Dynamics of an atomic Bose-Einstein condensation interacting with a laser field in a double-well potential

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For the case of an atomic Bose-Einstein condensate formed in a symmetrical double-well potential with well-separated minima, the dynamics of Bose-condensed atoms interacting with a single-mode quantized traveling-wave laser field are investigated in detail. By considering the system consisting of the Bose-condensed atoms and laser field with and without dissipation, the eigenstates and eigenvalues of the corresponding Hamiltonian for both cases are obtained. The time development of the Bose-condensed state plays a major role in the discussion. It is shown that the probability of finding all atoms in the Bose-condensed state displays an undamped oscillatory behavior in the absence of dissipation and a damped oscillatory behavior in the presence of dissipation. Moreover, it is found that the tunneling effect can increase the oscillatory frequency among the dressed bosonic states and affect the dynamics of the system. As an application of these results, a characteristic time that may be used to evaluate the number of condensed atoms is introduced.

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I. INTRODUCTION

The recent observations of Bose-Einstein condensation (BEC) in dilute and ultracold gases of neutral alkali-metal atoms using a combination of laser and evaporative cooling have attracted intense interest [1–5]. The impressive progress stimulated a large amount of theoretical and experimental work on the properties of the atomic BEC, and opened the way to a new and rich field of physics. Since the first experimental realization of atomic BEC in 1995 [1–3], much research on the optical properties [6–9], statistical properties [10], phase properties [9,11–16], tunneling effect [17–20], and interference in atomic BEC has been undertaken in order to understand and utilize this fascinating phenomenon. In particular, atomic BEC confined to a symmetrical or asymmetrical double-well potential (DWP) has become an interesting topic [17–23]. This kind of potential is realistic and provides a simple example of the traps used in the BEC experiments performed by Davis *et al.* [3] and Andrews *et al.* [21], where an off-resonant laser is applied to plug the bottom of the trap to prevent the atomic BEC from leaking out. This results in the formation of a DWP. As concluded in this work [17–23], the tunneling effect, interference, evolution of the relative phase, and other properties of the atomic BEC are closely related to this special potential, which can therefore be regarded as a simple model of the actual traps used in these experiments.

One of the main goals of the present work is to investigate the dynamics of Bose-condensed atoms interacting with a single-mode quantized traveling-wave laser field in a symmetrical DWP, where the tunneling effect of atomic BEC will occur. In essence, it is the dynamics of collective excitation of the Bose-condensed atom-field system that are studied. As is well known, one of the striking features of BEC is a macroscopic population in the ground state of the system. The treatment of such a system is a typical many-body problem and therefore it is necessary to solve the Gross-Pitaevskii equation, which takes the nonlinear interatomic interactions into account. However, as far as the present system is concerned, if the atomic BEC is considered to be so dilute that the interatomic collision interactions become very weak compared with the interaction of the atomic BEC with the laser field, then they can be neglected as a crude approximation. Hence, throughout this paper, the interatomic collision interactions are neglected and all atoms are considered as two-level quantum systems. The following discussion of the dynamics of an atomic BEC interacting with a traveling-wave laser field are based on the pioneering studies performed by Polizer [6], Lewenstein and co-workers [7,8], and Javanainen [9]. In addition, the theory of the Jordan-Schwinger bosonic realization of the SO(3) group [24,25] and the biorthonormal theory [26] are also used.

This article is organized as follows. In Sec. II, the quantum tunneling of atoms in DWPs is reviewed, and then an effective Hamiltonian describing the interaction of a single-mode traveling-wave laser field with those condensed atoms is derived under the electric dipole and rotating-wave approximations. In Sec. III, in the absence of dissipation, the eigenstates and eigenvalues of the effective Hamiltonian derived in Sec. II are obtained by means of the theory of Jordan-Schwinger bosonic realization of the SO(3) group, and the dynamics of this system are further discussed. In Sec. IV, taking a dissipation term into account, the dynamics of the Bose-condensed atom-field system are studied using the biorthonormal theory [26].

II. TUNNELING IN AN ATOMIC BEC IN A DWP

In this section an effective Hamiltonian will be derived to deal with the interaction of a single-mode traveling-wave laser field with an atomic BEC. In the first place, the tunneling effect of an atom confined in a symmetrical DWP is outlined. Secondly, the standard Jaynes-Cummings (JC) model [27] is used to treat the interaction between atoms and the laser field and an effective Hamiltonian for the case that a BEC phase transition occurs is given.

In this paper we consider that the atoms are trapped in a symmetrical potential denoted by $V(x)=\frac{1}{2}M\omega^2(|x|-d)^2$, which was initially used by Merzbacher [28]. This potential has two wells and hereafter they will be called the left (L) and right (R) well. For a single atom in such a DWP, the Hamiltonian of the center-of-mass (c.m.) of an atom, $H_{\text{c.m.}}$, has the form

$$H_{\text{c.m.}}=\frac{p^2}{2M}+\tfrac{1}{2}M\omega^2(|x|-d)^2, \tag{2.1}$$

in which p and x are the operators of momentum and position of the c.m. of the atom, ω is the angular frequency, d half the distance between the two minima of the potential $V(x)$, and M the atomic mass. It is obvious that tunneling will take place due to the symmetry of this DWP, and thus the c.m. Hamiltonian can be rewritten as

$$\begin{aligned}H_{\text{c.m.}}=\sum_n\,&[E_n(|Ln\rangle\langle Ln|+|Rn\rangle\langle Rn|)\\&+\hbar\Omega_n(|Ln\rangle\langle Rn|+|Rn\rangle\langle Ln|)],\end{aligned} \tag{2.2}$$

where $|Ln\rangle$ and $|Rn\rangle$ stand for the nth c.m. state $|n\rangle$ of the atom in the L and R wells, respectively, E_n is the energy of the c.m. state $|n\rangle$, and Ω_n is the tunneling frequency of the atom in state $|n\rangle$. Ω_n has the form [29]

$$\Omega_n=(E_{na}-E_{ns})/2\hbar \quad (n=0,1,2,\cdots), \tag{2.3}$$

where E_{ns} and E_{na} are the energies corresponding to the symmetrical and antisymmetrical eigenstates of the c.m. Hamiltonian.

Because the atom is now considered to be a two-level quantum system, we assume that the states $|e\rangle$ and $|g\rangle$ are two internal states separated by an energy interval $\hbar\omega_a$. When a single-mode quantized traveling-wave laser field is applied, according to the JC model [27] the Hamiltonian for the atom-field system can be expressed as

$$H=H_{\text{c.m.}}+\hbar\omega_a|e\rangle\langle e|+\hbar\omega_f a^\dagger a+H_I. \tag{2.4}$$

Here $a^\dagger$ and a are the creation and annihilation operators for a photon with energy $\hbar\omega_f$ and wave vector k; they fulfill the standard commutation relation $[a,a^\dagger]=1$. The interaction H_I, under electric dipole and rotating-wave approximations, takes the form

$$H_I=\hbar g(|e\rangle\langle g|ae^{-ikx}+|g\rangle\langle e|a^\dagger e^{ikx}) \tag{2.5}$$

in which the quantity g denotes the dipole coupling constant. Furthermore, Eq. (2.4) can be rewritten as

$$\begin{aligned}H=\sum_{n,j}\,&[(E_n+\hbar\omega_a)|ejn\rangle\langle ejn|+E_n|gjn\rangle\langle gjn|]\\&+\sum_n\hbar\Omega_n[(|eLn\rangle\langle eRn|+|gLn\rangle\langle gRn|)+\text{H.c.}]\\&+\hbar g\sum_{n,m,j}(W_{nm}|ejn\rangle\langle gjm|a+\text{H.c.})+\hbar\omega_f a^\dagger a,\end{aligned} \tag{2.6}$$

where the j sum corresponds to L and R and the terms

$$W_{nm}=\langle n|e^{-ikx}|m\rangle \tag{2.7}$$

are the matrix elements for the transition from the mth c.m. state to the nth c.m. state accompanied by the process of excitation of the atom from $|g\rangle$ to $|e\rangle$ through the atom-field interaction.

For the case that many atoms are confined to the present DWP, in the presence of the laser field and ignoring the interatomic collision interactions, the Hamiltonian governing the evolution of the system has the following second-quantization form:

$$\begin{aligned}H^m=\sum_{n,j}\,&[(E_n+\hbar\omega_a)e^\dagger_{jn}e_{jn}+E_n g^\dagger_{jn}g_{jn}]\\&+\sum_n\hbar\Omega_n[(e^\dagger_{Ln}e_{Rn}+g^\dagger_{Ln}g_{Rn})+\text{H.c.}]\\&+\hbar g\sum_{n,m,j}(W_{nm}e^\dagger_{jn}g_{jm}a+\text{H.c.})+\hbar\omega_f a^\dagger a,\end{aligned} \tag{2.8}$$

where $e^\dagger_{jn}$, e_{jn}, $g^\dagger_{jn}$, and g_{jn} are the creation and annihilation operators of atoms in the states $|e\rangle\otimes|n\rangle$ and $|g\rangle\otimes|n\rangle$ of the j well, respectively. They satisfy the relations

$$\begin{aligned}&[e_{in},e_{jm}]=[g_{in},g_{jm}]=[e^\dagger_{in},e^\dagger_{jm}]=[g^\dagger_{in},g^\dagger_{jm}]\\&\qquad=[g_{in},e^\dagger_{jm}]=[e_{in},g^\dagger_{jm}]=0,\\&[e_{in},e^\dagger_{jm}]=[g_{in},g^\dagger_{in}]=\delta_{nm}\delta_{ij}\quad(i,j=L,R).\end{aligned} \tag{2.9}$$

Furthermore, following the elegant treatment of Lewenstein and co-workers [7,8] and Javanainen [9] a set of collective operators

$$b_{jm}=\sum_n W_{nm}e_{jn},\quad b^\dagger_{jm}=\sum_n W^*_{nm}e^\dagger_{jn} \tag{2.10}$$

can be introduced, which are subject to the commutation relations $[b_{im},b^\dagger_{jn}]=\delta_{ij}\delta_{nm}$, where W^*_{mn} are complex conjugate quantities of W_{nm} defined by Eq. (2.7). Thus, under the approximations of neglecting the contributions of Doppler shift and photon recoil, the following equation can be obtained:

$$\sum_n E_n b^\dagger_{jn}b_{jn}=\sum_n E_n e^\dagger_{jn}e_{jn}. \tag{2.11}$$

Substituting the operators defined by Eq. (2.10) into Eq. (2.8) yields

$$H^m=\sum_{n,j}\,[(E_n+\hbar\omega_a)b_{jn}^\dagger b_{jn}+E_n g_{jn}^\dagger g_{jn}]+\sum_n\,\hbar\Omega_n[(b_{Ln}^\dagger b_{Rn}+g_{Ln}^\dagger g_{Rn})+\text{H.c.}]+\hbar g\sum_{m,j}\,(b_{jm}^\dagger g_{jm}a+\text{H.c.})+\hbar\omega_f a^\dagger a\quad (j=L,R), \tag{2.12}$$

in which the tunneling effect of atoms in all energy levels is explicitly involved. This Hamiltonian describes the interactions of all atoms trapped in the DWP with the traveling-wave laser field.

It is of great interest to study the interaction of an atomic BEC interacting with the laser field in the present paper. The following discussion will therefore focus on the dynamics for the case that an atomic BEC phase transition takes place. In this special case the Hamiltonian given by Eq. (2.12) may be further simplified. To this end, it is necessary to stress that the tunneling effect of atoms in the ground state will play a crucial role for the Bose-condensed atoms trapped in the symmetrical DWP mentioned above. The reason is twofold. The more important one is that a macroscopic number of the atoms denoted by N_c will condense in the ground state of the two wells as the BEC phase transition occurs, so the tunneling effect of atoms in the Bose-condensed state is dominant. On the other hand, we may suppose that the two lowest energy levels due to the tunneling splitting of the ground state are closely spaced and well separated from the other higher-energy levels. Thus, for the Bose-condensed state occupied by a large number of atoms, the operators $g_{j0}^\dagger$ and g_{j0} $(j=L,R)$ may be replaced by a c number $\sqrt{N_c}/2$ according to a customary treatment. Keeping this in mind, ignoring the noncondensed atoms, and letting $E_0=0$, Eq. (2.12) becomes the following desired form:

$$H_{\text{eff}}^m=\hbar[(\omega_a+\Omega_0)c^\dagger c+(\omega_a-\Omega_0)d^\dagger d+\omega_f a^\dagger a+g\sqrt{N_c}(c^\dagger a+a^\dagger c)], \tag{2.13}$$

where $c^\dagger$ and $d^\dagger$ are the Hermitian adjoint operators of c and d defined by

$$c=\sqrt{2}^{-1}(b_{L0}+b_{R0}),\quad d=\sqrt{2}^{-1}(b_{L0}-b_{R0}), \tag{2.14}$$

and Ω_0 is the tunneling frequency of the ground state with the form [28]

$$\Omega_0=\left(\frac{2V_0\omega}{\hbar\pi}\right)^{1/2}\exp\left(-\frac{2V_0}{\hbar\omega}\right) \tag{2.15}$$

in which V_0 $[V_0=V(0)=M\omega^2d^2/2]$ is the height of the potential at the origin.

The Hamiltonian given by Eq. (2.13) is the effective Hamiltonian that determines the dynamics and time evolution of the Bose-condensed atom-field system and, in essence, describes the same physical content as that of two coupled oscillators. The effective Hamiltonian is just the starting point to discuss the time development of the Bose-condensed state.

III. DYNAMICS OF AN ATOMIC BEC INTERACTING WITH A LASER FIELD IN A DWP WITHOUT DISSIPATION

In Sec. II, an effective Hamiltonian was obtained to describe the interaction of an atomic BEC with a single-mode traveling-wave laser field. We shall now study the dynamics of the system in the absence of dissipation.

Let us define operator $\hat{N}$ as

$$\hat{N}=a^\dagger a+c^\dagger c. \tag{3.1}$$

It is easy to find that the relations

$$[\hat{N},H_{\text{eff}}^m]=[d^\dagger d,H_{\text{eff}}^m]=0 \tag{3.2}$$

hold true. This means that the physical quantities corresponding to operators $\hat{N}$ and $d^\dagger d$ are two integrals of motion in this system. Furthermore, the operators

$$J_1=(a^\dagger c+c^\dagger a)/2,\quad J_2=(a^\dagger c-c^\dagger a)/2i,\quad J_3=(a^\dagger a-c^\dagger c)/2 \tag{3.3}$$

can be introduced as a set of new operators, which satisfy the relations

$$[J_\alpha,J_\beta]=i\epsilon_{\alpha\beta\gamma}J_\gamma, \tag{3.4}$$

where the symbol $\epsilon_{\alpha\beta\gamma}$ stands for Levi-Cività coefficients. Clearly, these new operators represent the Jordan-Schwinger bosonic realization of the SO(3) group [25]. Through the operators J_1, J_2, and J_3, the effective Hamiltonian in Eq. (2.13) can be transformed into

$$H_{\text{eff}}^m=\hbar[\Delta\hat{N}+2\sqrt{g^2N_c+\delta^2}e^{iJ_2\theta}J_3e^{-iJ_2\theta}+(\omega_a-\Omega_0)d^\dagger d], \tag{3.5}$$

where the quantities Δ, δ, and $\tan\theta$ are defined as

$$\Delta=(\omega_f+\omega_a+\Omega_0)/2,\quad \delta=(\omega_f-\omega_a-\Omega_0)/2,\quad \tan\theta=(g\sqrt{N_c}/\delta). \tag{3.6}$$

It can be seen from Eq. (3.5) that the dynamical symmetry of this Hamiltonian is the direct product of the SO(3) and SU(1) groups by writing SO(3)⊗SU(1). For the convenience of discussion, it is supposed that the integral of motion corresponding to the operator $\hat{N}$ is N, and $|0\rangle_a$, $|0\rangle_c$, and $|0\rangle_d$ stand for three ‘‘vacuum’’ states defined by $a|0\rangle_a=0$, $c|0\rangle_c=0$, and $d|0\rangle_d=0$, respectively. Based on SO(3) group theory, the mth eigenstate together with the eigenvalue of this effective Hamiltonian can be given by

$$|E_m(\theta)\rangle = e^{iJ_2\theta}|J,m\rangle_q \otimes |l\rangle_d$$
$$= \sum_{m'} d^J_{m',m}(\theta)|J,m'\rangle_q \otimes |l\rangle_d \quad (J=N/2), \tag{3.7}$$

$$E_m(\theta) = \hbar[2J\Delta + 2m\sqrt{\delta^2 + g^2 N_c} + (\omega_a - \Omega_0)l], \tag{3.8}$$

where the matrix elements $d^J_{m',m}(\theta)$ are defined as $d^J_{m',m}(\theta) \equiv {}_q\langle J,m'|\exp(iJ_2\theta)|J,m\rangle_q$ (sometimes called Wigner functions [25]), $|l\rangle_d = [(d^\dagger)^l/\sqrt{l!}]|0\rangle_d$, and

$$|J,m\rangle_q = |J+m\rangle_a \otimes |J-m\rangle_c = \frac{(a^\dagger)^{J+m}(c^\dagger)^{J-m}}{\sqrt{(J+m)!(J-m)!}}|0\rangle_a \otimes |0\rangle_c \tag{3.9}$$

is the simultaneous eigenstate of operators J^2 ($J^2 = J_1^2 + J_2^2 + J_3^2$) and J_3 satisfying the relations $J^2|J,m\rangle_q = J(J+1)|J,m\rangle_q$, $J_3|J,m\rangle_q = m|J,m\rangle_q$, and $\hat{N}|J,m\rangle_q = 2J|J,m\rangle_q$. In analogy with the theory of angular momentum, these states are considered to be the eigenstates of the "quasi-angular-momentum" operators. In obtaining Eqs. (3.7)–(3.9), the relation $J=N/2$ has been used.

In what follows the time development of this system will be investigated using the results given by Eqs. (3.7)–(3.9). For example, for an initial state possessing $N-n$ photons and another two kinds of excitation with the excitation numbers n and l, respectively, i.e.,

$$|\psi(t=0)\rangle = |N-n\rangle_a \otimes |n\rangle_c \otimes |l\rangle_d = |J,J-n\rangle_q \otimes |l\rangle_d$$
$$= \sum_{m'} d^J_{m',J-n}(\theta)|E_{m'}(\theta)\rangle, \tag{3.10}$$

at time t, this state will evolve into a superposition of states with different photon numbers, that is,

$$|\psi(t)\rangle = \sum_{m'}\sum_{n'} d^J_{m',J-n}(\theta) d^J_{m',J-n'}(\theta) \times \exp\left(-\frac{iE_{m'}(\theta)t}{\hbar}\right)|N-n'\rangle_a \otimes |n'\rangle_c \otimes |l\rangle_d. \tag{3.11}$$

For a final state $|\phi\rangle$ possessing $N-n_f$ photons, if we write $|\phi\rangle = |N-n_f\rangle_a \otimes |n_f\rangle_c \otimes |l\rangle_d$, through a straightforward calculation, the probability of finding $|\psi(t)\rangle$ evolving into the final state is found to be

$$|\langle\phi|\psi(t)\rangle|^2 = \left|\sum_{m'} d^J_{m',J-n}(\theta) d^J_{m'J-n_f}(\theta) \times \exp\left(-\frac{iE_{m'}(\theta)t}{\hbar}\right)\right|^2. \tag{3.12}$$

This expression is just the probability that the system originally in the state $|J,J-n\rangle_q \otimes |l\rangle_d$ makes a transition to the state $|J,J-n_f\rangle_q \otimes |l\rangle_d$ after a time interval t.

In particular, it is of great interest to study the time evolution of the atomic Bose-condensed state. This means that the atomic Bose-condensed state should be chosen as the initial and final states to discuss the problem of evolution. It can be seen from the notation in Eq. (3.10) that the state of atomic BEC can be written as $|J,J\rangle_q$, in which all atoms are condensed and there are N photons in the cavity. Note that, for the case that $N=2J$, the function $d^J_{m,m'}(\theta)$ is the standard d function in angular momentum theory [25]. The probability that the initial state $|J,J\rangle_q \otimes |l\rangle_d$ returns to itself at time t can therefore be expressed as

$$|({}_d\langle l| \otimes {}_q\langle J,J|)|\psi(t)\rangle|^2 = \left|\sum_{m'=-J}^{J} d^J_{m',J}(\theta) d^J_{m',J}(\theta) \times \exp\left(-\frac{iE_{m'}(\theta)t}{\hbar}\right)\right|^2$$
$$= (1 - \sin^2\theta \sin^2\sqrt{\delta^2 + g^2 N_c t})^N$$
$$(J=N/2). \tag{3.13}$$

This result indicates that the probability of finding all atoms in the Bose-condensed state displays an undamped oscillatory behavior with the oscillatory frequency $\sqrt{\delta^2 + g^2 N_c}$; in other words, the traveling-wave laser field can periodically excite the atomic BEC. In addition, from the expression of δ in Eq. (3.6), it should be emphasized that the oscillatory frequency $\sqrt{\delta^2 + g^2 N_c}$ is now relevant not only to the number of condensed atoms N_c and the coupling constant g, but also to the tunneling frequency Ω_0. In particular, for the resonant case that ω_a is equal to ω_f, it is evident that the oscillatory frequency $\sqrt{\Omega_0^2/4 + g^2 N_c}$ is explicitly related to the tunneling frequency. Thus the tunneling effect in an atomic BEC in a DWP results in an obvious increment in the oscillatory frequency, which shows that the influence of the tunneling effect on the dynamics of the condensed-atom–field system in a DWP is important.

IV. DYNAMICS OF THE SYSTEM WITH DISSIPATION

In Sec. III, the dynamics of the system of Bose-condensed atom and field was studied in the absence of dissipation, which, in fact, is an ideal case. However, as an atom is excited from the ground state $|g\rangle$ to the excited state $|e\rangle$ by the laser field, in quantum electrodynamics it always decays to the ground state $|g\rangle$ even if the laser field is empty. This is just the process of stimulated and spontaneous emission. Hence a dissipation mechanism such as spontaneous emission and other factors should be taken into account for a more real situation. In this section, the major aim is to study the dynamics of the system in the presence of the dissipation represented by

$$H' = -i\hbar\Gamma|e\rangle\langle e|, \tag{4.1}$$

where the quantity Γ denotes a strength parameter. Now let us turn our attention to discussing the dynamics governed by the new Hamiltonian $\tilde{H}$, which is constructed by adding the dissipation term to the Hamiltonian in Eq. (2.4), i.e.,

$$\tilde{H}=H^m+H'. \tag{4.2}$$

Note that this Hamiltonian is no longer a Hermitian operator because of the presence of the dissipation term H'.

Repeating the same procedure as in Secs. II and III, the new Hamiltonian $\tilde{H}$ therefore becomes

$$\tilde{H}=\hbar[(\tilde{\omega}_a+\Omega_0)c^\dagger c+(\tilde{\omega}_a-\Omega_0)d^\dagger d+\omega_f a^\dagger a+g\sqrt{N_c}(c^\dagger a+a^\dagger c)]. \tag{4.3}$$

Here the operators c, $c^\dagger$, d, and $d^\dagger$ have the same definitions as before. By means of the operators J_1, J_2, and J_3 in Eq. (3.4), this equation can be rewritten as

$$\tilde{H}=\hbar[\tilde{\Delta}\hat{N}+2\sqrt{g^2N_c+\tilde{\delta}^2}e^{iJ_2\tilde{\theta}}J_3e^{-iJ_2\tilde{\theta}}+(\tilde{\omega}_a-\Omega_0)d^\dagger d], \tag{4.4}$$

where the quantities $\tilde{\omega}_a$, $\tilde{\Delta}$, $\tilde{\delta}$, and $\tilde{\theta}$ are defined by

$$\tilde{\omega}_a=\omega_a-i\Gamma,\quad \tilde{\Delta}=(\omega_f+\tilde{\omega}_a+\Omega_0)/2,$$
$$\tilde{\delta}=(\omega_f-\tilde{\omega}_a-\Omega_0)/2,\quad \tan\tilde{\theta}=(g\sqrt{N_c}/\tilde{\delta}). \tag{4.5}$$

Likewise, the mth eigenstate and eigenvalue of the Hamiltonian $\tilde{H}$ are given by

$$|\tilde{E}_m(\tilde{\theta})\rangle=e^{iJ_2\tilde{\theta}}|J,m\rangle_q\otimes|l\rangle_d=\sum_{m'}d^J_{m',m}(\tilde{\theta})|J,m'\rangle_q\otimes|l\rangle_d$$
$$(J=N/2), \tag{4.6}$$

$$\tilde{E}_m(\tilde{\theta})=\hbar[2J\tilde{\Delta}+2m\sqrt{\tilde{\delta}^2+g^2N_c}+(\tilde{\omega}_a-\Omega_0)l], \tag{4.7}$$

where the function $d^J_{m',m}(\tilde{\theta})$ with complex argument $\tilde{\theta}$ is defined as

$$d^J_{m',m}(\tilde{\theta})=\sum_k\frac{(-1)^{k+m'-m}\sqrt{(J-m)!(J+m)!(J-m')!(J+m')!}}{k!(J-m'-k)!(J+m-k)!(k+m'-m)!}\left(\cos\frac{\tilde{\theta}}{2}\right)^{2(J-k)+m-m'}\left(\sin\frac{\tilde{\theta}}{2}\right)^{2k+m'-m}. \tag{4.8}$$

Although the Hamiltonian $\tilde{H}$ is non-Hermitian, the biorthonormal theory [26] still provides a useful method for discussion of the dynamics of the system. As an interesting application of the biorthonormal theory [26], the completeness relation and biorthonormal relations can be obtained as follows:

$$\sum_m|\overline{E}_m(\tilde{\theta})\rangle\langle\tilde{E}_m(\tilde{\theta})|=\sum_m|\tilde{E}_m(\tilde{\theta})\rangle\langle\overline{E}_m(\tilde{\theta})|=I, \tag{4.9}$$

$$\langle\overline{E}_m(\tilde{\theta})|\tilde{E}_n(\tilde{\theta})\rangle=\langle\tilde{E}_n(\tilde{\theta})|\overline{E}(\tilde{\theta})\rangle=\delta_{mn}, \tag{4.10}$$

where the state $|E_m(\tilde{\theta})\rangle$ with the eigenvalue $\overline{E}_m(\tilde{\theta})=[\tilde{E}_m(\tilde{\theta})]^*$ is defined as

$$|\overline{E}_m(\tilde{\theta})\rangle=|\overline{E}_m(\tilde{\theta}^*)\rangle=\sum_m d^J_{m,m'}(\tilde{\theta}^*)|J,m'\rangle_q\otimes|l\rangle_d, \tag{4.11}$$

which is an eigenstate of $\tilde{H}^\dagger$. Here $\tilde{H}^\dagger$ is the adjoint operator of $\tilde{H}$, namely,

$$\tilde{H}^\dagger=\hbar[\tilde{\Delta}^*\hat{N}+2\sqrt{g^2N_c+\tilde{\delta}^{*2}}e^{iJ_2\tilde{\theta}^*}J_3e^{iJ_2\tilde{\theta}^*}+(\tilde{\omega}_a^*-\Omega_0)d^\dagger d]. \tag{4.12}$$

Now, if the initial state of the system is chosen as

$$|\tilde{\psi}(t=0)\rangle=|N-n\rangle_a\otimes|n\rangle_c\otimes|l\rangle_d=|J,J-n\rangle_q\otimes|l\rangle_d$$
$$=\sum_{m'}d^J_{m',J-n}(\tilde{\theta})|\tilde{E}_{m'}(\tilde{\theta})\rangle \tag{4.13}$$

at time t, it evolves into the state

$$|\tilde{\psi}(t)\rangle=\sum_{m'}\sum_{n'}d^J_{m',J-n}(\tilde{\theta})d^J_{m',J-n'}(\tilde{\theta})\times\exp\left(-\frac{i\tilde{E}_{m'}(\tilde{\theta})t}{\hbar}\right)|N-n'\rangle_a\otimes|n'\rangle_c\otimes|l\rangle_d. \tag{4.14}$$

Thus, for a final state $|\tilde{\phi}\rangle$ possessing $N-n_f$ photons, i.e., $|\tilde{\phi}\rangle=|N-n_f\rangle_a\otimes|n_f\rangle_c\otimes|l\rangle_d$, the probability of finding $|\tilde{\psi}(t)\rangle$ in such a state can be expressed as

$$|\langle\tilde{\phi}|\tilde{\psi}(t)\rangle|^2=\left|\sum_{m'}d^J_{m',J-n}(\tilde{\theta})d^J_{m',J-n_f}(\tilde{\theta})\times\exp\left(-\frac{i\tilde{E}_{m'}(\tilde{\theta})t}{\hbar}\right)\right|^2. \tag{4.15}$$

Similarly, it is of interest to discuss the dynamics of the atomic Bose-condensed state in which all atoms are condensed. Substituting the Bose-condensed state into the left-hand side of Eq. (4.15) yields

$$|({}_d\langle l|\otimes {}_q\langle J,J|)|\psi(t)\rangle|^2 = \left|\sum_{m'=-J}^{J} d^J_{m',J}(\tilde{\theta}) d^J_{m',J}(\tilde{\theta}) \times \exp\left(-\frac{i\tilde{E}_{m'}(\tilde{\theta})t}{\hbar}\right)\right|^2 = |1-\sin^2\tilde{\theta}\sin^2\sqrt{\tilde{\delta}^2+g^2N_c}t|^N \times \exp[-2(N+l)\Gamma t]. \quad (4.16)$$

This result means that, when the initial state of the system is the atomic Bose-condensed state, after a time interval t the probability that all atoms are condensed displays a damping oscillatory pattern. That is, the state of the atomic BEC has a tendency to be broken due to the presence of dissipation in the system. Again, it can be seen from Eq. (4.16) that the tunneling effect in the present case can increase the oscillatory frequency and affects the dynamics of this system.

If a characteristic time defined as

$$\tau_c = [2(l+N)\Gamma]^{-1} \quad (4.17)$$

is introduced, determined by both the number of excitations and the strength parameter Γ, then from Eq. (4.16), the excitations induced by the interaction of the atomic BEC with the laser field will disappear rapidly after the characteristic time τ_c. Note that, in general, the strength parameter Γ in the dissipation term H' may be regarded as the coefficient of spontaneous emission and can be explicitly given [30]. Thus, by measurement of the characteristic time τ_c, the approximate number of total excitations and hence the number of Bose-condensed atoms can be calculated.

V. SUMMARY

In conclusion, for the case of an atomic BEC trapped in a symmetrical DWP, the dynamics of the atomic BEC interacting with a single-mode quantized traveling-wave laser field have been investigated in detail by considering cases with and without dissipation. The eigenstates and eigenvalues of the corresponding Hamiltonian are obtained for both cases. In particular, the time evolution of the atomic Bose-condensed state is discussed. It is shown that the probability of finding all atoms in a state of BEC displays an undamped oscillatory behavior in the absence of dissipation and a damped oscillatory behavior in the presence of dissipation. For both cases, tunneling can increase the oscillatory frequency among the dressed bosonic states and affects the dynamics of the system. Moreover, in the presence of dissipation, a characteristic time depending on the number of excitations and the parameter Γ is introduced. The characteristic time may be used to evaluate the number of condensed atoms. The present discussion on the dynamics of the condensed-atom–field system are expected to be helpful for further investigations in this field.

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