Three-level atoms inside a degenerate optical parametric oscillator: Steady-state behaviors

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We study a composite system consisting of N three-level Λ-type atoms and a degenerate optical parametric oscillator (DOPO). The standard DOPO threshold is modified by the presence of the three-level atoms. Bistability (in the intracavity field intensity versus pumping intensity for the DOPO) appears for certain atomic detunings and coupling field strengths between the intracavity field and one of the atomic transitions. The effects of the coupling field (for the auxiliary transition in the three-level atomic system) are discussed. The susceptibility of the three-level atoms is calculated for the lower and upper branches of the bistable curve. Both analytical (for certain special cases) and numerical (for more general cases) results are presented.

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I. INTRODUCTION

In the past few years, atomic coherence effects in multilevel atomic systems, such as electromagnetically induced transparency (EIT) [1,2] and gain without inversion (GWI) [3–6], have attracted great attention. Absorption and dispersion properties of near ideal three-level atomic systems in different configurations (Λ-type, *ladder*-type, and V-type) interacting with continuous wave (CW) diode lasers were studied both theoretically and experimentally [2,7,8]. Due to the atomic coherence effect, the absorption and dispersion properties of the weak probe field coupling to one of the atomic transitions can be controlled by the coupling field interacting with the other atomic transition. When the three-level atoms are placed inside an optical cavity, many interesting effects, such as optical bistability, frequency pulling, and cavity linewidth narrowing, can exist in the steady state and be controlled by the coupling field [9–11]. It has also been predicted that quantum noise in one quadrature of the output field may be suppressed in the system with three-level atoms inside an optical cavity [12].

A composite system consisting of a degenerate optical parametric oscillator (DOPO) [13] with N two-level atoms has been studied previously [14]. Optical bistability with interesting features at steady states is predicted and studied in detail. Due to the special properties of this composite system, i.e., the mean intensity of the lower branch of the bistability curve is always zero, one can analytically calculate the spectra of squeezing and the amount of intracavity squeezing [15,16]. It is predicted that the DOPO intracavity squeezing can be enhanced by the presence of the two-level atoms.

In this paper, we will study the composite system consisting of the DOPO and N Λ-type three-level atoms. Due to the EIT effect, when the intracavity field (or subharmonic field of the DOPO) is on resonance with the probe transition of the three-level atoms (which is coupled by a strong resonant coupling field at another transition), the atom will not interact with the cavity field. The maximum interaction occurs when the frequency of the cavity field is tuned to the dressed transition states due to the coupling field [17]. Steady-state behaviors, such as bistability of the intracavity field versus external pumping field for the DOPO and bistability of intracavity field versus the strong atomic coupling field, are studied. Due to the modified absorption and dispersion properties in the three-level atomic system, we explore the absorption and dispersion properties along the upper branch and lower branch of the bistable curve, respectively. Other than characterizing the bistable behaviors and conditions to realize such behaviors, we pay special attention to the controlling of such bistable behavior with the coupling field. The optical parametric process in this composite system makes different, interesting effects, such as zero mean-field intensity at the lower branch of the bistable curve and modification of threshold behaviors, which do not exist in previously studied systems with three-level atoms inside an optical cavity [9–12].

The paper is organized in the following way. Section II establishes the model and presents the basic equations for this composite system. Section III gives the analytical (for the case of a special detuning) and numerical (in general) calculations for the bistability as functions of various parameters. At the end of this section, we briefly look at the absorption and dispersion properties of the bistable curve. Section IV serves as a conclusion.

II. HAMILTONIAN AND EQUATIONS OF MOTION

We consider a composite system consisting of a set of N three-level atoms and a nonlinear crystal inside an optical cavity. The atoms have two lower states $|1\rangle$ and $|3\rangle$, and an excited state $|2\rangle$ (such a three-level atomic system is usually referred to as a Λ-type system), as illustrated in Fig. 1. With small modifications, the following treatment can easily apply

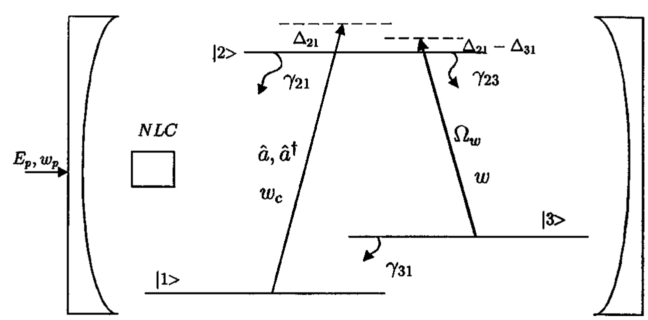

FIG. 1. Energy schematic for a three-level system inside an optical cavity with a nonlinear crystal. $|2\rangle\to|1\rangle$ is the lasing frequency.

to the cases of other types (*ladder*-type and *V*-type) of three-level atomic systems. Only transitions $|1\rangle \rightarrow |2\rangle$ and $|3\rangle \rightarrow |2\rangle$ are dipole allowed. The transition $|1\rangle$ to $|2\rangle$ is coupled to the cavity subharmonic field $\hat{a},\hat{a}^\dagger$ (frequency w_c), and the transition $|3\rangle$ to $|2\rangle$ is coupled by a strong classical coupling field E_w with frequency w. The cavity subharmonic field is produced through the optical parametric down-conversion process by an external pumping field with frequency $w_p = 2w_c$. The coherent coupling field interacting with transition $|3\rangle$ and $|2\rangle$ is treated classically, but the cavity field is quantized, which sets the stage for future studies of other properties (such as optical spectra and quantum statistical properties of the intracavity field) of this composite system [15,18].

The Hamiltonian of this system under the electric-dipole and rotating-wave approximations without considering the field decay rates is [13,14,19]

$$\hat{H} = \sum_{i=1}^{5} \hat{H}_i, \tag{1a}$$

$$\hat{H}_1 = \hbar w_c \hat{a}^\dagger \hat{a} + \hbar w_2 \hat{a}_2^\dagger \hat{a}_2 + \hbar \sum_\mu (w_{21}\hat{\sigma}_{22}^\mu + w_{31}\hat{\sigma}_{33}^\mu), \tag{1b}$$

$$\hat{H}_2 = i\hbar g \sum_\mu (\hat{\sigma}_{12}^\mu \hat{a}^\dagger - \hat{\sigma}_{21}^\mu \hat{a}), \tag{1c}$$

$$\hat{H}_3 = i\Omega_w \sum_\mu (\hat{\sigma}_{23}^\mu e^{-iwt} - \hat{\sigma}_{32}^\mu e^{iwt}), \tag{1d}$$

$$\hat{H}_4 = i\hbar \frac{\kappa}{2} (\hat{a}^{\dagger 2}\hat{a}_2 - \hat{a}^2 \hat{a}_2^\dagger), \tag{1e}$$

$$\hat{H}_5 = i\hbar(\epsilon_2 \hat{a}_2^\dagger e^{-iw_p t} - \epsilon_2^* \hat{a}_2 e^{iw_p t}), \tag{1f}$$

where $\hat{H}_1$ is the free energy of the fields and atoms, $\hat{H}_2$ is the interaction energy between the atoms and the subharmonic field, $\hat{H}_3$ is the interaction energy between the atoms and the classical field, $\hat{H}_4$ describes the DOPO process, and $\hat{H}_5$ is the pumping term to drive the DOPO. $\hat{a}_s^\dagger, \hat{a}_s$ are the creation and annihilation operators of the sth cavity-field mode. σ_{ij} denotes the atomic operators. w_{ij} is the transition frequency from state $|i\rangle$ to state $|j\rangle$. g is the coupling coefficient between the cavity field and the atomic transition $|1\rangle$ to $|2\rangle$. κ is the coupling of the nonlinear process of the crystal. ϵ_2 is the amplitude of the external pumping field. $2\Omega_w$ is the Rabi frequency of the classical coupling field E_w.

The Liouvillian relaxation terms necessary to introduce the effects of field decays and atomic decays are [19]

$$\mathcal{L}_1(\hat{\rho}) = \gamma_1([\hat{a}\hat{\rho},\hat{a}^\dagger] + [\hat{a},\hat{\rho}\hat{a}^\dagger]) + \gamma_2([\hat{a}_2\hat{\rho},\hat{a}_2^\dagger] + [\hat{a}_2,\hat{\rho}\hat{a}_2^\dagger]), \tag{2}$$

$$\mathcal{L}_2(\hat{\rho}) = \sum_\mu \Bigg\{ \frac{\gamma_{21}}{2}([\hat{\sigma}_{12}^\mu\hat{\rho},\hat{\sigma}_{21}^\mu] + [\hat{\sigma}_{12}^\mu,\hat{\rho}\hat{\sigma}_{21}^\mu]) + \frac{\gamma_{23}}{2}([\hat{\sigma}_{32}^\mu\hat{\rho},\hat{\sigma}_{23}^\mu] + [\hat{\sigma}_{32}^\mu,\hat{\rho}\hat{\sigma}_{23}^\mu]) + \frac{\gamma_{31}}{2}([\hat{\sigma}_{13}^\mu\hat{\rho},\hat{\sigma}_{31}^\mu] + [\hat{\sigma}_{13}^\mu,\hat{\rho}\hat{\sigma}_{31}^\mu]) \Bigg\},$$

where $\mathcal{L}_1$ includes the terms due to the field decay rates and $\mathcal{L}_2$ includes the terms due to the atomic decay rates. γ_1 is the cavity decay rate of the subharmonic field and γ_2 is the cavity decay rate of the pumping field. γ_{ij} is the spontaneous decay rate from state $|i\rangle$ to state $|j\rangle$.

From a Fokker-Planck equation derived from this Hamiltonian, all the operators can be translated into the corresponding c numbers defined by [19]

$$\begin{aligned}(\hat{a},\hat{a}^\dagger,\hat{a}_2,\hat{a}_2^\dagger,\hat{J}_{12},\hat{J}_{13},\hat{J}_{23},\hat{J}_{22},\hat{J}_{33},\hat{J}_{23}^\dagger,\hat{J}_{13}^\dagger,\hat{J}_{12}^\dagger)&\\ \rightarrow(\alpha,\alpha^*,\alpha_2,\alpha_2^*,J_{12},J_{13},J_{23},J_{22},J_{33},J_{23}^\dagger,J_{13}^\dagger,J_{12}^\dagger)&\\ \equiv \mathbf{u},&\end{aligned} \tag{3}$$

where $\hat{J}_{ij}$ are the collective atomic raising or lowering operators and $\hat{J}_{ii}$ is the population operator for the ith atomic energy level. The equations of motion for this composite system without fluctuations can be found from the drifting terms of the Fokker-Planck equation, and the final result is a set of 12 differential equations:

$$\frac{du_i}{dt} = A_i(\mathbf{u}), \quad i = 1,...,12 \tag{4}$$

with

$$\begin{aligned}
A_1(\mathbf{u}) &= gJ_{12} - \gamma_1\alpha + \kappa\alpha^*\alpha_2,\\
A_2(\mathbf{u}) &= gJ_{12}^\dagger - \gamma_1\alpha^* + \kappa\alpha\alpha_2^*,\\
A_3(\mathbf{u}) &= -\gamma_2\alpha_2 - \frac{\kappa}{2}\alpha^2 + \epsilon_2,\\
A_4(\mathbf{u}) &= -\gamma_2\alpha_2^* - \frac{\kappa}{2}\alpha^{*2} + \epsilon_2^*,\\
A_5(\mathbf{u}) &= (i\Delta_{21} - \gamma_\perp)J_{12} + \Omega J_{13} - g(J_{11} - J_{22})\alpha,\\
A_6(\mathbf{u}) &= i\Delta_{31}J_{13} - \Omega J_{12} + gJ_{23}\alpha,\\
A_7(\mathbf{u}) &= (i\Delta_{31} - i\Delta_{21} - \gamma_\perp)J_{23} + \Omega(J_{33} - J_{22}) - g\alpha^*J_{13},\\
A_8(\mathbf{u}) &= \Omega_w(J_{23} + J_{23}^\dagger) - g(\alpha J_{12}^\dagger + \alpha^* J_{12}) - 2\gamma_\perp J_{22},\\
A_9(\mathbf{u}) &= \Omega_w(J_{23} + J_{23}^\dagger) + \gamma_{23}J_{22},\\
A_{10}(\mathbf{u}) &= (i\Delta_{21} - i\Delta_{31} - \gamma_\perp)J_{23}^\dagger + \Omega(J_{33} - J_{22}) - g\alpha J_{13}^\dagger,\\
A_{11}(\mathbf{u}) &= -i\Delta_{31}J_{13}^\dagger - \Omega_w J_{12}^\dagger + gJ_{23}^\dagger\alpha^*,\\
A_{12}(\mathbf{u}) &= -(i\Delta_{21} + \gamma_\perp)J_{12}^\dagger + \Omega_w J_{13}^\dagger - g(J_{11} - J_{22})\alpha^*,
\end{aligned} \tag{5}$$

where $\Delta_{21}=(w_p/2)-w_{21}$ and $\Delta_{31}=(w_p/2)-w-w_{31}$. Here, we assumed a closed atomic system with $J_{11}+J_{22}+J_{33}=N$.

III. BISTABLE BEHAVIORS

The steady-state solutions of Eq. (4) can be obtained analytically by letting the time derivatives go to zero, but the general solutions are tedious. For that reason, we will look at some special cases and derive simple analytical results, and then we will present numerical results for more general situations.

A. Analytical results

In this section, we restrict our discussions to a special case, e.g., $\Delta_{21}=\Delta_{31}$, which means perfect resonance between the classical coupling field and the atomic transition $|2\rangle \leftrightarrow |3\rangle$. We assume a zero dephasing rate between state $|3\rangle$ and state $|1\rangle$ ($\gamma_{31}=0$), and take $\gamma_{23}=\gamma_{21}$. With these conditions the steady-state solutions are given in the following equation:

$$x^* \frac{\epsilon_2}{\epsilon_2^c}=\frac{N_s}{n_0}xX+x\left[\frac{A+2\Omega^2C\Delta^2+i(2\Omega^2C\Delta^3+\Omega^2XC\Delta+2\Omega^4C\Delta)}{A}\right], \tag{6}$$

$$\text{where}\quad A\equiv\frac{X^3}{8}+\frac{3\Omega^2X^2}{4}+\frac{X}{2}(\Delta^2+4\Omega^2\Delta^2+3\Omega^4)+\Omega^2[\Delta^2+(\Omega^2-\Delta^2)^2],$$

and the new normalized parameters are defined as

$$\gamma\equiv\gamma_{23}=\gamma_{21},\quad \Delta\equiv\frac{\Delta_{21}}{\gamma}=\frac{\Delta_{31}}{\gamma},\quad C\equiv\frac{Ng^2}{2\gamma\gamma_1},$$
$$N_s\equiv\frac{\gamma^2}{8g^2},\quad n_0\equiv\frac{2\gamma_1\gamma_2}{\kappa^2},\quad |\epsilon_2^c|\equiv\frac{\gamma_1\gamma_2}{\kappa},\quad \Omega\equiv\frac{\Omega_w}{\gamma}. \tag{7}$$

The normalized dimensionless field variables are

$$x\equiv\frac{\alpha}{\sqrt{N_s}},\quad X\equiv xx^*. \tag{8}$$

C is the atomic cooperative parameter, N_s is the single-atom saturation photon number, and n_0 is the saturation photon number for the DOPO system. ϵ_2^c is the pumping amplitude when the atoms are not present [13], so we can define a normalized pumping intensity $Y\equiv|\epsilon_2/\epsilon_2^c|$.

A plot of the intracavity subharmonic field intensity as a function of the pumping field intensity with a set of typical parameters is plotted in Fig. 2. The condition for the existence of the bistable behavior is given by values of C that satisfy $dY/dX>0$ at the threshold value $Y=Y_c$. C can be experimentally controlled due to its dependence on the atomic density. Notice that $X=0$ is always a solution for the steady-state equation, which gives the lower branch of the bistable curve. This result is very similar to the case with two-level atoms inside a DOPO as in Refs. [14,15], but very different from the standard atomic optical bistability (with atoms inside an optical cavity) as in Refs. [9,10].

When the pumping intensity Y is increased from zero, the system presents an extra loss due to the presence of the atoms (for no resonance case), which modifies the lower turning point (or DOPO threshold). Before the system reaches the unstable point A, the atoms absorb the intracavity subharmonic field generated by parametric down-conversion process until they reach their saturation point. Once the atoms cannot absorb more photons, the gain (due to parametric amplification) will exceed the loss (due to the atomic absorption), and a coherent intracavity field will appear in the cavity. The value of the intracavity field is given by the value (at point B) of the upper branch at the threshold point. The big jump in intracavity intensity is due to the fact that as soon as this field starts to oscillate, all the atoms will give up all their stored energy (in aligned dipole moments) due to stimulated emission. When the pumping intensity is reduced from above threshold (point A), the intracavity intensity (at the upper branch) will decrease until the upper turning point C', where the field will drop to zero at the point D. At the threshold (point B), the field will not jump to zero because the atoms have energy stored that is being given to the intracavity field.

Due to the coupling between the atoms and the fields, the threshold value (or lower turning point of the bistable curve) of this composite system becomes

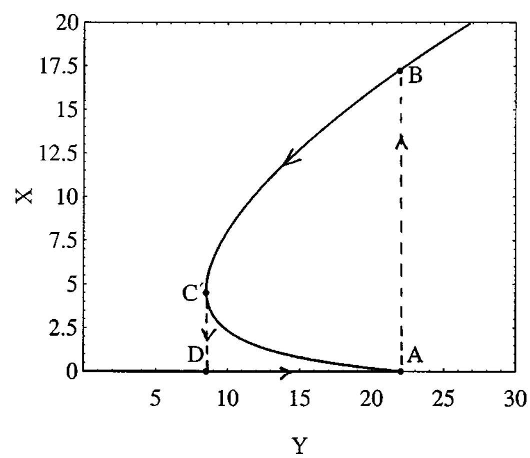

FIG. 2. Typical graph of intracavity subharmonic intensity X as a function of the driving intensity Y for $\Delta_{21}=8$, $\Delta_{31}=9$, $\Omega=10$, $C=7$, and $r=0.2$. The minimum value of C for the existence of bistability is $C=1.2$.

$$|\overline{\epsilon^c}| = |\epsilon^c| \frac{\{[(\Delta^2-\Omega^2)^2+\Delta^2(1+2C)]^2+4C^2\Delta^2(\Delta^2-\Omega^2)^2\}^{1/2}}{(\Delta^2-\Omega^2)^2+\Delta^2}. \tag{9}$$

Figure 3 shows a graph of the modified threshold value for this composite system as a function of coupling field strength Ω and atomic detuning Δ for $C=10$. Since we neglected the decay rate from level $|3\rangle$ to level $|1\rangle$ (dipole forbidden transition), when Δ is equal to zero, the susceptibility goes to zero (a perfect EIT situation) and the system behaves like a normal DOPO. Also, in the limit of $\Delta\rightarrow\infty$, the threshold value reduces to $|\overline{\epsilon^c}|=|\epsilon^c|$, which we expected since the atoms decouple from the cavity subharmonic field. It is between those two limits that the three-level atoms couple to the intracavity field and the system exhibits an increase in the threshold value. The maximum threshold value is obtained when $\Omega=\pm\Delta$, which correspond to the transitions from the Autler-Townes doublet (dressed states) to the state $|1\rangle$. The maximum value is

$$|\overline{\epsilon^c}| = |\epsilon^c|(1+2C), \tag{10}$$

which is exactly the same value as obtained in the case of a set of two-level atoms inside a DOPO cavity and on resonance [14]. C denotes the loss through atomic absorption over cavity loss. It is clear that if the Rabi frequency 2Ω is not on resonance with the dressed states, the region in which the system is bistable decreases.

1. Resonance case with coupling field Rabi frequency $\Delta=\pm\Omega$

We are interested in the case $\Delta=\pm\Omega$ because this corresponds to the case in which the interaction between the atoms and the field is maximum. The steady-state equations can be found to be

$$Y = \frac{N_s^2}{n_0^2}X^2 + \frac{2\frac{N_s}{n_0}X(8\Omega^4+16\Omega^4C+4\Omega^2X+28\Omega^4X+6\Omega^2X^2+X^3)}{8\Omega^4+4\Omega^2X+28\Omega^4X+6\Omega^2X^2+X^3} + \left[\frac{\Omega^6C^2X^2+(\Omega^4+2\Omega^4C+\Omega^2X/2+7\Omega^4X/2+3\Omega^2X^2/4+X^3/8)^2}{(\Omega^4+\Omega^2X/2+7\Omega^4X/2+3\Omega^2X^2/4+X^3/8)^2}\right], \tag{11a}$$

and

$$X=0, \tag{11b}$$

where $r\equiv N_s/n_0$ is a measure of the relative strength of the atomic system over the DOPO. These are the steady-state solutions for the system. In Fig. 4, we plot intracavity intensity X as a function of pumping intensity Y for different values of C, keeping r constant. Since the threshold value depends only on C, for each different value the lower turning point changes its location. If C is increased, the bistable region is also increased. Figure 5 shows the same graph as Fig. 4, but keeping C fixed and changing r. In this case, the lower turning point does not move, and only the upper branch of the bistable curve changes. As we increase r, the intracavity

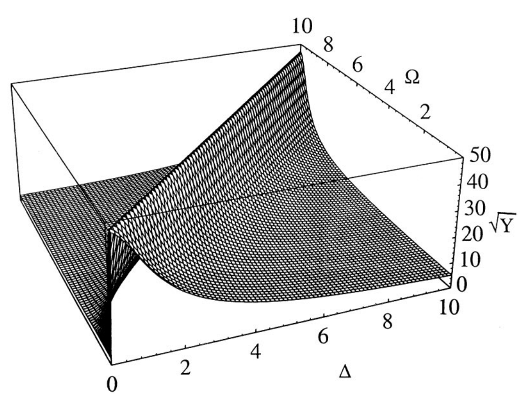

FIG. 3. The modified threshold value for the system vs Ω and Δ. Parameters are $C=10$, $r=0.1$.

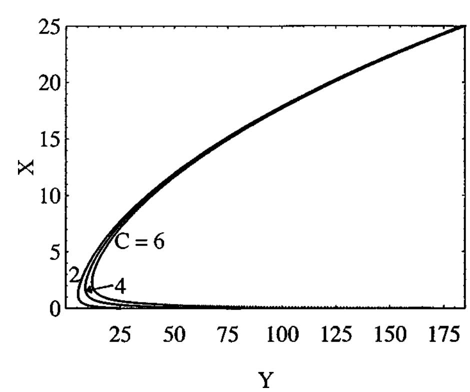

FIG. 4. Intracavity subharmonic intensity vs pumping intensity Y for different values $C=2$, 4, 6. Parameters are $r=0.5$, $\Omega=\Delta_{21}=\Delta_{31}=5$.

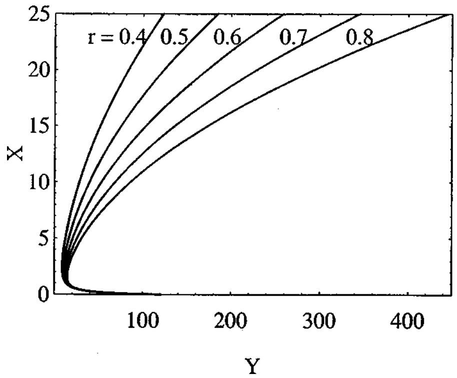

FIG. 5. Intracavity subharmonic intensity vs pumping intensity Y for different values $r=0.4$, 0.5, 0.6, 0.7, 0.8. Parameters are $C=5$, $\Omega=\Delta_{21}=\Delta_{31}=5$.

intensity becomes smaller, reflecting the fact that the atomic system is interacting more with the field. It is clear that in both cases the nature of the bistability is the same.

2. For cases of $\Delta \neq \pm\Omega$

In the case in which the system is not perfectly resonant with the dressed states, we can find a similar equation for the intracavity intensity as a function of the pumping intensity. However, such an expression is quite long, and we instead prefer to plot the results. Figure 6 is a graph of intracavity intensity for different values of detuning Δ. The threshold value reaches its maximum in the cases of $\Omega=\pm\Delta$. As we increase the detuning, the bistability region increases until it reaches the maximum value, and then it starts to decrease. It is interesting to notice that when $\Omega-\Delta=\delta$, the threshold value is bigger in the case of $\delta<0$ than in the case of $\delta>0$. This effect has a simple explanation: when $\delta>0$, the cavity subharmonic field has a frequency that is closer to the state $|2\rangle$ than in the case of $\delta<0$. In addition, the coupling field is in perfect resonance with the transition $|2\rangle\leftrightarrow|3\rangle$; therefore, coherent population trapping is more likely to occur in the case $\delta>0$ than when $\delta<0$.

B. Numerical results

When $\Delta_{21}\neq\Delta_{31}$, it is hard to obtain analytic solutions for the steady-state equations. We solve these equations numerically to explore the dependence of bistability behavior on the cavity detuning from the atomic transition between level $|1\rangle$ and level $|2\rangle$. Figure 7 exhibits the intracavity field intensity as a function of the pumping field intensity for different detuning Δ_{31} while keeping Ω and Δ_{21} equal. As before, maximum threshold value occurs while $\Delta_{31}=\Delta_{21}$. Above and below that value, the bistability region is smaller and highly sensitive to the detuning.

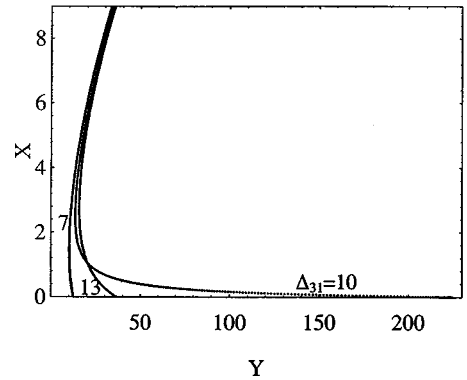

FIG. 7. Intracavity subharmonic intensity vs pumping intensity Y for different values of $\Delta_{31}=7$, 10, 13. Parameters are $C=7$, $\Omega=\Delta_{21}=10$, $r=0.5$.

Another way to change the bistability in this composite system is to use Ω as a control parameter. Due to the Autler-Townes splitting, when Ω is far from Δ, the bistability disappears. Figure 8 is a graph of intracavity field intensity as a function of pumping field intensity for different values of Ω. An interesting feature of this system is that it is possible to have bistable behavior in the intracavity subharmonic field as a function of Ω for certain values of parameters. Figure 9 exhibits this property for the case of $C=10$, $\Delta=3.5$, $r=0.1$, and $Y=6.5$. Once the value of the pumping field is fixed (when $\Omega=0$), and the value of Ω starts to increase, the field intensity in the cavity is going to be equal to zero. This effect is due to the fact that at low Ω (because of the consideration $\gamma_{31}=0$), all the atoms are pumped up by the cavity subharmonic field to the level $|2\rangle$, and then they decay to the level $|3\rangle$, where they remain. Once Ω is increased to a higher value close to the value of Δ, some of the atoms are going to

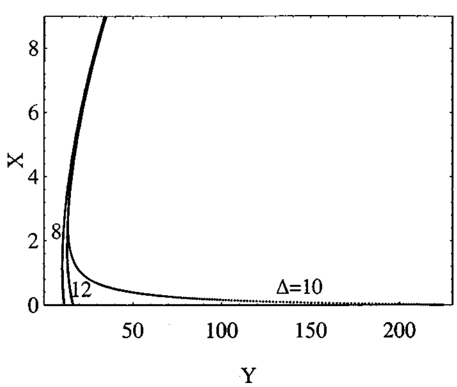

FIG. 6. Intracavity subharmonic intensity vs pumping intensity Y for different values of $\Delta=8$, 10, 12 in the nonresonant case. Parameters are $C=7$, $\Omega=10$, $r=0.5$.

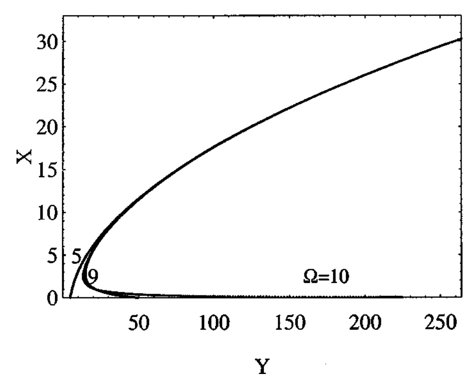

FIG. 8. Intracavity subharmonic intensity vs pumping intensity Y for different values of $\Omega=5$, 9, 10. Parameters are $C=7$, $\Delta_{21}=10$, $r=0.5$.

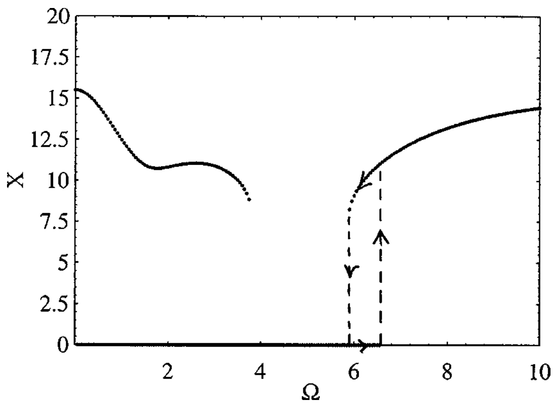

FIG. 9. Intracavity subharmonic intensity vs coupling field Rabi frequency Ω. Parameters are $C=10$, $\Delta=3.5$, $r=0.1$, $Y=6.5$.

interact with the intracavity field (the coupling field Ω tunes the atoms into resonance with the intracavity field through moving the dressed-state levels). However, in such a case, because the pumping field intensity is low, atoms absorb the intracavity subharmonic field without reaching saturation. If Ω is increased, the detuning of the intracavity field from the Autler-Townes doublet to the state $|1\rangle$ is going to increase, and the system is going to behave more like a normal DOPO system. In this case, the intensity will jump to a higher value because the medium is transparent to the field. When coming down from the upper branch, the intensity will not drop to zero at the same value because part of the energy is stored in the atoms and this atomic dipole energy will be released into the cavity field. Due to the atomic coherence affects in the three-level atomic system, there are many interesting effects in the absorption and dispersion properties [2,7,8,20]. We would like to look at these properties in the current composite system. The effect of EIT in an intracavity medium without the presence of the nonlinear crystal is to reduce the absorption at resonance where the dispersion can be large. When the nonlinear crystal is introduced into the cavity, the system is going to behave as a composite system affecting the results; the intracavity field especially can have bistable behaviors, which effect the absorption and dispersion properties.

From Eq. (5) we can calculate the susceptibility at the probe frequency in the case $\Delta_{21}=\Delta_{31}$ as

$$\chi\sim\Delta\frac{-\Delta+i\left(\Delta^2-\Omega^2-\frac{X}{2}\right)}{\frac{X^3}{8}+\frac{X\Delta^2}{2}+\Omega^2\Delta^4+2\Omega^2X\Delta^2+\frac{3\Omega^2X^2}{4}+\Omega^2\Delta^2-2\Omega^4\Delta^2+\frac{3\Omega^4X}{2}+\Omega^6}. \tag{12}$$

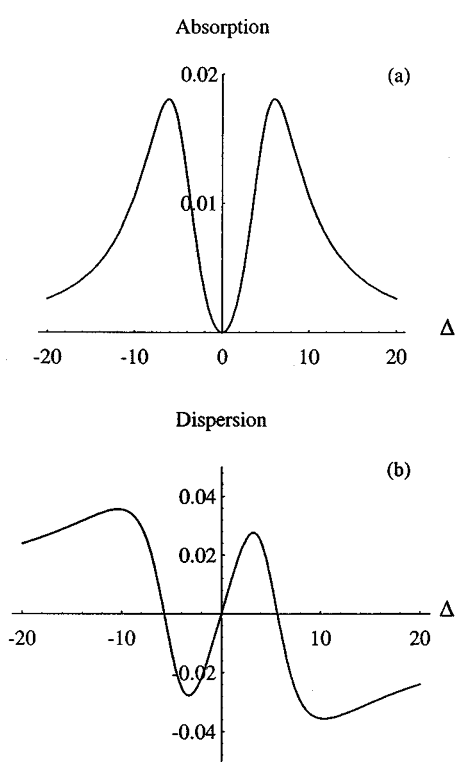

FIG. 10. Typical susceptibility as a function of Δ. Parameters are $\Omega=5$, $r=0.5$, and $X=15$. (a) Normalized absorption. (b) Normalized dispersion.

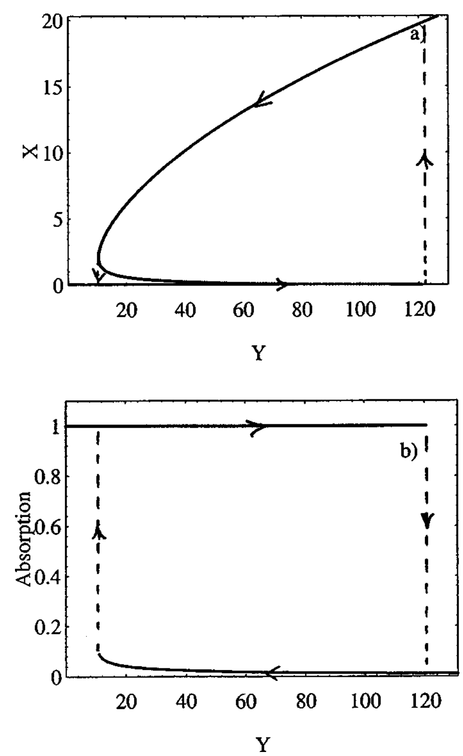

FIG. 11. (a) Intracavity intensity as a function of pumping intensity for $\Delta=\Omega=C=5$ and $r=0.5$. The threshold value is $Y_c=121$. (b) Normalized absorption as a function of pumping intensity using the same parameters.

The response of the medium depends on the real (χ') and the imaginary (χ'') parts of the susceptibility. The absorption coefficient is given by $\alpha = w_p n_0 \chi''/c$, and the dispersion coefficient is given by $\beta = w_p n_0 \chi'/2c$. Typical graphs of absorption and dispersion are given in Fig. 10 as a function of Δ for $\Omega=5$, $r=0.5$, and intracavity intensity $X=15$ (at the upper branch). Figure 11 exhibits a bistable curve with its respective absorption curve as a function of the pumping intensity Y for a specific set of values. Since the mean intracavity intensity at the lower branch of the bistable curve is always zero, the absorption remains 1 as the pumping intensity is increased. As soon as the threshold value is reached, the absorption jumps to a lower value because the atoms will not absorb more photons and the gain will exceed the losses in the cavity. As shown in Fig. 11(b), the new value of absorption can be very close to zero because the intracavity field will experience parametric gain through the DOPO process and the atomic absorption is saturated. When the pumping intensity decreases from above the threshold, the absorption remains very small due to the saturation until the atoms can start absorbing the field again (when the cavity field decreases, the energy stored in the atomic dipoles will be released into the cavity mode). At that value the absorption jumps to one (maximum absorption). Due to EIT effects, when $\Delta=0$, both absorption and dispersion are zero and the atoms decouple from the cavity field completely. When $\Delta_{21} \neq \Delta_{31}$, the region of bistability decreases and, therefore, a typical graph of absorption shows a jump that is not as big as before.

Due to the similarity between the bistable curves in the two-level system [14] and those in this three-level system, graphs of the absorption as a function of the pumping intensity in both systems are going to be very similar. However, due to the interesting effects of atomic coherence, the dispersion in the three-level system is much larger, and can be modified by changing the coupling field intensity and detuning [7,8,20].

IV. CONCLUSIONS

We have developed a model for the composite system consisting of N three-level Λ-type atoms inside a DOPO. This model can be easily modified to treat other three-level systems (such as ladder-type and V-type systems). Bistable behaviors in an intracavity field as a function of an external pumping field for the DOPO and in a coupling field for the three-level atoms are predicted, respectively. When this model is compared to the previously studied two-level atom system, there are some similarities, such as zero mean intensity at the lower branch and equal maximum threshold. However, this composite system with three-level atoms exhibits maximum interaction with the cavity field at the Autler-Townes doublets and no interaction when the atoms are on resonance with the cavity field due to EIT. The coupling field Ω can modify the bistability behaviors and tunes the atoms on or away from resonance with the cavity field. Such control can be used to make an optical switch. Recently, several theoretical papers were published about doing nonlinear optics at low light levels in multilevel atomic systems [21,22]. One can consider making such a low light level optical switch in this composite system, since the parametric amplification process could help to enhance the efficiency and contrast of the switching.

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