int} = \pi/2$. Bell states incident on this device will be disentangled as described in Sec. IV B.

With this device we are thus able to disentangle Bell states, or, for that matter, to entangle two incident particles. One may note that, when using the creation-annihilation operator formalism, the quantum statistics is automatically taken into account through commutation relations. The reason for there being no apparent difference between fermions and bosons in the schemes considered is that two particles never are incident on the same beam splitter. Thus the familiar quantum statistical effect that bosons emerge together and fermions at different output ports when two particles are directed into the two input ports of a beam splitter plays no role. The difference between fermions and bosons appears only when it comes to the particle-particle interaction in the phase gate, as discussed in Sec. II B.

V. DISCUSSION AND CONCLUSIONS

As potential applications of neutral-atom microtrap networks, we have considered a controlled-NOT gate and a Bell-state analyzer. Until now, what has been achieved experimentally is the propagation over a few centimeters of atomic clouds with a diameter of approximately 100 μm. For the devices we consider, it is desirable to achieve single-mode coherent transport of individual atoms, and also very precise timing. To preserve coherence, the various scattering processes, depending on the particular realization of the microtrap, have to be controlled. With present-day technology, trapping frequencies of a few kHz are readily achieved; even frequencies as high as some MHz may be reached. One gate operation is completed within a few oscillations; thus it should be feasible to perform a few steps of calculation. These trapping frequencies correspond to ground-state widths in the nanometer regime. Laser-induced dipole-dipole interactions, on the other hand, require the atoms to be confined to relative distances smaller than the optical wavelength.

If the qubits are encoded into the spatial degrees of freedom, as suggested in this paper, to perform the readout one has to determine in which output channel an atom is emerging. One advantage with the proposed scheme is that the output of the device may be used directly as the input to a subsequent stage of similar physical nature. A possibility to fill the single atom criterion could be to load atoms in optical lattices and then to combine these with other types of neutral atom guides. In microlens arrays, atoms located in different dipole traps may be accessed individually by lasers [37]. Launching Bose-condensed atoms into microtraps may also prove possible; it will be interesting to follow the development of the experimental realizations to come.

ACKNOWLEDGMENTS

The authors thank Professor Stig Stenholm for inspiring discussions. Partial support from the European Union TMR program is gratefully acknowledged.

[1] J. D. Weinstein and K. G. Libbrecht, Phys. Rev. A **52**, 4004 (1995).
[2] J. Schmiedmayer, Eur. Phys. J. D **4**, 57 (1998).
[3] G. K. Brennen, C. M. Caves, P. S. Jessen, and I. Deutsch, Phys. Rev. Lett. **82**, 1060 (1999).
[4] D. Jaksch, H.-J. Briegel, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **82**, 1975 (1999).
[5] T. Calarco, E. A. Hinds, D. Jaksch, J. Schmiedmayer, J. I. Cirac, and P. Zoller, Phys. Rev. A **61**, 22304 (2000).
[6] H.-J. Briegel, T. Calarco, D. Jaksch, J. I. Cirac, and P. Zoller, J. Mod. Opt. **47**, 415 (2000).
[7] E. Andersson, M. Fontenelle, and S. Stenholm, Phys. Rev. A **59**, 3841 (1999).
[8] E. Andersson and S. Stenholm (unpublished).
[9] E. Andersson and S. Stenholm (unpublished).
[10] R. C. Liu, B. Odom, Y. Yamamoto, and S. Tarucha, Nature (London) **391**, 263 (1998).
[11] E. Buks, R. Schuster, M. Heiblum, D. Mahalu, and V. Umansky, Nature (London) **391**, 871 (1998).
[12] T. Esslinger, M. Weidemüller, A. Hemmerich, and T. Hänsch, Opt. Lett. **18**, 450 (1993).
[13] W. Seifert, R. Kaiser, A. Aspect, and J. Mlynek, Opt. Commun. **111**, 566 (1994).
[14] T. M. Roach, H. Abele, M. G. Boshier, H. L. Grossman, K. P. Zetie, and E. A. Hinds, Phys. Rev. Lett. **75**, 629 (1995); I. C. Hughes, P. A. Barton, T. M. Roach, M. G. Boshier, and E. A. Hinds, J. Phys. B **30**, 647 (1997).
[15] J. Denschlag, D. Cassettari, and J. Schmiedmayer, Phys. Rev. Lett. **82**, 2014 (1999).
[16] J. Reichel, W. Hänsel, and T. W. Hänsch, Phys. Rev. Lett. **83**, 3398 (1999).
[17] D. Müller, D. Z. Anderson, R. J. Grow, P. D. D. Schwindt, and E. Cornell, Phys. Rev. Lett. **83**, 5194 (1999).
[18] M. Key, I. G. Hughes, W. Rooijakkers, B. E. Sauer, E. A. Hinds, D. J. Richardson, and P. G. Kazansky, Phys. Rev. Lett. **84**, 1371 (2000).
[19] N. H. Dekker, C. S. Lee, V. Lorent, J. H. Thywissen, S. P. Smith, M. Drndić, R. M. Westerwelt, and M. Prentiss, Phys. Rev. Lett. **84**, 1124 (2000).
[20] R. Folman, P. Krüger, D. Cassettari, B. Hessmo, T. Maier, and J. Schmiedmayer, Phys. Rev. Lett. **84**, 4749 (2000).
[21] D. Cassettari, A. Chenet, R. Folman, A. Haase, B. Hessmo, P. Krüger, T. Maier, S. Schneider, T. Calarco, and J. Scmiedmayer, Appl. Phys. B: Lasers Opt. **70**, 721 (2000).
[22] D. Cassettari, B. Hessmo, R. Folman, T. Maier, and J. Schmiedmayer, e-print quant-ph/0003135.
[23] O. Zobay and B. M. Garraway, Opt. Commun. **178**, 93 (2000).
[24] M. A. Olshanii, Y. B. Ovchinnikov, and V. S. Lethokov, Opt. Commun. **98**, 77 (1993).
[25] S. Marksteiner, C. M. Savage, P. Zoller, and S. L. Rolston, Phys. Rev. A **50**, 2680 (1994).
[26] M. J. Renn, D. Montgomery, O. Vdovin, D. Z. Anderson, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **75**, 3253 (1995); M. J. Renn, E. A. Donley, E. A. Cornell, C. E. Wieman, and D.

Z. Anderson, Phys. Rev. A **53**, R648 (1996); **55**, 3684 (1997).
[27] H. Ito, T. Nakata, K. Sakaki, M. Ohtsu, K. I. Lee, and W. Jhe, Phys. Rev. Lett. **76**, 4500 (1996).
[28] J. Yin, Y. Zhu, W. Wang, Y. Wang, and W. Jhe, J. Opt. Soc. Am. B **15**, 25 (1998).
[29] P. S. Jessen and I. H. Deutsch, Adv. At. Mol., Opt. Phys. **37**, 95 (1996).
[30] N. Lütkenhaus, J. Calsamiglia, and K.-A. Suominen, Phys. Rev. A **59**, 3295 (1999).
[31] S. M. Barnett and P. Radmore, *Methods in Theoretical Quantum Optics* (Oxford University Press, Oxford, 1997).
[32] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[33] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Winfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).
[34] D. Boschi, S. Branca, F. DeMartini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[35] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[36] K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. **76**, 4656 (1996).
[37] G. Birkl (private communication).