Nonlinear dynamical evolution of the driven two-photon Jaynes-Cummings model

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The driven two-photon Jaynes-Cummings model (JCM) exhibits a collapse and revival phenomenon in its mean photon number on a time scale much larger than the periodic revival time for two-state inversion in the usual two-photon JCM. The time scale of revivals for these oscillations is much larger than the corresponding time scale for the driven one-photon JCM and can be explained with the Hamiltonian of type $H_\eta = \pm\sqrt{(J_\eta^\dagger + \alpha P_\eta)^2 (J_\eta + \alpha P_\eta)^2}$ constructed explicitly for this purpose. The effect of Stark shifts is also studied and it is observed that the dynamics is strongly influenced by such Stark shifts both at early stages as well as at late stages of the evolution but in different ways, and thus Stark shifts play a significant role in determining the dynamical evolution of the system.

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I. INTRODUCTION

Both one- as well as two-photon Jaynes-Cummings models [1,2] describing interaction of a single-mode quantized electromagnetic field with a two-level or an effective two-level system have a very significant role not only in many theoretical predictions [3] but also in the explanation of experiments [4] in cavity quantum electrodynamics. These models exhibit collapse and revival phenomena of Rabi oscillations [5] on different time scales when interacting with a superposition of the Fock state field. A considerable amount of work on the various aspects of these models is available in the literature [6].

Recently, Chough and Carmichael [7] have discussed a situation in which the usual one-photon JCM driven by an external field, interacting with a single-mode cavity field (initially in the vacuum state), is considered. They have shown that it is possible to transform this kind of interaction Hamiltonian into a usual one-photon JCM Hamiltonian by using a displacement operator on the radiation mode. The most interesting aspects of this study are that the mean photon number not only oscillates like the orbital motion of bound mechanical system, but it collapses and revives on a larger time scale as compared to the time scale of the usual collapse-revival phenomenon of Rabi oscillations. Essentially, they have uncovered the nonlinear oscillator character of one-photon JCM not discussed earlier.

In this paper, we extend the work of Chough and Carmichael [7] for a two-photon JCM. The motivation for this work is threefold. (i) As the two-photon JCM describes a nonlinear interaction of atom and field, so it would be interesting to know how a driven two-photon JCM would behave and how the dynamical evolution of the mean photon number would take place. (ii) We would like to compare the nonlinear oscillator model of the one-photon JCM with that of the two-photon JCM. (iii) We would also examine the effect of Stark shifts arising due to the transitions to virtual levels which as we will see give rise to very interesting and novel changes in the dynamics. The rest of the paper is organized as follows. In Sec. II, we formulate the model and present results for the mean photon number and compare them with collapse and revival phenomena in the standard two-photon JCM. In order to have a physical understanding of the results, a nonlinear model has been explicitly constructed in Sec. III. We compare our results with that of Ref. [7] in Sec. IV. Some concluding remarks are presented in Sec. V.

II. MODEL AND RESULTS FOR THE MEAN PHOTON NUMBER

We consider a two-photon JCM consisting of an effective two-level atom (transition frequency ω_0) undergoing a nonresonant two-photon transition in a resonant single mode of the quantized radiation field having frequency ω such that $2\omega = \omega_0$. For simplicity of analysis, the interaction of the field and atom will be considered in an ideal and closed cavity so that field damping and the radiative damping can be ignored. We include here Stark splitting due to intermediate levels just to make our model more realistic. This system is also driven by a resonant undepleted external field. So, the atom is interacting with the sum of two fields, which leads to the following form of a Hamiltonian under the rotating-wave approximation (RWA),

$$H = 2\hbar\omega(\sigma_z/2) + \hbar\omega(a^\dagger a + 1/2) + \hbar\left(a^\dagger + \frac{\varepsilon_{\text{ext}}}{\sqrt{g}}\right)\left(a + \frac{\varepsilon_{\text{ext}}}{\sqrt{g}}\right) \times(\beta_1|e\rangle\langle e| + \beta_2|g\rangle\langle g|) + i\hbar g\left[\sigma_+\left(a + \frac{\varepsilon_{\text{ext}}}{\sqrt{g}}\right)^2 - \sigma_-\left(a^\dagger + \frac{\varepsilon_{\text{ext}}}{\sqrt{g}}\right)^2\right], \tag{1}$$

in which β_1, β_2 are the measures of Stark splitting, g is the coupling constant for two-photon transition in the cavity field, ε_{ext} is the real amplitude of the external driving field, and a ($a^\dagger$) is the annihilation (creation) operator. $\sigma_\pm$ are pseudospin operators which along with the operator σ_z are satisfying the usual commutation relationships $[\sigma_+, \sigma_-] = \sigma_z, [\sigma_\pm, \sigma_z] = \mp 2\sigma_\pm$, and $|e\rangle$ ($|g\rangle$) is the excited (ground) state of the atom. The Hamiltonian (1) can be written in the interaction picture as

$$H=\hbar\left(a^{\dagger}+\frac{\varepsilon_{\rm ext}}{\sqrt{g}}\right)\left(a+\frac{\varepsilon_{\rm ext}}{\sqrt{g}}\right)(\beta_1|e\rangle\langle e|+\beta_2|g\rangle\langle g|)+i\hbar g\left[\sigma_+\left(a+\frac{\varepsilon_{\rm ext}}{\sqrt{g}}\right)^2-\sigma_-\left(a^{\dagger}+\frac{\varepsilon_{\rm ext}}{\sqrt{g}}\right)^2\right]. \qquad (2)$$

The Glauber displacement operator is defined as

$$D(\alpha)=\exp[\alpha(a^{\dagger}-a)],$$
$$D^{\dagger}(\alpha)aD(\alpha)=a+\alpha, \qquad (3)$$

where $\alpha=\varepsilon_{\rm ext}/\sqrt{g}$.

We transform Hamiltonian (2) under the displacement operator (3) and get

$$\tilde{H}=D(\alpha)HD^{\dagger}(\alpha)=\hbar a^{\dagger}a(\beta_1|e\rangle\langle e|+\beta_2|g\rangle\langle g|)+i\hbar g(\sigma_+a^2-\sigma_-a^{\dagger 2}), \qquad (4)$$

which is the standard or usual two-photon Jaynes-Cummings model in the interaction picture with Stark splitting included. We can revert back to the noninteraction (Schrödinger) picture by including terms such as $\hbar\omega\sigma_z+\hbar\omega(a^{\dagger}a+1/2)$ in Eq. (4) and we refer to this Hamiltonian as $\tilde{H}_T$. The construction of the resonant two-photon JCM is such that the creation of two photons is accompanied by an atomic deexcitation and the annihilation of two photons is accompanied by an atomic excitation. Hence the operator $(a^{\dagger}a)/2+\sigma_+\sigma_-$ commuting with the total Hamiltonian $\tilde{H}_T$ is the constant of motion [2]. This operator represents the total excitation in the atom-field interacting two-photon JC system, which is a conserved quantity. In the case of the one-photon JCM, the equivalent constant of motion [1,6] is the operator $a^{\dagger}a+\sigma_+\sigma_-$. This kind of two-photon JC model is directly applicable to experiments of [8] describing the process of the two-photon transition between ${}^{44}S_{1/2}\longleftrightarrow{}^{43}S_{1/2}$ levels in Rydberg ^{85}Rb atoms through the relay level ${}^{39}P_{3/2}$. Also, a trapped and laser irradiated ion exhibits a Jaynes-Cummings dynamics in appropriate limits [9]. The center-of-mass motion of the ion is quantized in the trap potential and it is coupled to the internal degrees of freedom by a suitable laser. The required experimental hardware to realize JCM is available in the moment [10]. Recently, it has been proposed that the multiquantum JCM [similar to Eq. (4) above] could be realized for a trapped and laser irradiated ion, far from the Lamb-Dicke regime under an appropriate resonance condition [11]. The nonlinearity appearing in the vibronic coupling is huge and thus it opens the possibility of the experimental realization of the multiquantum JCM for a very long time limit. The time-dependent Schrödinger equation for the Hamiltonian (4) is

$$i\hbar\frac{\partial}{\partial t}|\tilde{\psi}\rangle=\tilde{H}|\tilde{\psi}\rangle. \qquad (5)$$

The wave function $|\tilde{\psi}\rangle$ is related to $|\psi\rangle$ of Hamiltonian (1) as follows:

$$|\tilde{\psi}(t)\rangle=D(\alpha)|\psi(t)\rangle, \qquad (6)$$

$$|\tilde{\psi}(0)\rangle=D(\alpha)|\psi(0)\rangle=|\alpha\rangle|e\rangle. \qquad (7)$$

We consider our effective two-state system to be consisting of the excited state $|e\rangle$ and the ground state $|g\rangle$ with the initial state of the total system of atom and electromagnetic field as $|\psi(0)\rangle=|0\rangle|e\rangle$. So, with the help of Eq. (7), we obtain $|\alpha\rangle=D(\alpha)|0\rangle$, a coherent state.

It is very straightforward to calculate the mean photon number:

$$\langle a^{\dagger}a\rangle=\langle\psi|a^{\dagger}a|\psi\rangle=\langle\tilde{\psi}|(a^{\dagger}-\alpha)(a-\alpha)|\tilde{\psi}\rangle=2\alpha(\alpha-\langle\tilde{\psi}|a|\tilde{\psi}\rangle)+2(1-P_e), \qquad (8)$$

where $P_e=\langle\tilde{\psi}|e\rangle\langle e|\tilde{\psi}\rangle$ is the occupation probability of the excited state $|e\rangle$ and we have used the fact that $(\langle\tilde{\psi}|a^{\dagger}a|\tilde{\psi}\rangle)/2+P_e$ is a conserved [2] quantity [in concurrence with the discussion after Eq. (4)], in which $\langle\tilde{\psi}|a^{\dagger}a|\tilde{\psi}\rangle$ represents the mean photon number as measured in the tilde state. With the help of the solution of the standard two-photon JCM [2], we can find

$$\langle\tilde{\psi}|a|\tilde{\psi}\rangle=\sum_{n=1}^{n=\infty}C_nC_{n-1}\left[\frac{1}{2}\left(\sqrt{n}+\frac{\sqrt{n}x_nx_{n-1}+\sqrt{n+2}y_ny_{n-1}}{\sqrt{x_n^2+y_n^2}\sqrt{x_{n-1}^2+y_{n-1}^2}}\right)\cos[(\sqrt{x_n^2+y_n^2}-\sqrt{x_{n-1}^2+y_{n-1}^2})gt]+\frac{1}{2}\left(\sqrt{n}-\frac{\sqrt{n}x_nx_{n-1}+\sqrt{n+2}y_ny_{n-1}}{\sqrt{x_n^2+y_n^2}\sqrt{x_{n-1}^2+y_{n-1}^2}}\right)\cos[(\sqrt{x_n^2+y_n^2}+\sqrt{x_{n-1}^2+y_{n-1}^2})gt]\right], \qquad (9)$$

$$P_e(t)=\frac{1}{2}\left(1+\frac{x_n^2}{x_n^2+y_n^2}+\frac{y_n^2}{x_n^2+y_n^2}\sum_{n=0}^{n=\infty}C_n^2\cos(2gt\sqrt{x_n^2+y_n^2})\right)$$

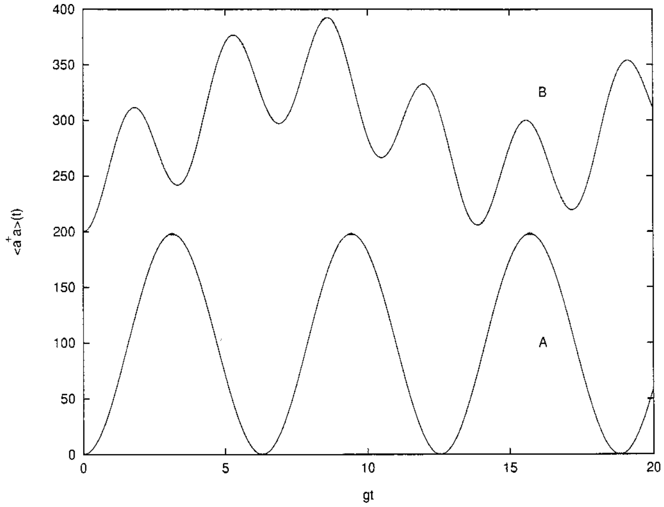

FIG. 1. The mean photon number $\langle a^\dagger a\rangle$ as a function of time for the driven two-photon JCM with $\varepsilon/\sqrt{g}=7$. Curve A is for without Stark shifts ($\beta_1/g=0,\beta_2=0$) while curve B is for with Stark shifts ($\beta_1/g=0.6,\beta_2/g=0.1$).

in which $x_n=(n\beta_1-(n+2)\beta_2)$, $y_n=\sqrt{(n+1)(n+2)}$, and

$$C_n=e^{-|\alpha|^2/2}\frac{\alpha^n}{\sqrt{n!}},\quad n=0,1,2,\ldots, \tag{10}$$

where C_n represents the expansion coefficient for a coherent state in terms of the Fock state such that $|C_n|^2$ is the Poissonian probability of photon number distribution and factor $\frac{1}{2}$ has been absorbed in β_L. We have depicted the evolution of the mean photon number $\langle a^\dagger a\rangle$ without Stark shifts ($\beta_1=0,\beta_2=0$) for $\varepsilon_{\mathrm{ext}}/\sqrt{g}=7$ in Fig. 1 (curve A), which clearly gives rise to orbital oscillations with the period of oscillations equal to $2\pi/g$. For the sake of comparison we have plotted the quantity $P_e(t)$ in Fig. 2 (curve A). We observe that there is synchronization in the orbital oscillations (Fig. 1, curve A) and the collapse and revival phenomena of Rabi oscillations (Fig. 2, curve A). The effect of nonzero Stark shifts ($\beta_1=0.6,\beta_2=0.1$) on $\langle a^\dagger a\rangle$ and $P_e(t)$ has been depicted in Fig. 1 (curve B) and Fig. 2 (curve B), respectively. Clearly, the nonzero Stark shift reduces the period of orbital oscillation to $2\pi/\sqrt{g^2+(\beta_1-\beta_2)^2}$ (approximately at intense fields) and introduces another time scale, i.e., modulation in the quantity $\langle a^\dagger a\rangle$. In $P_e(t)$ (Fig. 2) also, there is a reduction in the revival period from π/g (without Stark shift, curve A) to $\pi/\sqrt{g^2+(\beta_1-\beta_2)^2}$ (with Stark shift, curve B) and the magnitude of revivals also reduces. Thus both the orbital oscillations as well as collapse-revival phenomena in the excitation probability are strongly influenced by the Stark shift. However, the orbital motion exhibits a kind of collapse and revival phenomenon over a very long time period compared to the revival time of Rabi oscillations. As we will see in the following, the revival time increases to $\sim 8\pi\alpha^6/g$ without the Stark shift (Fig. 3, curve A) and to

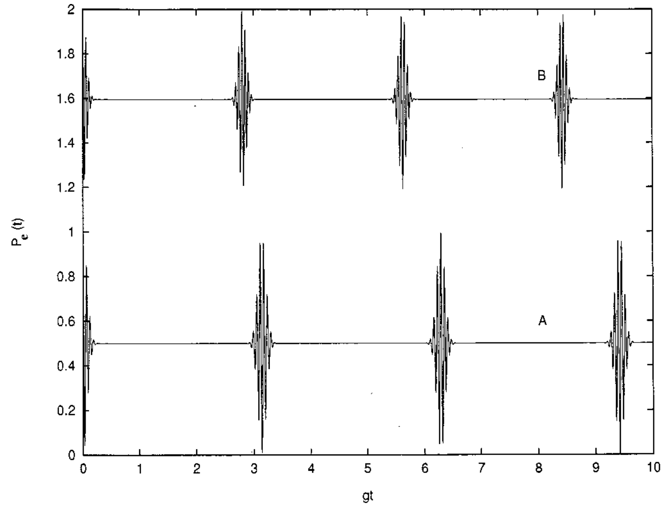

FIG. 2. The collapse and revival phenomenon in excitation probability $P_e(t)$ as a function of time for the same parameters as in Fig. 1. Curve A is for without Stark shifts ($\beta_1/g=0,\beta_2/g=0$) while curve B is for with Stark shifts ($\beta_1/g=0.6,\beta_2/g=0.1$).

$$\sim\frac{8\pi\alpha^6}{g}\left[\frac{\left(1+\dfrac{(\beta_1-\beta_2)^2}{g^2}\right)^{3/2}}{\left|1-\dfrac{8(\beta_1-\beta_2)^2}{g^2}\right|}\right]$$

with the Stark shift (Fig. 3, curve B). By analyzing Eq. (9) without Stark shifts, i.e., by keeping $\beta_1=\beta_2=0$, we observe that there are two sets of frequencies determining the time evolution of $\langle\tilde{\psi}|a|\tilde{\psi}\rangle$. These frequencies are the sum and difference of $\sqrt{(n+1)(n+2)g}$ and $\sqrt{n(n+1)g}$ and the corresponding amplitudes of these frequency terms are $(\sqrt{n+2}-\sqrt{n})/2$ and $(\sqrt{n+2}+\sqrt{n})/2$, respectively. In the intense cavity field when the photon field is large, we find that

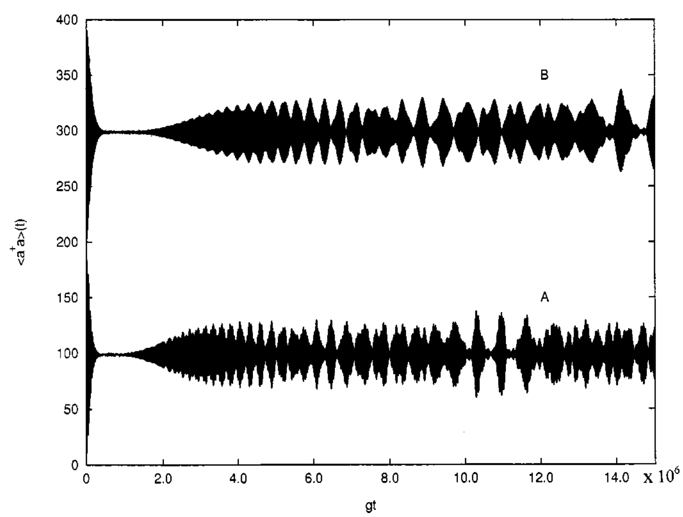

FIG. 3. The same as Fig. 1 but for large timings. Curve A is for without Stark shifts ($\beta_1/g=0,\beta_2=0$) while curve B is for with Stark shifts ($\beta_1/g=0.6,\beta_2/g=0.1$).

the difference frequency term dominates the time evolution and the contribution from the sum frequency is significantly small. Thus we obtain

$$\langle a^\dagger a\rangle = 2\alpha^2[1-\cos(gt)] + \tfrac{1}{2}, \tag{11}$$

where $\langle P_e(t)\rangle = \frac{1}{2}$ represents the time average of $P_e(t)$ in Eq. (9). Clearly, the orbital motion is represented by Eq. (11) and the period of orbital oscillation is given by

$$gt_{\text{orbit}} = 2\pi = 2gT_{\text{rev}}, \tag{12}$$

in which T_{rev} is the revival time for the two-state inversion.

When Stark shifts are included, we find that $\langle a^\dagger a\rangle$ and the orbital period satisfy the following equations for the intense fields:

$$\begin{aligned}\langle a^\dagger a\rangle &= 2\alpha^2\,[1-\cos[(\beta_1+\beta_2)t]\cos(gt)] + \frac{1}{2}\left(1+\frac{(\beta_1-\beta_2)^2}{(\beta_1-\beta_2)^2+g^2}\right)\\ &\quad\times\sqrt{g^2+(\beta_1-\beta_2)^2}\,t_{\text{orbit}}\\ &= 2\pi = 2\sqrt{g^2+(\beta_1-\beta_2)^2}\,T_{\text{rev}}.\end{aligned} \tag{13}$$

This confirms our numerical plots represented in Fig. 1. Note that the orbital oscillations are completely sinusoidal in nature (with period $2\pi/g$, no Stark shift). On the other hand, if we retain higher-order terms in the expansion of the expression $\sqrt{(n+1)(n+2)}-\sqrt{n(n+1)}$, then the approximate closed-form expression [2] of (in the absence of Stark shift) is given by

$$\begin{aligned}\langle\tilde{\Psi}|a|\tilde{\Psi}\rangle &\approx \alpha\{e^{-\alpha^2[1-\cos(gt/4\alpha^6)]}\\ &\quad\times\cos[\alpha^2\sin(gt/4\alpha^6)+gt]\}.\end{aligned} \tag{14}$$

Clearly, there is collapse and revival of the orbital motion in this situation. The more detailed analysis can be carried out as in [6]. The collapse time can be calculated as a time when the frequency components of one standard deviation of the photon number distribution dephases from its central frequency, i.e.,

$$gt_{\text{collapse}} = 4\pi\alpha^5. \tag{15}$$

Similarly, the revival time can be estimated as the time when the two neighboring oscillations acquire a 2π phase difference,

$$gt_{\text{rev}} = 8\pi\alpha^6. \tag{16}$$

When Stark shifts are taken into account, then we can rewrite the modified Eqs. (15) and (16) in the following way provided $(\beta_1-\beta_2)/g \ll 1$:

$$\begin{aligned}gt_{\text{collapse}} &= 4\pi\alpha^5 \bigg/ \left(1-\frac{19}{2}\,\frac{(\beta_1-\beta_2)^2}{g^2}\right),\\ gt_{\text{rev}} &= 8\pi\alpha^6 \bigg/ \left(1-\frac{19}{2}\,\frac{(\beta_1-\beta_2)^2}{g^2}\right).\end{aligned} \tag{17}$$

Note that both the collapse time and the revival time are elongated with the inclusion of Stark shifts. This result is in marked contrast to the results obtained for $\langle a^\dagger a\rangle$ and $P_e(t)$ at the early stages of their evolution (Figs. 1 and 2), where the inclusion of Stark shifts reduces both the period of orbital motion as well as the revival time of the collapse-revival phenomenon of Rabi oscillations.

III. CONSTRUCTION OF THE NONLINEAR MODEL

To have a better physical picture for the results of Eqs. (8)–(10), we look again at the problem from the point of view of constructing semiclassical as well as quantized oscillators. We ignore the Stark shift at this stage just for simplicity and include it at the end. For semiclassical construction we can write, using the work of [7] and [12],

$$\begin{aligned}2\langle\tilde{\psi}|a|\tilde{\psi}\rangle &= \chi_u(t)+\chi_l(t),\\ \chi_{u,l}(t) &= \alpha e^{\mp igt},\end{aligned} \tag{18}$$

in which $\chi_{u,l}$ is a complex number satisfying the equation

$$\begin{aligned}\dot{\chi}_{u,l} &= \mp ig\chi_{u,l},\\ \chi_{u,l}(0) &= \alpha.\end{aligned} \tag{19}$$

For further details of such construction, refer to [7,12].

Next, we will discuss the dressed eigenstates and energies for the model under consideration. The dressed eigenstates of

$\tilde{H}_T$, i.e., Hamiltonian (4) along with terms $\hbar\omega(a^\dagger a+1/2+\sigma_z)$ (where ω=field frequency, ω_0=atomic resonance frequency, such that $\omega_0=2\omega$) can easily be obtained as

$$|u_n\rangle=(|n-2\rangle|e\rangle-i|n\rangle|g\rangle/\sqrt{2},$$
$$|l_n\rangle=(|n-2\rangle|e\rangle+i|n\rangle|g\rangle)/\sqrt{2}, \tag{20}$$

with $n=2,3,4,...$ and the eigenenergies are

$$E_{u_n}=n\hbar\omega+\hbar\sqrt{n(n-1)}g,$$
$$E_{l_n}=n\hbar\omega-\hbar\sqrt{n(n-1)}g. \tag{21}$$

At high energies or at high photon number, the frequency shift varies as

$$(E_{u_n,l_n}-E_{u_{n-2},l_{n-2}})/\hbar-2\omega\approx\pm g(2+1/4n^2). \tag{22}$$

In fact, when n is large, we can neglect the term $1/4n^2$ in Eq. (22) and thus the frequency shift becomes constant approximately. This kind of constant frequency shift reminds us of the frequency shift in the levels of a simple harmonic oscillator exhibiting sinusoidal oscillations. In semiclassical pictures this oscillator conserves the magnitude of the complex field but a phase shift is accumulated relative to the harmonic motion [as in Eq. (19)]. So the orbital motion exhibits sinusoidal oscillations.

There is no collapse and revival phenomenon in the evolution of mean photon number in the semiclassical picture. It should also be noticed [from Eq. (22)] that the departure from even spacing is smaller (involves higher orders in $1/\sqrt{n}$) for the two-photon JCM than for the one-photon JCM [7] and hence the revival time is longer in the former case.

It would be interesting to have a quantum analog of the semiclassical picture discussed above for the driven two-photon JCM and rediscover the orbital motion. We follow Ref. [7] closely for this purpose and rewrite the Hamiltonian (4) in a slightly different manner,

$$\tilde{H}=\hbar g\sum_{n=2}^{n=\infty}\sqrt{n}\sqrt{n-1}(|u_n\rangle\langle u_n|-|l_n\rangle\langle l_n|). \tag{23}$$

The above form is known as the energy representation. We can now define the lowering and raising operators (i.e., ladder operators),

$$\hat{J}_\eta=|g_0\rangle\langle g_1|+\sqrt{2}|g_1\rangle\langle\eta_2|+\sum_{n=3}^{n=\infty}\sqrt{n}|\eta_{n-1}\rangle\langle\eta_n|, \tag{24}$$

where $|g_0\rangle=1/\sqrt{2}|0,g\rangle$, $|g_1\rangle=1/\sqrt{2}|1,g\rangle$, $\eta=u,l$, and the commutator relationship is

$$[\hat{J}_\eta,\hat{J}_\eta^\dagger]=\hat{P}_\eta+|g_1\rangle\langle g_1|+|g_0\rangle\langle g_0|,$$
$$\hat{P}_\eta=\sum_{n=2}^{\infty}|\eta_n\rangle\langle\eta_n|. \tag{25}$$

The projection operators $\hat{P}_u$ and $\hat{P}_l$ are defined for the orthogonal subspaces by ladder u and l, respectively. However, the ground state is shared by the orthogonal subspaces so the orthogonality is not complete. The operators $\hat{J}_u,\hat{J}_u^\dagger$ and $\hat{J}_l,\hat{J}_l^\dagger$ are thus limited to their respective subspaces and so $\hat{J}_u\hat{J}_l=\hat{J}_l\hat{J}_u=0$ and

$$\hat{J}_u\hat{J}_l^\dagger=\hat{J}_l\hat{J}_u^\dagger=|g_0\rangle\langle g_0|,$$
$$\hat{J}_u^\dagger\hat{J}_l=|g_1\rangle\langle g_1|+2|u_2\rangle\langle l_2|, \tag{26}$$
$$\hat{J}_l^\dagger\hat{J}_u=|g_1\rangle\langle g_1|+2|l_2\rangle\langle u_2|.$$

Note that Eq. (26) above is quite different from that of Eq. (23) obtained in [7] and hence the two-photon JCM is nontrivial from the one-photon JCM in this sense.

Now we can represent Hamiltonian (23) using ladder operators in the following form:

$$\hat{H}=\hbar g(\sqrt{J_u^\dagger J_u^\dagger J_u J_u}-\sqrt{J_l^\dagger J_l^\dagger J_l J_l}). \tag{27}$$

Clearly, the Hamiltonian (27) has got two orthogonal excited-state ladders in it. It is also possible to write the wave function $|\tilde{\psi}(t)\rangle$ as a sum of two orthogonal states,

$$|\tilde{\psi}(t)\rangle=[|\tilde{U}(t)\rangle+|\tilde{L}(t)\rangle]/2, \tag{28}$$

with

$$|\tilde{\Theta}(t)\rangle=\sqrt{2}\exp[\mp igt\sqrt{J_\eta^\dagger J_\eta^\dagger J_\eta J_\eta}]\hat{P}_\eta|\alpha\rangle|e\rangle, \tag{29}$$

and $\Theta\sim U,L$. The ladder operators $J_\eta^\dagger$ and J_η are creation and annihilation operators for the entangled excitation of the field+atomic system. We can find dynamics of both field and atom with two orthogonal states defined by $|\tilde{\Theta}(t)\rangle$. For example, the occupation probability of the excited states is given by

$$P_e=(1+\mathrm{Re}\langle\tilde{U}|\hat{P}_e|\tilde{L}\rangle)/2, \tag{30}$$

where $\hat{P}_e=|e\rangle\langle e|$, and by substituting it in Eq. (30) we can get back the same expression as in Eq. (9). Note that in Eq. (9), $P_e=\langle\tilde{\psi}|\hat{P}_e|\tilde{\psi}\rangle$. From this we infer the localization of the orthogonal wave functions, however at a certain time they overlap and thus the two-state inversion and the orbital oscillations are synchronized. Finally, we calculate $\langle a^\dagger a\rangle$ in terms of these orthogonal states [7],

$$\langle a^\dagger a\rangle=\frac{1}{2}\sum_{\eta=u,l}\langle\Theta(t)|\hat{J}_\eta^\dagger\hat{J}_\eta|\Theta(t)\rangle$$
$$=\frac{1}{2}\sum_{\eta=u,l}\alpha[\alpha-\mathrm{Re}\langle\Theta(t)|\hat{J}_\eta|\Theta(t)\rangle]+\frac{1}{2}, \tag{31}$$

from which we are able to reproduce the $\langle a^\dagger a\rangle$ plot in Fig. 1 but without periodic dots because the $P_e(t)$ term is now absent. Note that the time evolution of $P_e(t)$ is periodic having a very compact wave-packet-like structure in time (see Fig. 2) and its magnitude is much smaller than the magnitude of $\langle a^\dagger a\rangle$ for the parameters selected here in this study, so it appears (periodically) as tiny dots riding on the peaks in the plot of $\langle a^\dagger a\rangle$ as shown in Fig. 1 (*A*). Such dots are absent when we plot Eq. (31) because $P_e(t)$ is absent here.

It is not difficult to write the undisplaced Hamiltonian as

$$H=H_u+H_l, \tag{32}$$

where

$$H_\eta=\pm\hbar g\sqrt{(\hat{J}_\eta^\dagger+\alpha\hat{P}_\eta)^2(\hat{J}_\eta+\alpha\hat{P}_\eta)^2}, \tag{33}$$

which is the constructed quantized anharmonic oscillator (but behaves as the linear harmonic oscillator for high *n*) for the driven two-photon JCM completely explaining all its features.

If we include Stark shifts, then it is very easy to show that

$$H=H_u^s+H_l^s+H_u+H_l, \tag{34}$$

in which H_η are as defined above [Eq. (33)] and H_η^s is given by

$$H_\eta^s=[(\hat{J}_\eta^\dagger+\alpha\hat{P}_\eta)(\hat{J}_\eta+\alpha\hat{P}_\eta)][\beta_1|e\rangle\langle e|+\beta_2|g\rangle\langle g|]. \tag{35}$$

The above Hamiltonian Eq. (32) is the quantized anharmonic oscillator model of the driven two-photon JCM explaining the features of collapse and revivals in the mean photon number. We have noticed (as discussed at the end of Sec. II) that there were no collapses and revivals of the mean photon number in the semiclassical approximation. It is possible to recover the semiclassical picture of ''orbital'' oscillations in the short time limit of the above model [Eq. (32)]. Also, this Hamiltonian may turn out to be useful as a starting point for further studies.

IV. COMPARISION OF THE DRIVEN TWO-PHOTON JCM WITH THE DRIVEN ONE-PHOTON JCM

The important similarity between our results for the driven two-photon JCM and that of the driven one-photon JCM (Ref. [7]) is synchronization of orbital oscillations and the collapse and revival phenomenon of Rabi oscillations but on different time scales for the two models. The major difference between our work and the work of Ref. [7] is due to the Stark shift in the former. The Stark shift influences both the orbital oscillations as well as the collapse and revival phenomenon in the excitation probability. At short times, the Stark shift introduces modulation in the orbital oscillations in the two-photon JCM. The orbital oscillations exhibit a kind of collapse and revival phenomenon over a very long time period in both one- and two-photon JCM's under discussion. However, the revival times for the long time collapse and revival phenomenon are $8\pi\alpha^6/g$ for the two-photon JCM and $8\pi\alpha^3/g$ for the one-photon JCM, respectively. These revival times clearly depend on α. Note that the revival time for the collapse and revival phenomenon of Rabi oscillations in the usual two-photon JCM is simply π/g. Further, the revival-time period for the driven two-photon JCM can elongate with the inclusion of the Stark shift.

V. CONCLUSIONS

In this work we have proceeded to look for the dynamics of a nonlinear two-photon JCM driven by an external field and we encountered a situation of a linear harmonic oscillator exhibiting sinusoidal orbital motion semiclassically. Quantum mechanically, the usual two-photon JCM undergoes collapse and revivals of Rabi oscillations and so does the orbital motion. However, the time scale of the latter is far larger than the former. The revival time ($\sim\pi/g$) of the phenomenon of collapse and revival of Rabi oscillations in the two-photon JCM is independent of the cavity field amplitude and the revivals are compact and complete, which is in marked contrast to the collapse and revival phenomenon of the orbital motion for the driven two-photon JCM. We have also shown that the Stark shifts arising from the transitions to virtual levels can strongly affect the orbital motion for the driven two-photon JCM. At early stages of evolution, Stark shifts cause reduction in the time period of oscillations of orbital motion as well as the revival time of the collapse-revival phenomenon of Rabi oscillations. For very large timings when orbital motion shows the collapse-revival phenomenon, the dynamic Stark shifts can elongate the revival time of this phenomenon and thus further reduce the nonlinear nature of the two-photon JCM. These results clearly indicate that the nonlinear behavior of the driven two-photon JCM is very much different from that of the driven one-photon JCM as well as from the standard nonlinear two-photon JCM. Thus the Stark shift arising from the transition to virtual levels plays a very significant role in determining the orbital motion of the driven two-photon JCM. It can affect the characteristic oscillation/revival time of the orbital motion in the opposite manner during its early and late stages of evolution. Hence the intrinsic nonlinear nature of the driven two-photon JCM becomes quite sensitive to the Stark effect and must be carefully determined in the course of its dynamical evolution. Only under very special circumstances, i.e., when $\beta_1=\beta_2$ or the strength of couplings of the lower and upper level to the intermediate level is equal, the effect of the Stark shifts is not pronounced on the dynamical evolution of the above system. It is quite likely that in determining other features of the driven JC model, e.g., statistical properties (intensity-intensity correlations, etc.) as well as in any experimental realization (as discussed in Sec. II) of such a model, the Stark shift can bring about quantitative changes in general and hence a proper estimation may be essential. Also, it would be quite interesting to know the dynamical evolution of the driven *n*-photon JCM in general along with the effects of the Stark shift.

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