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Low-energy collective excitations in a superfluid trapped Fermi gas

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We study low-energy collective excitations in a trapped superfluid Fermi gas. These excitations originate from slow variations of the phase of the superfluid order parameter and, in this sense, are similar to the Bogolyubov sound in a homogeneous superfluid Fermi gas. Well below the critical temperature the eigenfrequencies of the lowest collective excitations are of the order of the trap frequency, and these modes manifest themselves in the density oscillations under periodic modulations of the trap frequencies. This gives a clear signature of the presence of a superfluid phase.

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The search for novel quantum phenomena in trapped ultracold gases has attracted a lot of attention since the discovery of Bose-Einstein condensation in trapped clouds of bosonic alkali-metal atoms [1–3]. The studies of trapped Bose-condensed gases have revealed a number of phenomena originating from the interparticle interaction and quantum statistics, such as collective oscillations, Bose enhancement of kinetic processes, etc. (see Ref. [4] for a review). Theoretical work on degenerate (nonsuperfluid) Fermi gases was mostly related to the influence of the Pauli exclusion principle on their optical [5–7] and collisional properties [8,9] and to the issue of collective oscillations [10,11]. The recent success [12,13] in cooling the trapped fermionic sample of ^{40}K below the temperature of quantum degeneracy T_F (Fermi energy ε_F) is stimulating interest in identifying and studying a superfluid phase transition in trapped Fermi gases. Possible versions of this transition originating from atomic Cooper pairing have been discussed in Refs. [14–17], and the shape of the order parameter in trapped gases has been analyzed in Refs. [18–20]. Although the transition temperature $T_c \ll \varepsilon_F$ and the pairing strongly influence only a small fraction ($\sim T_c/\varepsilon_F \ll 1$) of quantum states in the vicinity of ε_F, the presence of the superfluid order parameter governs the response of the entire system to small external perturbations. As has been found in Ref. [19] and confirmed by numerical calculations [21], the superfluid pairing in a harmonically trapped gas smears out the resonance in the density oscillations induced by periodic modulations of the trap frequencies. (See also Refs. [22,23] for the discussion of other possible ways of detecting the BCS pairing.)

In this paper we study low-energy collective excitations in a superfluid Fermi gas trapped in an isotropic harmonic potential. These excitations are related to the phase fluctuations of the order parameter $\Delta(\mathbf{r})$ and, at intermediate temperatures $T<T_c$, are overdamped. However, well below the transition temperature the damping rate is small, and one has well-defined eigenmodes. The lowest eigenfrequencies are of the order of the trap frequency, and are quite different from the eigenfrequencies of the gas above T_c. The collective excitations manifest themselves in the density oscillations of the gas under periodic modulations of the trap frequencies. The dependence of the oscillation amplitude on the modulation frequency has resonances at frequencies of collective oscillations. The observation of this resonance structure, which is different from the one in a nonsuperfluid gas, will give a clear indication of the presence of a superfluid phase.

The properties of collective modes in the superfluid phase depend on the structure of the order parameter and, hence, on the type of pairing. For a singlet s-wave pairing the order parameter is a complex function, and there are two branches of collective excitations. One of them corresponds to the phase fluctuations of the order parameter, and the other branch to the fluctuations of the modulus. For a triplet p-wave pairing the order parameter is a complex 3×3 matrix, and, hence, there will be additional branches of collective modes (see, e.g., Ref. [24]). However, in both cases the lowest branch is related to the fluctuations of the phase of the order parameter (Bogolyubov sound in the homogeneous case). We will study this mode for a trapped superfluid Fermi gas, considering for simplicity the case of the ''singlet'' s-wave pairing. This implies the presence of two hyperfine components and attractive interaction between them. Possible experimental realizations include the gas of ^{6}Li atoms in a magnetic trap, where the interatomic interaction is characterized by a large and negative triplet s-wave scattering length $a \approx -1140$Å [25], and also ^{40}K [26].

The Hamiltonian of a two-component gas of fermionic atoms (labeled as α and β) trapped in an isotropic harmonic potential reads ($\hbar = 1$)

$$H = \sum_{i=\alpha,\beta} \int d\mathbf{r} \psi_i^\dagger(\mathbf{r}) H_0 \psi_i(\mathbf{r}) + V \int d\mathbf{r} \psi_\alpha^\dagger(\mathbf{r}) \psi_\alpha(\mathbf{r}) \psi_\beta^\dagger(\mathbf{r}) \psi_\beta(\mathbf{r}). \quad (1)$$

Here $\psi_i(\mathbf{r})$ are the field operators of the α and β components that are assumed to have equal concentrations, $H_0 = -\nabla^2/2m + m\Omega^2 r^2/2 - \mu$, Ω is the trap frequency, m the atom mass, and μ the chemical potential. The second term in Eq. (1) corresponds to attractive short-range interaction between the α and β atoms (s-wave scattering length $a<0$), with the coupling constant $V = 4\pi a/m$. In the Hamiltonian (1) we neglect the $\alpha\alpha$ and $\beta\beta$ interactions originating in the case of fermions only from the scattering with orbital angular momenta $l \geqslant 1$. The presence of attractive intercomponent interaction in the s-wave scattering channel leads to a superfluid phase transition, and the critical temperature $T_c \ll \mu$ [14]. We assume that T_c is much larger than Ω and, hence, the value of the critical temperature in the trap is very close [18] to the critical temperature $T_c^{(0)}$ of the spatially homogeneous gas with density equal to the maximum density n_0 of the trapped gas [27]:

$$T_c^{(0)} = 0.28 \varepsilon_F \exp\{-1/\lambda\},$$

where $\lambda = 2|a|p_F/\pi \ll 1$ is a small parameter of the theory, $p_F = (3\pi^2 n_0)^{1/3}$ is the Fermi momentum, and $\varepsilon_F = p_F^2/2m \approx \mu$ is the Fermi energy.

The superfluid phase is characterized by the equilibrium order parameter $\Delta_0(\mathbf{r}) = |V|\langle \psi_\alpha(\mathbf{r})\psi_\beta(\mathbf{r})\rangle$ that can be considered as a real function. For a trapped gas, the spatial form of $\Delta(\mathbf{r})$ was studied analytically in Ref. [19] and numerically in Ref. [20]. The appearance of $\Delta(\mathbf{r})$ strongly influences only the quantum states with energies in the interval of order T_c near ε_F. As a result, the Thomas-Fermi density profile of the gas $n(\mathbf{r}) = n_0(1-(r/R_{TF})^2)^{3/2}$ changes only slightly ($R_{TF} = p_F/m\Omega$ is the Thomas-Fermi radius of the gas cloud). The interparticle interaction leads to corrections in this formula. However, the leading corrections originating from the mean-field interparticle interaction [28] are proportional to the small parameter λ and, hence, will be neglected below.

For a superfluid gas, the evolution of small deviation from equilibrium can be studied by using the time-dependent Bogolyubov–de Gennes equations for the u,v functions:

$$i\frac{\partial}{\partial t}\begin{pmatrix} u_\nu(\mathbf{r},t) \\ v_\nu(\mathbf{r},t)\end{pmatrix} = H_0\begin{pmatrix} u_\nu(\mathbf{r},t) \\ -v_\nu(\mathbf{r},t)\end{pmatrix} - \Delta(\mathbf{r},t)\begin{pmatrix} v_\nu(\mathbf{r},t) \\ u_\nu(\mathbf{r},t)\end{pmatrix}, \quad (2)$$

together with the self-consistency condition

$$\Delta(\mathbf{r},t) = |V|\sum_\nu u_\nu(\mathbf{r},t)v_\nu^*(\mathbf{r},t)\tanh \varepsilon_\nu/2T. \quad (3)$$

For $t \to -\infty$ the time-dependent order parameter $\Delta(\mathbf{r},t) \to \Delta_0(\mathbf{r})$ and $(u_\nu(\mathbf{r},t), v_\nu(\mathbf{r},t)) \to (u_\nu(\mathbf{r}), v_\nu(\mathbf{r}))\exp(-i\varepsilon_\nu t)$, where $u_\nu(\mathbf{r}), v_\nu(\mathbf{r})$ are the u,v functions of single-particle excitations with eigenenergies $\varepsilon_\nu \geqslant 0$ [solutions of the stationary Bogolyubov–de Gennes equations with $\Delta = \Delta_0(\mathbf{r})$].

Low-energy collective excitations correspond to small fluctuations of the phase of the order parameter. In this case one has $\Delta(\mathbf{r},t) = \Delta_0(\mathbf{r})\exp[2i\varphi(\mathbf{r},t)] \approx \Delta_0(\mathbf{r})[1+2i\varphi(\mathbf{r},t)]$, where $\varphi(\mathbf{r},t) \ll 1$ is a real function slowly varying in space and time. Equations (2) can be solved perturbatively, and after substituting these solutions into Eq. (3) one gets the following equation for the Fourier transform $\varphi_\omega(\mathbf{r}) = \int dt\, \varphi(\mathbf{r},t)\exp(i\omega t)$ of the phase fluctuations:

$$\Delta_0(\mathbf{r})\varphi_\omega(\mathbf{r}) = |V|\sum_{\nu,\nu_1}\left\{\frac{u_{\nu_1}(\mathbf{r})v_\nu^*(\mathbf{r})}{\omega - \varepsilon_{\nu_1} + \varepsilon_\nu + i0}M_{\nu_1\nu}^{(1)}(\omega)\left[\tanh\frac{\varepsilon_{\nu_1}}{2T} - \tanh\frac{\varepsilon_\nu}{2T}\right] + \left[\frac{u_{\nu_1}(\mathbf{r})u_\nu(\mathbf{r})}{\omega - \varepsilon_{\nu_1} - \varepsilon_\nu + i0}M_{\nu_1\nu}^{(2)*}(-\omega) - \frac{v_{\nu_1}^*(\mathbf{r})v_\nu^*(\mathbf{r})}{\omega + \varepsilon_{\nu_1} + \varepsilon_\nu + i0}M_{\nu_1\nu}^{(2)}(\omega)\right]\tanh\frac{\varepsilon_\nu}{2T}\right\}, \quad (4)$$

where $M_{\nu_1\nu}^{(2)}(\omega) = \int_{\mathbf{r}}\Delta_0\varphi_\omega(u_{\nu_1}u_\nu + v_{\nu_1}v_\nu)$ and $M_{\nu_1\nu}^{(1)}(\omega) = \int_{\mathbf{r}}\Delta_0\varphi_\omega(u_{\nu_1}^* v_\nu - v_{\nu_1}^* u_\nu) = -M_{\nu\nu_1}^{(1)*}(-\omega)$.

The analysis of this equation at arbitrary temperature below T_c is rather lengthy. Therefore, we discuss here only two limiting cases: $(T_c - T)/T_c \ll 1$ and $T \ll T_c$.

For $(T_c - T)/T_c \ll 1$ the order parameter is small: $\Delta_0(\mathbf{r}) \ll T_c$. As a result, one can substitute for u_ν, v_ν, and ε_ν their values in the normal phase: $u_\nu = \phi_\nu$, $v_\nu = 0$ for $\xi_\nu = \varepsilon_\nu > 0$, and $u_\nu = 0$, $v_\nu = \phi_\nu^*$ for $\xi_\nu = -\varepsilon_\nu < 0$, where ϕ_ν and ξ_ν are the eigenfunctions and eigenvalues of the Hamiltonian H_0: $H_0\phi_\nu = \xi_\nu\phi_\nu$. Then, Eq. (4) can be rewritten in the form

$$\Delta_0(\mathbf{r})\varphi_\omega(\mathbf{r}) - \frac{|V|}{2}\sum_{\nu_1,\nu_2}\frac{\tanh \xi_{\nu_1}/2T + \tanh \xi_{\nu_2}/2T}{\xi_{\nu_1} + \xi_{\nu_2}} \times \phi_{\nu_1}(\mathbf{r})\phi_{\nu_2}(\mathbf{r})\int_{\mathbf{r}'}\Delta_0\varphi_\omega\phi_{\nu_1}^*\phi_{\nu_2}^* = \frac{|V|}{2}\sum_{\nu_1,\nu_2}\frac{-\omega(\tanh \xi_{\nu_1}/2T + \tanh \xi_{\nu_2}/2T)}{(\xi_{\nu_1} + \xi_{\nu_2})(\omega + \xi_{\nu_1} + \xi_{\nu_2} + i0)} \times \phi_{\nu_1}(\mathbf{r})\phi_{\nu_2}(\mathbf{r})\int_{\mathbf{r}'}\Delta_0\varphi_\omega\phi_{\nu_1}^*\phi_{\nu_2}^*, \quad (5)$$

where the left-hand side coincides with the (time-independent) Ginzburg-Landau equation (see Ref. [18]) for $\Delta_0\varphi$. The presence of the small frequency ω in the right-hand side of Eq. (5) allows us to write $\int_{\mathbf{r}'}\Delta_0\varphi_\omega\phi_{\nu_1}^*\phi_{\nu_2}^*$ as $\Delta_0(\mathbf{r})\varphi_\omega(\mathbf{r})\delta_{\nu_1\nu_2}$. Then the sum over $\nu = \nu_1 = \nu_2$ can be replaced by the integral over ξ, where the main contribution comes from small ξ (from the states near the Fermi energy). Accordingly, Eq. (5) transforms to

$$-\frac{7\zeta(3)}{6\pi^3}\frac{\Omega^2}{T_c}\left(\frac{1}{\sqrt{1-R^2}}\nabla_{\mathbf{R}}[(1-R^2)^{3/2}\nabla_{\mathbf{R}}\varphi(\mathbf{R})] + 2(1-R^2)\nabla_{\mathbf{R}}\ln\Delta_0\nabla_{\mathbf{R}}\varphi(\mathbf{R})\right) = i\omega\varphi(\mathbf{R}), \quad (6)$$

where $\mathbf{R} = \mathbf{r}/R_{TF}$ and $\zeta(z)$ is the Riemann zeta function. Equation (6) shows that at $T \approx T_c$ the eigenfrequencies ω are purely imaginary. This means that collective modes rapidly decay into pairs of single-particle excitations.

For $T \ll T_c$, one can neglect the contribution of the thermal component in Eq. (4) and put $\tanh \varepsilon_\nu/2T \approx 1$. Taking into account various relations between u_ν and v_ν, which follow from the time-independent Bogolyubov–de Gennes equations and from unitarity of the Bogolyubov transformation, Eq. (4) can be reduced to the form

$$-\frac{\Omega^2}{3}\frac{1}{\sqrt{1-R^2}}\nabla_{\mathbf{R}}[(1-R^2)^{3/2}\nabla_{\mathbf{R}}\varphi]=\omega^2\varphi. \tag{7}$$

In Eq. (7) we only keep leading terms in the gradient and frequency expansions. This equation gives real frequencies of collective modes, which are of the order of the trap frequency Ω. Being excited at $T \ll T_c$, the collective modes result in oscillations of the (superfluid) current $\mathbf{j}=(i/m)\Sigma_\nu(v_\nu^*\nabla v_\nu - v_\nu\nabla v_\nu^*)=(n/m)\nabla\varphi$ and density $n=2\Sigma_\nu|v_\nu|^2=n_0+\delta n$, which are related to each other by the continuity equation $\partial\delta n/\partial t+\mathrm{div}\,\mathbf{j}=0$ following directly from Eqs. (2) and (3). As a result, the entire gas sample oscillates:

$$n(\mathbf{r},t)=n_0(\mathbf{r})+\delta n(\mathbf{r},t)\approx\left[1+\frac{1}{m}\nabla^2 f\right]n_0\left(\mathbf{r}+\frac{1}{m}\nabla f\right),$$

where $f(\mathbf{r},t)=\int^t\varphi(\mathbf{r},t')dt'$.

The damping of the collective modes is not present in Eq. (7). This damping is mostly provided by inelastic scattering of low-energy in-gap single-particle excitations (see Ref. [19]) from a given collective mode or by the decay of the collective mode into two in-gap single-particle excitations [29]. In these processes the energy of the collective mode is transfered to the normal component in the outer part of the gas sample. As the wave function of in-gap single-particle excitations decays exponentially in the central part of the sample [19], where the order parameter is essentially nonzero, the coupling between the fluctuations of the order parameter and the in-gap excitations is exponentially weak [$\sim\exp(-T_c/\Omega)$]. Therefore, one expects a very small damping rate.

Equation (7) can also be obtained in the hydrodynamic approach for a superfluid Fermi gas. If the superfluid velocity $\mathbf{v}_s=m^{-1}\nabla\varphi$ and the deviation δn of the particle density from its equilibrium value $n_0(r)$ are small, the corresponding Hamiltonian has the form

$$H_{\mathrm{h}}=\int d\mathbf{r}\left\{\frac{1}{2}mn\mathbf{v}_s^2+U(n)\right\}$$
$$\approx\int d\mathbf{r}\left\{\frac{1}{2m}n_0(\nabla\varphi)^2+\frac{1}{2}U''(n_0)\delta n^2+U(n_0)\right\}, \tag{8}$$

where $U(n)$ is the density-dependent part of the energy. The equilibrium density n_0 is defined by the condition $U'(n_0)=0$. In the Thomas-Fermi approximation we have

$$U(n)=\frac{3}{10}(3\pi^2)^{2/3}\frac{n^{5/3}}{m}+\left(\frac{m\Omega^2r^2}{2}-\mu\right)n, \tag{9}$$

where the first term results from the filled Fermi sphere, and the equilibrium density profile is $n_0(r)=(p_F^3/3\pi^2)[1-(r/R_{TF})^2]^{3/2}$. In Eq. (9) we omit the effects of the mean-field interaction and superfluid pairing because they are proportional to small parameters λ and $(T_c/\varepsilon_F)^2$, respectively. For the quantity $U''(n_0)$ in Eq. (8) one now has $U''(n_0)=(3\pi^2)^{-2/3}N(r)^{-1}$, with $N(r)=(mp_F/\pi^2)\sqrt{1-(r/R_{TF})^2}$ being the density of states on the local Fermi surface. Then the standard commutation relation $[\delta n(\mathbf{r}_1),\varphi(\mathbf{r}_2)]=i\delta(\mathbf{r}_1-\mathbf{r}_2)$ leads to

$$\partial\varphi/\partial t=i[H_{\mathrm{h}},\varphi]=U''(n_0)\delta n,$$

$$\partial(\delta n)/\partial t=i[H_{\mathrm{h}},\delta n]=-\nabla(n_0\nabla\varphi).$$

This immediately gives Eq. (7) for the phase φ and

$$\frac{\partial^2}{\partial t^2}\delta n+\frac{\Omega^2}{3}\nabla_{\mathbf{R}}\left[(1-R^2)^{3/2}\nabla_{\mathbf{R}}\frac{\delta n}{\sqrt{1-R^2}}\right]=0 \tag{10}$$

for the density fluctuations.

Equation (7) [or (10)], together with the condition that φ (or δn) is finite at any R, provides us with the energy spectrum of collective modes:

$$(\omega_{nl}/\Omega)^2=l+\tfrac{4}{3}n(n+l+2),\quad n=0,1,2,\ldots \tag{11}$$

and the corresponding eigenfunctions

$$\varphi_{nl}(\mathbf{R})\propto R^l\,{}_2F_1(-n,n+l+2;\tfrac{3}{2}+l;R^2)Y_{lm}(\theta,\phi), \tag{12}$$

where ${}_2F_1$ is the hypergeometric function, l is the angular momentum of the collective mode, and n is an integer ($n=0,1,2,\ldots$ for nonzero l, and $n=1,2,\ldots$ for $l=0$). The eigenfunctions (12) are orthogonal with the weight $1/\sqrt{1-R^2}$.

The spectrum (11) coincides with that of a trapped normal Fermi gas in the hydrodynamic regime [10]. However, for realistic parameters, the trapped gas just above T_c is likely to be in the collisionless regime. The corresponding criterion assumes that the oscillation period in the trap, $2\pi/\Omega$, is much smaller than the characteristic collisional frequency in the degenerate Fermi gas. This frequency is given by $\tau^{-1}\sim na^2v_F(T_c/\varepsilon_F)^2\sim\lambda^2T_c^2/\varepsilon_F$, where na^2v_F is the classical collisional frequency and the factor $(T_c/\varepsilon_F)^2$ results from the Pauli blocking. As a result, the collisionless criterion reads $\Omega\tau\sim\lambda^{-2}(\Omega/T_c)\exp(1/\lambda)\gg1$.

We now compare the eigenfrequencies of the collisionless normal gas just above T_c with the eigenfrequencies of the superfluid gas at $T\ll T_c$. Of particular interest are the lowest eigenmodes, as they can be excited by modulating the trap frequencies [a small external perturbation $V_{\mathrm{ext}}\exp(-i\omega t)$ results in an extra term $-i\omega V_{\mathrm{ext}}\exp(-i\omega t)$ in the right-hand side of Eq. (7)]. For the superfluid phase, as follows from Eq. (11), the lowest eigenfrequency ω_{10} for the monopole breathing mode ($l=0,n=1$) is equal to 2Ω (this result can be obtained on the basis of the sum rules [30]), and one has the anticipated result $\omega_{01}=\Omega$ for the dipole mode ($l=1,n=0$). These eigenfrequencies coincide with those calculated for the collisionless normal Fermi gas in Ref. [11]. On the other hand, for the lowest quadrupole mode Eq. (11) gives $\omega_{02} = \sqrt{2}\Omega$, whereas in the collisionless regime (at $T > T_c$) this mode has frequency 2Ω [11]. Experimentally, the quadrupole mode can be excited by a small out-of-phase modulation of the trap frequency in, for example, the x and y directions: $V_{\rm ext}(\mathbf{r},t) = (m\Omega^2/2)(x^2 - y^2)\zeta\cos(\omega t)$ with $\zeta \ll 1$. The response of the gas sample will be characterized by the presence of resonances in the amplitude of the density oscillations. For $T > T_c$ the resonance will be at frequency 2Ω, and for $T \ll T_c$ at frequency $\sqrt{2}\Omega$.

In conclusion, we have found the low-energy collective modes of the superfluid trapped Fermi gas. These modes are related to the fluctuations of the phase of the superfluid order parameter, and, hence, describe the motion of the superfluid component. Just below the critical temperature of the superfluid phase transition the eigenenergies of the collective modes are purely imaginary, and these modes describe a diffusive relaxation of superfluid fluctuations. For temperatures well below T_c, the eigenenergies are of the order of the trap frequency, and the damping is small. Therefore, these modes can manifest themselves as eigenmodes of the density oscillations. The oscillations can be observed experimentally and serve as an indication of the superfluid phase transition.

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