Quantum theory of time refraction

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The concept of time refraction is introduced to describe the effects of a sudden change of the optical properties of a dielectric medium. This can be seen as the most elementary process associated with photon acceleration and frequency upshifting. The quantum theory of such a process shows that the initial wave splits into time-transmitted and time-reflected waves propagating in opposite directions after the occurrence of a time discontinuity of the refractive index. The time refraction laws, analogous to the well known Fresnel formulas and Snell's law, are also derived. It is shown that, in quantum terms, time refraction is equivalent to a squeezing transformation.

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I. INTRODUCTION

In recent years, the concept of photon acceleration has been explored in the context of plasma physics, and it is now well understood, in both theory [1–4] and experiments [5–7]. This is a general concept that can be used to describe a large number of optical phenomena, in plasmas and in other optical media [8]. It can be defined as a nonresonant process of interaction of waves in a space- and time-dependent medium, which results in a change of the energy and momentum of the propagating wave packets.

The development of intense laser systems has allowed for many possibilities of photon acceleration, among which we distinguish the frequency shift induced by relativistic ionization fronts [5,7], flash ionization [9], nonlinear perturbations, and wake fields [10]. In general, photon acceleration can be perceived as a space-time refraction [8]. By space-time refraction we mean the phenomenologies of light crossing space- and time-dependent boundaries between two different optical media. Pure space refraction is well known from classical optics and has been recently studied in quantum optics [11,12]. Related work on the quantum theory of spatial beam splitters should also be mentioned [13,14].

In this work we shall focus on pure time refraction, which appears as a natural extension of the refraction concept. The idea of time refraction [8] explores the formal analogies between photon acceleration and ordinary refraction, which helps us to gain insight into the elementary aspects of the photon dynamics. During ordinary refraction a beam of photons suffers deflection as it crosses the stationary boundary between two optical media, i.e., the photon wave vector is altered due to the space variation of the optical properties of the medium. Likewise, when the optical properties of the medium change with time but remain constant in space, we expect the photon light frequency to be changed. This effect of frequency shifting is as universal as ordinary refraction. Associated with time refraction we can find expressions analogous to the Fresnel formulas and Snell's law, which relate the electromagnetic field before and after the time perturbation of the optical properties of the medium [8].

Until now, the concept of photon acceleration has been described only in the context of classical theory. The quantum theory was never explored, even if somewhat related work on time-dependent dielectrics in cavities [15–17] has thrown some light on the processes of time-varying dielectric media. The interest of this kind of system is the possibility of creation of photons [17,18], which has been interpreted as a nonadiabatic distortion of the electromagnetic vacuum state.

The main aim of this work is to establish the quantum theory of time refraction, due to the sudden change of the dielectric constant of an infinite medium. We choose an infinite medium because, unlike most cases studied in quantum optics, photon acceleration can take place in free space. Also, in this case, the analogies between the usual space refraction and time refraction can be more clearly stated. Following the quantization method of Glauber [11] and Cirone, Rgazewski, and Mostowski [17], we use creation and annihilation operators that no longer describe the usual photons in empty space, but the elementary excitations of the radiation field in the presence of matter. When the dielectric constant changes in time (or in space), the photon gains a new meaning, resulting in a change of the photon energy (or momentum).

In Sec. II we begin by recalling the quantum theory of space refraction, in order to define the relevant field operators and to settle the basis for comparison. In Sec. III, we establish the quantum theory of time refraction and derive the time equivalents of Fresnel's and Snell's laws for the field operators. Such laws are formally identical to those derived by one of us [8] using the classical approach. The quantum theory confirms, as expected, the classical results, but it also shows that purely quantum effects can occur. Finally, in Sec. IV, we state our conclusions.

II. SPACE REFRACTION

Let us consider a sharp boundary between two stationary, semi-infinite, nondispersive, and nondissipative dielectric media. For simplicity, we assume that the two media, with dielectric constants equal to ϵ_1 and ϵ_2, have the boundary at $x=0$, and that the waves propagate along the x axis. This can be described by the dielectric function $\epsilon(x)=\epsilon_1 H(-x)+\epsilon_2 H(x)$, where $H(t)$ is the Heaviside function. We assume that ϵ_1 and ϵ_2 are well known values, which are not explicitly calculated by a microscopic theory, but correspond to a phenomenological description of the media.

If photons with frequency ω and initial wave vector $\vec{k}_i$ are propagating in medium 1 and interact with the boundary, we can write for the associated electric field operator, valid in the semi-infinite region $x<0$, and for a given polarization ($\lambda=1$ or 2),

$$\vec{E}(x,t)=\vec{E}_i(x,t)+\vec{E}_r(x,t),$$

where the incident and reflected field operators are

$$\vec{E}_i(x,t)=i\sqrt{\frac{\hbar\omega}{2\epsilon_1}}[a_1(\vec{k}_i,t)e^{ik_ix}-a_1^\dagger(\vec{k}_i,t)e^{-ik_ix}]\vec{e}(\vec{k}_i), \tag{1}$$

$$\vec{E}_r(x,t)=i\sqrt{\frac{\hbar\omega}{2\epsilon_1}}[a_1(\vec{k}_r,t)e^{ik_rx}-a_1^\dagger(\vec{k}_r,t)e^{-ik_rx}]\vec{e}(k_r), \tag{2}$$

with $k_i=\omega n_1/c$ and $k_r=k_i$. We have introduced here the time-dependent destruction and creation operators [11,17]

$$a(\vec{k},t)=a(\vec{k})e^{-i\omega_k t},\quad a^\dagger(\vec{k},t)=a^\dagger(\vec{k})e^{i\omega_k t}, \tag{3}$$

which satisfy the following commutation relations: $[\hat{a}_\lambda(k),\hat{a}_\lambda^\dagger(k)]=\delta_{kk'}\delta_{\lambda\lambda'}$. These field operators are strictly equivalent to those obtained for propagation in vacuum, the only difference being the replacement of the vacuum permittivity ϵ_0 by the appropriate dielectric constant of the medium, ϵ_1.

In the second medium ($x>0$), we can define the electric field operator associated with the transmitted wave as

$$\vec{E}_t(x,t)=i\sqrt{\frac{\hbar\omega}{2\epsilon_2}}[a_2(\vec{k}_t,t)e^{ik_tx}-a_2^\dagger(\vec{k}_t,t)e^{-ik_tx}]\vec{e}(\vec{k}_t) \tag{4}$$

with $k_t=\omega n_2/c=k_i(n_2/n_1)$. The corresponding magnetic field operators are determined by $\vec{B}(x,t)=-i\omega^{-1}[\vec{\nabla}\times\vec{E}(x,t)]$.

The quantization of the electromagnetic field is based on the assumption that the field operators satisfy Maxwell's equations. This means that the boundary conditions for these operators have to be formally identical to those for the classical fields. Both the electric and magnetic fields are tangent to the boundary between the two media; thus we know, from the classical theory, that they are continuous:

$$E_i(0,t)+E_r(0,t)=E_t(0,t),\quad B_i(0,t)+B_r(0,t)=B_t(0,t). \tag{5}$$

Equating separately the terms with the same time dependence, we obtain

$$a_2(\vec{k}_t)=\frac{2}{1+\alpha^2}a_1(\vec{k}_i),\quad a_1(\vec{k}_r)=\frac{\alpha^2-1}{1+\alpha^2}a_1(\vec{k}_i) \tag{6}$$

with $\alpha=(n_1/n_2)^{1/2}$.

These expressions can be seen as the Fresnel formulas relating the transmission and refraction destruction operators. It can easily be drawn that the same expressions are valid for the creation operators. We can derive from Eq. (6) a more familiar version of these Fresnel formulas, by multiplying them by $\exp(-i\omega t)$ and adding their Hermitian conjugates. We get

$$E_i(0,t)+E_r(0,t)=E_t(0,t),\quad E_i(0,t)-E_r(0,t)=\frac{1}{\alpha^2}E_t(0,t). \tag{7}$$

These new operator relations are formally identical to the Fresnel formulas for classical fields.

III. TIME REFRACTION

Let us consider an infinite, nondispersive, and nondissipative dielectric medium, characterized by a real dielectric constant ϵ_1. We then assume a time discontinuity at time $t=0$, where the dielectric constant suddenly changes from this initial value ϵ_1 to a new value ϵ_2. This can be described by a time-dependent dielectric constant $\epsilon(t)=\epsilon_1H(-t)+\epsilon_2H(t)$, where again $H(t)$ is the Heaviside function.

The electric field operator for a single mode $\vec{k}$ and a given polarization ($\lambda=1$ or $\lambda=2$) in the medium is written as

$$\vec{E}(\vec{r},t)=i\vec{e}_k\sqrt{\frac{\hbar}{2\epsilon_j}\omega_j}[a_j(\vec{k},t)e^{i\vec{k}\cdot\vec{r}}-a_j^\dagger(\vec{k},t)e^{-i\vec{k}\cdot\vec{r}}]. \tag{8}$$

We use the index $j=1$ for $t<0$ and $j=2$ for $t>0$. The displacement vector and the magnetic field operators are determined by

$$\vec{D}(\vec{k},t)=\epsilon_j\vec{E}(x,t),\quad \vec{B}(x,t)=-i\omega_j^{-1}[\vec{\nabla}\times\vec{E}(x,t)]. \tag{9}$$

The new pair of operators a_2 and $a_2^\dagger$ are different from the old ones a_1 and $a_1^\dagger$, because the meaning of a photon (an elementary excitation of the field) changes with the refractive index at $t=0$. In order to relate them, we use the classical continuity conditions for the fields in time [17]:

$$\vec{D}(\vec{r},t=0^-)=\vec{D}(\vec{r},t=0^+),\quad \vec{B}(\vec{r},t=0^-)=\vec{B}(\vec{r},t=0^+). \tag{10}$$

These equalities are independent of $\vec{r}$, which means that the wave number $|k|$ is conserved and Eqs. (10) can easily be reduced to the following relations between the new and old operators:

$$a_1(\vec{k})=Aa_2(\vec{k})-Ba_2^\dagger(-\vec{k}), \tag{11}$$

$$a_1^\dagger(-\vec{k})=Aa_2^\dagger(-\vec{k})-Ba_2(\vec{k}), \tag{12}$$

where $A=(1+\alpha^2)/2\alpha, B=(1-\alpha^2)/2\alpha$, and $\alpha=(n_1/n_2)^{1/2}$. These equations are the time equivalents for the Fresnel formulas, and can be called the Fresnel formulas for time refraction.

We should note that $A^2-B^2=1$, which is the fundamental relation for hyperbolic functions. So we can define the squeezing parameter as $r=\arg\cosh(A)=\arg\sinh(B)$. Then, the above equations can be rewritten with the help of a double mode squeezing operator

$$S=\exp[r(a(\vec{k})a(-\vec{k})-a^\dagger(\vec{k})a^\dagger(-\vec{k}))], \tag{13}$$

which couples both modes $\vec{k}$ and $-\vec{k}$, in the form

$$a_2(\vec{k})=Sa_1(\vec{k})S^{\dagger}, \quad a_2^{\dagger}(\vec{k})=Sa_1^{\dagger}(\vec{k})S^{\dagger}. \tag{14}$$

So, at the quantum level, we conclude that time refraction can be described as a squeezing transformation. This shows that each field mode existing for $t<0$ with a given wave vector $\vec{k}$ will be coupled with two modes existing for $t>0$ with wave vectors $\vec{k}$ and $-\vec{k}$. This is responsible for the existence of two sets of photons associated with transmitted and reflected waves, in a way similar to the usual space refraction.

Another aspect of time refraction is that it is accompanied by a shift of the photon frequency, because the photon wave number $|k|$ is conserved but it satisfies a different dispersion relation, due to the change of the refractive index. From the wave number conservation we can then easily extract a relation between the initial and the final photon frequencies: $\omega_1 n_1=\omega_2 n_2$. This relation can be called the Snell's law for time refraction [8].

Let us now consider, as an example, the transformation of an initial Fock state corresponding to n photons propagating along a given direction, with wave vector $\vec{k}$. In the process of time discontinuity, the modes $\vec{k}$ and $-\vec{k}$ become coupled, as shown by the above Fresnel formulas for time reflection. It is then useful to introduce symmetric Fock vectors, $|n,n'\rangle_j \equiv |n_k,n'_{-k}\rangle_j=|n_k\rangle_j|n'_{-k}\rangle_j$, where the index $j=1$ refers to $t<0$, and $j=2$ to $t>0$.

Using well known procedures [19,20], we derive

$$|n,0\rangle_1=\sum_{s=0}^{\infty} b_s(n)|n+s,s\rangle_2 \tag{15}$$

with

$$b_s(n)=\sqrt{1-(B/A)^2}\sqrt{\frac{(s+n)!}{s!n!}}B^s A^{-(n+s)}. \tag{16}$$

This result shows that, after the occurrence of a time discontinuity at $t=0$ in the value of the refractive index of an infinite dielectric medium, there is a probability $p(n,0)\neq 1$ of observing the state $|n,0\rangle_2$ with the same number of photons and the same wave vector but with a shifted frequency. On the other hand, there is also a finite probability $p(n,s)\neq 0$ of observing a number $n+s>n$ of photons propagating with the initial wave vector $\vec{k}$, and a number $s>0$ propagating in the opposite direction $-\vec{k}$ (see Fig. 1 for a numerical example). These two simultaneous groups of photons can be interpreted as the time-transmitted and time-reflected waves. Obviously, by ''time-reflected'' waves we do not mean reflected in time but rather waves propagating in the opposite direction resulting from a time discontinuity. We could proceed in a similar way for initial coherent states.

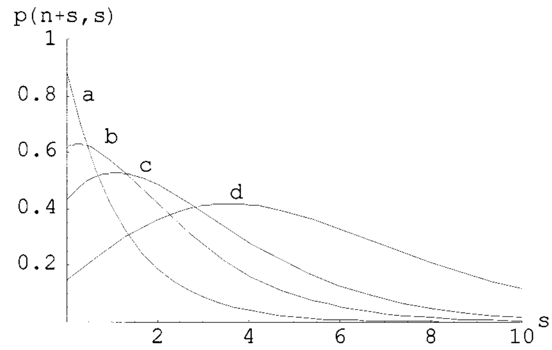

FIG. 1. Final photon probability distribution $|b_s(n)|^2$, for initial vacuum (a) and Fock states with $n=3$ (b), $n=6$ (c), and $n=15$ (d). We have assumed $r=0.5$, corresponding to $n_2=1.65n_1$.

IV. CONCLUSIONS

In this work we have developed a quantum theory of time refraction, which results from a sudden change of the refractive index of a dielectric medium, and can be considered as the elementary process associated with photon acceleration. This is a natural extension of the theory of the usual space refraction, and leads to the derivation of the Fresnel formulas and Snell's law for time refraction. It was also shown that time refraction can be described by a squeezing transformation.

Our theoretical model allowed us to confirm and to justify at the elementary quantum level the main features of time refraction already known from classical theory [8], namely, photon frequency shifting, and the production of a reflected wave (or wave propagating in the opposite sense) after the occurrence of a time discontinuity of the refractive index of a medium. The existence of this reflected wave is ultimately justified by a specific quantum effect, namely, the emission of pairs of photons in opposite directions from out of the vacuum. These pairs of real photons are created from out of the vacuum fluctuations by the time perturbation, and can appear as isolated or can be added to the initially existing photons. Actually, the existing photons also change in nature, because they get a new frequency and obey a new dispersion relation after the occurrence of a time perturbation.

The present theory can be improved in at least two distinct aspects. One is to replace the nondispersive medium by a dispersive and dissipative medium. This could be useful to examine the effects associated with the creation of a plasma from an initial neutral gas. The other corresponds to the more general case of a moving perturbation of the refractive index, or a moving boundary between two different media, which would lead to space-time refraction. Such a boundary could be due to an ionization front. These two aspects are known from the classical theory but have never been formulated in quantum terms, and will be the object of future work.

Finally, it should be noted that a varying dielectric medium is optically equivalent to a varying gravitational field [21]. It is then obvious that the creation of pairs of photons in vacuum by the mechanism of time refraction is connected with the emission of Unruh radiation by an accelerated dielectric boundary [22], and with the Hawking mechanism of black-hole evaporation [23]. Such connections will be discussed in future work.

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