Radiative processes in a confined Fermi sea

Bereket Berhane and T. A. B. Kennedy
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332-0430
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We investigate the combined effects of quantum statistics, resonant dipole-dipole atomic interactions, and reabsorption of radiation on the spontaneous emission of excited atoms in a confined Fermi sea. Even for dilute gases, reabsorption may be qualitatively important in determining the rate and angular distribution of emitted radiation. A temporal crossover behavior in the angular distribution is illustrated, in which the characteristic signatures of Fermi statistics are qualitatively changed by reabsorption within the gas.

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Following successful experiments to cool and trap bosonic alkali metal vapors below the Bose-Einstein condensation (BEC) temperature [1], there is now a great deal of interest in the observation of the effects of quantum degeneracy in trapped fermionic gases [2]. These include the possibility of Cooper pairing and radiative inhibition effects caused by the Fermi sea [3–5]. In this paper we investigate some fundamental features of radiative emission into the Fermi sea, including resonant atomic dipole-dipole interactions responsible for reabsorption of radiation within the sample. In the nondegenerate limit, our work relates to the old problems of line broadening, radiation trapping, and collective effects [6], although since we work in the dilute gas limit, we do not consider Dicke-superradiance effects here [7]. In recent times reabsorption has been noted as an important limitation on the ability of all-optical cooling methods to achieve BEC [8,9]. In this paper our primary focus is the qualitative influence of reabsorption effects on the rate and angular distribution of radiation emitted by excited atoms into a Fermi sea of confined ground-state atoms. In the absence of reabsorption effects the spatially anisotropic Fermi sea produces certain characteristic signatures which have recently been considered theoretically [5].

We consider a gas of fermionic atoms, spatially confined in a harmonic trap. The atoms are modeled as a two level system with Zeeman degeneracy, although we will shortly simplify our description. The mutual interaction of the atoms is mediated by the resonant electromagnetic field, and we ignore any other atomic interaction assuming a zero temperature spin-polarized sample in which *s*-wave fermionic interactions can be ignored as a consequence of the antisymmetry of the wave function. The Hamiltonian for the system of interacting atoms and field is given by $\hat{H}=\hat{H}_{\text{atoms}}+\hat{H}_{\text{photons}}+\hat{H}_I$, where $\hat{H}_{\text{atoms}}$ and $\hat{H}_{\text{photons}}$ are the usual free atomic and electromagnetic field Hamiltonians, respectively. In the electric dipole approximation the interaction Hamiltonian for the atoms and electromagnetic field is given by

$$\hat{H}_I=-\frac{1}{\epsilon_o}\int d^3r\hat{\mathbf{D}}(\mathbf{r})\cdot\hat{\mathbf{P}}(\mathbf{r}), \tag{1}$$

where $\hat{\mathbf{D}}(\mathbf{r})$ is the electric displacement operator and the polarization density of the atoms is given by

$$\hat{\mathbf{P}}(\mathbf{r})=d\sum_{ijMq}\mathcal{C}(J_g1J_e;Mq)\mathbf{e}_q^*\psi_i^*(\mathbf{r})\phi_j(\mathbf{r})\hat{g}_{iM}^\dagger\hat{e}_{jM+q}+\text{H.c.}, \tag{2}$$

where M is a magnetic quantum number, d is the reduced dipole matrix element, $\mathcal{C}$ is a Clebsh-Gordan coefficient, and $\{\mathbf{e}_q, q=-1,0,1\}$ are the standard spherical basis vectors. The annihilation and creation operators for the ground $(\hat{g})$ electronic-vibrational and excited $(\hat{e})$ electronic-vibrational atomic states, satisfy the usual anticommutation relations for fermions. Here, ψ_i and ϕ_j are vibrational eigenfunctions of a trapped atom in the ground and excited electronic states, respectively. We assume for simplicity that the excited atom experiences the same harmonic trapping potential as the Fermi sea of ground-state atoms [5]. However, as we discuss later, the results are not very sensitive to this assumption for appropriately prepared initial states.

The radiative emission and reabsorption is conveniently treated by master equation methods. For the purpose of numerical calculations it is necessary to simplify to a two electronic-state system $(M-m=0)$. While the theoretical derivations can be carried forward including atomic degeneracy, in order to simplify the presentation we will not do so here. We of course fully expect the simplified model will preserve the important qualitative features of quantum statistics and radiative interactions in a real degenerate atom.

The master equation for the atomic density matrix $\rho(t)$ is obtained by standard methods, treating the radiation field as reservoir initially in the vacuum state

$$\dot{\rho}(t)=(\mathcal{S}+\mathcal{J})\rho(t), \tag{3}$$

where

$$\mathcal{S}\rho(t)=-i[H_{\text{eff}}\rho(t)-\rho(t)H_{\text{eff}}^\dagger], \tag{4}$$

$$\mathcal{J}\rho(t)=\Gamma_A\sum\text{Re}[\mathcal{G}(i,j,i',j')]\hat{g}_{i'}^\dagger\hat{e}_{j'}\rho(t)\hat{e}_i^\dagger\hat{g}_j \tag{5}$$

and the effective (non-Hermitian) Hamiltonian

$$H_{\text{eff}}=-i\frac{\Gamma_A}{2}\sum\mathcal{G}(i,j,i',j')\hat{e}_j^\dagger\hat{g}_i\hat{g}_{i'}^\dagger\hat{e}_{j'}, \tag{6}$$

where the summations are taken over the twelve vibrational quantum numbers labeled by i, i', j, j'. Further, $\Gamma_A = d^2 k_A^3/3\hbar\epsilon_o\pi$ is the single atom spontaneous emission rate and $k_A = \omega_A/c$. The resonant dipole-dipole interaction among the atoms is mediated by the coupling coefficient

$$\mathcal{G}(i,j,i',j') = \left[-\frac{3i}{2k_A}\right]\int d^3r \int d^3r' \times \psi_i^*(\mathbf{r})\phi_j(\mathbf{r})\psi_{i'}(\mathbf{r}')\phi_{j'}^*(\mathbf{r}') \times \mathbf{e}_0\cdot\mathbf{G}(\mathbf{r}-\mathbf{r}',k_A)\cdot\mathbf{e}_0 \tag{7}$$

the real part of which, $\mathrm{Re}(\mathcal{G})$, modifies the radiative coupling rate, and the imaginary part, $\mathrm{Im}(\mathcal{G})$, is the dipole-dipole interaction potential for the atoms. The dyadic Green function is given by

$$\mathbf{G}(\mathbf{r}-\mathbf{r}',k_A) = \left[\mathbf{1} + \frac{\boldsymbol{\nabla}\boldsymbol{\nabla}}{k_A^2}\right]\frac{\exp\{ik_A|\mathbf{r}-\mathbf{r}'|\}}{4\pi|\mathbf{r}-\mathbf{r}'|}, \tag{8}$$

relating the field observed at $\mathbf{r}$ to a source dipole oscillator at $\mathbf{r}'$. To evaluate $\mathcal{G}$ it is useful to express $\mathbf{G}$ in a separable form with respect to its arguments. In spherical coordinates this is readily achieved, for example by using addition theorems. For the anisotropic traps also of interest here, cylindrical or even Cartesian coordinates are more convenient. The separation of $\mathbf{G}$ is then facilitated by the use of vector wave functions [10]. We will not discuss the details further here, but note that the calculations presented have been greatly simplified by employing this approach.

We proceed by expanding the solution to Eq. (3) in the form [11]

$$\rho(t) = \left\{ e^{\mathcal{S}t} + \int_0^t dt' e^{\mathcal{S}(t-t')}\mathcal{J}e^{\mathcal{S}t'} + \int_0^t dt' \int_0^{t'} dt'' e^{\mathcal{S}(t-t')}\mathcal{J}e^{\mathcal{S}(t'-t'')}\mathcal{J}e^{\mathcal{S}t''} + \cdots \right\}\rho(0). \tag{9}$$

The expansion describes all photon emission, absorption, reemission, ..., processes the atoms undergo as they evolve in time. The decomposition given has the property that each occurrence of $\mathcal{J}$ is associated with a *detected* photon, i.e, one which has exited the sample. All of the reabsorption processes are contained in $\mathcal{S}$, and are discussed further below.

We now focus our discussion on spontaneous emission and treat initial states in which only one atom is electronically excited, while the other ground-state atoms are in a confined zero temperature Fermi distribution. In this case only one photon can leave the sample, so that Eq. (9) reduces to

$$\rho(t) = \left\{ e^{\mathcal{S}t} + \int_0^t dt' e^{\mathcal{S}(t-t')}\mathcal{J}e^{\mathcal{S}t'} \right\}\rho(0). \tag{10}$$

An emitted photon may be reabsorbed by an atom in the Fermi sea before being detected. In order to discuss such emission-absorption cycles we apply perturbation theory. We separate from $\mathcal{S}$ the terms that are responsible for reabsorption within the system, i.e., $\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_I$, with

$$\mathcal{S}_0\rho = -\frac{\Gamma}{2}\sum [\mathcal{G}(i,j,i,j)\hat{e}_j^\dagger\hat{g}_i\hat{g}_i^\dagger\hat{e}_j\rho + \mathrm{H.c.}], \tag{11}$$

$$\mathcal{S}_I\rho = -\frac{\Gamma}{2}\sum{}' [\mathcal{G}(i,j,i',j')\hat{e}_j^\dagger\hat{g}_i\hat{g}_{i'}^\dagger\hat{e}_{j'}\rho + \mathrm{H.c.}]. \tag{12}$$

and where all of the effects of reabsorption are contained in $\mathcal{S}_I$: the prime on the summation indicates a sum over all indices except when $i=i'$ and $j=j'$ are simultaneously satisfied. By contrast $\mathcal{S}_0$ contains exchange effects which lead to inhibited spontaneous emission into the Fermi sea. The term $\mathcal{S}_0$ was recently considered by Busch et al using Fermi's golden rule [5]. We are now able to investigate how radiative reabsorption within the sample modifies these quantum statistical effects. Of course the influence of quantum statistics is also correctly incorporated into our treatment of reabsorption. Using the separation of $\mathcal{S}$ we make the following expansion:

$$e^{\mathcal{S}t} = e^{\mathcal{S}_0 t} + \int_0^t dt' e^{\mathcal{S}_0(t-t')}\mathcal{S}_I e^{\mathcal{S}_0 t'} + \int_0^t dt' \int_0^{t'} dt'' e^{\mathcal{S}_0(t-t')}\mathcal{S}_I e^{\mathcal{S}_0(t'-t'')}\mathcal{S}_I e^{\mathcal{S}_0 t''} + \cdots \tag{13}$$

and retain terms up to second order in $\mathcal{S}_I$. The result is back-substituted into Eq. (9). Physically we allow for at most one emission-absorption-reemission cycle before the photon is detected outside the system. This is reasonable for the dilute gas limit we consider here.

The excited-state population is given by $P_e(t) = \mathrm{Tr}\{\Sigma_j \hat{e}_j^\dagger\hat{e}_j\rho(t)\}$. If reabsorption is ignored, we see the influence of purely quantum statistical effects ($\mathcal{S}_0$) on the spontaneous emission

$$P_e(t) = \exp\{-\Gamma_A t[1-f_0]\}, \tag{14}$$

where $f_0 = \Sigma_i \mathrm{Re}[\mathcal{G}(i,j_0,i,j_0)]$ is the Fermi blocking factor. Here i runs over all initially occupied ground-state levels while j_0 denotes the initially occupied excited-state level. In general $0 \leq f_0 \leq 1$. For a single atom regardless of the trap shape $f_0 = 0$, whereas if all ground-state levels are occupied $f_0 = 1$. The presence of ground-state atoms inhibits spontaneous emission leading to a reduced fluoresecence rate. The dependence of f_0 on the parameter E_F/E_R, where E_F and $E_R = \hbar^2 k_A^2/2M$ (M is the atomic mass) are the Fermi and recoil energies, respectively is given in Ref. [5]. Clearly, more dramatic results are obtained as E_F/E_R increases. A zero temperature Fermi gas has average interparticle separation $R_0 \approx \lambda_A\sqrt{E_R/E_F}$ [12]. First generation experiments will probably be limited to the regime $E_F/E_R < 1$, and thus we restrict our attention to dilute gases $R_0 > \lambda_A$.

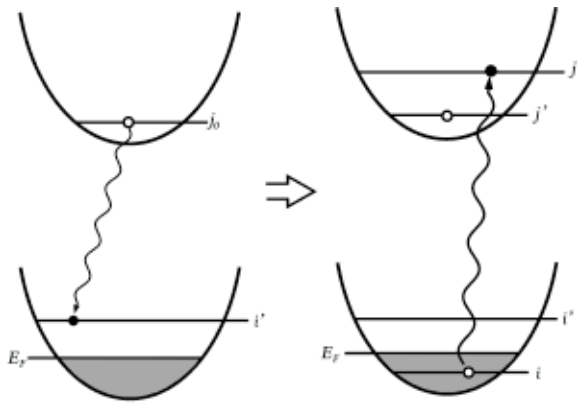

FIG. 1. Schematic representation of the reabsorption process.

We will now consider the effects of reabsorption on the emission rate. The first correction to the excited state population comes from a term quadratic in $\mathcal{S}_I$, which includes all processes involving a single reabsorption in the Fermi sea, before the photon is detected. The result is to good approximation given by

$$P_e(t)=e^{-\Gamma_A t[1-f_0]}\left\{1+\frac{\alpha}{2}(\Gamma_A t)^2\right\}, \tag{15}$$

where the influence of reabsorption is entirely contained in the parameter $\alpha=\Sigma'_{iji'}\mathrm{Re}[\mathcal{G}(i,j,i',j_0)]^2$. Higher order corrections involving sequences of two or more reabsorption processes are neglected, these become important for $\alpha\Gamma_A t/2\gtrsim 1$. The parameter α is a measure of the radiative trapping probability for the process $j_0\to i'\rightsquigarrow i\to j$, in which the excited atom in vibrational state j_0 makes a transition to an unoccupied ground-state level i', emitting a photon which is reabsorbed by an atom in level i of the Fermi sea causing a transition to the excited electronic-vibrational state j (see Fig. 1). In the summation, i ranges over the initially occupied ground state levels of the Fermi sea, while i' ranges over the complementary set, consistent with the Pauli exclusion principle. Alternatively, the leading approximation to α can be expressed in the form

$$\alpha\approx\left(\frac{9}{4k_A^2}\right)\int d^3r\int d^3r'\,\rho_g(\mathbf{r})\mathrm{Re}[\mathbf{G}(\mathbf{r}-\mathbf{r}')]^2\rho_e(\mathbf{r}'), \tag{16}$$

where $\rho_\ell(\mathbf{r})$ and $\ell=g,\ e$ is the density of particles in electronic state ℓ. This term is readily understood if one considers the resonant dipole-dipole interaction between two atoms. Perturbation theory then leads to an equation of exactly this form if we replace the self-consistent densities by the corresponding single particle atomic wave functions: $\rho_g(\mathbf{r})\to|\phi_g(\mathbf{r})|^2$ and $\rho_e(\mathbf{r})\to|\phi_e(\mathbf{r}')|^2$.

In Fig. 2 we show the excited-state population as a function of time, with and without reabsorption effects, for a cylindrically symmetric harmonic trap with trap frequencies $\Omega_x=\Omega_y\equiv\Omega_r=\Omega_z/\lambda$, with $\lambda=0.1$ and $E_F/E_R=4/5(\alpha\approx 0.47)$. This illustrates that even for a highly anisotropic

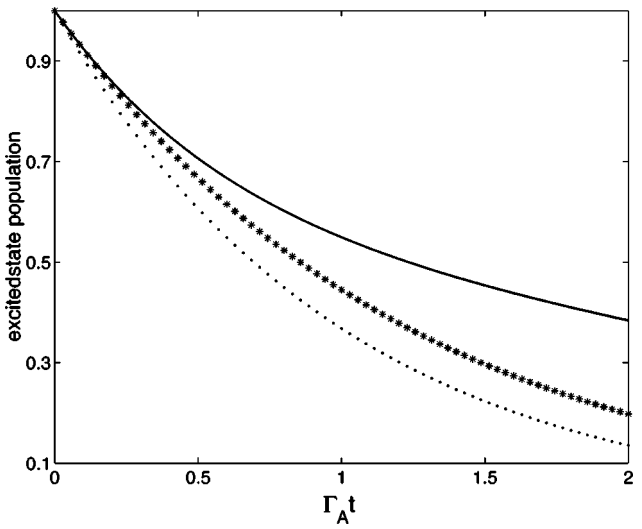

FIG. 2. The excited-state population for a cylindrically symmetric trap with trap frequency Ω, $E_R=25\hbar\Omega(\Omega=\sqrt[3]{\Omega_x\Omega_y\Omega_z})$, $E_F/E_R=4/5$, and $\lambda=0.1$. The dotted curve is for a single atom, the asterisks include exchange interactions only ($\mathcal{S}_0$), solid curve includes reabsorption effects ($\mathcal{S}_0+\mathcal{S}_I$).

trap, reabsorption effects can be very significant, since they cause population of excited vibrational states which would otherwise be unoccupied.

The angular distribution of the spontaneous emission by a single atom in a trap is a typical dipole pattern given in terms of the dipole matrix element $\mathbf{d}$ by $\{1-|\hat{\mathbf{d}}\cdot\hat{k}|^2\}$ regardless of the shape of the trap. However, the presence of identical ground-state atoms in an anisotropic trap modifies the angular distribution [5]. The detection of spontaneous emission is governed by the jump operator $\mathcal{J}\rho(t)$ which can be written as

$$\mathcal{J}\rho(t)=\Gamma_A\int d\Omega_k\{1-|\hat{\mathbf{d}}\cdot\hat{k}|^2\}p^+(k_A\hat{\mathbf{k}})\rho(t)p^-(k_A\hat{\mathbf{k}}), \tag{17}$$

where $\mathbf{P}^\pm(\mathbf{k})=p^\pm(\mathbf{k})\mathbf{d}$ is the Fourier transform of the positive/negative frequency component of the polarization density operator. Given a photon is emitted, the probability that it is emitted in the $\hat{\mathbf{k}}$ direction is proportional to $\{1-|\hat{\mathbf{d}}\cdot\hat{\mathbf{k}}|^2\}\langle p^-(\hat{\mathbf{k}})p^+(\hat{\mathbf{k}})\rangle$. Since $\{1-|\hat{\mathbf{d}}\cdot\hat{\mathbf{k}}|^2\}$ appears as a factor in all cases, we will define an emission profile by

$$I(\hat{\mathbf{k}},t)=\frac{\langle p^-(\hat{\mathbf{k}})p^+(\hat{\mathbf{k}})\rangle}{\int d\Omega_k\langle p^-(\hat{\mathbf{k}})p^+(\hat{\mathbf{k}})\rangle}, \tag{18}$$

which is a normalized measure of the angular distribution of photons detected at time t. In the absence of reabsorption $I(\hat{\mathbf{k}},t)$ is independent of time t. In addition for classical particle statistics, I is independent of $\hat{\mathbf{k}}$. We again consider cylindrically symmetric traps. The emission profile can be computed numerically from the density matrix. In Fig. 3 we plot $I(\theta,t)(\theta=\hat{\mathbf{k}}\cdot\hat{z}$ is defined with respect to the rotational

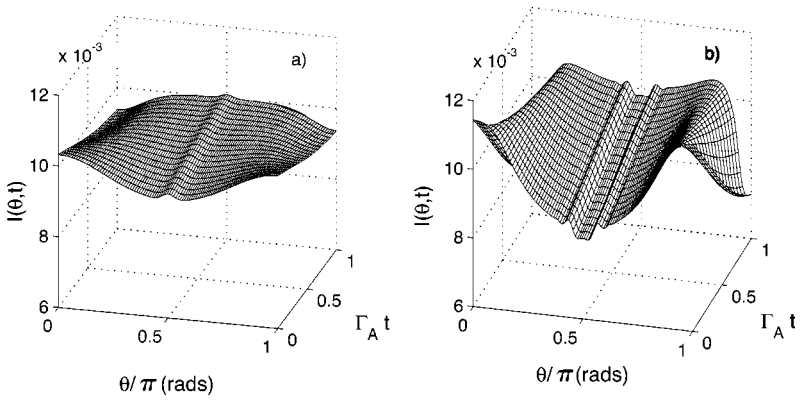

FIG. 3. The emission profile $[I(\theta,t)]$ as a function of time for (a) $\Omega_z/\Omega_r=0.5$ and (b) $\Omega_z/\Omega_r=0.1$.

axis of symmetry, z). When $E_F/E_R<1$, the effect of Fermi statistics in the absence of reabsorption is to inhibit photon emission in the radial direction [5]. By contrast it is intuitively obvious that reabsorption of a photon is more probable axially than radially as light propagating in the axial direction encounters more atoms. Exchange and reabsorption effects thus tend to counteract one another in this limit. (In the limit $E_F/E_R>>1$ this may not be the case [5].) Figure 3(a) shows that when $\Gamma_A t\approx 1$ radiative reabsorption has qualitatively changed the initial profile, while in the highly deformed trap [Fig. 3(b)] the changes are mainly quantitative except along the axial direction. It has been shown previously that deforming the trap reduces the reabsorption probability [9]. To observe appreciable anisotropy in the emission profile due to quantum statistics, as opposed to reabsorption, one needs significant trap anisotropy and to limit the photon counting time to $t<1/\Gamma_A$; in this case temporal resolution is not necessary. With good temporal resolution however, our emission profiles illustrate how the character of the emission changes as a function of time from quantum statistics dominated to reabsorption dominated.

In our treatment we have assumed the excited-state potential is identical to that which confines the Fermi sea. We wish to make some remarks about the effect of the excited-state potential on our results. Since $\Gamma_A>>\Omega_{r,z}$ an excited atom cannot move significantly with respect to the Fermi sea within its natural lifetime provided the initial excited wave packet has vibrational energy less than about $E_F(\Gamma_A/\Omega)^2>>E_F$. Equivalently, in the absence of an excited state potential the same condition would be necessary for the kinetic energy of an excited atom. The latter argument ignores the dipole-dipole interaction potential which scales as $V_{\rm dip}\approx\hbar\Gamma_A/(k_aR_0)^3$. In the dilute limit $k_AR_0>1$, and in our examples $k_AR_0\approx 7$. Thus $V_{\rm dip}<<\hbar\Gamma_A$. An excited atom cannot leave the vicinity of the Fermi sea in a spontaneous lifetime as a result of $V_{\rm dip}$ provided that $\Omega/\Gamma_A<<E_R^{3/2}/(\hbar\Omega E_F)$, which is satisfied in the dilute limit considered here. The numerical results presented here include the dipole interaction potential, and are rather insensitive to the excited-state potential.

In conclusion, we have discussed the radiative emission process in the vicinity of the Fermi sea, including quantum statistical effects and the resonant dipole-dipole interaction responsible for reabsorption within the gas. Even in the dilute gas limit, reabsorption is qualitatively important in determining the radiative emission rate and in the cross over behavior of the time resolved radiative angular distribution.

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