Quantum topological phase of an electric dipole circulating around a ferromagnetic wire

V. M. Tkachuk*
Department of Theoretical Physics, Ivan Franko National University of Lviv, 12 Drahomanov Street, Lviv UA–79005, Ukraine
(Received 8 February 2000; published 19 October 2000)

The quantum topological phase of electric dipole circulating around the line of magnetic charges (monopoles) is discussed. We propose to mimic the line of magnetic monopoles using a ferromagnetic wire in which the magnetization is parallel to the wire and the magnitude of magnetization changes linearly along the wire. The phase shift in the proposed scheme is within the reach of present experimental techniques and may be observed in atomic or molecular interferometry.

PACS number(s): 03.65.Bz, 14.80.Hv, 39.20.+q

In the famous work of Aharonov and Bohm (AB) it was predicted that a charged particle circulating around a magnetic-flux line should accumulate a quantum topological phase that should not depend on the particle path [1]. The experimental confirmation of the AB effect was first obtained in Ref. [2] and later in a series of experiments culminating in Ref. [3]. Twenty-five years after the AB prediction, Aharonov and Casher (AC) showed that a magnetic dipole circulating around an electric charged straight line also accumulates a quantum topological phase [4]. The AC effect was observed in a neutron interferometer [5] and in a neutral atomic Ramsey interferometer [6].

Recently He and McKellar and independently Wilkens (HMW) predicted the existence of a topological phase for the electric dipole circulating around the line of the magnetic charge [7,8]. As there are no magnetic monopoles in nature, Wilkens proposed to mimic the line of magnetic monopoles using a magnetic sheet with two parallel edges. These edges effectively realize two oppositely charged lines of magnetic monopoles. Close to one of the edges a small hole is left. Then an interferometric path that passes through this hole and encircles only one of the edges experiences the topological phase shift. This incited the discussion between Wilkens and Spavieri around the question of whether or not the proposed schemes should work even in principle [9] (see also Refs. [10,11]). As a result of this discussion Spavieri derived a quantum topological phase that differs from that proposed by Wilkens by the presence of the extra term (see Eq. 6). Just this difference was the subject of debate.

The authors of paper in Ref. [12] derived the quantum phase accompanying a neutral particle moving in the area where a nonuniform electric field and uniform magnetic field are simultaneously applied. In particular they considered particles circulating around charged wire in the uniform magnetic field parallel to the wire. The difficulties associated with the $1/r^2$ interaction of the neutral particle with a charged wire was also the subject of discussion in Ref. [13]. In the Wei *et al.* reply it was explained that the difficulties with negative $1/r^2$ potential are purely of a theoretical nature and come from the assumption of an infinitely thin charged wire. Actually, the wire has a finite radius. It was shown that the atoms with a large velocity do not get to the region of the wire and collision between the atoms and the wire is well avoided. In connection with the paper in Ref. [12] let us note the papers in Refs. [14–16] where the scattering of neutral atoms on charged wire was studied.

As was shown in Ref. [17] the topological quantum phase is a generic feature of any multipole moment moving in an electromagnetic field. Note also a very recent paper [18] where the Maxwell electromagnetic duality relation between the AB, AC, and HMW topological phases was discussed and the description of all three phenomena was unified.

The aim of the present paper is to calculate the quantum topological phase for electric dipole circulating around a ferromagnetic wire and to consider some questions posed in the discussion between Wilkens and Spavieri. The latter author obtained an additional term in the quantum phase, compared to that obtained by Wilkens. Wilkens argues that this additional term is not physical. Note, that in order to calculate this term explicitly it is necessary to know the vector potential. But the calculation of the vector potential for Wilkens' configuration is a difficult problem. Therefore we propose to mimic the line of magnetic monopoles using a ferromagnetic wire, with the magnetization parallel to the wire and the magnitude of magnetization changing linearly along it. For this configuration it is possible to calculate the vector potential explicitly. We will show that for our configuration the phase derived by Wilkens coincides with the phase derived by Spavieri, i.e., in our case the additional term derived by Spavieri is equal to zero. This of course does not resolve completely the original issues debated by Wilkens and Spavieri. Nevertheless we think that it is important that, for our configuration, the phase derived by Wilkens coincide with the phase derived by Spavieri.

We will also show that the phase shift in the proposed scheme is within the reach of the present experimental technique and may be observed in atomic or molecular interferometry. Recently we showed that a ferromagnetic wire can be used for trapping and guiding the laser cooled neutral atoms [19].

Spavieri proposed a simple way to derive the quantum phase for the dipole directly from the AB phase considering the electric dipole as composed of two charges $\pm e$ [10,11]. The AB phase reads

$$\phi_{AB}=\frac{e}{\hbar c}\int \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}. \quad (1)$$

*Email address: tkachuk@ktf.franko.lviv.ua

Then the phase for an electric dipole consists in summing the AB phase for two charges $\pm e$

$$\phi_S = \frac{e}{\hbar c}\int \mathbf{A}(\mathbf{r}')\cdot d\mathbf{r}' - \frac{e}{\hbar c}\int \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}, \qquad (2)$$

where $\mathbf{r}$ is the radius vector of the charge $-e$, $\mathbf{r}'=\mathbf{r}+\mathbf{a}$ is the radius vector of the charge $+e$, and $\mathbf{a}$ is the small vector between the $\pm e$ charges. In the dipole approximation (linear approximation over $\mathbf{a}$) the quantum phase can be written as follows

$$\phi_S = \frac{1}{\hbar c}\int (\mathbf{d}\cdot\nabla)[\mathbf{A}(\mathbf{r})\cdot\mathbf{v}]dt, \qquad (3)$$

where $\mathbf{d}=e\mathbf{a}$ is the electric dipole moment that is constant in our case $d\mathbf{d}/dt=0$, $\mathbf{v}$ is the velocity of the particle, and the vector potential $\mathbf{A}=\mathbf{A(r)}$ is a function of $\mathbf{r}$ and does not depend on the time t.

Phase (3) can be transformed to the following form

$$\phi_S = \frac{1}{\hbar c}\int (\mathbf{d}\cdot\boldsymbol{\nabla})\mathbf{A}\cdot d\mathbf{r} = \frac{1}{\hbar c}\int \boldsymbol{\nabla}(\mathbf{d}\cdot\mathbf{A})\cdot d\mathbf{r} + \phi_W, \qquad (4)$$

where ϕ_W is the quantum phase derived by Wilkens in Ref. [8]

$$\phi_W = \frac{1}{\hbar c}\int [\mathbf{B}\times\mathbf{d}]\cdot d\mathbf{r}, \qquad (5)$$

where $\mathbf{B}=\mathrm{rot}\mathbf{A}$ is the magnetic field. Equation (4) gives the relation between the phases derived by Wilkens ϕ_W and by Spavieri ϕ_S.

Note that a measurable quantity is the phase shift $\Delta\phi$ that is given by the closed contour integral. Then from Eq. (4) we have the relation between phase shifts $\Delta\phi_S$ and $\Delta\phi_W$

$$\Delta\phi_S = \frac{1}{\hbar c}\oint \boldsymbol{\nabla}(\mathbf{d}\cdot\mathbf{A})\cdot d\mathbf{r} + \Delta\phi_W, \qquad (6)$$

where $\Delta\phi_S$ and $\Delta\phi_W$ are given by Eqs. (3), (4), (5), in which $\int$ is replaced by $\oint$. Just the difference between $\Delta\phi_S$ and $\Delta\phi_W$ and the question of whether or not the schemes proposed by Wilkens should work even in principle, was the subject of the debate between Wilkens and Spavieri [9] (see also Refs. [10,11]).

In the present paper, in order to avoid the problem of the Wilken's configuration, we propose to mimic the line of magnetic monopoles using a ferromagnetic wire with the magnetization parallel to the wire, and the magnitude of magnetization changing linearly along it. We will calculate explicitly the vector potential and magnetic field and will show that in our case $\Delta\phi_S=\Delta\phi_W$.

Let us consider the wire directed along the z axis with the magnetization parallel to the wire. The magnitude of the line magnetization M is a linear function of z and is given by

$$M(z) = -qz, \qquad (7)$$

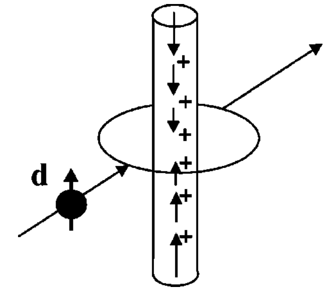

FIG. 1. Schematic configuration for quantum topological phase of electric dipole **d** circulating around and parallel to the ferromagnetic wire. The ferromagnetic wire with the magnetization parallel to the wire and the magnitude of magnetization changing linearly along it, is used to mimic the line of magnetic charges (monopoles). The magnetization of the wire is denoted by vertical arrows. Various sizes of arrows show that the magnetization changes along the wire. The magnetic charges are denoted by ''+.'' In the interferometer experiment two split coherent beams encircle the wire.

where $-dM(z)/dz=q$ can be treated as a linear density of the magnetic charge. Each end of the wire contains the charge $Q=-qL/2$ (L being the length of the wire) so the full magnetic charge of the wire is obviously equal to zero. When the wire is sufficiently long, the magnetic field in the region around the central part of the wire is

$$\mathbf{B} = \frac{2q}{\rho}\mathbf{n}_\rho, \quad \mathbf{n}_\rho = \frac{\boldsymbol{\rho}}{\rho}, \qquad (8)$$

where $\boldsymbol{\rho}=x\mathbf{i}+y\mathbf{j}$ is the two-dimensional vector in the $x-y$ plane. Here we consider $\rho\ll L$ and the contribution of the magnetic field in the region around the central part of the wire created by the charges placed on the ends of the wire is of the order of $Q/(L/2)^2=2q/L$. Therefore its contribution can be neglected.

Inside a thin ferromagnetic wire the magnetic field can be written as follows

$$B_z = -4\pi qz\,\delta(x)\,\delta(y) \qquad (9)$$

and can be treated as that of ''Dirac strings'' that connect opposite magnetic charges. Including this field guarantees that $\oint\mathbf{B}\cdot d\mathbf{S}=0$.

Now let us calculate the vector potential of a ferromagnetic wire. It can be done considering the latter as composed of magnetic dipoles. Then summing the vector potentials of magnetic dipoles along the wire we obtain

$$A_x = 2q\frac{yz}{\rho^2}, \quad A_y = -2q\frac{xz}{\rho^2}, \quad A_z = 0. \qquad (10)$$

We can easily check that this vector potential gives the magnetic field (8), and (9).

It is important to note that in our case the vector potential is a monovalued function. Rewriting the contour integral in Eq. (6) in the form $\oint\nabla(\mathbf{d}\cdot\mathbf{A})\cdot d\mathbf{r}=\oint d(\mathbf{d}\cdot\mathbf{A})$ we see that for the monovalued vector potential $\mathbf{A}$, this contour integral is equal to zero. Therefore, the first term in Eq. (6) is equal to zero and thus it follows that $\Delta\phi_S=\Delta\phi_W$.

For the electric dipole d circulating around and parallel to the ferromagnetic wire (which mimics the line of magnetic charge with the linear density q) (Fig. 1) the quantum topological phase reads

$$\Delta\phi=\Delta\phi_S=\Delta\phi_W=-\frac{4\pi q d}{\hbar c}. \tag{11}$$

This phase shift can be observed in an interferometer experiment (Fig. 1). The incoming beam is split before reaching the ferromagnetic wire into two coherent beams with the same dipoles and travelling on the opposite sides of the wire. The particles moving on the opposite sides of the wire accumulate the opposite phases and the phases of outcoming beams are shifted by the observable $\Delta\phi$.

In order to calculate the phase shift, let us consider the dipole moment $d=|e|a_0$ (e is the charge of electron, a_0 is the Bohr radius) circulating around a ferromagnetic wire with the following parameters. The radius of the wire is r, the length of the wire is L, the mean distance between the magnetic atoms is a, and the magnetic moment of the atom is μ_f. Let us suppose that the line magnetization changes in a linear way from M_{max} on the left end of the wire to $-M_{max}$ on the right end of it. Then the phase shift can be represented in the following form

$$\Delta\phi=-4\pi\alpha^2\pi\frac{\mu_f}{\mu_0}\left(\frac{a_0}{a}\right)^2\frac{r^2}{La}, \tag{12}$$

where α is the fine-structure constant and μ_0 is the Bohr magneton. Note that some atom interferometers can detect the phases of 0.1 rad [20] and atomic beam splitters may reach the supermillimeter range [21,22]. For the iron ferromagnetic wire with the radius $r=0.2$ mm and the length $L=0.1$ m, we obtain $\Delta\phi\simeq-0.14\pi$. Thus we can conclude that the phase shift in the proposed scheme is within the reach of the present experimental technique and may be observed in atomic or molecular interferometry.

[1] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 380 (1959).
[2] R. G. Chambers, Phys. Rev. Lett. **5**, 3 (1960).
[3] A. Tonomura, N. Osakabe, T. Matsuda, T. Kawasaki, J. Endo, S. Yano, and H. Yamada, Phys. Rev. Lett. **56**, 792 (1986).
[4] Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).
[5] A. Cimmino, G. T. Opat, A. G. Klein, H. Kaiser, S. A. Werner, M. Arif, and R. Clothier, Phys. Rev. Lett. **63**, 380 (1989).
[6] K. Sangster, E. A. Hinds, S. M. Barnett, and E. Riis, Phys. Rev. Lett. **71**, 3641 (1993); K. Sangster *et al.*, Phys. Rev. A **51**, 1776 (1995).
[7] X. G. He and B. H. J. McKellar, Phys. Rev. A **47**, 3424 (1993).
[8] M. Wilkens, Phys. Rev. Lett. **72**, 5 (1994).
[9] G. Spavieri, Phys. Rev. Lett. **81**, 1533 (1998); M. Wilkens, *ibid.* **81**, 1534 (1998).
[10] G. Spavieri, Phys. Rev. Lett. **82**, 3932 (1999).
[11] G. Spavieri, Phys. Rev. A **59**, 3194 (1999).
[12] H. Wei, R. Han, and X. Wei, Phys. Rev. Lett. **75**, 2071 (1995).
[13] C. R. Hagen, Phys. Rev. Lett. **77**, 1656 (1996); H. Wei, X. Wei, and R. Han, *ibid.* **77**, 1657 (1996).
[14] U. Leonhardt and M. Wilkens, Europhys. Lett. **42**, 365 (1998).
[15] J. Audretsch and V. D. Skarzhinsky, Phys. Lett. A **241**, 7 (1998).
[16] J. Audretsch and V. D. Skarzhinsky, Phys. Rev. A **60**, 1854 (1999).
[17] Chia-Chu Chen, Phys. Rev. A **51**, 2611 (1995).
[18] J. P. Dowling, C. P. Williams, and J. D. Franson, Phys. Rev. Lett. **83**, 2486 (1999).
[19] V. M. Tkachuk, Phys. Rev. A **60**, 4715 (1999).
[20] D. W. Keith, C. R. Ekstrom, Q. A. Turchette, and D. E. Pritchard, Phys. Rev. Lett. **66**, 2693 (1991).
[21] T. Pfau, Ch. Kurtsiefer, C. S. Adams, M. Sigel, and J. Mlynek, Phys. Rev. Lett. **71**, 3427 (1993).
[22] T. Pfau, C. S. Adams, and J. Mlyneck, Europhys. Lett. **21**, 439 (1993).