Consistent histories, the quantum Zeno effect, and time of arrival

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We present a decomposition of the general quantum-mechanical evolution operator that corresponds to the path decomposition expansion, and interpret its constituents in terms of the quantum Zeno effect (QZE). This decomposition is applied to a finite-dimensional example and to the case of a free particle in the real line, where the possibility of boundary conditions more general than those hitherto considered in the literature is shown. We reinterpret the assignment of consistent probabilities to different regions of space-time in terms of the QZE. The comparison of the approach of consistent histories to the problem of time of arrival with the solution provided by the probability distribution of Kijowski shows the strength of the latter point of view.

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I. INTRODUCTION

The theoretical treatment of ''time observables'' is an important loose end of quantum mechanics. An example of the problems encountered was formulated by Misra and Sudarshan in the form of a paradox [1]. They sought the probability that an unstable particle decays at some time during an interval $\Delta = [0,t]$. This has to be distinguished from, and in general differs from, the standard quantum probability that the particle is found decayed at the instant t. More generally, they also looked for the probability that a quantum system makes a transition from a preassigned subspace of states to the orthonormal subspace during a given period of time, further examples being the dissociation of a diatomic molecule or the arrival of a particle at a region of space. Classically, we can ask whether a particle moving on a line is always to the same side of the point $x=0$, be it to the right or the left (but always to the right or always to the left), or if it crosses the $x=0$ point during $\Delta = [0,t]$. What are the probabilities for the particle being always to the same side during Δ or for the particle crossing, according to quantum mechanics?

Since many experiments deal with such topics, and provide answers for them, we may expect that quantum mechanics should provide an unambiguous recipe to compute these probabilities. However, the standard formalism, as found in all textbooks, tells us only how to evaluate expectation values and probabilities for a given instant of time, so these questions seem to pose the need for some extension of the standard rules. Misra and Sudarsan attempted an apparently natural procedure: they modeled the continuous observation implied in these issues by a repetition of ideal first kind of measurements in the limit of infinite frequency. The consequence of such an interpretation of the continuous measurement, however, is that the system never abandons the original subspace (the quantum Zeno effect). Misra and Sudarshan considered the contradiction between the theoretical prediction and actual experiments detecting time distributions (of arrival, of decay, in general of occurrence of events) as paradoxical. For these authors such a mathematical result was physically inacceptable: it was merely an indication that the assumed procedure was not adequate to provide the probabilities they were looking for. The completeness of the quantum theory was therefore pending until a *trustworthy algorithm* could be found. In fact, most of the many publications on the Zeno effect have been devoted to the analysis or implementation of the repeated measurement scheme, overlooking the origin of the paradox, namely the need to find a trustworthy algorithm for time distributions.

Later on, the formulation of nonrelativistic quantum mechanics in terms of sum-over-histories opened up the possibility that some questions, even though lying outside the realm of the standard rules of quantum mechanics, could be sensibly posed. One such question, for example, is whether it is possible to define probabilities for alternative regions of space-time from amplitudes built as sums over restricted classes of paths. This was indeed discussed by Feynman himself [2]. Hartle [3] and Yamada and Takagi [4,5] studied the possibility of defining a probability for crossing or not crossing $x=0$ in an interval Δ for a free particle on a line. Their conclusion was that it is not possible to define such probabilities, because the interference term between the possibilities (the ''decoherence functional'' of the literature) generically does not vanish. Yamada and Takagi, however, pointed out that for antisymmetric initial wave functions it was indeed possible to define the probabilities with the result that there was no crossing of the point $x=0$ whatsoever. Another exception pointed out by Halliwell was the particle coupled to a bath. However, the resulting probabilities depend on the nature of the bath and the coupling, i.e., no ideal distribution emerges.

In this paper we shall show how the class of states that allows for a positive answer within the consistent histories framework can be considerably enlarged. This is done by means of a generalization of the PDX (path decomposition expansion). The idea of summing over classes of (Feynman) paths is of course much more general than just its application to the example mentioned, and leads to many different interesting aspects. One of particular interest to us, because of its possible relationship to the question of times of tunneling [6] or arrival [7], is the path decomposition expansion (PDX), formulated by Auerbach and Kivelson [8] to study tunneling problems with several spatial dimensions. We find the rather striking fact that, although hard-wall boundary conditions have been assumed in all derivations of the decomposition

formulas for the propagators, which is the central result obtained so far from the PDX, other boundary conditions could be imposed on the restricted propagator without impairing the validity of the expression.

We shall start with an operator derivation of the PDX which is a generalization of the ones proposed by Halliwell [9] and Muga and Leavens [10]. As a simple illustration we shall apply it to a two-state system. We shall then see that there is a set of exclusive alternatives for which the formalism of consistent histories [11–15] cannot generically give a set of probabilities. This will be understood in terms of the quantum Zeno effect for the two-state system (which is actually the one that pertains to the proposal of Cook [16] and has been realized experimentally [17]). Even though the example corresponds to a finite-dimensional Hilbert space, the derivation of the PDX holds formally for infinite-dimensional Hilbert spaces as well. However, topological considerations come into play, and we show the need to specify boundary conditions for the restricted propagator. We then analyze the Yes/No question formulated by Hartle and Yamada and Takagi, and show that it is possible to define probabilities consistently for a much wider class of initial conditions than the antisymmetric one put forward by Yamada and Takagi. We explain the result by analogy to the finite-dimensional example given previously.

This extension, however, falls short of the broad generality that can be attributed to other conventional approaches, in particular to the definition of probabilities by means of positive operator valued measures: the time of arrival distribution of Kijowski is perfectly well defined for free particles on the line. Our aim in the final discussion is to solve this apparent contradiction.

II. OPERATOR DERIVATION OF THE PDX

Halliwell [9] obtained an operator derivation of the PDX which is closely related to the point of view of consistent or decoherent histories [11–15]. Let P be a projector and Q its complementary projector, $Q = 1 - P$. Define $P(t) = \exp(iHt/\hbar)P\exp(-iHt/\hbar) = U^\dagger(t)PU(t)$, and similarly $Q(t)$. It follows that if H is self-adjoint, $P(t) + Q(t) = 1$ for every real t. There exists a generalized decomposition of unity, given by

$$1 = P + \sum_{k=1}^{n} P(t_k)Q(t_{k-1})Q(t_{k-2})\cdots Q(t_1)Q$$

$$+ Q(t_n)Q(t_{n-1})\cdots Q(t_1)Q \tag{1}$$

for any set of real numbers $\{t_1, t_2, \ldots, t_{n-1}, t_n\}$. Assume that $t_k = k\,\delta t$, with δt small. Rewrite $P(t_k)$ as

$$P(t_k) = P(t_{k-1}) + \delta t\,\dot{P}(t_{k-1}) + O(\delta t^2)$$

$$= P(t_{k-1}) + \delta t\,U^\dagger(t_{k-1})\dot{P}U(t_{k-1}) + O(\delta t^2), \tag{2}$$

where $\dot{P}$ is simply $(i/\hbar)[H,P]$. Multiply Eq. (1) from the left with $U(t_n)$, and use Eq. (2). We obtain the following decomposition of the propagation operator:

$$U(t_n) = U(t_n)P + \sum_{k=1}^{n} \delta t\,U(t_n - t_{k-1})$$

$$\times \dot{P}U(t_{k-1})Q(t_{k-1})Q(t_{k-2})\ldots Q$$

$$+ U(t_n)Q(t_n)Q(t_{n-1})\ldots Q + O(\delta t^2). \tag{3}$$

We define the following ''restricted'' propagation operator

$$U_r(t) := \lim_{n\to\infty,\,\delta t = t/n} U(n\,\delta t)Q(n\,\delta t)Q[(n-1)\,\delta t]\ldots Q. \tag{4}$$

Taking the limit $\delta t \to 0$ in expression (3) we arrive at the generalized form of the PDX proposed by Halliwell [see [9], expression (2.19)],

$$U(t) = U(t)P + \int_0^t ds\,U(t-s)\dot{P}U_r(s) + U_r(t). \tag{5}$$

Notice that it can be further generalized without complication to time-dependent Hamiltonians.

A. Two-state example

Consider the two-state Hamiltonian

$$H = \hbar\omega\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Let

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

and $Q = 1 - P$. The unitary evolution matrix is easily computed to be

$$U(t) = \begin{pmatrix} \cos(\omega t) & -i\sin(\omega t) \\ -i\sin(\omega t) & \cos(\omega t) \end{pmatrix}. \tag{6}$$

It follows that $U_r(t) = Q$. Since

$$U(t)P = \begin{pmatrix} \cos(\omega t) & 0 \\ -i\sin(\omega t) & 0 \end{pmatrix}$$

and

$$\dot{P} = \omega\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

we see that each of the terms in Eq. (5) is different from zero: the operator form of the PDX is not a trivial identity.

To interpret each of these terms, observe that $U_r(t)$, the restricted propagation operator, corresponds to the continuous limit of a series of preparations of the system in the subspace of states invariant under Q. These preparations are equally spaced in time, and are von Neumann collapses onto the eigenspace of Q. It is to be expected, therefore, that this term is the propagator for a system that is continuously ob-

served in the eigenspace of Q, and this is, in fact, the purport of the analysis of Misra and Sudarshan [1] of the quantum Zeno effect.

B. Quantum Zeno effect

If the initial state were in the eigenspace of Q, the term $U(t)P$ would not contribute to the later evolution of the system. We understand therefore that the convolution integral is the term required to retain a probability that the initial quantum state in the eigenspace of Q does indeed jump at some point in time to the eigenspace of P. It is immediate to observe that the sum of the convolution integral and the restricted propagator preserves the norm of a state initially in the eigenspace of Q. The quantum Zeno effect can be understood in this case, therefore, as the decomposition of the unitary evolution in the whole Hilbert space of an eigenstate of Q in two terms: on the one hand, the restricted propagator, which is unitary in the eigenspace of Q, but nonunitary over the whole Hilbert space, and on the other hand, the crossing term, necessary to recover unitarity over the Hilbert space, and which accounts for transitions out of the initial eigenspace.

Let us now pose the following questions: given a time interval t and a particle initially prepared with spin down [i.e., in the state $|\downarrow\rangle = \binom{0}{1}$], what is the probability that it has always stayed with spin down in the interval? What is the probability that it has switched spin at some instant? We can answer the first one by looking at the restricted propagator $U_r(t)$: the probability amplitude that it has always stayed with spin down is $\langle\downarrow|U_r(t)|\downarrow\rangle = 1$. However, notice that $\langle\uparrow|\int_0^t ds\, U(t-s)\dot{P}U_r(s)|\downarrow\rangle = -i\sin(\omega t)$ and $\langle\downarrow|\int_0^t ds\, U(t-s)\dot{P}U_r(s)|\downarrow\rangle = \cos(\omega t) - 1$. It follows that we cannot assign probabilities consistently to the exclusive events (i) staying with spin down during the whole interval t and (ii) having flipped spin at some instant of the interval. The histories into which we have decomposed the *unitary* evolution of the particle with initial spin down are not consistent histories.

In terms of operators, the operator associated with the continuous measurement of being in the eigenspace of Q and the operator associated with, at some point, jumping to the eigenspace of P do not commute and give rise to a crossing term: they cannot be measured simultaneously.

More explicitly, the history operator associated with the particle always being in the eigenspace of Q is $C_1 = \lim_{n\to\infty,\delta t=t/n} Q(n\delta t)Q[(n-1)\delta t]\cdots Q$, i.e., a product of succeeding projectors. The complementary operator is $C_2 = 1 - C_1$. The decoherence functional is $d(i,j) = \text{Tr}(C_i\rho C_j^\dagger)$, and the inconsistency of probability assignments is reflected in the fact that, in the case portrayed above, $\text{Re}[d(1,2)] \neq 0$. Notice that the history operator C_1 is related to the restricted propagator defined above through the following expression: $C_1 = U^\dagger(t)U_r(t)$.

It is relevant at this point to mention the ''spectral decomposition'' approach of Pascazio and Namiki [18], similar to the idea of the generalized PDX presented above. Additionally, notice that the models in the literature that attempt to

obtain the quantum Zeno effect as a consequence of decoherence are in fact canceling out the crossing term. In other words, if the pointer basis for a decoherence process is adequately aligned with the eigenspaces of P and Q, the quantum Zeno effect will be immediately obtained as a consequence of decoherence.

Insofar as the quantum Zeno effect is a paradox (see [19] for a general discussion), it is a paradox in that what seem to be exclusive and consistent events for assignments of probability in classical mechanics cannot be assigned quantum-mechanical probabilities in a consistent manner. It should be stressed, however, that this is no logical internal contradiction of quantum mechanics. Rather, this simply reflects the fact that statements about quantum events have to be much more precisely enunciated, and that classical language and presuppositions do not always translate readily into the quantum world.

III. HISTORIES ON THE REAL LINE

The derivation of Eq. (5) presented above is formal, with no attention being paid to topological issues. In order to highlight the difficulties, consider the case of a free particle of mass m that moves on a line. By simple integration by parts one can realize that PHQ need not be zero, since

$$(PHQ\psi)(x) = \frac{-\hbar^2}{2m}[1 - \theta(x)]\partial_x^2[\theta(x)\psi(x)]. \quad (7)$$

It therefore behooves us to analyze the meaning of $U_r(t)$. It is obtained as a time-ordered limit of products of QHQ terms. The operator QHQ, however, is not self-adjoint: it admits a continuous one-parameter family of self-adjoint extensions. Therefore, unless a particular self-adjoint extension is chosen, $U_r(t)$ will not be unitary in the eigenspace of the projector Q. Imagine now that a particular extension has been chosen. The meaning of PHQ is subservient to the extension chosen, since what we actually require is $PHQ + QHQ = HQ$. If the meaning of QHQ is modified, so should the meaning of PHQ be modified.

This observation can be strengthened by applying the theorem of Misra and Sudarshan concerning the quantum Zeno effect [1] to this case of the free particle. The Hamiltonian of the free particle is self-adjoint and semibounded (the first assumption of the theorem), and there exists a time-reversal operator, which commutes with the projectors onto spatial regions (second assumption). Suppose now that the limit defining $U_r(t)$ exists; actually assume that it exists in the strong topology. It is clear in our case that if it does, its limit when $t\to 0$ is Q. It follows from Theorem 1 of Ref. [1] that $U_r(t)$ then can be written as $Q\exp(-iBt/\hbar)Q$, with B self-adjoint, and such that $QB = BQ = B$. The meaning of this result is that the *existence* of $U_r(t)$ implies the existence of a self-adjoint operator to which it can be related, which can be understood as a self-adjoint Hamiltonian acting on the eigenspace of Q. Therefore, the validity of the operator form of the PDX hinges on choosing a specific self-adjoint extension of the original Hamiltonian when restricted to the Q

eigenspace and considering the unitary evolution in that sub-space with this new Hamiltonian.

Profiting from the simplicity of the example at hand, let us be more specific. The self-adjoint extensions of the free-particle Hamiltonian on the half-line are parametrized by a real parameter β, and the domain of the extension H_β is the set of the square integrable, absolutely continuous functions on the half-line, whose derivative is square integrable and that fulfill the condition $\psi(0) = \beta \psi'(0)$.

Thus the term $\dot{P} U_r(t)$ can be understood in terms of integration by parts, as follows. We define (formally) the propagator $g(x,y,t) = \langle x | U(t) | y \rangle$ and the restricted propagator $g_r^\beta(x,y,t) = \langle x | U_r^\beta(t) | y \rangle$, where $U_r^\beta(t) = \exp(-iH_\beta t/\hbar) Q$. The convolution integral in Eq. (5) is then written as

$$\langle x | \int_0^t ds\, U(t-s) \dot{P} U_r^\beta(s) | y \rangle$$

$$= \int_0^t ds \int_{-\infty}^{+\infty} d\xi\, g(x,\xi,t-s)\, \theta(-\xi) \left(\frac{-i\hbar}{2m} \right) \partial_\xi^2 g_r^\beta(\xi,y,s)$$

$$= \left(\frac{-i\hbar}{2m} \right) \int_0^t ds\, g(x,\xi,t-s) \overleftrightarrow{\partial}_\xi g_r^\beta(\xi,y,s) \Bigg|_{\xi=0},$$

where $f(\xi) \overleftrightarrow{\partial}_\xi g(\xi) = f(\xi) g'(\xi) - f'(\xi) g(\xi)$. It is important to stress that this derivation is valid for all real β, not just for $\beta = 0$, which is the case analyzed in the literature.

Using the formalism of path integrals, Auerbach and Kivelson [8] arrive at this symmetric form (with $\overleftrightarrow{\partial}_\xi$ instead of ∂_ξ) from the consideration that there is a change of variable in the path integral, trading $x_\sigma(s)$ for the time s after which the path is confined to one side of $x = 0$, and that the Jacobian associated with this change of variables leads to the symmetric operation $\overleftrightarrow{\partial}_\xi$. However, they do not consider general boundary conditions of the form stated here because they do not seem to appear in their derivation of the PDX in terms of a skeletonization of the path. Other alternative derivations [20,21] use Wick rotation and study the Wiener integral associated with a diffusion process with a totally reflecting boundary for the restricted propagator. However, both in the quantum-mechanical and in the Brownian path integral, restricted to the half-line, it is feasible to consider alternative boundary conditions, as was shown by Clark, Mekinoff, and Sharp [22]. In this paper, they use an ''elastic barrier'' at the boundary, that is to say, in the language of a diffusive process, that the Brownian particle has a probability of being reflected and a probability of being absorbed at the boundary. This is translated, by analytic continuation, into the boundary conditions that preserve probability (no absorption of probability) for the quantum-mechanical problem, i.e., the boundary conditions that lead to self-adjoint restricted Hamiltonians. From the point of view of quantum-mechanical path integrals, the restriction that the particle not cross the boundary is not enough to describe completely all possible paths: how is it to be reflected? It is this freedom that leads to the possible alternatives.

Yet another derivation of the PDX is given by Hartle (see [3], subsection 6.c and note 27), who is rather cautious in his analysis of Trotter's formula, which is basically what underlies the definition of the restricted propagator, but is misled by the uniqueness results available for the associated diffusion equation. Yamada [7] derives the PDX decomposition out of a postulated integral equation, and imposes a particular choice of boundary conditions, also missing out the alternatives highlighted in the discussion above.

A. Consistent probabilities

Let us now ask the question posed by Hartle [3] and, independently, Yamada and Takagi [4]. Is it possible to assign consistently probabilities to the following exclusive events: (i) that a free particle moving on the line stays always to the same side of $x = 0$ during a time interval t; (ii) that it crosses $x = 0$ once or more during the same time interval? To make the discussion easier, imagine first an initial wave-function restricted to the positive half-line. Under the restricted evolution $U_r^\beta(t)$, this wave-function stays always in the positive half-line with no loss of probability: $U_r^\beta(t)$ is unitary when acting on $L^2(\mathbf{R}^+)$. However, when we try to understand $U_r^\beta(t)$ as extended to an operator on the whole real line, it is no longer unitary: the convolution integral is required to guarantee the unitary evolution of the initial one-sided state in the whole Hilbert space. There is therefore a crossing term, and this prevents the consistent assignment of probabilities to the exclusive events mentioned. As we see it, the requirement that a particle always has to be to one side of the $x = 0$ point is, in a way, imposed by constantly monitoring that the particle is to one side, thus preventing the classical exclusive events from being consistently exclusive also from the quantum point of view. In other words, we again run into the quantum Zeno paradox.

Having said this, there *is* an example of initial conditions, as pointed out by Yamada and Takagi [4], for which the probability assignments are consistent: the antisymmetric case. Antisymmetric wave functions preserve this characteristic under evolution with the free-particle Hamiltonian, or, in other words, the parity operator commutes with the free-particle Hamiltonian. This can also be understood with regard to the restricted propagators as follows: the evolution of an antisymmetric wave-function under the whole Hamiltonian is identical to the direct sum of the evolution in each of the half-lines under the half-line free-particle propagator with hard-wall boundary conditions. There is no probability flow from one half-line to the other under free-particle evolution if the initial condition is antisymmetric. This implies that in this case the interference term is zero, and that the probability of always staying to the same side during any time interval is unity: for any given instant there is no probability of crossing $x = 0$.

Given this point of view, it is immediate to generalize the example of Yamada and Takagi to other instances: the meaning of the boundary conditions that correspond to self-adjoint extensions of the free-particle Hamiltonian when restricted to the half-line is that they prevent probability flowing out of the half-line. So for each real β we see that the wave-functions that fulfill $\psi(0) = \beta \psi'(0)$ have no transfer of probability from one half-line to the other. Alternatively, the evo-

lution under the whole Hamiltonian of a wave function obeying this condition is identical to the independent evolution of the parts of the wave function in each of the half-lines under the half-line free-particle propagator with the corresponding boundary conditions. Thus we see that, for these initial wave functions, the assignment of probability one to always staying to one side of the origin, and zero probability to crossing the origin once or more during a time interval, is indeed a consistent assignment of quantum probabilities.

B. Arrival probabilities

As seen above, only in some rather special circumstances can we make consistent assignments of probability using a decomposition of possible paths for the alternatives considered. This does not mean, though, that there is no consistent prescription within the realm of standard quantum mechanics for the probability of having crossed a given point, $x=0$, say, in a particular time interval. Misra and Sudarshan, in their seminal paper [1], already point out that the existence of such a probability would imply the existence of a generalized resolution of the identity (in their language; a positive operator valued measure, or POVM, in modern parlance) for a time of arrival operator. In fact, we now have at our disposal such a POVM for the case of a free particle; the associated probability density is, for a pure state ψ,

$$\Pi_K(t,\psi) = \left| \int_0^\infty dp \left(\frac{p}{2\pi m\hbar} \right)^{1/2} e^{-ip^2 t/2m\hbar} \psi(p) \right|^2$$
$$+ \left| \int_{-\infty}^0 dp \left(\frac{-p}{2\pi m\hbar} \right)^{1/2} e^{-ip^2 t/2m\hbar} \psi(p) \right|^2,$$

where we have used the momentum representation. This is actually the probability density proposed by Kijowski from an axiomatic point of view [23], which is related to the time of arrival operator of Aharonov and Bohm [24] (see [25] for details of the relationship between the two objects).

Given this distribution, it is sensible to ask whether a similar construction could hold for the finite-dimensional example given above. Unfortunately, the answer is negative. Imagine that indeed there exists a distribution of probability for the time of first shifting from $|\downarrow\rangle$ to $|\uparrow\rangle$. The existence of this distribution would imply the existence of a POVM [which in this finite-dimensional example would have to be a projection valued measure (PVM)], whose first operator moment T would be a self-adjoint operator (in this finite-dimensional case, all symmetric operators are self-adjoint). Since this operator would have a "time" interpretation, it would have to be canonically conjugate to the Hamiltonian, $[H,T]=i\hbar$. In the example at hand, H is proportional to σ_1, and all operators, such as T, can be written as $\alpha + \vec{\beta} \cdot \vec{\sigma}$, where the σ_i matrices are Pauli's matrices. There are no four numbers $(\alpha, \vec{\beta})$ such that a canonically conjugate T can be obtained. Therefore, there is no analog of Kijowski's distribution for this finite-dimensional example, and, in fact, there is no analog of Kijowski's distribution for any finite-dimensional example.

IV. CONCLUSIONS

The operator derivation of the PDX formula we have presented here has allowed us to identify the paradoxical aspects of the quantum Zeno effect of Misra and Sudarshan as being due to incompatible assignments of probability to inconsistent histories. We have explicitly separated the crossing term that leads to this inconsistency.

Feeding the well-known results of Misra and Sudarshan back onto the PDX formula, it also obtains that in cases such as that of a free-particle moving on the line, there are several different PDX expressions, each one corresponding to a particular partial isometry, i.e., to a particular self-adjoint extension of the restricted Hamiltonian. Furthermore, we have analyzed for which cases the PDX probability assignments for the alternatives of having or not crossed a given point are consistent, extending the result of Yamada and Takagi to all instances of boundary conditions for which there is no probability flow through that point. In spite of this extension, no time-of-arrival probability could be assigned to the overwhelming majority of possible states within the consistent histories approach.

We remark that there is a different, fully consistent prescription for the probability of having crossed a given point in a certain time interval, given by Kijowski's distribution in the free case. Notice that Kijowski's distribution is obtained in the context of (almost) completely standard quantum mechanics, the only extension needed thereof being that POVMs are accepted to describe observables. How is this distribution compatible with the negative results obtained within the framework of consistent histories? The consistent histories approach is actually much more demanding since it requires the absence of interferences between the space-time histories to attribute to them a classical-like status as alternatives that actually occur with certain probabilities. Instead, the distribution of Kijowski should be regarded, from the perspective of the standard interpretation, as a "potentiality," a distribution that a properly designed apparatus could measure. Therefore no association with noninterfering histories is claimed or required. The apparatus would actually be the "best" one, in the sense of providing a covariant distribution with minimum variance. Of course a less than perfect apparatus would provide convolutions or deformed versions of Π_K. Werner has described the family of covariant distributions, each representing a potentiality associated with a different measurement device, for states with positive momentum components [26]. From a more technical point of view, the difference can be associated with the fact that Kijowski's distribution at time t is the expectation value for $\psi(t)$ of a certain operator, a quantum version of the positive flux minus the negative flux [10]. It is thus *not* related to expectation values of strings of operators that depend on different instants of time. In a slightly facetious way, we might say that standard, old-fashioned quantum mechanics has the upper hand on the consistent histories formalism for this particular case. While Kijowski's distribution is "ideal," in the sense of depending only on the state of the particle, there are other approaches in which additional degrees of freedom for the apparatus and or the environment are included, which

provide operational time-of-arrival distributions [10]. Again, these results are found without demanding any noninterference condition. Halliwell in particular [27] has compared the distribution derived from an irreversible detector model with the one associated with consistent histories in the presence of a bath coupled to the particle, and has shown how in the decoherent histories approach the coupling with the environment destroys far more interference than is really needed in order to define the arrival time with the irreversible detector.

For most cases of practical interest Π_K is approximately equal to the current density J. The challenge now is to perform experiments able to realize the ''potentiality'' of Kijowski's distribution in ''quantum'' regimes where it differs significantly from the current density. In general one may expect to obtain convolutions depending on the particular apparatus response [28], see [10] for a more detailed discussion of the interpretation of Π_K.

One may wonder if Kijowski's distribution is the key to the ''trustworthy algorithm'' sought by Misra and Sudarsan for arbitrary problems where a time distribution for the passage between complementary subspaces is required. Indeed, the existence of Kijowski's distribution opens up the possibility that similar constructions might be feasible for other situations where the histories analysis has not been able to live up to its full promise. However, we have proved that no analog of Kijowski's distribution can be constructed in the case of finite-dimensional Hilbert spaces. The question as to the existence of ''trustworthy'' analogs of Kijowski's distribution for infinite-dimensional situations remains an open question, which we hope will be settled in the affirmative in the future (see [29] for an extension of Kijowski's distribution in the case of one-dimensional motion with potentials).

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