Broadband teleportation

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Quantum teleportation of an unknown broadband electromagnetic field is investigated. The continuous-variable teleportation protocol by Braunstein and Kimble [Phys. Rev. Lett. **80**, 869 (1998)] for teleporting the quantum state of a single mode of the electromagnetic field is generalized for the case of a multimode field with finite bandwith. We discuss criteria for continuous-variable teleportation with various sets of input states and apply them to the teleportation of broadband fields. We first consider as a set of input fields (from which an independent state preparer draws the inputs to be teleported) arbitrary pure Gaussian states with unknown coherent amplitude (squeezed or coherent states). This set of input states, further restricted to an alphabet of coherent states, was used in the experiment by Furusawa *et al.* [Science **282**, 706 (1998)]. It requires unit-gain teleportation for optimizing the teleportation fidelity. In our broadband scheme, the excess noise added through unit-gain teleportation due to the finite degree of the squeezed-state entanglement is just twice the (entanglement) source's squeezing spectrum for its ''quiet quadrature.'' The teleportation of one half of an entangled state (two-mode squeezed vacuum state), i.e., ''entanglement swapping,'' and its verification are optimized under a certain nonunit gain condition. We will also give a broadband description of this continuous-variable entanglement swapping based on the single-mode scheme by van Loock and Braunstein [Phys. Rev. A **61**, 10 302 (2000)].

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I. INTRODUCTION

Teleportation of an unknown quantum state is its disembodied transport through a classical channel, followed by its reconstitution, using the quantum resource of entanglement. Quantum information cannot be transmitted reliably via a classical channel alone, as this would allow us to replicate the classical signal and so produce copies of the initial state, thus violating the no-cloning theorem [1]. More intuitively, any attempted measurement of the initial state only obtains partial information due to the Heisenberg uncertainty principle and the subsequently collapsed wave packet forbids information gain about the original state from further inspection. Attempts to circumvent this disability with more generalized measurements also fail [2].

Quantum teleportation was first proposed to transport an unknown state of any discrete quantum system, e.g., a spin-$\frac{1}{2}$ particle [3]. In order to accomplish the teleportation, classical and quantum methods must go hand in hand. A part of the information encoded in the unknown input state is transmitted via the quantum correlations between two separated subsystems in an entangled state shared by the sender and the receiver. In addition, classical information must be sent via a conventional channel. For the teleportation of a spin-$\frac{1}{2}$-particle state, the entangled state required is a pair of spins in a Bell state [4]. The classical information that has to be transmitted contains two bits in this case.

Important steps toward the experimental implementation of quantum teleportation of single-photon polarization states have already been accomplished [5,6]. However, a complete realization of the original teleportation proposal [3] has not been achieved in these experiments, as either the state to be teleported is not independently coming from the outside [6] or destructive detection of the photons in the teleported state is employed as part of the protocol [5]. In the latter case, a teleported state did not emerge for subsequent examination or exploitation. This situation has been termed ''*a posteriori* teleportation,'' being accomplished via post selection of photoelectric counting events [7]. Without postselection, the fidelity would not have exceeded the value $\frac{2}{3}$ required.

The teleportation of continuous quantum variables such as position and momentum of a particle [8] relies on the entanglement of the states in the original Einstein, Podolsky, and Rosen (EPR) paradox [9]. In quantum optical terms, the observables analogous to the two conjugate variables position and momentum of a particle are the quadrature amplitudes of a single mode of the electromagnetic field [10]. By considering the finite (nonsingular) degree of correlation between these quadratures in a two-mode squeezed state [10], a realistic implementation for the teleportation of continuous quantum variables was proposed [11]. Based on this proposal, in fact, quantum teleportation of arbitrary coherent states has been achieved with a fidelity $F=0.58\pm0.02$ [12]. Without using entanglement, by purely classical communication, an average fidelity of 0.5 is the best that can be achieved if the set of input states contains all coherent states [13]. The scheme with continuous quadrature amplitudes of a single mode enables an *a priori* (or ''unconditional'') teleportation with high efficiency [11], as reported in Refs. [14,12]. In this experiment, three criteria necessary for quantum teleportation were achieved: (1) An unknown quantum state enters the sending station for teleportation. (2) A teleported state emerges from the receiving station for subsequent evaluation or exploitation. (3) The degree of overlap between the input and the teleported states is higher than that which could be achieved if the sending and the receiving stations were linked only by a classical channel.

In continuous-variable teleportation, the teleportation process acts on an infinite-dimensional Hilbert space instead of the two-dimensional Hilbert space for the discrete spin variables. However, an arbitrary electromagnetic field has an infinite number of modes, or in other words, a finite bandwidth containing a continuum of modes. Thus, the teleportation of the quantum state of a broadband electromagnetic field requires the teleportation of a quantum state which is defined in the tensor product space of an infinite number of infinite-dimensional Hilbert spaces. The aim of this paper is to extend the treatment of Ref. [11] to the case of a broadband field, and thereby to provide the theoretical foundation for laboratory investigations as in Refs. [14,12]. In particular, we demonstrate that the two-mode squeezed state output of a nondegenerate optical parametric amplifier (NOPA) [15] is a suitable EPR ingredient for the efficient teleportation of a broadband electromagnetic field.

In the three above mentioned teleportation experiments, in Innsbruck [5], in Rome [6], and in Pasadena [12], the nonorthogonal input states to be teleported were single-photon polarization states (qubits) [5,6] and coherent states [12]. From a true quantum teleportation device, however, we would also require the capability of teleporting the entanglement source itself. This teleportation of one half of an entangled state (entanglement swapping [16]) means to entangle two quantum systems that have never directly interacted with each other. For discrete variables, a demonstration of entanglement swapping with single photons has been reported by Pan *et al.* [17]. For continuous variables, experimental entanglement swapping has not yet been realized in the laboratory, but there have been several theoretical proposals of such an experiment. Polkinghorne and Ralph [18] suggested teleporting polarization-entangled states of single photons using squeezed-state entanglement where the output correlations are verified via Bell inequalities. Tan [19] and van Loock and Braunstein [20] considered the unconditional teleportation (without postselection of ''successful'' events by photon detections) of one half of a two-mode squeezed state using different protocols and verification. Based on the single-mode scheme of Ref. [20], we will also present a broadband description of continuous-variable entanglement swapping.

II. TELEPORTATION OF A SINGLE MODE

In the teleportation scheme of a single mode of the electromagnetic field (for example, representing a single pulse or wave packet), the shared entanglement is a two-mode squeezed vacuum state [11]. For infinite squeezing, this state contains exactly analogous quantum correlations as does the state described in the original EPR paradox, where the quadrature amplitudes of the two modes play the roles of position and momentum [11]. The entangled state is sent in two halves: one to ''Alice'' (the teleporter or sender) and the other one to ''Bob'' (the receiver), as illustrated in Fig. 1. In order to perform the teleportation, Alice has to couple the

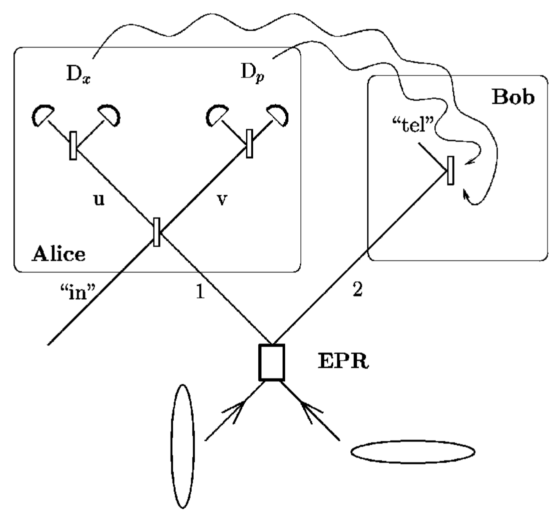

FIG. 1. Teleportation of a single mode of the electromagnetic field as in Ref. [11]. Alice and Bob share the entangled state of modes 1 and 2. Alice combines the mode ''in'' to be teleported with her half of the EPR state at a beam splitter. The homodyne detectors D_x and D_p yield classical photocurrents for the quadratures x_u and p_v, respectively. Bob performs phase-space displacements of his half of the EPR state depending on Alice's classical results.

input mode she wants to teleport with her ''EPR mode'' at a beam splitter. The ''Bell detection'' of the x quadrature at one beam splitter output, and of the p quadrature at the other output, yields the classical results to be sent to Bob via a classical communication channel. In the limit of an infinitely squeezed EPR source, these classical results contain no information about the mode to be teleported. This is analogous to the Bell-state measurement of the spin-$\frac{1}{2}$-particle pair by Alice for the teleportation of a spin-$\frac{1}{2}$-particle state. The measured Bell state of the spin-$\frac{1}{2}$-particle pair determines whether the particles have equal or different spin projections. The spin projection of the individual particles, i.e., Alice's EPR particle and her unknown input particle, remains completely unknown [3]. According to this analogy, we call Alice's quadrature measurements for the teleportation of the state of a single mode (and of a multimode field in the following sections) ''Bell detection.'' Due to this Bell detection, the entanglement between Alice's ''EPR mode'' and Bob's ''EPR mode'' means that suitable phase-space displacements of Bob's mode convert it into a replica of Alice's unknown input mode (a perfect replica for infinite squeezing). In order to perform these displacements, Bob needs the classical results of Alice's Bell measurement.

The previous protocol for the quantum teleportation of continuous variables used the Wigner distribution and its convolution formalism [11]. The teleportation of a single mode of the electromagnetic field can also be recast in terms of Heisenberg equations for the quadrature amplitude operators, which is the formalism that we employ in this paper. For that purpose, the Wigner function $W_{\rm EPR}$ describing the entangled state shared by Alice and Bob [11] is replaced by equations for the quadrature amplitude operators of a two-mode squeezed vacuum state. Two independently squeezed vacuum modes can be described by [10]

$$\hat{x}_1 = e^{r}\hat{x}_1^{(0)}, \quad \hat{p}_1 = e^{-r}\hat{p}_1^{(0)},$$
$$\hat{x}_2 = e^{-r}\hat{x}_2^{(0)}, \quad \hat{p}_2 = e^{r}\hat{p}_2^{(0)}, \tag{1}$$

where a superscript (0) denotes initial vacuum modes and r is the squeezing parameter. Superimposing the two squeezed modes at a 50/50 beam splitter yields the two output modes

$$\hat{x}_1 = \frac{1}{\sqrt{2}} e^{r}\hat{x}_1^{(0)} + \frac{1}{\sqrt{2}} e^{-r}\hat{x}_2^{(0)},$$
$$\hat{p}_1 = \frac{1}{\sqrt{2}} e^{-r}\hat{p}_1^{(0)} + \frac{1}{\sqrt{2}} e^{r}\hat{p}_2^{(0)},$$
$$\hat{x}_2 = \frac{1}{\sqrt{2}} e^{r}\hat{x}_1^{(0)} - \frac{1}{\sqrt{2}} e^{-r}\hat{x}_2^{(0)}, \tag{2}$$
$$\hat{p}_2 = \frac{1}{\sqrt{2}} e^{-r}\hat{p}_1^{(0)} - \frac{1}{\sqrt{2}} e^{r}\hat{p}_2^{(0)}.$$

The output modes 1 and 2 are now entangled to a finite degree in a two-mode squeezed vacuum state. In the limit of infinite squeezing, $r \to \infty$, both output modes become infinitely noisy, but also the EPR correlations between them become ideal: $(\hat{x}_1 - \hat{x}_2) \to 0$, $(\hat{p}_1 + \hat{p}_2) \to 0$. Now mode 1 is sent to Alice and mode 2 is sent to Bob. Alice's mode is then superimposed at a 50/50 beam splitter with the input mode ''in'':

$$\hat{x}_u = \frac{1}{\sqrt{2}}\hat{x}_{\rm in} - \frac{1}{\sqrt{2}}\hat{x}_1, \quad \hat{p}_u = \frac{1}{\sqrt{2}}\hat{p}_{\rm in} - \frac{1}{\sqrt{2}}\hat{p}_1,$$
$$\hat{x}_v = \frac{1}{\sqrt{2}}\hat{x}_{\rm in} + \frac{1}{\sqrt{2}}\hat{x}_1, \quad \hat{p}_v = \frac{1}{\sqrt{2}}\hat{p}_{\rm in} + \frac{1}{\sqrt{2}}\hat{p}_1. \tag{3}$$

Using Eqs. (3) we will find it useful to write Bob's mode 2 as

$$\hat{x}_2 = \hat{x}_{\rm in} - (\hat{x}_1 - \hat{x}_2) - \sqrt{2}\hat{x}_u = \hat{x}_{\rm in} - \sqrt{2}e^{-r}\hat{x}_2^{(0)} - \sqrt{2}\hat{x}_u,$$
$$\hat{p}_2 = \hat{p}_{\rm in} + (\hat{p}_1 + \hat{p}_2) - \sqrt{2}\hat{p}_v = \hat{p}_{\rm in} + \sqrt{2}e^{-r}\hat{p}_1^{(0)} - \sqrt{2}\hat{p}_v. \tag{4}$$

Alice's Bell detection yields certain classical values x_u and p_v for $\hat{x}_u$ and $\hat{p}_v$. The quantum variables $\hat{x}_u$ and $\hat{p}_v$ become classically determined, random variables. We indicate this by turning $\hat{x}_u$ and $\hat{p}_v$ into x_u and p_v. The classical probability distribution of x_u and p_v is associated with the quantum statistics of the previous operators [11]. Now, due to the entanglement, Bob's mode 2 collapses into states that for $r \to \infty$ differ from Alice's input state only in (random) classical phase-space displacements. After receiving Alice's classical results x_u and p_v, Bob displaces his mode

$$\hat{x}_2 \to \hat{x}_{\rm tel} = \hat{x}_2 + \Gamma\sqrt{2}x_u,$$
$$\hat{p}_2 \to \hat{p}_{\rm tel} = \hat{p}_2 + \Gamma\sqrt{2}p_v, \tag{5}$$

thus accomplishing the teleportation [11]. The parameter Γ describes a normalized gain for the transformation from classical photocurrent to complex field amplitude. For $\Gamma = 1$, Bob's displacement eliminates x_u and p_v appearing in Eqs. (4) after the collapse of $\hat{x}_u$ and $\hat{p}_v$ due to the Bell detection. The teleported field then becomes

$$\hat{x}_{\rm tel} = \hat{x}_{\rm in} - \sqrt{2}e^{-r}\hat{x}_2^{(0)},$$
$$\hat{p}_{\rm tel} = \hat{p}_{\rm in} + \sqrt{2}e^{-r}\hat{p}_1^{(0)}. \tag{6}$$

For an arbitrary gain Γ, we obtain

$$\hat{x}_{\rm tel} = \Gamma\hat{x}_{\rm in} - \frac{\Gamma - 1}{\sqrt{2}} e^{r}\hat{x}_1^{(0)} - \frac{\Gamma + 1}{\sqrt{2}} e^{-r}\hat{x}_2^{(0)},$$
$$\hat{p}_{\rm tel} = \Gamma\hat{p}_{\rm in} + \frac{\Gamma - 1}{\sqrt{2}} e^{r}\hat{p}_2^{(0)} + \frac{\Gamma + 1}{\sqrt{2}} e^{-r}\hat{p}_1^{(0)}. \tag{7}$$

Note that these equations take no Bell detector inefficiencies into account.

Consider the case $\Gamma = 1$. For infinite squeezing $r \to \infty$, Eqs. (6) describe perfect teleportation of the quantum state of the input mode. On the other hand, for the classical case of $r = 0$, i.e., no squeezing and hence no entanglement, each of the teleported quadratures has *two* additional units of vacuum noise compared to the original input quadratures. These two units are so-called quantum duties or ''quduties'' which have to be paid when crossing the border between quantum and classical domains [11]. The two quduties represent the minimal tariff for every ''classical teleportation'' scheme [13]. One quduty, the unit of vacuum noise due to Alice's detection, arises from her attempt to simultaneously measure the two conjugate variables $x_{\rm in}$ and $p_{\rm in}$ [21]. This is the standard quantum limit for the detection of both quadratures [22] when attempting to gain as much information as possible about the quantum state of a light field [23]. The standard quantum limit yields a product of the measurement accuracies which is twice as large as the Heisenberg minimum uncertainty product. This product of the measurement accuracies contains the intrinsic quantum limit (Heisenberg uncertainty of the field to be detected) plus an additional unit of vacuum noise due to the detection [22]. The second quduty arises when Bob uses the information of Alice's detection to generate the state at amplitude $\sqrt{2}x_u + i\sqrt{2}p_v$ [11]. It can be interpreted as the standard quantum limit imposed on state broadcasting.

III. TELEPORTATION CRITERIA

The teleportation scheme with Alice and Bob is complete without any further measurement. The quantum state teleported remains unknown to both Alice and Bob and need not be demolished in a detection by Bob as a final step. How-

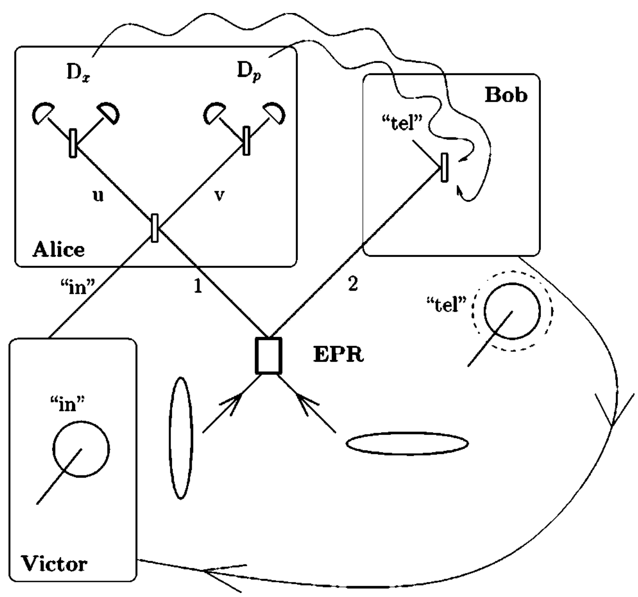

FIG. 2. Verification of quantum teleportation. The verifier ''Victor'' is independent of Alice and Bob. Victor prepares the input states which are known to him, but unknown to Alice and Bob. After a supposed quantum teleportation from Alice to Bob, the teleported states are given back to Victor. Due to his knowledge of the input states, Victor can compare the teleported states with the input states.

ever, maybe Alice and Bob are cheating. Instead of using an EPR channel, they try to get away without entanglement and use only a classical channel. In particular, for the realistic experimental situation with finite squeezing and inefficient detectors where perfect teleportation is unattainable, how may we verify that successful quantum teleportation has taken place? To make this verification we shall introduce a third party, ''Victor'' (the verifier), who is independent of Alice and Bob (Fig. 2). We assume that he prepares the initial input state (drawn from a fixed set of states) and passes it on to Alice. After accomplishing the supposed teleportation, Bob sends the teleported state back to Victor. Victor's knowledge about the input state and detection of the teleported state enable Victor to verify if quantum teleportation has really taken place. For that purpose, however, Victor needs some measure that helps him to assess when the similarity between the teleported state and the input state exceeds a boundary that is only exceedable with entanglement.

A. Teleporting Gaussian states with a coherent amplitude

The single-mode teleportation scheme from Ref. [11] works for arbitrary input states, described by any Wigner function $W_{\rm in}$. Teleporting states with a coherent amplitude as reliably as possible requires unit-gain teleportation (unit gain in Bob's final displacement). Only in this case, the coherent amplitudes of the teleported mode always match those of the input mode when Victor draws states with different amplitudes from the set of input states in a sequence of trials. For this unit-gain teleportation, the teleported state $W_{\rm tel}$ is a convolution of the input $W_{\rm in}$ with a complex Gaussian of variance e^{-2r}. Classical teleportation with $r=0$ then means the teleported mode has an excess noise of two units of vacuum $\frac{1}{2}+\frac{1}{2}$ compared to the input, as also discussed in the previous section. Any $r>0$ beats this classical scheme, i.e., if the input state is always recreated with the right amplitude and less than two units of vacuum excess noise, we may call this already quantum teleportation. Let us derive this result using the least noisy model for classical communication. For the input quadratures of Alice's sending station and the output quadratures at Bob's receiving station, the least noisy (linear) model if Alice and Bob are only classically communicating can be written as

$$\begin{aligned} \hat{x}_{{\rm out},j} &= \Gamma_x \hat{x}_{\rm in} + \Gamma_x s_a^{-1} \hat{x}_a^{(0)} + s_{b,j}^{-1} \hat{x}_{b,j}^{(0)}, \\ \hat{p}_{{\rm out},j} &= \Gamma_p \hat{p}_{\rm in} - \Gamma_p s_a \hat{p}_a^{(0)} + s_{b,j} \hat{p}_{b,j}^{(0)}. \end{aligned} \tag{8}$$

This model takes into account that Alice and Bob can only communicate via classical signals, since arbitrarily many copies of the output mode can be made by Bob where the subscript j labels the jth copy. In addition, it ensures that the output quadratures satisfy the commutation relations

$$\begin{aligned} [\hat{x}_{{\rm out},j}, \hat{p}_{{\rm out},k}] &= (i/2)\,\delta_{jk}, \\ [\hat{x}_{{\rm out},j}, \hat{x}_{{\rm out},k}] &= [\hat{p}_{{\rm out},j}, \hat{p}_{{\rm out},k}] = 0. \end{aligned} \tag{9}$$

Since we are only interested in one single copy of the output we drop the label j. The parameter s_a is given by Alice's measurement strategy and determines the noise penalty due to her homodyne detections. The gains Γ_x and Γ_p can be manipulated by Bob as well as the parameter s_b determining the noise distribution of Bob's original mode. The set of input states may contain pure Gaussian states with a coherent amplitude, described by $\hat{x}_{\rm in} = \langle \hat{x}_{\rm in} \rangle + s_v^{-1} \hat{x}^{(0)}$ and $\hat{p}_{\rm in} = \langle \hat{p}_{\rm in} \rangle + s_v \hat{p}^{(0)}$, where Victor can choose in each trial the coherent amplitude and if and to what extent the input is squeezed (parameter s_v). Since Bob always wants to reproduce the input amplitude, he is restricted to unit gain, symmetric in both quadratures $\Gamma_x = \Gamma_p = 1$. First, after obtaining the output states from Bob, Victor verifies if their amplitudes match the corresponding input amplitudes. If not, all the following considerations concerning the excess noise are redundant, because Alice and Bob can always manipulate this noise by fiddling the gain (less than unit gain reduces the excess noise). If Victor finds overlapping amplitudes in all trials (at least within some error range), he looks at the excess noise in each trial. For that purpose, let us define the normalized variance

$$V_{\rm out,in}^{\hat{x}} \equiv \frac{\langle \Delta(\hat{x}_{\rm out} - \hat{x}_{\rm in})^2 \rangle}{\langle \Delta \hat{x}^2 \rangle_{\rm vacuum}}, \tag{10}$$

and analogously $V_{\rm out,in}^{\hat{p}}$ with $\hat{x} \to \hat{p}$ throughout $[\langle \Delta \hat{0}^2 \rangle \equiv {\rm var}(\hat{0})]$. Using Eqs. (8) with unit gain, we obtain the product

$$V_{\rm out,in}^{\hat{x}} V_{\rm out,in}^{\hat{p}} = (s_a^{-2} + s_b^{-2})(s_a^2 + s_b^2). \tag{11}$$

It is minimized for $s_a=s_b$, yielding $V^{\hat{x}}_{\text{out,in}}V^{\hat{p}}_{\text{out,in}}=4$. The optimum value of 4 is exactly the result we obtain for what we may call classical teleportation $V^{\hat{x}}_{\text{tel,in}}(r=0)V^{\hat{p}}_{\text{tel,in}}(r=0)=4$, using Eqs. (6) with subscript out→tel in Eq. (10). Thus, we can write our first ''fundamental'' limit for teleporting states with a coherent amplitude as

$$V^{\hat{x}}_{\text{out,in}}V^{\hat{p}}_{\text{out,in}}\geq V^{\hat{x}}_{\text{tel,in}}(r=0)V^{\hat{p}}_{\text{tel,in}}(r=0)=4. \tag{12}$$

If Victor, comparing the output states with the input states, always finds violations of this inequality, he may already have big confidence in Alice's and Bob's honesty (i.e., that they indeed have used entanglement). Equation (12) may also enable us already to assess if a scheme or protocol is capable of quantum teleportation. Alternatively, instead of looking at the products $V^{\hat{x}}_{\text{out,in}}V^{\hat{p}}_{\text{out,in}}$, we could also use the sums $V^{\hat{x}}_{\text{out,in}}+V^{\hat{p}}_{\text{out,in}}=s_a^{-2}+s_b^{-2}+s_a^2+s_b^2$ that are minimized for $s_a=s_b=1$. Then we find the classical boundary $V^{\hat{x}}_{\text{out,in}}+V^{\hat{p}}_{\text{out,in}}\geq 4$.

However, taking into account all the assumptions made for the derivation of Eq. (12), this boundary appears to be less fundamental. First, we have only assumed a linear model. Secondly, we have only considered the variances of two conjugate observables and a certain kind of measurement of these. An entirely rigorous criterion for quantum teleportation should take into account all possible variables, measurements and strategies that can be used by Alice and Bob. Another ''problem'' of our boundary Eq. (12) is that the variances $V_{\text{out,in}}$ are not directly measurable, because the input state is destroyed by the teleportation process. However, for Gaussian input states, Victor can combine his knowledge of the input variances V_{in} with the detected variances V_{out} in order to infer $V_{\text{out,in}}$. With a more specific set of Gaussian input states, namely coherent states, the least noisy model for classical communication allows us to determine the directly measurable ''fundamental'' limit for the normalized variances of the output states

$$V^{\hat{x}}_{\text{out}}V^{\hat{p}}_{\text{out}}\geq 9. \tag{13}$$

But still we need to bear in mind that we did not consider all possible strategies of Alice and Bob. Also for arbitrary s_v (set of input states contains all coherent and squeezed states), Eq. (13) represents a classical boundary, as

$$V^{\hat{x}}_{\text{out}}V^{\hat{p}}_{\text{out}}=(s_v^{-2}+s_a^{-2}+s_b^{-2})(s_v^2+s_a^2+s_b^2) \tag{14}$$

is minimized for $s_v=s_a=s_b$, yielding $V^{\hat{x}}_{\text{out}}V^{\hat{p}}_{\text{out}}=9$. However, since s_v is unknown to Alice and Bob in every trial, they can attain this classical minimum only by accident. For s_v fixed, e.g., $s_v=1$ (set of input states contains ''only'' coherent states), Alice and Bob knowing this s_v can always satisfy $V^{\hat{x}}_{\text{out}}V^{\hat{p}}_{\text{out}}=9$ in the classical model. Alternatively, the sums $V^{\hat{x}}_{\text{out}}+V^{\hat{p}}_{\text{out}}=s_v^{-2}+s_a^{-2}+s_b^{-2}+s_v^2+s_a^2+s_b^2$ are minimized with $s_a=s_b=1$. In this case, we obtain the s_v-dependent boundary $V^{\hat{x}}_{\text{out}}+V^{\hat{p}}_{\text{out}}\geq s_v^{-2}+s_v^2+4$. Without knowing s_v, Alice and Bob can always attain this minimum in the classical model. In every trial, Victor must combine his knowledge of s_v with the detected output variances in order to find violations of this sum inequality.

Ralph and Lam [24] define the classical boundaries

$$V^{\hat{x}}_{\text{c}}+V^{\hat{p}}_{\text{c}}\geq 2 \tag{15}$$

and

$$T^{\hat{x}}_{\text{out}}+T^{\hat{p}}_{\text{out}}\leq 1, \tag{16}$$

using the conditional variance

$$V^{\hat{x}}_{\text{c}}\equiv\frac{\langle\Delta\hat{x}^2_{\text{out}}\rangle}{\langle\Delta\hat{x}^2\rangle_{\text{vacuum}}}\left(1-\frac{|\langle\Delta\hat{x}_{\text{out}}\Delta\hat{x}_{\text{in}}\rangle|^2}{\langle\Delta\hat{x}^2_{\text{out}}\rangle\langle\Delta\hat{x}^2_{\text{in}}\rangle}\right), \tag{17}$$

and analogously for $V^{\hat{p}}_{\text{c}}$ with $\hat{x}\to\hat{p}$ throughout, and the transfer coefficient

$$T^{\hat{x}}_{\text{out}}\equiv\frac{\mathcal{S}^{\hat{x}}_{\text{out}}}{\mathcal{S}^{\hat{x}}_{\text{in}}}, \tag{18}$$

and analogously $T^{\hat{p}}_{\text{out}}$ with $\hat{x}\to\hat{p}$ throughout. Here, $\mathcal{S}$ denotes the signal to noise ratio for the square of the mean amplitudes, namely $\mathcal{S}^{\hat{x}}_{\text{out}}=\langle\hat{x}_{\text{out}}\rangle^2/\langle\Delta\hat{x}^2_{\text{out}}\rangle$.

Alice and Bob using only classical communication are not able to violate *either* of the two inequalities Eq. (15) and Eq. (16). In fact, these boundaries are two independent limits, each of them unexceedable in a classical scheme. However, Alice and Bob can simultaneously approach $V^{\hat{x}}_{\text{c}}+V^{\hat{p}}_{\text{c}}=2$ and $T^{\hat{x}}_{\text{out}}+T^{\hat{p}}_{\text{out}}=1$ using either an asymmetric classical detection and transmission scheme with coherent-state inputs or a symmetric classical scheme with squeezed-state inputs [24]. For quantum teleportation, Ralph and Lam [24] require their classical limits be simultaneously exceeded, $V^{\hat{x}}_{\text{c}}+V^{\hat{p}}_{\text{c}}<2$ and $T^{\hat{x}}_{\text{out}}+T^{\hat{p}}_{\text{out}}>1$. This is only possible using more than 3 dB squeezing in the entanglement source [24]. Apparently, these criteria determine a classical boundary different from ours in Eq. (12). For example, in unit-gain teleportation, our inequality Eq. (12) is violated for any nonzero squeezing $r>0$. Let us briefly explain why we encounter this discrepancy. We have a priori assumed unit gain in our scheme to achieve outputs and inputs overlapping in their mean values. This assumption is, of course, motivated by the assessment that good teleportation means good similarity between input and output *states* (here, to be honest, we already have something in mind similar to the fidelity, introduced in the next section). First, Victor has to check the match of the amplitudes before looking at the variances. Ralph and Lam permit arbitrary gain, because they are not interested in the similarity of input and output *states*, but in certain correlations that manifest separately in the individual quadratures [25]. This point of view originates from the context of quantum nondemolition (QND) measurements [26], which are focused on a single QND variable while the conjugate variable is not of interest. For arbitrary gain, an inequality as in Eq. (16), containing the input and output mean values, has to be added to an inequality only for variances as in Eq. (15). Ralph and Lam's *best* classical protocol permits output states completely different from the input states, e.g., via asymmetric detection where the lack of information in one quadrature leads on average to output states with amplitudes completely different from the input states. The asymmetric scheme means that Alice is *not* attempting to gain as much information about the *quantum state* as possible, as in an Arthurs-Kelly measurement [21]. The Arthurs-Kelly measurement, however, is exactly what Alice should do in our *best* classical protocol, i.e., classical teleportation. Therefore, our best classical protocol always achieves output states already pretty similar to the input states. Apparently, ''the best'' that can be classically achieved has a different meaning from Ralph and Lam's point of view and from ours. Then it is no surprise that the classical boundaries differ as well. Apart from these differences, however, Ralph and Lam's criteria do have something in common with our criterion given by Eq. (12): they also do not satisfy the rigor we require from criteria for quantum teleportation taking into account everything Alice and Bob can do. By limiting the set of input states to coherent states, we are able to present such a rigorous criterion in the next section.

B. The fidelity criterion for coherent-state teleportation

The rigorous criterion we are looking for to determine the best classical teleportation and to quantify the distinction between classical and quantum teleportation relies on the fidelity F, for an arbitrary input state $|\psi_{\rm in}\rangle$ defined by [13]

$$F \equiv \langle \psi_{\rm in}|\hat{\rho}_{\rm out}|\psi_{\rm in}\rangle. \tag{19}$$

It is an excellent measure for the similarity between the input and the output state and equals one only if $\hat{\rho}_{\rm out} = |\psi_{\rm in}\rangle\langle\psi_{\rm in}|$. Now Alice and Bob know that Victor draws his states $|\psi_{\rm in}\rangle$ from a fixed set, but they do not know which particular state is drawn in a single trial. Therefore, an average fidelity should be considered [13],

$$F_{\rm av} = \int P(|\psi_{\rm in}\rangle)\langle\psi_{\rm in}|\hat{\rho}_{\rm out}|\psi_{\rm in}\rangle d|\psi_{\rm in}\rangle, \tag{20}$$

where $P(|\psi_{\rm in}\rangle)$ is the probability of drawing a particular state $|\psi_{\rm in}\rangle$, and the integral runs over the entire set of input states. If the set of input states contains simply all possible quantum states in an infinite-dimensional Hilbert space (i.e., the input state is completely unknown apart from the Hilbert-space dimension), the best average fidelity achievable without entanglement is zero. If the set of input states is restricted to coherent states of amplitude $\alpha_{\rm in} = x_{\rm in} + ip_{\rm in}$ and $F = \langle\alpha_{\rm in}|\hat{\rho}_{\rm out}|\alpha_{\rm in}\rangle$, on average, the fidelity achievable in a purely classical scheme (when averaged across the entire complex plane) is bounded by [13]

$$F_{\rm av} \leqslant \frac{1}{2}. \tag{21}$$

Let us illustrate these nontrivial results with our single-mode teleportation equations. Up to a factor π, the fidelity $F = \langle\alpha_{\rm in}|\hat{\rho}_{\rm tel}|\alpha_{\rm in}\rangle$ is the Q function of the teleported mode evaluated for $\alpha_{\rm in}$:

$$F = \pi Q_{\rm tel}(\alpha_{\rm in}) = \frac{1}{2\sqrt{\sigma_x\sigma_p}}\exp\left[-(1-\Gamma)^2\left(\frac{x_{\rm in}^2}{2\sigma_x} + \frac{p_{\rm in}^2}{2\sigma_p}\right)\right], \tag{22}$$

where Γ is the gain from the previous sections and σ_x and σ_p are the variances of the Q function of the teleported mode for the corresponding quadratures. These variances are according to Eqs. (7) for a coherent-state input and $\langle\Delta\hat{x}^2\rangle_{\rm vacuum} = \langle\Delta\hat{p}^2\rangle_{\rm vacuum} = \frac{1}{4}$ given by

$$\sigma_x = \sigma_p = \frac{1}{4}(1+\Gamma^2) + \frac{e^{2r}}{8}(\Gamma-1)^2 + \frac{e^{-2r}}{8}(\Gamma+1)^2. \tag{23}$$

For classical teleportation $(r=0)$ and $\Gamma=1$, we obtain $\sigma_x = \sigma_p = \frac{1}{2} + \frac{1}{4}V^{\hat{x}}_{\rm tel,in}(r=0) = \frac{1}{2} + \frac{1}{4}V^{\hat{p}}_{\rm tel,in}(r=0) = \frac{1}{2} + \frac{1}{2} = 1$ and indeed $F = F_{\rm av} = \frac{1}{2}$. In order to obtain a better fidelity, entanglement is necessary. Then, if $\Gamma=1$, we obtain $F = F_{\rm av} > \frac{1}{2}$ for any $r>0$. For $r=0$, the fidelity drops to zero as $\Gamma \to \infty$ since the mean amplitude of the teleported state does not match that of the input state and the excess noise increases. For $r=0$ and $\Gamma=0$, the fidelity becomes $F = \exp(-|\alpha_{\rm in}|^2)$. Upon averaging over all possible coherent-state inputs, this fidelity also vanishes. Assuming nonunit gain, it is crucial to consider the average fidelity $F_{\rm av} \neq F$. When averaging across the entire complex plane, any nonunit gain yields $F_{\rm av}=0$. This is exactly why Victor should first check the match of the amplitudes for different input states. If Alice and Bob are cheating and fiddle the gain in a classical scheme, a sufficiently large input amplitude reveals the truth. These considerations also apply to the asymmetric classical detection and transmission scheme with a coherent-state input [24] discussed in the previous section. Of course, the asymmetric scheme does not provide an improvement in the fidelity. In fact, the average fidelity drops to zero, if Alice detects only one quadrature (and gains complete information about this quadrature) and Bob obtains the full information about the measured quadrature, but no information about the second quadrature. In an asymmetric classical scheme, Alice and Bob stay far within the classical domain $F_{\rm av} < \frac{1}{2}$. The best classical scheme with respect to the fidelity is the symmetric one (''classical teleportation'') with $F_{\rm av} = \frac{1}{2}$.

The supposed limitation of the fidelity criterion that the set of input states contains ''only'' coherent states is compensated by having an entirely rigorous criterion. Of course, the fidelity criterion does not limit the possible input states for which the presented protocol works. It does not mean we can only teleport coherent states (as we will clearly see in the next section). However, so far, it is the only criterion that enables the experimentalist to rigorously verify quantum teleportation. That is why Furusawa *et al.* [12] were happy to have used coherent-state inputs, because they could rely on a

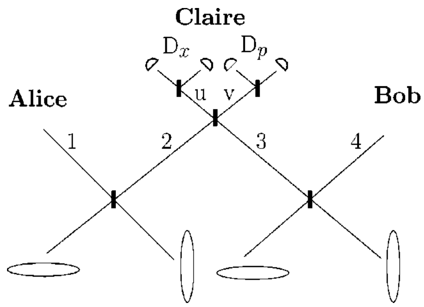

FIG. 3. Entanglement swapping using the two entangled two-mode squeezed vacuum states of modes 1 and 2 (shared by Alice and Claire) and of modes 3 and 4 (shared by Claire and Bob) as in Ref. [20].

strict and rigorous criterion (and not only because coherent states are the most readily available source for the state preparer Victor).

C. Teleporting entangled states: entanglement swapping

From a true quantum teleportation device, we require that it can not only teleport nonorthogonal states very similar to classical states (such as coherent states), but also extremely nonclassical states such as entangled states. When teleporting one half of an entangled state (''entanglement swapping''), we are certainly much more interested in the preservation of the inseparability than in the match of any input and output amplitudes. We can say that entanglement swapping is successful, if the initially unentangled modes become entangled via the teleportation process (even, if this is accompanied by a decrease of the quality of the initial entanglement). In Ref. [20] has been shown, that the single-mode teleportation scheme enables entanglement swapping for any nonzero squeezing ($r>0$) in the two initial entangled states (of which one provides the teleporter's input and the other one the EPR channel or vice versa).

Let us introduce ''Claire'' who performs the Bell detection of modes 2 and 3 (Fig. 3). Before her measurement, mode 1 (Alice's mode) is entangled with mode 2, and mode 3 is entangled with mode 4 (Bob's mode) [20]. Due to Claire's detection, mode 1 and 4 are projected on entangled states. Entanglement is teleported in every single projection (for every measured value of x_u and p_v) without any further local displacement [27]. How can we verify that entanglement swapping was successful? Simply, by verifying that Alice and Bob, who initially did not share any entanglement, are able to perform quantum teleportation using mode 1 and 4 after entanglement swapping [20]. But then we urgently need a rigorous criterion for quantum teleportation that unambigously recognizes when Alice and Bob have used entanglement and when they have not. Now, again, we can rely on the fidelity criterion for coherent-state teleportation. Alice and Bob again have to convince Victor that they are using entanglement and are not cheating. Of course, this is only a reliable verification scheme of entanglement swapping, if one can be sure that Alice and Bob did not share entanglement prior to entanglement swapping and that Claire is not allowed to perform unit-gain displacements (or that Claire is not allowed to receive any classical information). Otherwise, Victor's coherent-state input could be teleported step by step from Alice to Claire (with unit gain) and from Claire to Bob (with unit gain). This protocol, however, requires more than 3 dB squeezing in both entanglement sources (if equally squeezed) to ensure $F_{\rm av}>\frac{1}{2}$ [20]. Using entanglement swapping, Alice and Bob can achieve $F_{\rm av}>\frac{1}{2}$ for any squeezing, but one of them has to perform local displacements based on Claire's measurement results. Any gain is allowed in these displacements, since in entanglement swapping, we are not interested in the transfer of coherent amplitudes (and the two initial two-mode squeezed states are vacuum states anyway). But only the optimum gain $\Gamma_{\rm swap}=\tanh 2r$ ensures $F_{\rm av}>\frac{1}{2}$ for any squeezing and provides the optimum fidelity [20]. Unit gain $\Gamma_{\rm swap}=1$ in entanglement swapping would require more than 3 dB squeezing in both entanglement sources (if equally squeezed) to achieve $F_{\rm av}>\frac{1}{2}$ [20], or to confirm the teleportation of entanglement via detection of the combined entangled modes [19].

We will also give a broadband protocol of entanglement swapping as a ''nonunit-gain teleportation.'' The verification of entanglement swapping via the fidelity criterion for coherent-state teleportation demonstrates how useful this criterion is. Less rigorous criteria, as presented in Sec. III A, cannot reliably tell us if Alice and Bob use entanglement emerging from entanglement swapping. Furthermore, the entanglement swapping scheme demonstrates that a two-mode squeezed state enables *true* quantum teleportation for any nonzero squeezing. Requiring more than 3 dB squeezing, as it is necessary for quantum teleportation according to Ralph and Lam [24], is not necessary for the teleporation of entanglement.

IV. BROADBAND ENTANGLEMENT

In this section, we demonstrate that the EPR state required for broadband teleportation can be generated either directly by nondegenerate parametric down conversion or by combining two independently squeezed fields produced via degenerate down conversion or any other nonlinear interaction.

First, we review the results of Ref. [15] based on the input-output formalism of Collett and Gardiner [28] where a nondegenerate optical parametric amplifier in a cavity (NOPA) is studied. We will see that the upper and lower sidebands of the NOPA output have correlations similar to those of the two-mode squeezed state in Eqs. (2). The optical parametric oscillator is considered polarization nondegenerate but frequency ''degenerate'' (equal center frequency for the orthogonally polarized output modes). The interaction between the two modes is due to the nonlinear $\chi^{(2)}$ medium (in a cavity) and may be described by the interaction Hamiltonian

$$\hat{H}_I=i\hbar\kappa(\hat{a}_1^\dagger\hat{a}_2^\dagger e^{-2i\omega_0 t}-\hat{a}_1\hat{a}_2 e^{2i\omega_0 t}). \qquad (24)$$

The undepleted pump field amplitude at frequency $2\omega_0$ is described as a c number and has been absorbed into the coupling κ which also contains the $\chi^{(2)}$ susceptibility. Without loss of generality κ can be taken to be real. The dynam-

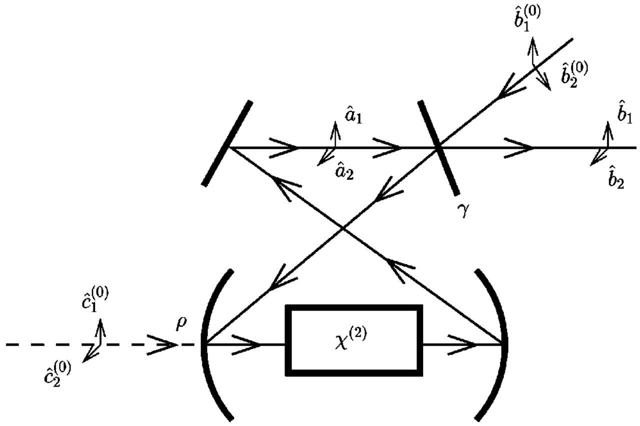

FIG. 4. The NOPA as in Ref. [15]. The two cavity modes $\hat{a}_1$ and $\hat{a}_2$ interact due to the nonlinear $\chi^{(2)}$ medium. The modes $\hat{b}_1^{(0)}$ and $\hat{b}_2^{(0)}$ are the external vacuum input modes, $\hat{b}_1$ and $\hat{b}_2$ are the external output modes, $\hat{c}_1^{(0)}$ and $\hat{c}_2^{(0)}$ are the vacuum modes due to cavity losses, γ is a damping rate and ρ is a loss parameter of the cavity.

ics of the two cavity modes $\hat{a}_1$ and $\hat{a}_2$ are governed by the above interaction Hamiltonian, and input-output relations can be derived relating the cavity modes to the external vacuum input modes $\hat{b}_1^{(0)}$ and $\hat{b}_2^{(0)}$, the external output modes $\hat{b}_1$ and $\hat{b}_2$, and two unwanted vacuum modes $\hat{c}_1^{(0)}$ and $\hat{c}_2^{(0)}$ describing cavity losses (Fig. 4). Recall, the superscript (0) refers to vacuum modes. We define uppercase operators in the rotating frame about the center frequency ω_0,

$$\hat{O}(t)=\hat{o}(t)e^{i\omega_0 t}, \tag{25}$$

with $\hat{O}=[\hat{A}_{1,2};\hat{B}_{1,2};\hat{B}_{1,2}^{(0)};\hat{C}_{1,2}^{(0)}]$ and the full Heisenberg operators $\hat{o}=[\hat{a}_{1,2};\hat{b}_{1,2};\hat{b}_{1,2}^{(0)};\hat{c}_{1,2}^{(0)}]$. By the Fourier transformation

$$\hat{O}(\Omega)=\frac{1}{\sqrt{2\pi}}\int dt\,\hat{O}(t)e^{i\Omega t}, \tag{26}$$

the fields are now described as functions of the modulation frequency Ω with commutation relation $[\hat{O}(\Omega),\hat{O}^\dagger(\Omega')]=\delta(\Omega-\Omega')$ for $\hat{B}_{1,2}$, $\hat{B}_{1,2}^{(0)}$ and $\hat{C}_{1,2}^{(0)}$ since $[\hat{O}(t),\hat{O}^\dagger(t')]=\delta(t-t')$. Expressing the outgoing modes in terms of the incoming vacuum modes, one obtains [15]

$$\hat{B}_j(\Omega)=G(\Omega)\hat{B}_j^{(0)}(\Omega)+g(\Omega)\hat{B}_k^{(0)\dagger}(-\Omega)+\overline{G}(\Omega)\hat{C}_j^{(0)}(\Omega)+\overline{g}(\Omega)\hat{C}_k^{(0)\dagger}(-\Omega), \tag{27}$$

where $k=3-j$, $j=1,2$ (so k refers to the opposite mode to j), and with coefficients to be specified later. The two cavity modes have been assumed to be both on resonance with half the pump frequency at ω_0.

Let us investigate the lossless case where the output fields become

$$\hat{B}_j(\Omega)=G(\Omega)\hat{B}_j^{(0)}(\Omega)+g(\Omega)\hat{B}_k^{(0)\dagger}(-\Omega), \tag{28}$$

with the functions $G(\Omega)$ and $g(\Omega)$ of Eq. (27) simplifying to

$$G(\Omega)=\frac{\kappa^2+\gamma^2/4+\Omega^2}{(\gamma/2-i\Omega)^2-\kappa^2},\qquad g(\Omega)=\frac{\kappa\gamma}{(\gamma/2-i\Omega)^2-\kappa^2}. \tag{29}$$

Here, the parameter γ is a damping rate of the cavity (Fig. 4) and is assumed to be equal for both polarizations. Equation (28) represents the input-output relations for a lossless NOPA.

Following Ref. [29], we introduce frequency resolved quadrature amplitudes given by

$$\begin{aligned}\hat{X}_j(\Omega)&=\frac{1}{2}[\hat{B}_j(\Omega)+\hat{B}_j^\dagger(-\Omega)],\\ \hat{P}_j(\Omega)&=\frac{1}{2i}[\hat{B}_j(\Omega)-\hat{B}_j^\dagger(-\Omega)],\\ \hat{X}_j^{(0)}(\Omega)&=\frac{1}{2}[\hat{B}_j^{(0)}(\Omega)+\hat{B}_j^{(0)\dagger}(-\Omega)],\\ \hat{P}_j^{(0)}(\Omega)&=\frac{1}{2i}[\hat{B}_j^{(0)}(\Omega)-\hat{B}_j^{(0)\dagger}(-\Omega)],\end{aligned} \tag{30}$$

provided $\Omega\ll\omega_0$. Using them Eq. (28) becomes

$$\begin{aligned}\hat{X}_j(\Omega)&=G(\Omega)\hat{X}_j^{(0)}(\Omega)+g(\Omega)\hat{X}_k^{(0)}(\Omega),\\ \hat{P}_j(\Omega)&=G(\Omega)\hat{P}_j^{(0)}(\Omega)-g(\Omega)\hat{P}_k^{(0)}(\Omega).\end{aligned} \tag{31}$$

Here, we have used $G(\Omega)=G^*(-\Omega)$ and $g(\Omega)=g^*(-\Omega)$.

At this juncture, we show that the output quadratures of a lossless NOPA in Eqs. (31) correspond to two independently squeezed modes coupled to a two-mode squeezed state at a beam splitter. The operational significance of this fact is that the EPR state required for broadband teleportation can be created either by nondegenerate parametric down conversion as described by the interaction Hamiltonian in Eq. (24), or by combining at a beam splitter two independently squeezed fields generated via degenerate down conversion [30] (as done in the teleportation experiment of Ref. [12]).

Let us thus define the superpositions of the two output modes (barred quantities)

$$\begin{aligned}\hat{\bar{B}}_1&\equiv\frac{1}{\sqrt{2}}(\hat{B}_1+\hat{B}_2),\\ \hat{\bar{B}}_2&\equiv\frac{1}{\sqrt{2}}(\hat{B}_1-\hat{B}_2),\end{aligned} \tag{32}$$

and of the two vacuum input modes

$$\hat{\bar{B}}_1^{(0)} \equiv \frac{1}{\sqrt{2}}(\hat{B}_1^{(0)}+\hat{B}_2^{(0)}),$$
$$\hat{\bar{B}}_2^{(0)} \equiv \frac{1}{\sqrt{2}}(\hat{B}_1^{(0)}-\hat{B}_2^{(0)}). \tag{33}$$

In terms of these superpositions, Eq. (28) becomes

$$\hat{\bar{B}}_1(\Omega)=G(\Omega)\hat{\bar{B}}_1^{(0)}(\Omega)+g(\Omega)\hat{\bar{B}}_1^{(0)\dagger}(-\Omega),$$
$$\hat{\bar{B}}_2(\Omega)=G(\Omega)\hat{\bar{B}}_2^{(0)}(\Omega)-g(\Omega)\hat{\bar{B}}_2^{(0)\dagger}(-\Omega). \tag{34}$$

In Eqs. (34), the initially coupled modes of Eq. (28) are decoupled, corresponding to two independent degenerate parametric amplifiers.

In the limit $\Omega\to 0$, the two modes of Eqs. (34) are each in the same single-mode squeezed state as the two modes in Eqs. (1). More explicitly, by setting $G(0)=\cosh r$ and $g(0)=\sinh r$, the annihilation operators

$$\hat{\bar{B}}_1=\cosh r\,\hat{\bar{B}}_1^{(0)}+\sinh r\,\hat{\bar{B}}_1^{(0)\dagger},$$
$$\hat{\bar{B}}_2=\cosh r\,\hat{\bar{B}}_2^{(0)}-\sinh r\,\hat{\bar{B}}_2^{(0)\dagger}, \tag{35}$$

have the quadrature operators

$$\hat{\bar{X}}_1=e^{r}\hat{\bar{X}}_1^{(0)},\quad \hat{\bar{P}}_1=e^{-r}\hat{\bar{P}}_1^{(0)},$$
$$\hat{\bar{X}}_2=e^{-r}\hat{\bar{X}}_2^{(0)},\quad \hat{\bar{P}}_2=e^{r}\hat{\bar{P}}_2^{(0)}. \tag{36}$$

From the alternative perspective of superimposing two independently squeezed modes at a 50/50 beam splitter to obtain the EPR state, we must simply invert the transformation of Eqs. (32) and recouple the two modes

$$\hat{B}_1=\frac{1}{\sqrt{2}}(\hat{\bar{B}}_1+\hat{\bar{B}}_2)=\frac{1}{\sqrt{2}}[\cosh r(\hat{\bar{B}}_1^{(0)}+\hat{\bar{B}}_2^{(0)})+\sinh r(\hat{\bar{B}}_1^{(0)\dagger}-\hat{\bar{B}}_2^{(0)\dagger})]=\cosh r\,\hat{B}_1^{(0)}+\sinh r\,\hat{B}_2^{(0)\dagger},$$
$$\hat{B}_2=\frac{1}{\sqrt{2}}(\hat{\bar{B}}_1-\hat{\bar{B}}_2)=\frac{1}{\sqrt{2}}[\cosh r(\hat{\bar{B}}_1^{(0)}-\hat{\bar{B}}_2^{(0)})+\sinh r(\hat{\bar{B}}_1^{(0)\dagger}+\hat{\bar{B}}_2^{(0)\dagger})]=\cosh r\,\hat{B}_2^{(0)}+\sinh r\,\hat{B}_1^{(0)\dagger}, \tag{37}$$

and

$$\hat{X}_1=\frac{1}{\sqrt{2}}(\hat{\bar{X}}_1+\hat{\bar{X}}_2)=\frac{1}{\sqrt{2}}(e^{r}\hat{\bar{X}}_1^{(0)}+e^{-r}\hat{\bar{X}}_2^{(0)}),$$
$$\hat{P}_1=\frac{1}{\sqrt{2}}(\hat{\bar{P}}_1+\hat{\bar{P}}_2)=\frac{1}{\sqrt{2}}(e^{-r}\hat{\bar{P}}_1^{(0)}+e^{r}\hat{\bar{P}}_2^{(0)}),$$
$$\hat{X}_2=\frac{1}{\sqrt{2}}(\hat{\bar{X}}_1-\hat{\bar{X}}_2)=\frac{1}{\sqrt{2}}(e^{r}\hat{\bar{X}}_1^{(0)}-e^{-r}\hat{\bar{X}}_2^{(0)}),$$
$$\hat{P}_2=\frac{1}{\sqrt{2}}(\hat{\bar{P}}_1-\hat{\bar{P}}_2)=\frac{1}{\sqrt{2}}(e^{-r}\hat{\bar{P}}_1^{(0)}-e^{r}\hat{\bar{P}}_2^{(0)}), \tag{38}$$

as the two-mode squeezed state in Eqs. (2). The coupled modes in Eqs. (37) expressed in terms of $\hat{B}_1^{(0)}$ and $\hat{B}_2^{(0)}$ are the two NOPA output modes of Eq. (28), if $\Omega\to 0$ and $G(0)=\cosh r$, $g(0)=\sinh r$.

More generally, for $\Omega\neq 0$, the quadratures corresponding to Eqs. (34),

$$\hat{\bar{X}}_1(\Omega)=[G(\Omega)+g(\Omega)]\hat{\bar{X}}_1^{(0)}(\Omega),$$
$$\hat{\bar{P}}_1(\Omega)=[G(\Omega)-g(\Omega)]\hat{\bar{P}}_1^{(0)}(\Omega),$$
$$\hat{\bar{X}}_2(\Omega)=[G(\Omega)-g(\Omega)]\hat{\bar{X}}_2^{(0)}(\Omega),$$
$$\hat{\bar{P}}_2(\Omega)=[G(\Omega)+g(\Omega)]\hat{\bar{P}}_2^{(0)}(\Omega), \tag{39}$$

are coupled to yield

$$\hat{X}_1(\Omega)=\frac{1}{\sqrt{2}}[G(\Omega)+g(\Omega)]\hat{\bar{X}}_1^{(0)}(\Omega)+\frac{1}{\sqrt{2}}[G(\Omega)-g(\Omega)]\hat{\bar{X}}_2^{(0)}(\Omega),$$
$$\hat{P}_1(\Omega)=\frac{1}{\sqrt{2}}[G(\Omega)-g(\Omega)]\hat{\bar{P}}_1^{(0)}(\Omega)+\frac{1}{\sqrt{2}}[G(\Omega)+g(\Omega)]\hat{\bar{P}}_2^{(0)}(\Omega),$$
$$\hat{X}_2(\Omega)=\frac{1}{\sqrt{2}}[G(\Omega)+g(\Omega)]\hat{\bar{X}}_1^{(0)}(\Omega)-\frac{1}{\sqrt{2}}[G(\Omega)-g(\Omega)]\hat{\bar{X}}_2^{(0)}(\Omega),$$
$$\hat{P}_2(\Omega)=\frac{1}{\sqrt{2}}[G(\Omega)-g(\Omega)]\hat{\bar{P}}_1^{(0)}(\Omega)-\frac{1}{\sqrt{2}}[G(\Omega)+g(\Omega)]\hat{\bar{P}}_2^{(0)}(\Omega). \tag{40}$$

The quadratures in Eqs. (40) are precisely the NOPA output quadratures of Eqs. (31) as anticipated. With the functions $G(\Omega)$ and $g(\Omega)$ of Eqs. (29), we obtain

$$G(\Omega)-g(\Omega)=\frac{\gamma/2-\kappa+i\Omega}{\gamma/2+\kappa-i\Omega},$$
$$G(\Omega)+g(\Omega)=\frac{(\gamma/2+\kappa)^2+\Omega^2}{(\gamma/2-i\Omega)^2-\kappa^2}. \tag{41}$$

For the limits $\Omega\to 0$, $\kappa\to\gamma/2$ (the limit of infinite squeezing), we obtain $[G(\Omega)-g(\Omega)]\to 0$ and $[G(\Omega)+g(\Omega)]\to\infty$. If $\Omega\to 0$, $\kappa\to 0$ (the classical limit of no squeezing), then $[G(\Omega)-g(\Omega)]\to 1$ and $[G(\Omega)+g(\Omega)]\to 1$. Thus for $\Omega\to 0$, Eqs. (40) in the above-mentioned limits correspond to Eqs. (38) in the analogous limits $r\to\infty$ (infinite squeezing) and $r\to 0$ (no squeezing). For large squeezing, apparently the individual modes of the ''broadband two-mode squeezed state'' in Eqs. (40) are very noisy. In general, the input vacuum modes are amplified in the NOPA, resulting in output modes with large fluctuations. But the correlations between the two modes increase simultaneously, so that $[\hat{X}_1(\Omega)-\hat{X}_2(\Omega)]\to 0$ and $[\hat{P}_1(\Omega)+\hat{P}_2(\Omega)]\to 0$ for $\Omega\to 0$ and $\kappa\to\gamma/2$.

The squeezing spectra of the independently squeezed modes can be derived from Eqs. (39) and are given by the spectral variances

$$\langle\Delta\hat{\tilde{X}}_1^\dagger(\Omega)\Delta\hat{\tilde{X}}_1(\Omega')\rangle=\langle\Delta\hat{\tilde{P}}_2^\dagger(\Omega)\Delta\hat{\tilde{P}}_2(\Omega')\rangle=\delta(\Omega-\Omega')|S_+(\Omega)|^2\langle\Delta\hat{X}^2\rangle_{\text{vacuum}}, \tag{42}$$
$$\langle\Delta\hat{\tilde{X}}_2^\dagger(\Omega)\Delta\hat{\tilde{X}}_2(\Omega')\rangle=\langle\Delta\hat{\tilde{P}}_1^\dagger(\Omega)\Delta\hat{\tilde{P}}_1(\Omega')\rangle=\delta(\Omega-\Omega')|S_-(\Omega)|^2\langle\Delta\hat{X}^2\rangle_{\text{vacuum}},$$

here with $|S_+(\Omega)|^2=|G(\Omega)+g(\Omega)|^2$ and $|S_-(\Omega)|^2=|G(\Omega)-g(\Omega)|^2$ $(\langle\Delta\hat{X}^2\rangle_{\text{vacuum}}=\frac{1}{4})$. In general, Eqs. (42) may define arbitrary squeezing spectra of two statistically identical but independent broadband squeezed states. The two corresponding squeezed modes

$$\hat{\tilde{X}}_1(\Omega)=S_+(\Omega)\hat{\tilde{X}}_1^{(0)}(\Omega),\quad \hat{\tilde{P}}_1(\Omega)=S_-(\Omega)\hat{\tilde{P}}_1^{(0)}(\Omega),$$
$$\hat{\tilde{X}}_2(\Omega)=S_-(\Omega)\hat{\tilde{X}}_2^{(0)}(\Omega),\quad \hat{\tilde{P}}_2(\Omega)=S_+(\Omega)\hat{\tilde{P}}_2^{(0)}(\Omega), \tag{43}$$

where $S_-(\Omega)$ refers to the quiet quadratures and $S_+(\Omega)$ to the noisy ones, can be used as EPR source for the following broadband teleportation scheme when they are combined at a beam splitter:

$$\hat{X}_1(\Omega)=\frac{1}{\sqrt{2}}S_+(\Omega)\hat{\tilde{X}}_1^{(0)}(\Omega)+\frac{1}{\sqrt{2}}S_-(\Omega)\hat{\tilde{X}}_2^{(0)}(\Omega),$$
$$\hat{P}_1(\Omega)=\frac{1}{\sqrt{2}}S_-(\Omega)\hat{\tilde{P}}_1^{(0)}(\Omega)+\frac{1}{\sqrt{2}}S_+(\Omega)\hat{\tilde{P}}_2^{(0)}(\Omega),$$
$$\hat{X}_2(\Omega)=\frac{1}{\sqrt{2}}S_+(\Omega)\hat{\tilde{X}}_1^{(0)}(\Omega)-\frac{1}{\sqrt{2}}S_-(\Omega)\hat{\tilde{X}}_2^{(0)}(\Omega),$$
$$\hat{P}_2(\Omega)=\frac{1}{\sqrt{2}}S_-(\Omega)\hat{\tilde{P}}_1^{(0)}(\Omega)-\frac{1}{\sqrt{2}}S_+(\Omega)\hat{\tilde{P}}_2^{(0)}(\Omega). \tag{44}$$

Before obtaining this ''broadband two-mode squeezed vacuum state,'' the squeezing of the two initial modes may be generated by any nonlinear interaction, e.g., apart from the OPA, also by four-wave mixing in a cavity [31].

V. TELEPORTATION OF A BROADBAND FIELD

For the teleportation of an electromagnetic field with finite bandwidth, the EPR state shared by Alice and Bob should be a broadband two-mode squeezed state, as discussed in the previous section. The incoming electromagnetic field to be teleported $\hat{E}_{\text{in}}(z,t)=\hat{E}_{\text{in}}^{(+)}(z,t)+\hat{E}_{\text{in}}^{(-)}(z,t)$, traveling in positive-z direction and having a single polarization, can be described by the positive-frequency part

$$\hat{E}_{\text{in}}^{(+)}(z,t)=[\hat{E}_{\text{in}}^{(-)}(z,t)]^\dagger=\int_{\text{W}}d\omega\frac{1}{\sqrt{2\pi}}\left(\frac{u\hbar\omega}{2cA_{\text{tr}}}\right)^{1/2}\hat{b}_{\text{in}}(\omega)e^{-i\omega(t-z/c)}. \tag{45}$$

The integral runs over a relevant bandwidth W centered on ω_0, A_{tr} represents the transverse structure of the field and u is a units-dependent constant (in Gaussian units $u=4\pi$) [29]. The annihilation and creation operators $\hat{b}_{\text{in}}(\omega)$ and $\hat{b}_{\text{in}}^\dagger(\omega)$ satisfy the commutation relations $[\hat{b}_{\text{in}}(\omega),\hat{b}_{\text{in}}(\omega')]=0$ and $[\hat{b}_{\text{in}}(\omega),\hat{b}_{\text{in}}^\dagger(\omega')]=\delta(\omega-\omega')$. The incoming electromagnetic field may now be described in a rotating frame as

$$\hat{B}_{\text{in}}(t)=\hat{X}_{\text{in}}(t)+i\hat{P}_{\text{in}}(t)=[\hat{x}_{\text{in}}(t)+i\hat{p}_{\text{in}}(t)]e^{i\omega_0 t}=\hat{b}_{\text{in}}(t)e^{i\omega_0 t}, \tag{46}$$

as in Eq. (25) with

$$\hat{B}_{\text{in}}(\Omega)=\frac{1}{\sqrt{2\pi}}\int dt\hat{B}_{\text{in}}(t)e^{i\Omega t}, \tag{47}$$

as in Eq. (26) and commutation relations $[\hat{B}_{\text{in}}(\Omega),\hat{B}_{\text{in}}(\Omega')]=0$, $[\hat{B}_{\text{in}}(\Omega),\hat{B}_{\text{in}}^\dagger(\Omega')]=\delta(\Omega-\Omega')$.

Of course, the unknown input field is not completely arbitrary. In the case of an EPR state from the NOPA, we will see that for successful quantum teleportation, the center of the input field's spectral range W should be around the NOPA center frequency ω_0 (half the pump frequency of the NOPA). Further, as we shall see, its spectral width should be small with respect to the NOPA bandwidth to benefit from the EPR correlations of the NOPA output. As for the transverse structure and the single polarization of the input field, we assume that both are known to all participants.

In spite of these complications, the teleportation protocol is performed in a fashion almost identical to the zero-bandwidth case. The EPR state of modes 1 and 2 is produced either directly as the NOPA output or by the superposition of two independently squeezed beams, as discussed in the preceding section. Mode 1 is sent to Alice and mode 2 is sent to Bob (see Fig. 1) where for the case of the NOPA, these modes correspond to two orthogonal polarizations. Alice arranges to superimpose mode 1 with the unknown input field at a 50/50 beam splitter, yielding for the relevant quadratures

$$\hat{X}_u(\Omega)=\frac{1}{\sqrt{2}}\hat{X}_{\rm in}(\Omega)-\frac{1}{\sqrt{2}}\hat{X}_1(\Omega),$$
$$\hat{P}_v(\Omega)=\frac{1}{\sqrt{2}}\hat{P}_{\rm in}(\Omega)+\frac{1}{\sqrt{2}}\hat{P}_1(\Omega). \tag{48}$$

Using Eqs. (48) we will find it useful to write the quadrature operators of Bob's mode 2 as

$$\begin{aligned}\hat{X}_2(\Omega)&=\hat{X}_{\rm in}(\Omega)-[\hat{X}_1(\Omega)-\hat{X}_2(\Omega)]-\sqrt{2}\hat{X}_u(\Omega)\\&=\hat{X}_{\rm in}(\Omega)-\sqrt{2}S_-(\Omega)\hat{X}_2^{(0)}(\Omega)-\sqrt{2}\hat{X}_u(\Omega),\\ \hat{P}_2(\Omega)&=\hat{P}_{\rm in}(\Omega)+[\hat{P}_1(\Omega)+\hat{P}_2(\Omega)]-\sqrt{2}\hat{P}_v(\Omega)\\&=\hat{P}_{\rm in}(\Omega)+\sqrt{2}S_-(\Omega)\hat{P}_1^{(0)}(\Omega)-\sqrt{2}\hat{P}_v(\Omega).\end{aligned} \tag{49}$$

Here we have used Eqs. (44). How is Alice's ''Bell detection'' which yields classical photocurrents performed? The photocurrent operators for the two homodyne detections, $\hat{i}_u(t)\propto|E_{\rm LO}^X|\hat{X}_u(t)$ and $\hat{i}_v(t)\propto|E_{\rm LO}^P|\hat{P}_v(t)$, can be written (without loss of generality we assume $\Omega>0$) as

$$\hat{i}_u(t)\propto|E_{\rm LO}^X|\int_{\rm W}d\Omega\, h_{\rm el}(\Omega)[\hat{X}_u(\Omega)e^{-i\Omega t}+\hat{X}_u^\dagger(\Omega)e^{i\Omega t}],$$
$$\hat{i}_v(t)\propto|E_{\rm LO}^P|\int_{\rm W}d\Omega\, h_{\rm el}(\Omega)[\hat{P}_v(\Omega)e^{-i\Omega t}+\hat{P}_v^\dagger(\Omega)e^{i\Omega t}], \tag{50}$$

with a noiseless, classical local oscillator (LO) and $h_{\rm el}(\Omega)$ representing the detectors' responses within their electronic bandwidths $\Delta\Omega_{\rm el}$: $h_{\rm el}(\Omega)=1$ for $\Omega\leqslant\Delta\Omega_{\rm el}$ and zero otherwise. We assume that the relevant bandwidth W ($\sim$MHz) is fully covered by the electronic bandwidth of the detectors ($\sim$GHz). Therefore, $h_{\rm el}(\Omega)\equiv1$ in Eqs. (50). Continuously in time, these photocurrents are measured and fed-forward to Bob via a classical channel with sufficient RF bandwidth. Each of them must be viewed as complex quantities in order to respect the RF phase. The whole feedforward process, continuously performed in the time domain (i.e., performed every inverse-bandwidth time), includes Alice's detections, her classical transmission and corresponding amplitude and phase modulations of Bob's EPR beam. Any *relative* delays between the classical information conveyed by Alice and Bob's EPR beam must be such that $\Delta t\ll1/\Delta\Omega$ with the inverse bandwidth of the EPR source $1/\Delta\Omega$ (for an EPR state from the NOPA: $\Delta t\ll\gamma^{-1}$). Expressed in the frequency domain, the final modulations can be described by the classical ''displacements''

$$\hat{X}_2(\Omega)\to\hat{X}_{\rm tel}(\Omega)=\hat{X}_2(\Omega)+\Gamma(\Omega)\sqrt{2}X_u(\Omega),$$
$$\hat{P}_2(\Omega)\to\hat{P}_{\rm tel}(\Omega)=\hat{P}_2(\Omega)+\Gamma(\Omega)\sqrt{2}P_v(\Omega). \tag{51}$$

The parameter $\Gamma(\Omega)$ is again a suitably normalized gain (now, in general, depending on Ω).

For $\Gamma(\Omega)=1$, Bob's displacements from Eqs. (51) exactly eliminate $\hat{X}_u(\Omega)$ and $\hat{P}_v(\Omega)$ in Eqs. (49). The same applies to the Hermitian conjugate versions of Eqs. (49) and Eqs. (51). We obtain the teleported field

$$\hat{X}_{\rm tel}(\Omega)=\hat{X}_{\rm in}(\Omega)-\sqrt{2}S_-(\Omega)\hat{X}_2^{(0)}(\Omega),$$
$$\hat{P}_{\rm tel}(\Omega)=\hat{P}_{\rm in}(\Omega)+\sqrt{2}S_-(\Omega)\hat{P}_1^{(0)}(\Omega). \tag{52}$$

For an arbitrary gain $\Gamma(\Omega)$, the teleported field becomes

$$\begin{aligned}\hat{X}_{\rm tel}(\Omega)&=\Gamma(\Omega)\hat{X}_{\rm in}(\Omega)-\frac{\Gamma(\Omega)-1}{\sqrt{2}}S_+(\Omega)\hat{X}_1^{(0)}(\Omega)\\&\quad-\frac{\Gamma(\Omega)+1}{\sqrt{2}}S_-(\Omega)\hat{X}_2^{(0)}(\Omega),\\ \hat{P}_{\rm tel}(\Omega)&=\Gamma(\Omega)\hat{P}_{\rm in}(\Omega)+\frac{\Gamma(\Omega)-1}{\sqrt{2}}S_+(\Omega)\hat{P}_2^{(0)}(\Omega)\\&\quad+\frac{\Gamma(\Omega)+1}{\sqrt{2}}S_-(\Omega)\hat{P}_1^{(0)}(\Omega).\end{aligned} \tag{53}$$

In general, these equations contain non-Hermitian operators with nonreal coefficients. Let us assume an EPR state from the NOPA, $S_\pm(\Omega)=G(\Omega)\pm g(\Omega)$. In the zero-bandwidth limit, the quadrature operators are Hermitian and the coefficients in Eqs. (52) and Eqs. (53) are real. For $\Omega\to0$ and $\Gamma(\Omega)=1$, the teleported quadratures computed from the above equations are, in agreement with the zero-bandwidth results, given by $\hat{X}_{\rm tel}=\hat{X}_{\rm in}$ and $\hat{P}_{\rm tel}=\hat{P}_{\rm in}$, if $\kappa\to\gamma/2$ and hence $[G(\Omega)-g(\Omega)]\to0$ (infinite squeezing). Thus, for zero bandwidth and an infinite degree of EPR correlations, Alice's unknown quantum state of mode ''in'' is exactly reconstituted by Bob after generating the output mode ''tel'' through unit-gain displacements. However, we are particularly interested in the physical case of finite bandwidth. Apparently, in unit-gain teleportation, the complete disappearance of the two classical quduties for perfect teleportation requires $\Omega=0$ (with an EPR state from the NOPA). Does this mean an increasing bandwidth always leads to deteriorating quantum teleportation? In order to make quantitative statements about this issue, we consider input states with a coherent amplitude (unit-gain teleportation) and calculate the spectral variances of the teleported quadratures for a coherent-state input to obtain a "fidelity spectrum."

A. Teleporting broadband Gaussian fields with a coherent amplitude

Let us employ teleportation equations for the real and imaginary parts of the non-Hermitian quadrature operators. In order to achieve a nonzero average fidelity when teleporting fields with a coherent amplitude, we assume $\Gamma(\Omega)=1$. According to Eqs. (52), the real and imaginary parts of the teleported quadratures are

$$\begin{aligned}
\mathrm{Re}\,\hat{X}_{\mathrm{tel}}(\Omega)&=\mathrm{Re}\,\hat{X}_{\mathrm{in}}(\Omega)-\sqrt{2}\,\mathrm{Re}[S_{-}(\Omega)]\mathrm{Re}\,\hat{\bar{X}}_{2}^{(0)}(\Omega)\\
&\quad+\sqrt{2}\,\mathrm{Im}[S_{-}(\Omega)]\mathrm{Im}\,\hat{\bar{X}}_{2}^{(0)}(\Omega),\\
\mathrm{Re}\,\hat{P}_{\mathrm{tel}}(\Omega)&=\mathrm{Re}\,\hat{P}_{\mathrm{in}}(\Omega)+\sqrt{2}\,\mathrm{Re}[S_{-}(\Omega)]\mathrm{Re}\,\hat{\bar{P}}_{1}^{(0)}(\Omega)\\
&\quad-\sqrt{2}\,\mathrm{Im}[S_{-}(\Omega)]\mathrm{Im}\,\hat{\bar{P}}_{1}^{(0)}(\Omega),\\
\mathrm{Im}\,\hat{X}_{\mathrm{tel}}(\Omega)&=\mathrm{Im}\,\hat{X}_{\mathrm{in}}(\Omega)-\sqrt{2}\,\mathrm{Im}[S_{-}(\Omega)]\mathrm{Re}\,\hat{\bar{X}}_{2}^{(0)}(\Omega)\\
&\quad-\sqrt{2}\,\mathrm{Re}[S_{-}(\Omega)]\mathrm{Im}\,\hat{\bar{X}}_{2}^{(0)}(\Omega),\\
\mathrm{Im}\,\hat{P}_{\mathrm{tel}}(\Omega)&=\mathrm{Im}\,\hat{P}_{\mathrm{in}}(\Omega)+\sqrt{2}\,\mathrm{Im}[S_{-}(\Omega)]\mathrm{Re}\,\hat{\bar{P}}_{1}^{(0)}(\Omega)\\
&\quad+\sqrt{2}\,\mathrm{Re}[S_{-}(\Omega)]\mathrm{Im}\,\hat{\bar{P}}_{1}^{(0)}(\Omega).
\end{aligned}\tag{54}$$

Their only nontrivial commutators are

$$\begin{aligned}
[\mathrm{Re}\,\hat{X}_j(\Omega),\mathrm{Re}\,\hat{P}_j(\Omega')]&=[\mathrm{Im}\,\hat{X}_j(\Omega),\mathrm{Im}\,\hat{P}_j(\Omega')]\\
&=(i/4)\,\delta(\Omega-\Omega'),
\end{aligned}\tag{55}$$

where we have used Eqs. (30) and $[\hat{B}_j(\Omega),\hat{B}_j^{\dagger}(\Omega')]=\delta(\Omega-\Omega')$.

We define spectral variances similar to Eq. (10),

$$\frac{\langle\Delta[\mathrm{Re}\,\hat{X}_{\mathrm{tel}}(\Omega)-\mathrm{Re}\,\hat{X}_{\mathrm{in}}(\Omega)]\Delta[\mathrm{Re}\,\hat{X}_{\mathrm{tel}}(\Omega')-\mathrm{Re}\,\hat{X}_{\mathrm{in}}(\Omega')]\rangle}{\langle\Delta\,\mathrm{Re}\,\hat{X}^2\rangle_{\mathrm{vacuum}}}\equiv\delta(\Omega-\Omega')V_{\mathrm{tel,in}}^{\mathrm{Re}\,\hat{X}}(\Omega).\tag{56}$$

We analogously define $V_{\mathrm{tel,in}}^{\mathrm{Re}\,\hat{P}}(\Omega)$, $V_{\mathrm{tel,in}}^{\mathrm{Im}\,\hat{X}}(\Omega)$, and $V_{\mathrm{tel,in}}^{\mathrm{Im}\,\hat{P}}(\Omega)$ with $\mathrm{Re}\,\hat{X}\to\mathrm{Re}\,\hat{P}$, etc., throughout.

From Eqs. (54), we obtain

$$V_{\mathrm{tel,in}}^{\mathrm{Re}\,\hat{X}}(\Omega)=V_{\mathrm{tel,in}}^{\mathrm{Re}\,\hat{P}}(\Omega)=V_{\mathrm{tel,in}}^{\mathrm{Im}\,\hat{X}}(\Omega)=V_{\mathrm{tel,in}}^{\mathrm{Im}\,\hat{P}}(\Omega)=2|S_{-}(\Omega)|^2.\tag{57}$$

Here we have used that

$$\begin{aligned}
\langle\Delta\,\mathrm{Re}\,\hat{\bar{X}}_j^{(0)}(\Omega)\Delta\,\mathrm{Re}\,\hat{\bar{X}}_j^{(0)}(\Omega')\rangle&=\delta(\Omega-\Omega')\langle\Delta\,\mathrm{Re}\,\hat{X}^2\rangle_{\mathrm{vacuum}}\\
&=\langle\Delta\,\mathrm{Im}\,\hat{\bar{X}}_j^{(0)}(\Omega)\Delta\,\mathrm{Im}\,\hat{\bar{X}}_j^{(0)}(\Omega')\rangle=\delta(\Omega-\Omega')\\
&\quad\times\langle\Delta\,\mathrm{Im}\,\hat{X}^2\rangle_{\mathrm{vacuum}},
\end{aligned}\tag{58}$$

and analogously for the other quadrature, and

$$\begin{aligned}
&\langle\Delta\,\mathrm{Re}\,\hat{\bar{X}}_j^{(0)}(\Omega)\Delta\,\mathrm{Im}\,\hat{\bar{X}}_j^{(0)}(\Omega')\rangle\\
&\quad=\langle\Delta\,\mathrm{Re}\,\hat{\bar{P}}_j^{(0)}(\Omega)\Delta\,\mathrm{Im}\,\hat{\bar{P}}_j^{(0)}(\Omega')\rangle=0.
\end{aligned}\tag{59}$$

Thus, for unit-gain teleportation at all frequencies, it turns out that the variance of each teleported quadrature is given by the variance of the input quadrature plus twice the squeezing spectrum of the quiet quadrature of a decoupled mode in a "broadband squeezed state" as in Eqs. (43). The excess noise in each teleported quadrature after the teleportation process is, relative to the vacuum noise, *twice* the squeezing spectrum $|S_{-}(\Omega)|^2$ from Eqs. (42).

We also obtain these results by directly defining

$$\frac{\langle\Delta[\hat{X}_{\mathrm{tel}}^{\dagger}(\Omega)-\hat{X}_{\mathrm{in}}^{\dagger}(\Omega)]\Delta[\hat{X}_{\mathrm{tel}}(\Omega')-\hat{X}_{\mathrm{in}}(\Omega')]\rangle}{\langle\Delta\hat{X}^2\rangle_{\mathrm{vacuum}}}\equiv\delta(\Omega-\Omega')V_{\mathrm{tel,in}}^{\hat{X}}(\Omega).\tag{60}$$

We analogously define $V_{\mathrm{tel,in}}^{\hat{P}}(\Omega)$ with $\hat{X}\to\hat{P}$ throughout. Using Eqs. (52), these variances become for $\Gamma(\Omega)=1$

$$V_{\mathrm{tel,in}}^{\hat{X}}(\Omega)=V_{\mathrm{tel,in}}^{\hat{P}}(\Omega)=2|S_{-}(\Omega)|^2.\tag{61}$$

We calculate some limits for $V_{\mathrm{tel,in}}^{\hat{X}}(\Omega)$ of Eq. (61), assuming an EPR state from the NOPA, $S_{-}(\Omega)=G(\Omega)-g(\Omega)$. Since $V_{\mathrm{tel,in}}^{\hat{X}}(\Omega)=V_{\mathrm{tel,in}}^{\hat{P}}(\Omega)$ and $\Gamma(\Omega)=1$, we can name the limits according to the criterion of Eq. (12).

Classical teleportation, $\kappa\to0$. $V_{\mathrm{tel,in}}^{\hat{X}}(\Omega)=2$, which is independent of the modulation frequency Ω.

Zero-bandwidth quantum teleportation, $\Omega\to0$, $\kappa>0$. $V_{\mathrm{tel,in}}^{\hat{X}}(\Omega)=2[1-2\kappa\gamma/(\kappa+\gamma/2)^2]$, and in the ideal case of infinite squeezing $\kappa\to\gamma/2$: $V_{\mathrm{tel,in}}^{\hat{X}}(\Omega)=0$.

Broadband quantum teleportation, $\Omega>0$, $\kappa>0$. $V_{\mathrm{tel,in}}^{\hat{X}}(\Omega)=2\{1-2\kappa\gamma/[(\kappa+\gamma/2)^2+\Omega^2]\}$, and in the ideal case $\kappa\to\gamma/2$: $V_{\mathrm{tel,in}}^{\hat{X}}(\Omega)=2[\Omega^2/(\gamma^2+\Omega^2)]$. So it turns out that also for finite bandwidth ideal quantum teleportation can be approached provided $\Omega\ll\gamma$.

We can express $V_{\mathrm{tel,in}}^{\hat{X}}(\Omega)$ in terms of experimental parameters relevant to the NOPA. For this purpose, we use the dimensionless quantities from Ref. [15],

$$\epsilon=\frac{2\kappa}{\gamma+\rho}=\sqrt{\frac{P_{\mathrm{pump}}}{P_{\mathrm{thres}}}},\qquad\omega=\frac{2\Omega}{\gamma+\rho}=\frac{\Omega}{2\pi}\,\frac{2F_{\mathrm{cav}}}{\nu_{\mathrm{FSR}}}.\tag{62}$$

Here, P_{pump} is the pump power, P_{thres} is the threshold value, F_{cav} is the measured finesse of the cavity, ν_{FSR} is its free spectral range, and the parameter ρ describes cavity losses (see Fig. 4). Note that we now use ω as a normalized modulation frequency in contrast to Eq. (45) and the following commutators where it was the frequency of the field operators in the nonrotating frame.

The spectral variances for the lossless case $(\rho=0)$ can be written as a function of ϵ and ω, namely,

$$V_{\text{tel,in}}^{\hat{X}}(\epsilon,\omega)=V_{\text{tel,in}}^{\hat{P}}(\epsilon,\omega)=2\left[1-\frac{4\epsilon}{(\epsilon+1)^2+\omega^2}\right]. \quad (63)$$

Now, the classical limit is $\epsilon\to 0$ ($V_{\text{tel,in}}^{\hat{X}}=2$, independent of ω) and the ideal case is $\epsilon\to 1$ $[V_{\text{tel,in}}^{\hat{X}}(\epsilon,\omega)=2\omega^2/(4+\omega^2)]$. Obviously, perfect quantum teleportation is achieved for $\epsilon\to 1$ and $\omega\to 0$. In fact, this limit can also be approached for finite $\Omega\neq 0$ provided $\omega\ll 1$ or $\Omega\ll\gamma$. Note that this condition is not specific to broadband teleportation, but is simply the condition for broadband squeezing, i.e., for the generation of highly squeezed quadratures at nonzero modulation frequencies Ω.

Let us now assume coherent-state inputs with $\langle\Delta\hat{X}_{\text{in}}^{\dagger}(\Omega)\Delta\hat{X}_{\text{in}}(\Omega')\rangle = \langle\Delta\hat{P}_{\text{in}}^{\dagger}(\Omega)\Delta\hat{P}_{\text{in}}(\Omega')\rangle = \frac{1}{4}\delta(\Omega-\Omega')$ $[\langle\Delta\,\text{Re}\,\hat{X}_{\text{in}}(\Omega)\Delta\,\text{Re}\,\hat{X}_{\text{in}}(\Omega')\rangle=\frac{1}{8}\delta(\Omega-\Omega')$ etc.], at all frequencies Ω in the relevant bandwidth W. In order to obtain a spectrum of the fidelities in Eq. (22) with $\Gamma\to\Gamma(\Omega)=1$, we need the spectrum of the Q functions of the teleported field with the spectral variances $\sigma_x(\Omega)=\sigma_p(\Omega)=\frac{1}{2}+\frac{1}{4}V_{\text{tel,in}}^{\hat{X}}(\Omega)$. We obtain the ''fidelity spectrum''

$$F(\Omega)=\frac{1}{1+|S_-(\Omega)|^2}. \quad (64)$$

Finally, with the new quantities ϵ and ω, the fidelity spectrum for quantum teleportation of arbitrary broadband coherent states using broadband entanglement from the NOPA ($\rho=0$) is given by

$$F(\epsilon,\omega)=\left[2-\frac{4\epsilon}{(\epsilon+1)^2+\omega^2}\right]^{-1}. \quad (65)$$

For different ϵ values, the spectrum of fidelities is shown in Fig. 5. From the single-mode protocol (with ideal detectors), we know that any nonzero squeezing enables quantum teleportation and coherent-state inputs can be teleported with $F=F_{\text{av}}>\frac{1}{2}$ for any $r>0$. Correspondingly, the fidelity from Eq. (65) exceeds $\frac{1}{2}$ for any nonzero ϵ at all finite frequencies, as, provided $\epsilon>0$, there is no squeezing at all only when $\omega\to\infty$. However, we had assumed [see after Eqs. (30): $\Omega\ll\omega_0$] modulation frequencies Ω much smaller than the NOPA center frequency ω_0. In fact, for $\Omega\to\omega_0$, squeezing becomes impossible at the frequency Ω [29]. But also within the region $\Omega\ll\omega_0$, effectively, the squeezing bandwith is limited and hence as well the bandwith of quantum teleportation $\Delta\omega\equiv 2\omega_{\max}$ where $F(\omega)\approx\frac{1}{2}$ (<0.51) for all $\omega>\omega_{\max}$ and $F(\omega)>\frac{1}{2}$ (≥ 0.51) for all $\omega\leq\omega_{\max}$. According to Fig. 5, we could say that the ''effective teleportation bandwidth'' is just about $\Delta\omega\approx 5.8$ $(\epsilon=0.1)$, $\Delta\omega\approx 8.6$ $(\epsilon=0.2)$, $\Delta\omega\approx 12.4$ $(\epsilon=0.4)$, $\Delta\omega\approx 15.2$ $(\epsilon=0.6)$, and $\Delta\omega\approx 19.6$ $(\epsilon=1)$. The maximum fidelities at frequency $\omega=0$ are $F_{\max}\approx 0.6$ $(\epsilon=0.1)$, $F_{\max}\approx 0.69$ $(\epsilon=0.2)$, $F_{\max}\approx 0.84$ $(\epsilon=0.4)$, $F_{\max}\approx 0.94$ $(\epsilon=0.6)$, and, of course, $F_{\max}=1$ $(\epsilon=1)$.

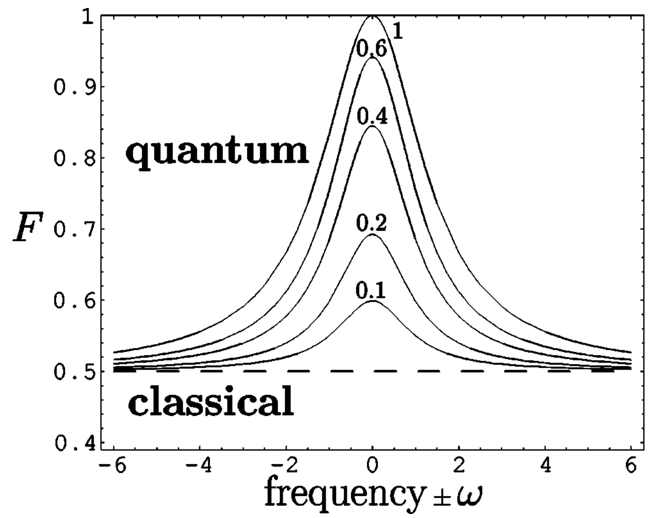

FIG. 5. Fidelity spectrum of coherent-state teleportation using entanglement from the NOPA. The fidelities here are functions of the normalized modulation frequency $\pm\omega$ for different parameter ϵ $(=0.1,\ 0.2,\ 0.4,\ 0.6,$ and $1)$.

B. Broadband entanglement swapping

As discussed in Sec. III, we particularly want our teleportation device to be capable of teleporting entanglement. We will present now the broadband theory of this entanglement swapping for continuous variables, as it was proposed in Ref. [20] for single modes. Before any detections (see Fig. 3), Alice (mode 1) and Claire (mode 2) share the broadband two-mode squeezed state from Eqs. (44), whereas Claire (mode 3) and Bob (mode 4) share the corresponding entangled state of modes 3 and 4 given by

$$\begin{aligned}
\hat{X}_3(\Omega)&=\frac{1}{\sqrt{2}}S_+(\Omega)\hat{X}_3^{(0)}(\Omega)+\frac{1}{\sqrt{2}}S_-(\Omega)\hat{X}_4^{(0)}(\Omega),\\
\hat{P}_3(\Omega)&=\frac{1}{\sqrt{2}}S_-(\Omega)\hat{P}_3^{(0)}(\Omega)+\frac{1}{\sqrt{2}}S_+(\Omega)\hat{P}_4^{(0)}(\Omega),\\
\hat{X}_4(\Omega)&=\frac{1}{\sqrt{2}}S_+(\Omega)\hat{X}_3^{(0)}(\Omega)-\frac{1}{\sqrt{2}}S_-(\Omega)\hat{X}_4^{(0)}(\Omega),\\
\hat{P}_4(\Omega)&=\frac{1}{\sqrt{2}}S_-(\Omega)\hat{P}_3^{(0)}(\Omega)-\frac{1}{\sqrt{2}}S_+(\Omega)\hat{P}_4^{(0)}(\Omega).
\end{aligned} \quad (66)$$

Let us interpret the entanglement swapping here as quantum teleportation of mode 2 to mode 4 using the entanglement of modes 3 and 4. This means we want Bob to perform ''displacements'' based on the classical results of Claire's Bell detection, i.e., the classical determination of $\hat{X}_u(\Omega) = [\hat{X}_2(\Omega) - \hat{X}_3(\Omega)]/\sqrt{2}, \hat{P}_v(\Omega) = [\hat{P}_2(\Omega) + \hat{P}_3(\Omega)]/\sqrt{2}$. These final ''displacements'' (amplitude and phase modulations) of mode 4 are crucial in order to reveal the entanglement from entanglement swapping and, for verification, to finally exploit it in a second round of quantum teleportation using the previously unentangled modes 1 and 4 [20]. The entire teleportation process with arbitrary gain $\Gamma(\Omega)$ that led to Eqs. (53), yields now, for the teleportation of mode 2 to mode 4, the teleported mode 4′ [where in Eqs. (53) simply $\hat{X}_{\rm tel}(\Omega) \to \hat{X}'_4(\Omega)$, $\hat{P}_{\rm tel}(\Omega) \to \hat{P}'_4(\Omega)$, $\hat{X}_{\rm in}(\Omega) \to \hat{X}_2(\Omega)$, $\hat{P}_{\rm in}(\Omega) \to \hat{P}_2(\Omega)$, $\hat{X}_1^{(0)}(\Omega) \to \hat{X}_3^{(0)}(\Omega)$, $\hat{P}_1^{(0)}(\Omega) \to \hat{P}_3^{(0)}(\Omega)$, $\hat{X}_2^{(0)}(\Omega) \to \hat{X}_4^{(0)}(\Omega)$, $\hat{P}_2^{(0)}(\Omega) \to \hat{P}_4^{(0)}(\Omega)$, and $\Gamma(\Omega) \to \Gamma_{\rm swap}(\Omega)$],

$$\begin{aligned}\hat{X}'_4(\Omega) = {}& \frac{\Gamma_{\rm swap}(\Omega)}{\sqrt{2}}[S_+(\Omega)\hat{X}_1^{(0)}(\Omega) - S_-(\Omega)\hat{X}_2^{(0)}(\Omega)] \\ & - \frac{\Gamma_{\rm swap}(\Omega)-1}{\sqrt{2}} S_+(\Omega)\hat{X}_3^{(0)}(\Omega) \\ & - \frac{\Gamma_{\rm swap}(\Omega)+1}{\sqrt{2}} S_-(\Omega)\hat{X}_4^{(0)}(\Omega),\end{aligned} \tag{67}$$

$$\begin{aligned}\hat{P}'_4(\Omega) = {}& \frac{\Gamma_{\rm swap}(\Omega)}{\sqrt{2}}[S_-(\Omega)\hat{P}_1^{(0)}(\Omega) - S_+(\Omega)\hat{P}_2^{(0)}(\Omega)] \\ & + \frac{\Gamma_{\rm swap}(\Omega)+1}{\sqrt{2}} S_-(\Omega)\hat{P}_3^{(0)}(\Omega) \\ & + \frac{\Gamma_{\rm swap}(\Omega)-1}{\sqrt{2}} S_+(\Omega)\hat{P}_4^{(0)}(\Omega).\end{aligned}$$

Provided entanglement swapping is successful, Alice and Bob can use their modes 1 and 4′ for a further quantum teleportation. Assuming unit gain in this ''second teleportation,'' where the unknown input state $\hat{X}_{\rm in}(\Omega)$, $\hat{P}_{\rm in}(\Omega)$ is to be teleported, the teleported field becomes

$$\begin{aligned}\hat{X}_{\rm tel}(\Omega) = {}& \hat{X}_{\rm in}(\Omega) + \frac{\Gamma_{\rm swap}(\Omega)-1}{\sqrt{2}} S_+(\Omega)\hat{X}_1^{(0)}(\Omega) \\ & - \frac{\Gamma_{\rm swap}(\Omega)+1}{\sqrt{2}} S_-(\Omega)\hat{X}_2^{(0)}(\Omega) \\ & - \frac{\Gamma_{\rm swap}(\Omega)-1}{\sqrt{2}} S_+(\Omega)\hat{X}_3^{(0)}(\Omega) \\ & - \frac{\Gamma_{\rm swap}(\Omega)+1}{\sqrt{2}} S_-(\Omega)\hat{X}_4^{(0)}(\Omega),\end{aligned}$$

$$\begin{aligned}\hat{P}_{\rm tel}(\Omega) = {}& \hat{P}_{\rm in}(\Omega) + \frac{\Gamma_{\rm swap}(\Omega)+1}{\sqrt{2}} S_-(\Omega)\hat{P}_1^{(0)}(\Omega) \\ & - \frac{\Gamma_{\rm swap}(\Omega)-1}{\sqrt{2}} S_+(\Omega)\hat{P}_2^{(0)}(\Omega) \\ & + \frac{\Gamma_{\rm swap}(\Omega)+1}{\sqrt{2}} S_-(\Omega)\hat{P}_3^{(0)}(\Omega) \\ & + \frac{\Gamma_{\rm swap}(\Omega)-1}{\sqrt{2}} S_+(\Omega)\hat{P}_4^{(0)}(\Omega).\end{aligned} \tag{68}$$

We calculate a fidelity spectrum for coherent-state inputs and obtain

$$\begin{aligned}F(\Omega) = \{ & 1 + [\Gamma_{\rm swap}(\Omega)-1]^2|S_+(\Omega)|^2/2 \\ & + [\Gamma_{\rm swap}(\Omega)+1]^2|S_-(\Omega)|^2/2\}^{-1}.\end{aligned} \tag{69}$$

The optimum gain, depending on the amount of squeezing, that maximizes this fidelity [20] at different frequencies turns out to be

$$\Gamma_{\rm swap}(\Omega) = \frac{|S_+(\Omega)|^2 - |S_-(\Omega)|^2}{|S_+(\Omega)|^2 + |S_-(\Omega)|^2}. \tag{70}$$

Let us now assume that the broadband entanglement comes from the NOPA (two NOPA's with equal squeezing spectra), $|S_-(\Omega)|^2 \to |S_-(\epsilon,\omega)|^2 = 1 - 4\epsilon/[(\epsilon+1)^2+\omega^2]$, $|S_+(\Omega)|^2 \to |S_+(\epsilon,\omega)|^2 = 1 + 4\epsilon/[(\epsilon-1)^2+\omega^2]$. The optimized fidelity then becomes

$$F_{\rm opt}(\epsilon,\omega) = \left\{ 1 + 2\frac{[(\epsilon+1)^2+\omega^2][(\epsilon-1)^2+\omega^2]}{[(\epsilon+1)^2+\omega^2]^2 + [(\epsilon-1)^2+\omega^2]^2} \right\}^{-1}. \tag{71}$$

The spectrum of these optimized fidelities is shown in Fig. 6 for different ϵ values. Again, we know from the single-mode protocol [20] with ideal detectors that any nonzero squeezing in both initial entanglement sources is sufficient for entanglement swapping to occur. In this case, mode 1 and 4′ enable quantum teleportation and coherent-state inputs can be teleported with $F = F_{\rm av} > \frac{1}{2}$. The fidelity from Eq. (71) is $\frac{1}{2}$ for $\epsilon = 0$ and becomes $F_{\rm opt}(\epsilon,\omega) > \frac{1}{2}$ for any $\epsilon > 0$, provided that ω does not become infinite (however, we had assumed $\Omega \ll \omega_0$). In this sense, the squeezing or entanglement bandwidth is preserved through entanglement swapping. At each frequency where the initial states were squeezed and entangled, also the output state of modes 1 and 4′ is entangled, but with less squeezing and worse quality of entanglement (unless we had infinite squeezing in the initial states so that the entanglement is perfectly teleported) [32]. Correspondingly, at frequencies with initially very small entanglement, the entanglement becomes even smaller after entanglement swapping (but never vanishes completely). Thus, the effective bandwidth of squeezing or entanglement decreases through entanglement swapping. Then, compared to the teleportation bandwidth using broadband two-mode squeezed

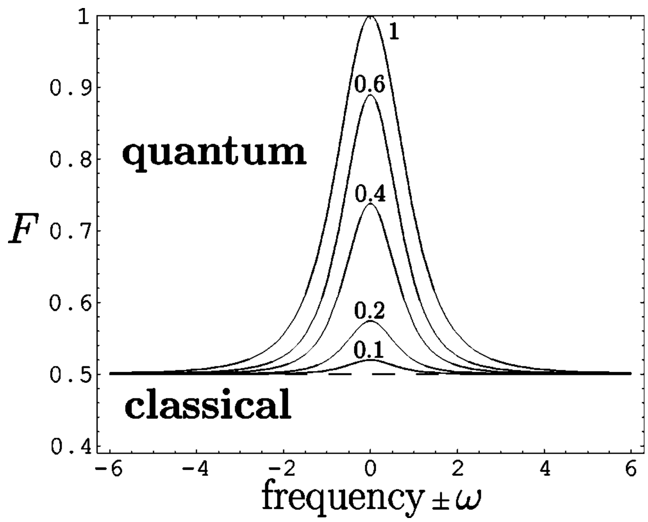

FIG. 6. Fidelity spectrum of coherent-state teleportation using the output of entanglement swapping with two equally squeezed (entangled) NOPA's. The fidelities here are functions of the normalized modulation frequency $\pm\omega$ for different parameter ϵ $(=0.1,\ 0.2,\ 0.4,\ 0.6, \text{and } 1)$.

states without entanglement swapping, the bandwidth of teleportation using the output of entanglement swapping is effectively smaller. The spectrum of the fidelities from Eq. (71) is narrower and the ''effective teleportation bandwidth'' is now about $\Delta\omega\approx 1.2$ $(\epsilon=0.1)$, $\Delta\omega\approx 2.6$ $(\epsilon=0.2)$, $\Delta\omega\approx 4.2$ $(\epsilon=0.4)$, $\Delta\omega\approx 5.2$ $(\epsilon=0.6)$, and $\Delta\omega\approx 6.8$ $(\epsilon=1)$. The maximum fidelities at frequency $\omega=0$ are $F_{\max}\approx 0.52$ $(\epsilon=0.1)$, $F_{\max}\approx 0.57$ $(\epsilon=0.2)$, $F_{\max}\approx 0.74$ $(\epsilon=0.4)$, $F_{\max}\approx 0.89$ $(\epsilon=0.6)$, and, still, $F_{\max}=1$ $(\epsilon=1)$.

VI. CAVITY LOSSES AND BELL DETECTOR INEFFICIENCIES

We extend the previous calculations and include losses for the particular case of the NOPA cavity and inefficiencies in Alice's Bell detection. For this purpose, we use Eq. (27) for the outgoing NOPA modes. We consider losses and inefficiencies for unit-gain teleportation (teleportation of Gaussian states with a coherent amplitude). For the case of entanglement swapping (nonunit-gain teleportation), detector inefficiencies have been included in the single-mode treatment of Ref. [20]. By superimposing the unknown input mode with the NOPA mode 1, the relevant quadratures from Eqs. (48) now become

$$\hat{X}_u(\Omega)=\frac{\eta}{\sqrt{2}}\hat{X}_{\rm in}(\Omega)-\frac{\eta}{\sqrt{2}}\hat{X}_1(\Omega)+\sqrt{\frac{1-\eta^2}{2}}\hat{X}_D^{(0)}(\Omega)+\sqrt{\frac{1-\eta^2}{2}}\hat{X}_E^{(0)}(\Omega),$$

$$\hat{P}_v(\Omega)=\frac{\eta}{\sqrt{2}}\hat{P}_{\rm in}(\Omega)+\frac{\eta}{\sqrt{2}}\hat{P}_1(\Omega)+\sqrt{\frac{1-\eta^2}{2}}\hat{P}_F^{(0)}(\Omega)+\sqrt{\frac{1-\eta^2}{2}}\hat{P}_G^{(0)}(\Omega). \tag{72}$$

The last two terms in each quadrature in Eqs. (72) represent additional vacua due to homodyne detection inefficiencies (the detector amplitude efficiency η is assumed to be constant over the bandwidth of interest). Using Eqs. (72) it is useful to write the quadratures of NOPA mode 2 corresponding to Eq. (27) as

$$\hat{X}_2(\Omega)=\hat{X}_{\rm in}(\Omega)-[G(\Omega)-g(\Omega)][\hat{X}_1^{(0)}(\Omega)-\hat{X}_2^{(0)}(\Omega)]-[\overline{G}(\Omega)-\overline{g}(\Omega)][\hat{X}_{C,1}^{(0)}(\Omega)-\hat{X}_{C,2}^{(0)}(\Omega)]+\sqrt{\frac{1-\eta^2}{\eta^2}}\hat{X}_D^{(0)}(\Omega)+\sqrt{\frac{1-\eta^2}{\eta^2}}\hat{X}_E^{(0)}(\Omega)-\frac{\sqrt{2}}{\eta}\hat{X}_u(\Omega), \tag{73}$$

$$\hat{P}_2(\Omega)=\hat{P}_{\rm in}(\Omega)+[G(\Omega)-g(\Omega)][\hat{P}_1^{(0)}(\Omega)+\hat{P}_2^{(0)}(\Omega)]+[\overline{G}(\Omega)-\overline{g}(\Omega)][\hat{P}_{C,1}^{(0)}(\Omega)+\hat{P}_{C,2}^{(0)}(\Omega)]+\sqrt{\frac{1-\eta^2}{\eta^2}}\hat{P}_F^{(0)}(\Omega)+\sqrt{\frac{1-\eta^2}{\eta^2}}\hat{P}_G^{(0)}(\Omega)-\frac{\sqrt{2}}{\eta}\hat{P}_v(\Omega),$$

where now [15]

$$G(\Omega)=\frac{\kappa^2+\left(\frac{\gamma-\rho}{2}+i\Omega\right)\left(\frac{\gamma+\rho}{2}-i\Omega\right)}{\left(\frac{\gamma+\rho}{2}-i\Omega\right)^2-\kappa^2},$$

$$g(\Omega)=\frac{\kappa\gamma}{\left(\frac{\gamma+\rho}{2}-i\Omega\right)^2-\kappa^2},$$

$$\overline{G}(\Omega)=\frac{\sqrt{\gamma\rho}\left(\frac{\gamma+\rho}{2}-i\Omega\right)}{\left(\frac{\gamma+\rho}{2}-i\Omega\right)^2-\kappa^2},$$

$$\overline{g}(\Omega)=\frac{\kappa\sqrt{\gamma\rho}}{\left(\frac{\gamma+\rho}{2}-i\Omega\right)^2-\kappa^2}, \tag{74}$$

still with $G(\Omega)=G^*(-\Omega)$, $g(\Omega)=g^*(-\Omega)$, and also $\overline{G}(\Omega)=\overline{G}^*(-\Omega)$, $\overline{g}(\Omega)=\overline{g}^*(-\Omega)$. The quadratures $\hat{X}_{C,j}^{(0)}(\Omega)$ and $\hat{P}_{C,j}^{(0)}(\Omega)$ are those of the vacuum modes $\hat{C}_j^{(0)}(\Omega)$ in Eq. (27) according to Eqs. (30).

Again, $\hat{X}_u(\Omega)$ and $\hat{P}_v(\Omega)$ in Eqs. (73) can be considered as classically determined quantities $X_u(\Omega)$ and $P_v(\Omega)$ due to Alice's measurements. The appropriate amplitude and phase modulations of mode 2 by Bob depending on the classical results of Alice's detections are described by

$$\hat{X}_2(\Omega)\rightarrow\hat{X}_{\rm tel}(\Omega)=\hat{X}_2(\Omega)+\Gamma(\Omega)\frac{\sqrt{2}}{\eta}X_u(\Omega),$$
$$\hat{P}_2(\Omega)\rightarrow\hat{P}_{\rm tel}(\Omega)=\hat{P}_2(\Omega)+\Gamma(\Omega)\frac{\sqrt{2}}{\eta}P_v(\Omega). \tag{75}$$

For $\Gamma(\Omega)=1$, the teleported quadratures become

$$\hat{X}_{\rm tel}(\Omega)=\hat{X}_{\rm in}(\Omega)-[G(\Omega)-g(\Omega)][\hat{X}_1^{(0)}(\Omega)-\hat{X}_2^{(0)}(\Omega)]-[\overline{G}(\Omega)-\overline{g}(\Omega)][\hat{X}_{C,1}^{(0)}(\Omega)-\hat{X}_{C,2}^{(0)}(\Omega)]+\sqrt{\frac{1-\eta^2}{\eta^2}}\hat{X}_D^{(0)}(\Omega)+\sqrt{\frac{1-\eta^2}{\eta^2}}\hat{X}_E^{(0)}(\Omega),$$

$$\hat{P}_{\rm tel}(\Omega)=\hat{P}_{\rm in}(\Omega)+[G(\Omega)-g(\Omega)][\hat{P}_1^{(0)}(\Omega)+\hat{P}_2^{(0)}(\Omega)]+[\overline{G}(\Omega)-\overline{g}(\Omega)][\hat{P}_{C,1}^{(0)}(\Omega)+\hat{P}_{C,2}^{(0)}(\Omega)]+\sqrt{\frac{1-\eta^2}{\eta^2}}\hat{P}_F^{(0)}(\Omega)+\sqrt{\frac{1-\eta^2}{\eta^2}}\hat{P}_G^{(0)}(\Omega). \tag{76}$$

We calculate again spectral variances and obtain with the dimensionless variables of Eqs. (62)

$$V_{\rm tel,in}^{\hat{X}}(\epsilon,\omega)=V_{\rm tel,in}^{\hat{P}}(\epsilon,\omega)=2\left[1-\frac{4\epsilon\beta}{(\epsilon+1)^2+\omega^2}\right]+2\frac{1-\eta^2}{\eta^2}, \tag{77}$$

where $\beta=\gamma/(\gamma+\rho)$ is a "cavity escape efficiency" which contains losses [15]. With the spectral Q-function variances of the teleported field $\sigma_x(\Omega)=\sigma_p(\Omega)=\frac{1}{2}+\frac{1}{4}V_{\rm tel,in}^{\hat{X}}(\Omega)$, now for coherent-state inputs, we find the fidelity spectrum (unit gain)

$$F(\epsilon,\omega)=\left[2-\frac{4\epsilon\beta}{(\epsilon+1)^2+\omega^2}+\frac{1-\eta^2}{\eta^2}\right]^{-1}. \tag{78}$$

Using the values $\epsilon=0.77$, $\omega=0.56$, and $\beta=0.9$, the measured values in the EPR experiment of Ref. [15] for maximum pump power (but still below threshold), and a Bell detector efficiency $\eta^2=0.97$ (as in the teleportation experiment of Ref. [12]), we obtain $V_{\rm tel,in}^{\hat{X}}=V_{\rm tel,in}^{\hat{P}}=0.453$ and a fidelity $F=0.815$. The measured value for the "normalized analysis frequency" $\omega=0.56$ corresponds to the measured finesse $F_{\rm cav}=180$, the free spectral range $\nu_{\rm FSR}=790$ MHz and the spectrum analyzer frequency $\Omega/2\pi=1.1$ MHz [15].

In the teleportation experiment of Ref. [12], the teleported states described fields at modulation frequency $\Omega/2\pi=2.9$ MHz within a bandwidth $\pm\Delta\Omega/2\pi=30$ kHz. Due to technical noise at low modulation frequencies, the nonclassical fidelity was achieved at these higher frequencies Ω.

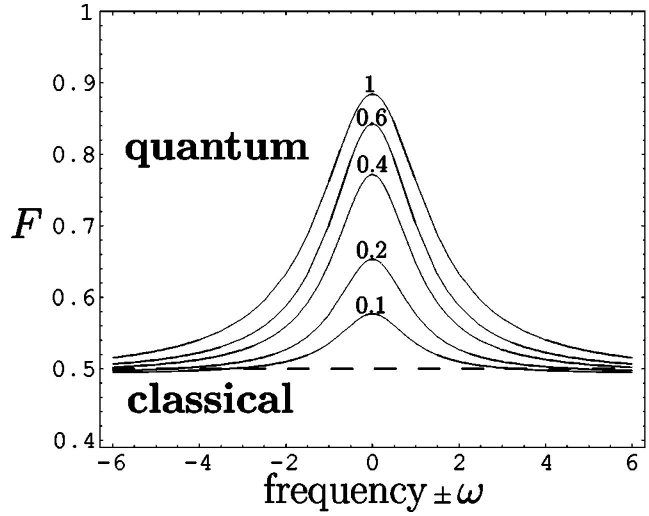

FIG. 7. Fidelity spectrum of coherent-state teleportation using entanglement from the NOPA. The fidelities here are functions of the normalized modulation frequency $\pm\omega$ for different parameter ϵ $(=0.1,\ 0.2,\ 0.4,\ 0.6,\text{ and }1)$. Bell detector efficiencies $\eta^2=0.97$ and cavity losses with $\beta=0.9$ have been included here.

The amount of squeezing at these frequencies was about 3 dB. The spectrum of the fidelities from Eq. (78) is shown in Fig. 7 for different ϵ values.

VII. SUMMARY AND CONCLUSIONS

We have presented the broadband theory for quantum teleportation using squeezed-state entanglement. Our scheme allows the broadband transmission of nonorthogonal quantum states. We have discussed various criteria determining the boundary between classical teleportation (i.e., measuring the state to be transmitted as well as quantum theory permits and classically conveying the results) and quantum teleportation (i.e., using entanglement for the state transfer). Depending on the set of input states, different criteria can be applied that are best met with the optimum gain used by Bob for the phase-space displacements of his EPR beam. Given an alphabet of arbitrary Gaussian states with unknown coherent amplitudes, on average, the optimum teleportation fidelity is attained with unit gain at all relevant frequencies. Optimal teleportation of an entangled state (entanglement swapping) requires a squeezing-dependent, and hence frequency-dependent, nonunit gain. Effectively, also with optimum gain, the bandwidth of entanglement becomes smaller after entanglement swapping compared to the bandwidth of entanglement of the initial states, as the quality of the entanglement deteriorates at each frequency for finite squeezing.

In the particular case of the NOPA as the entanglement source, the best quantum teleportation occurs in the frequency regime close to the center frequency (half the NOPA's pump frequency). In general, a suitable EPR source for broadband teleportation can be obtained by combining two independent broadband squeezed states at a beam splitter (actually, even one squeezed state split at a beam splitter is sufficient to create entanglement for quantum teleportation

[33,20]). Provided ideal Bell detection, unit-gain teleportation will then in general produce an excess noise in each teleported quadrature of twice the squeezing spectrum of the quiet quadrature in the corresponding broadband squeezed state (for the NOPA, cavity loss appears in the squeezing spectrum). Thus, good broadband teleportation requires good broadband squeezing. However, the entanglement source's squeezing spectrum for its quiet quadrature need not be a minimum near the center frequency ($\Omega=0$) as for the optical parametric oscillator. In general, it might have large excess noise there and be quiet at $\Omega\neq 0$ as for four-wave mixing in a cavity [31]. The spectral range to be teleported $\Delta\Omega$ always should be in the ''quiet region'' of the squeezing spectrum.

The scheme presented here allows very efficient teleportation of broadband quantum states: the quantum state at the input (a coherent, a squeezed, an entangled or any other state), describing the input field at modulation frequency Ω within a bandwidth $\Delta\Omega$, is teleported on each and every trial (where the duration of a single trial is given by the inverse-bandwidth time $1/\Delta\Omega$). Every inverse-bandwidth time, a quantum state is teleported with nonclassical fidelity or previously unentangled fields become entangled. Also the output of entanglement swapping can therefore be used for efficient quantum teleportation, succeeding every inverse-bandwidth time.

In contrast, the discrete-variable schemes involving weak down conversion enable only relatively rare transfers of quantum states. For the experiment of Ref. [5], a fourfold coincidence (i.e., ''successful'' teleportation [7]) at a rate of 1/40 Hz and a UV pulse rate of 80 MHz [34] yield an overall efficiency of 3×10^{-10} (events per pulse). Note that due to filtering and collection difficulties the photodetectors in this experiment operated with an effective efficiency of 10% [34].

The theory presented in this paper applies to the experiment of Ref. [12] where coherent states were teleported using the entanglement built from two squeezed fields generated via degenerate down conversion. The experimentally determined fidelity in this experiment was $F=0.58\pm 0.02$ (this fidelity was achieved at higher frequencies $\Omega\neq 0$ due to technical noise at low modulation frequencies) which proved the quantum nature of the teleportation process by exceeding the classical limit $F\leqslant\frac{1}{2}$. Our analysis was also intended to provide the theoretical foundation for the teleportation of quantum states that are more nonclassical than coherent states, e.g., squeezed states or, in particular, entangled states (two-mode squeezed states). This is yet to be realized in the laboratory.

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[1] W. K. Wootters and W. H. Zurek, Nature (London) **299**, 802 (1982).
[2] K. Kraus, *States, Effects, and Operations* (Springer-Verlag, Berlin, 1983).
[3] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
[4] S. L. Braunstein, A. Mann, and M. Revzen, Phys. Rev. Lett. **68**, 3259 (1992).
[5] D. Bouwmeester *et al.*, Nature (London) **390**, 575 (1997).
[6] D. Boschi *et al.*, Phys. Rev. Lett. **80**, 1121 (1998).
[7] S. L. Braunstein and H. J. Kimble, Nature (London) **394**, 840 (1998); D. Bouwmeester *et al.*, *ibid.* **394**, 841 (1998); P. Kok and S. L. Braunstein, Phys. Rev. A **61**, 042304 (2000); D. Bouwmeester *et al.*, e-print quant-ph/9910043.
[8] L. Vaidman, Phys. Rev. A **49**, 1473 (1994).
[9] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[10] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1994).
[11] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. **80**, 869 (1998).
[12] A. Furusawa *et al.*, Science **282**, 706 (1998).
[13] S. L. Braunstein, C. A. Fuchs, and H. J. Kimble, J. Mod. Opt. **47**, 267 (2000).
[14] S. L. Braunstein *et al.*, *International Quantum Electronics Conference*, Vol. 7 of the 1998 OSA Technical Digest Series (Optical Society of America, Washington, DC, 1998), p. 133.
[15] Z. Y. Ou, S. F. Pereira, and H. J. Kimble, Appl. Phys. B: Photophys. Laser Chem. **55**, 265 (1992).
[16] M. Zukowski *et al.*, Phys. Rev. Lett. **71**, 4287 (1993).
[17] J.-W. Pan *et al.*, Phys. Rev. Lett. **80**, 3891 (1998).
[18] R. E. S. Polkinghorne and T. C. Ralph, Phys. Rev. Lett. **83**, 2095 (1999).
[19] S. M. Tan, Phys. Rev. A **60**, 2752 (1999).
[20] P. van Loock and S. L. Braunstein, Phys. Rev. A **61**, 10 302 (2000).
[21] E. Arthurs and J. L. Kelly, Jr., Bell Syst. Tech. J. **44**, 725 (1965).
[22] Y. Yamamoto *et al.* (unpublished).
[23] U. Leonhardt, *Measuring the Quantum State of Light* (Cambridge University Press, Cambridge, England, 1997).
[24] T. C. Ralph and P. K. Lam, Phys. Rev. Lett. **81**, 5668 (1998).
[25] T. C. Ralph, R. E. S. Polkinghorne, and P. K. Lam, quant-ph/9903003 (unpublished).
[26] C. M. Caves *et al.*, Rev. Mod. Phys. **52**, 341 (1980); M. J. Holland *et al.*, Phys. Rev. A **42**, 2995 (1990).
[27] P. van Loock and S. L. Braunstein (in preparation).
[28] M. J. Collett and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984); **31**, 3761 (1985).
[29] C. M. Caves and B. L. Schumaker, Phys. Rev. A **31**, 3068 (1985).

[30] H. J. Kimble, in *Fundamental Systems in Quantum Optics, Les Houches, Session LIII, 1990*, edited by J. Dalibard, J. M. Raimond, and J. Zinn-Justin (Elsevier Science Publishers, Amsterdam, 1992), pp. 549–674.

[31] R. E. Slusher *et al.*, Phys. Rev. Lett. **55**, 2409 (1985).

[32] That indeed after entanglement swapping, accomplished by appropriate final displacements, the outgoing (average or ensemble) state of modes 1 and $4'$ is again a *pure* two-mode squeezed state with less squeezing than in the initial states is explained in more detail for single modes in a future publication [27].

[33] P. van Loock and S. L. Braunstein, Phys. Rev. Lett. **84**, 3482 (2000).

[34] H. Weinfurter (private communication).