Quantum teleportation of an entangled state

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We consider the teleportation of entangled two-particle and multiparticle states and present a scheme for the teleportation that may be suitable for both entangled atomic states or field states inside high-Q cavities.

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I. INTRODUCTION

The notions of coherent superposition and entanglement in quantum mechanics lie at the conceptual foundation of quantum mechanics as evident through fundamental contributions like bell inequalities [1] and Greenberger-Horne-Zeilinger (GHZ) equalities [2]. These alternative concepts are finding interesting and useful applications in the field of quantum computing and quantum information.

One of the key problems in quantum communication is how to transmit a quantum state from one observer to another and keep the received state exactly the same as that sent without necessitating the movement of an information carrier. This can be accomplished in two steps. First, the sender ''disassembles'' information of a quantum state into two parts, one of which is sent through a quantum channel run by the nonlocal correlation between two entangled quantum entities, and the other of which is sent through a classical channel. Second, the receiver ''reconstructs'' the state on the basis of information from both the quantum and classical channels. Because in this process a quantum state to be transmitted is destroyed in one place and later reconstructed in another place, this transmission is termed as teleportation of a quantum state. Bennett *et al.* [3] proposed a scheme for the teleportation of an unknown quantum state from one observer to another through dual Einstein-Podolsky-Rosen (EPR) and classical channels.

Since this proposal was made, a number of experimentally feasible schemes have been suggested for the teleportation of two-level atomic states [4–13] and field states [14–16] for two-dimensional states to N-dimensional states [17]. Most of these schemes rely on methods based on cavity quantum electrodynamics in which two identical high-Q cavities are initially prepared in an entangled state. Quantum teleportation was experimentally verified by producing pairs of entangled photons through the process of parametric down-conversion [18]. Recently, a scheme has been presented that exploits the cavity decay for the teleportation of the atomic state of an atom trapped in a leaky cavity [19]. In addition to these schemes of discrete variables, much progress has also been made for the quantum teleportation of states of dynamical variables with continuous spectra [20–22]. The teleportation of a coherent state of the radiation field [23] and the teleportation of a superposition of chiral amplitudes have also been reported [24].

All these schemes are for the teleportation of single-qubit states. In many potential applications of quantum computing, such as factorizing a very large number [25] or searching an unordered quantum database [26], one needs the system of many-qubit states. It is therefore an interesting question whether we can teleport a multiqubit state. In this paper, we present a scheme for the teleportation of a two-particle (two-qubit) entangled state from a pair of high-Q cavities to another pair of high-Q cavities using methods based on cavity quantum electrodynamics. This scheme is then generalized for the teleportation of the N-qubit field state.

II. QUANTUM TELEPORTATION OF AN ENTANGLED STATE

In this section, we consider the teleportation of a two-qubit entangled state of the radiation field in two separated high-Q cavities A_1 and A_2 to another pair of high-Q cavities C_1 and C_2. The entangled state of the radiation field is assumed to be

$$|\psi(A_1,A_2)\rangle=\sum_{p_1,p_2=0}^{1} C_{p_1p_2}|p_1,p_2\rangle. \tag{1}$$

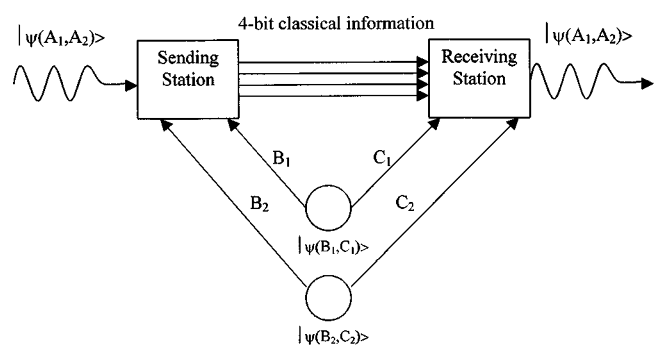

FIG. 1. Quantum teleportation of the two-qubit state $|\psi(A_1A_2)\rangle=\Sigma_{n_1,n_2=0}^{1}C_{n_1,n_2}|n_1,n_2\rangle$. $|\psi(B_1C_1)\rangle$ and $|\psi(B_2C_2)\rangle$ are two entangled states. Cavities B_1 and B_2 belong to the sending station while cavities C_1 and C_2 belong to the receiving station. A four-bit piece of classical information transmitted from the sending station to the receiving station enables the receiver to reconstruct the original state.

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It may be pointed out that this scheme also corresponds to the teleportation of entangled two-level atomic states because the atomic entanglement can be transferred to the two cavities by passing them through the two cavities with π pulse. As usual, the teleportation of state (1) can be carried out in three steps, as shown in Fig. 1.

In the first step, we consider the other two sets of cavities, B_1, C_1 and B_2, C_2, prepared in entangled states:

$$|\psi(B_1C_1)\rangle=\frac{1}{\sqrt{2}}(|0_{B_1},1_{C_1}\rangle+|1_{B_1},0_{C_1}\rangle), \tag{2}$$

$$|\psi(B_2C_2)\rangle=\frac{1}{\sqrt{2}}(|0_{B_2},1_{C_2}\rangle+|1_{B_2},0_{C_2}\rangle). \tag{3}$$

We then have

$$|\psi(B_1B_2C_1C_2)\rangle=\frac{1}{2}[|0_{B_1},0_{B_2},1_{C_1},1_{C_2}\rangle+|0_{B_1},1_{B_2},1_{C_1},0_{C_2}\rangle +|1_{B_1},0_{B_2},0_{C_1},1_{C_2}\rangle+|1_{B_1},1_{B_2},0_{C_1},0_{C_2}\rangle]. \tag{4}$$

It is important to note here that for the teleportation of a two-qubit quantum state we do not need to prepare an entangled state of four qubits. Instead, we need two entangled states of two qubits each. The combined state of the fields in the cavities A_1, A_2, B_1, B_2, C_1, and C_2 is therefore given as

$$|\psi(A_1A_2B_1B_2C_1C_2)\rangle=\tfrac{1}{2}\sum_{p_1,p_2=0}^{1} C_{p_1p_2}|p_1\rangle_{A_1}|p_2\rangle_{A_2} \times(|0_{B_1},0_{B_2},1_{C_1},1_{C_2}\rangle+|0_{B_1},1_{B_2},1_{C_1},0_{C_2}\rangle+|1_{B_1},0_{B_2},0_{C_1},1_{C_2}\rangle+|1_{B_1},1_{B_2},0_{C_1},0_{C_2}\rangle). \tag{5}$$

Next we define the basis states for the $A_1A_2B_1B_2$ system:

$$|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle=\tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_1},1_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},1_{B_1},0_{B_2}\rangle +e^{i\pi j_1}|1_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle+e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle), \tag{6}$$

$$|\psi_{j_1,j_2,0,1}(A_1A_2B_1B_2)\rangle=\tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_1},0_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},1_{B_1},1_{B_2}\rangle +e^{i\pi j_1}|1_{A_1},0_{A_2},0_{B_1},0_{B_2}\rangle+e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_1},1_{B_2}\rangle), \tag{7}$$

$$|\psi_{j_1,j_2,1,0}(A_1A_2B_1B_2)\rangle=\tfrac{1}{2}(|0_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle +e^{i\pi j_1}|1_{A_1},0_{A_2},1_{B_1},1_{B_2}\rangle+e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},1_{B_1},0_{B_2}\rangle), \tag{8}$$

$$|\psi_{j_1,j_2,1,1}(A_1A_2B_1B_2)\rangle=\tfrac{1}{2}(|0_{A_1},0_{A_2},0_{B_1},0_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_1},1_{B_2}\rangle +e^{i\pi j_1}|1_{A_1},0_{A_2},1_{B_1},0_{B_2}\rangle+e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},1_{B_1},1_{B_2}\rangle), \tag{9}$$

where $j_1,j_2=0,1$. We therefore have 16 basis states. The combined state $|\psi(A_1A_2B_1B_2C_1C_2)\rangle$ can be rewritten as a linear superposition of the basis states $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ of the $A_1A_2B_1B_2$ system as follows:

$$\begin{aligned}|\psi(A_1A_2B_1B_2C_1C_2)\rangle=\sum_{j_1,j_2=0}^{1}&|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle(C_{00}|0_{C_1},0_{C_2}\rangle+C_{01}e^{i\pi j_2}|0_{C_1},1_{C_2}\rangle+C_{10}e^{i\pi j_1}|1_{C_1},0_{C_2}\rangle\\ &+C_{11}e^{i\pi(j_1+j_2)}|1_{C_1},1_{C_2}\rangle)+|\psi_{j_1,j_2,0,1}(A_1A_2B_1B_2)\rangle(C_{00}|0_{C_1},1_{C_2}\rangle+C_{01}e^{i\pi j_2}|0_{C_1},0_{C_2}\rangle\\ &+C_{10}e^{i\pi j_1}|1_{C_1},1_{C_2}\rangle+C_{11}e^{i\pi(j_1+j_2)}|1_{C_1},0_{C_2}\rangle)\\ &+|\psi_{j_1,j_2,1,0}(A_1A_2B_1B_2)\rangle(C_{00}|1_{C_1},0_{C_2}\rangle+C_{01}e^{i\pi j_2}|1_{C_1},1_{C_2}\rangle+C_{10}e^{i\pi j_1}|0_{C_1},0_{C_2}\rangle\\ &+C_{11}e^{i\pi(j_1+j_2)}|0_{C_1},1_{C_2}\rangle)+|\psi_{j_1,j_2,1,1}(A_1A_2B_1B_2)\rangle(C_{00}|1_{C_1},1_{C_2}\rangle+C_{01}e^{i\pi j_2}|1_{C_1},0_{C_2}\rangle\\ &+C_{10}e^{i\pi j_1}|0_{C_1},1_{C_2}\rangle+C_{11}e^{i\pi(j_1+j_2)}|0_{C_1},0_{C_2}\rangle).\end{aligned} \tag{10}$$

In the second step, we make a measurement of the $A_1A_2B_1B_2$ system. A detection of the $A_1A_2B_1B_2$ system in the state $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ projects the field state in the cavities C_1C_2 into

$$|\psi(C_1C_2)\rangle=\sum_{p_1,p_2=0}^{1} e^{i\pi(j_1p_1+j_2p_2)}C_{p_1p_2}|(k_1+p_1)\text{mod}2\rangle_{C_1}\times|(k_2+p_2)\text{mod}2\rangle_{C_2}. \tag{11}$$

The field state in the cavities C_1C_2 has thus been projected to a state that has all the information about the amplitudes $C_{p_1p_2}$.

In the third and final step of the quantum teleportation, a manipulation of the cavities C_1C_2 needs to be done to bring state (11) to from (1). In the following sections we give the details of these three steps.

A. Preparation of entangled states

In the first step, we prepare two pairs of cavities B_1,C_1 and B_2,C_2 in entangled states (2) and (3). This can be done by passing a two-level atom initially in the excited state through the two resonant cavities. The interaction times of an atom with two cavities are chosen to be such that we have a $\pi/2$ pulse in the first cavity and a π pulse in the second cavity [6]. Initially, the two cavities B_1 and C_1 are taken in a vacuum and the two-level atom is taken in an excited state $|a\rangle$. When the atom has undergone a $\pi/2$ pulse in the first cavity, the second cavity is still empty and the atom-field system is in a state that corresponds to a linear superposition with equal weights of $|a\rangle$ and $|b\rangle$ atomic states correlated to zero and one photon, respectively, in cavity B_1 as

$$|\psi(B_1C_1)\rangle=\frac{1}{\sqrt{2}}(|b,1_{B_1}\rangle+|a,0_{B_1}\rangle)\otimes|0_{C_1}\rangle. \tag{12}$$

If the atom is still in an excited state $|a\rangle$ after leaving cavity B_1 in its vacuum state, it will, with unit probability, be transferred to $|b\rangle$ by the π pulse in cavity C_1 and leave a photon in the second cavity. If it emits a photon in cavity B_1 and exits it in level $|b\rangle$, it will be unaffected by its coupling with the vacuum in cavity C_1 and will leave the second cavity empty. Thus the atom always exits from second cavity C_1 in state $|b\rangle$, while the field is left in the entangled state (2). Similarly, we prepare another pair of cavities, B_2C_2, in entangled state (3).

B. Measurement of $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$

The second step of the teleportation is the measurement of the Bell states $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ of the $A_1A_2B_1B_2$ system. The subscripts j_1, j_2, k_1, and k_2 have the values 0 and 1. Among these, k_1 and k_2 can be determined by the number of photons inside the four cavities, while j_1 and j_2 can be determined from the relative phase of the states. Thus the state $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ can be determined in two sets of measurements, the first determining k_1 and k_2 via the total number of photons inside the cavities, and the second determining j_1 and j_2 via the relative phase. It is clear that the cavities in state $|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle$ have two photons, those in states $|\psi_{j_1,j_2,0,1}(A_1A_2B_1B_2)\rangle$ and $|\psi_{j_1,j_2,1,0}(A_1A_2B_1B_2)\rangle$ have one or three photons, while in state $|\psi_{j_1,j_2,1,1}(A_1A_2B_1B_2)\rangle$ they have zero, two, or four photons.

There are a number of ways to determine the number of photons inside the cavities. We propose the use of Ramsey interferometry. In this scheme, we consider two-level atoms initially prepared in ground state $|b\rangle$ that are off-resonant with the radiation field inside the cavities. The cavities are placed between two classical microwave fields (Ramsey zones R_1 and R_2) driving the $|a\rangle\rightarrow|b\rangle$ transition. When an atom passes from the first zone R_1 with a microwave field tuned at frequency ω_{ab}, it is prepared in a coherent superposition of states $(|a\rangle+|b\rangle)/\sqrt{2}$. This atom is then passed through the two selected cavities with the same interaction time θ in each cavity. During the passage through the cavities, a phase shift proportional to the photon number s in the two cavities is introduced as a phase of the state $|b\rangle$ [27]. The resulting state of the atom then becomes

$$\frac{1}{\sqrt{2}}[|a\rangle+e^{is\theta}|b\rangle]. \tag{13}$$

The atom is then passed through the second zone R_2, again resonant with ω_{ab}. The interaction time and the coupling parameters are chosen such that $|a\rangle\rightarrow(|a\rangle+|b\rangle)/\sqrt{2}$ and $|b\rangle\rightarrow(|a\rangle-|b\rangle)/\sqrt{2}$. The final atomic state is

$$e^{is\theta/2}[\cos(s\theta/2)|a\rangle-i\sin(s\theta/2)|b\rangle]. \tag{14}$$

The complete atom-field state is entangled and rather complicated. We have therefore not reproduced it here. It is, however, clear that a measurement of the atom in state $|a\rangle$ or $|b\rangle$ would reduce the fields inside the cavities to states with only an appropriate number of total photons in the two cavities.

The first atom is sent through the two cavities A_1 and B_1 with the interaction time $\theta=\pi$ in each cavity. It follows from Eq. (13) that if the atom is found in the excited state $|a\rangle$, the total number of photons in the two cavities is even, i.e., 0, 2. This implies $k_1=1$, $k_2=0$ or $k_1=1$, $k_2=1$. If the atom is detected in state $|b\rangle$, then the total number of photons in the two cavities is odd and $k_1=0$, $k_2=0$ or $k_1=0$, $k_2=1$. In the next step we make a measurement in the cavities A_2 and B_2 only with the same interaction time. A detection of an atom in either the excited state $|a\rangle$ or the ground state $|b\rangle$ completely determines the values of k_1 and k_2 according to the following sequence:

$$|a\rangle|a\rangle\Rightarrow|\psi_{j_1,j_2,1,1}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|b\rangle\Rightarrow|\psi_{j_1,j_2,1,0}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle\Rightarrow|\psi_{j_1,j_2,0,1}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle\Rightarrow|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle.$$

For the determination of phase factors j_1 and j_2 we make measurements in the cavities A_1 and A_2 only after first evacuating the cavities B_1 and B_2. However, during the process of "emptying" the cavities B_1 and B_2, the relative phase between the component states in the resulting state $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$ may change. There are a number of ways to remove the photons from the cavities B_1 and B_2.

Here we consider two two-level atoms initially in their ground states $|b\rangle$. One of the atoms is sent through the cavity B_1 and the other through cavity B_2. After the passage, the atomic internal states $|a\rangle$ and $|b\rangle$ are mixed by a classical field such that $|a\rangle\rightarrow(|a\rangle+|b\rangle)/\sqrt{2}$ and $|b\rangle\rightarrow(|a\rangle-|b\rangle)/\sqrt{2}$. A subsequent detection of these atoms in states $|a\rangle$ or $|b\rangle$ introduces phase factors. To see this clearly, we take $k_1=k_2=0$ for the sake of simplicity. Similar arguments will, however, apply for other values of k_1 and k_2. First we consider the passage of atoms through cavity B_1 only. The initial state is therefore

$$\begin{aligned}|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle\otimes|\text{atom}\rangle\\ =\tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_1},1_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},1_{B_1},0_{B_2}\rangle\\ +e^{i\pi j_1}|1_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle\\ +e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle)\otimes|b\rangle.\end{aligned}\tag{15}$$

The removal of a photon from B_1 followed by mixing of the atomic levels by the classical field yields

$$\begin{aligned}|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle\otimes|\text{atom}\rangle\\ =\tfrac{1}{2}[(|0_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle\\ +e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle)\otimes\frac{1}{\sqrt{2}}(|a\rangle+|b\rangle)\\ +(e^{i\pi j_1}|1_{A_1},0_{A_2},0_{B_1},1_{B_2}\rangle\\ +e^{i\pi j_2}|1_{A_1},1_{A_2},0_{B_1},0_{B_2}\rangle)\otimes\frac{1}{\sqrt{2}}(|a\rangle-|b\rangle)].\end{aligned}\tag{16}$$

The detection of the atom in level $|a\rangle$ gives

$$\begin{aligned}|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle\\ =\tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_2}\rangle+e^{i\pi j_1}|1_{A_1},0_{A_2},1_{B_2}\rangle\\ +e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_2}\rangle)\otimes|0_{B_1}\rangle,\end{aligned}\tag{17}$$

whereas the detection of the atom in level $|b\rangle$ gives

$$\begin{aligned}|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle\\ =\tfrac{1}{2}(|0_{A_1},0_{A_2},1_{B_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2},0_{B_2}\rangle\\ -e^{i\pi j_1}|1_{A_1},0_{A_2},1_{B_2}\rangle-e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2},0_{B_2}\rangle)\otimes|0_{B_1}\rangle.\end{aligned}\tag{18}$$

By a similar procedure, the photon can be removed from the cavity B_2 and the resulting cavity field state will have phase factors according to the final outcome of the atomic state. Here we summarize the final outcome depending upon the sequence of atom states for the removal of photons from cavity B_1 and cavity B_2:

$$\begin{aligned}|a\rangle|a\rangle\rightarrow\tfrac{1}{2}(|0_{A_1},0_{A_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2}\rangle+e^{i\pi j_1}|1_{A_1},0_{A_2}\rangle\\ +e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2}\rangle)\otimes|0_{B_1},0_{B_2}\rangle,\\ |a\rangle|b\rangle\rightarrow\tfrac{1}{2}(|0_{A_1},0_{A_2}\rangle-e^{i\pi j_2}|0_{A_1},1_{A_2}\rangle+e^{i\pi j_1}|1_{A_1},0_{A_2}\rangle\\ -e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2}\rangle)\otimes|0_{B_1},0_{B_2}\rangle,\\ |b\rangle|a\rangle\rightarrow\tfrac{1}{2}(|0_{A_1},0_{A_2}\rangle+e^{i\pi j_2}|0_{A_1},1_{A_2}\rangle-e^{i\pi j_1}|1_{A_1},0_{A_2}\rangle\\ -e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2}\rangle)\otimes|0_{B_1},0_{B_2}\rangle,\\ |b\rangle|b\rangle\rightarrow\tfrac{1}{2}(|0_{A_1},0_{A_2}\rangle-e^{i\pi j_2}|0_{A_1},1_{A_2}\rangle-e^{i\pi j_1}|1_{A_1},0_{A_2}\rangle\\ +e^{i\pi(j_1+j_2)}|1_{A_1},1_{A_2}\rangle)\otimes|0_{B_1},0_{B_2}\rangle.\end{aligned}\tag{19}$$

This completes the procedure of evacuating cavities B_1 and B_2. The resulting state can have different but known phase factors between the constituent states. The net effect is equivalent to a transformation to a different basis. Next we make measurements in the cavities A_1 and A_2 in order to determine the phase factors j_1 and j_2. For simplicity's sake, we assume that the first two atoms are detected in state $|a\rangle$.

We now remove photons from cavities A_1 and A_2 by a similar procedure, i.e., by sending two-level atoms in their ground state $|b\rangle$ followed again by a classical field that mixes the levels such that $|a\rangle\rightarrow(|a\rangle+|b\rangle)/\sqrt{2}$ and $|b\rangle\rightarrow(|a\rangle-|b\rangle)/\sqrt{2}$:

$$\begin{aligned}|\psi_{j_1,j_2,0,0}(A_1A_2B_1B_2)\rangle\otimes|\text{atom}\rangle\\ =\frac{1}{2\sqrt{2}}(|0_{A_2}\rangle+e^{i\pi j_2}|1_{A_2}\rangle)\\ \times[(1+e^{i\pi j_1})|a\rangle-(1-e^{i\pi j_1})|b\rangle]\otimes|0_{A_1},0_{B_1},0_{B_2}\rangle.\end{aligned}\tag{20}$$

If the atom is detected in $|a\rangle$ then $j_1=0$, and if atom is detected in $|b\rangle$ then $j_1=1$. The resulting cavity field state is

$$\begin{aligned}|\psi_{j_2,0,0}(A_1A_2B_1B_2)\rangle=\frac{1}{\sqrt{2}}(|0_{A_2}\rangle+e^{i\pi j_2}|1_{A_2}\rangle)\\ \otimes|0_{A_1},0_{B_1},0_{B_2}\rangle.\end{aligned}\tag{21}$$

Finally, we send the atom through the cavity A_2 and repeat the same procedure. If the atom is found in state $|a\rangle$ then $j_2=0$, and if the atom is detected in $|b\rangle$ then $j_2=1$. So by making measurements only in cavities A_1 and A_2 by first removing one photon from cavity A_1 and then removing one photon from cavity A_2, detection of the atom in different states yields the different values of j_1 and j_2 as

$$|a\rangle|a\rangle|a\rangle|a\rangle\rightarrow j_1=0,\quad j_2=0\Rightarrow|\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|a\rangle|a\rangle|b\rangle\rightarrow j_1=0,\quad j_2=1\Rightarrow|\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|a\rangle|b\rangle|a\rangle\rightarrow j_1=1,\quad j_2=0\Rightarrow|\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|a\rangle|b\rangle|b\rangle\rightarrow j_1=1,\quad j_2=1\Rightarrow|\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle.$$

If we have other sequences of detection of the first two atoms, then by doing the same process detection of the atom in different states gives the different values of j_1 and j_2 as shown below:

$$|a\rangle|b\rangle|a\rangle|a\rangle\rightarrow j_1=0,\quad j_2=1\Rightarrow|\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|b\rangle|a\rangle|b\rangle\rightarrow j_1=0,\quad j_2=0\Rightarrow|\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|b\rangle|b\rangle|a\rangle\rightarrow j_1=1,\quad j_2=1\Rightarrow|\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|a\rangle|b\rangle|b\rangle|b\rangle\rightarrow j_1=1,\quad j_2=0\Rightarrow|\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle|a\rangle|a\rangle\rightarrow j_1=1,\quad j_2=0\Rightarrow|\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle|a\rangle|b\rangle\rightarrow j_1=1,\quad j_2=1\Rightarrow|\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle|b\rangle|a\rangle\rightarrow j_1=0,\quad j_2=0\Rightarrow|\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|a\rangle|b\rangle|b\rangle\rightarrow j_1=0,\quad j_2=1\Rightarrow|\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle|a\rangle|a\rangle\rightarrow j_1=1,\quad j_2=1\Rightarrow|\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle|a\rangle|b\rangle\rightarrow j_1=1,\quad j_2=0\Rightarrow|\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle|b\rangle|a\rangle\rightarrow j_1=0,\quad j_2=1\Rightarrow|\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle,$$

$$|b\rangle|b\rangle|b\rangle|b\rangle\rightarrow j_1=0,\quad j_2=0\Rightarrow|\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle.$$

We can summarize from the above equations that if the order of detection of the first two atoms is the same as the last two, then we have $j_1=0$ and $j_2=0$ and the state is $|\psi_{0,0,k_1,k_2}(A_1A_2B_1B_2)\rangle$. If the detection of atomic states is the same for the first and third atom and detection of the fourth atom is reversed with respect to the second atom, then $j_1=0$ and $j_2=1$ and the state is $|\psi_{0,1,k_1,k_2}(A_1A_2B_1B_2)\rangle$. If the detection of the atomic states is the same for the second and fourth atom and detection of the third atom is reversed with respect to the first atom, then $j_1=1$ and $j_2=0$ and the state is $|\psi_{1,0,k_1,k_2}(A_1A_2B_1B_2)\rangle$. If the order of detection of atomic states for the third and fourth atom is reversed with respect to the first and second atom, respectively, then $j_1=1$ and $j_2=1$ and the state is $|\psi_{1,1,k_1,k_2}(A_1A_2B_1B_2)\rangle$.

A determination of the entangled state of the field inside the cavities A_1, A_2, B_1, and B_2, say, in state $|\psi_{j_1,j_2,k_1,k_2}(A_1A_2B_1B_2)\rangle$, projects the state of the field in cavities C_1 and C_2 into the state $|\psi(C_1C_2)\rangle$ as given by Eq. (11). In the final step of the teleportation, we transform this state into the original state (1).

C. Transformation

The transformation of state $|\psi(C_1C_2)\rangle$ given by Eq. (11) into that given by Eq. (1) involves two steps. One is the removal of phases $\exp(i\pi j_1)$ and $\exp(i\pi j_2)$ and the other is an appropriate transformation of photon numbers.

First we consider the transformation of phase only. For the sake of simplicity, we take $k_1=0$ and $k_2=0$. We then have

$$|\psi(C_1C_2)\rangle=C_{00}|0_{C_1},0_{C_2}\rangle+C_{01}e^{i\pi j_2}|0_{C_1},1_{C_2}\rangle+C_{10}e^{i\pi j_1}|1_{C_1},0_{C_2}\rangle+C_{11}e^{i\pi(j_1+j_2)}|1_{C_1},1_{C_2}\rangle. \tag{22}$$

(i) If $j_1=0$ and $j_2=0$, then the state $|\psi(A_1A_2)\rangle$ is recovered.

(ii) If $j_1=0$ and $j_2=1$, then

$$|\psi(C_1C_2)\rangle=C_{00}|0_{C_1},0_{C_2}\rangle+C_{01}e^{i\pi}|0_{C_1},1_{C_2}\rangle+C_{10}|1_{C_1},0_{C_2}\rangle+C_{11}e^{i\pi}|1_{C_1},1_{C_2}\rangle. \tag{23}$$

An atom in a superposition state $[|a\rangle+|b\rangle]/\sqrt{2}$ is passed through the cavity C_2 only in such a way that the ground state $|b\rangle$ picks the phase $\exp(ip\pi)$ (p being the number of photons inside the cavity C_2) while the excited state $|a\rangle$ does not pick any additional phase. We then have

$$|\psi(C_1C_2)\rangle=\frac{1}{\sqrt{2}}(C_{00}|0_{C_1},0_{C_2}\rangle-C_{01}|0_{C_1},1_{C_2}\rangle+C_{10}|1_{C_1},0_{C_2}\rangle-C_{11}|1_{C_1},1_{C_2}\rangle)|a\rangle+\frac{1}{\sqrt{2}}(C_{00}|0_{C_1},0_{C_2}\rangle+C_{01}|0_{C_1},1_{C_2}\rangle+C_{10}|1_{C_1},0_{C_2}\rangle+C_{11}|1_{C_1},1_{C_2}\rangle)|b\rangle. \tag{24}$$

If the atom is detected in $|b\rangle$ after the passage through cavity C_2 then the state $|\psi(A_1A_2)\rangle$ is recovered. If the atom is detected in state $|a\rangle$ then repeat the process until the atom is detected in $|b\rangle$.

(iii) For $j_1=1$ and $j_2=0$,

$$|\psi(C_1C_2)\rangle=C_{00}|0_{C_1},0_{C_2}\rangle+C_{01}|0_{C_1},1_{C_2}\rangle+C_{10}e^{i\pi}|1_{C_1},0_{C_2}\rangle+C_{11}e^{i\pi}|1_{C_1},1_{C_2}\rangle. \tag{25}$$

We carry out the same process again, but this time we pass the atom through cavity C_1 only. With the detection of the atom in state $|b\rangle$, we recover the required state.

(iv) For $j_1=1$ and $j_2=1$,

$$|\psi(C_1C_2)\rangle = C_{00}|0_{C_1},0_{C_2}\rangle + C_{01}e^{i\pi}|0_{C_1},1_{C_2}\rangle + C_{10}e^{i\pi}|1_{C_1},0_{C_2}\rangle + C_{11}|1_{C_1},1_{C_2}\rangle. \quad (26)$$

Again, the same procedure is repeated except that the atom passes through both cavities. As before, the detection of the atom in state $|b\rangle$ will recover the required state; otherwise, we repeat the process until it is detected in state $|b\rangle$.

Next we consider the transformation of photon numbers in the cavities. As phase is removed by the method discussed above, we take $j_1=j_2=0$ for simplicity's sake:

(i) For $k_1=0$ and $k_2=0$,

$$|\psi(C_1C_2)\rangle = C_{00}|0_{C_1},0_{C_2}\rangle + C_{01}|0_{C_1},1_{C_2}\rangle + C_{10}|1_{C_1},0_{C_2}\rangle + C_{11}|1_{C_1},1_{C_2}\rangle, \quad (27)$$

and the original state is recovered and we do nothing further.

(ii) For $k_1=0$ and $k_2=1$,

$$|\psi(C_1C_2)\rangle = C_{00}|0_{C_1},1_{C_2}\rangle + C_{01}|0_{C_1},0_{C_2}\rangle + C_{10}|1_{C_1},1_{C_2}\rangle + C_{11}|1_{C_1},0_{C_2}\rangle. \quad (28)$$

In order to recover the original state (1), we should interchange the state between zero and one photon in cavity C_2. For this purpose, we pass a two-level atom in its ground state $|b\rangle$ through cavity C_2 with a π pulse followed by its passage through a classical field again with a π pulse ($|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |a\rangle$) and finally through an empty cavity C_2' such that the atom in excited state $|a\rangle$ leaves the cavity in ground state $|b\rangle$ while leaving one photon inside the cavity and the atom in ground state $|b\rangle$ leaves the cavity in the ground state with no photon inside the cavity. This leads to the field states in the cavities C_1 and C_2' in the entangled state (1) and the teleportation is complete.

(iii) For $k_1=1$ and $k_2=0$,

$$|\psi(C_1C_2)\rangle = C_{00}|1_{C_1},0_{C_2}\rangle + C_{01}|1_{C_1},1_{C_2}\rangle + C_{10}|0_{C_1},0_{C_2}\rangle + C_{11}|0_{C_1},1_{C_2}\rangle. \quad (29)$$

We carry out the same procedure as above with the only difference being that the atom is passed through cavity C_1.

(iv) For $k_1=1$ and $k_2=1$,

$$|\psi(C_1C_2)\rangle = C_{00}|1_{C_1},1_{C_2}\rangle + C_{01}|1_{C_1},0_{C_2}\rangle + C_{10}|0_{C_1},1_{C_2}\rangle + C_{11}|0_{C_1},0_{C_2}\rangle. \quad (30)$$

Here we carry out the above procedure independently for the two cavities C_1 and C_2.

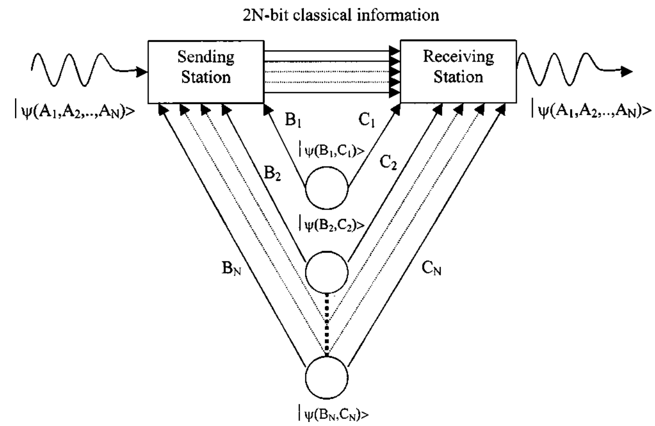

FIG. 2. Quantum teleportation of the N-qubit state $|\psi(A_1A_2\ldots A_N)\rangle = \Sigma^1_{n_1,n_2,\ldots,n_N=0} C_{n_1,n_2,\ldots,n_N}|n_1,n_2,\ldots,n_N\rangle$. $|\psi(B_iC_i)\rangle$ are N entangled states. Cavities B_i ($i=1,2,\ldots,N$) belong to sender while cavities C_i ($i=1,2,\ldots,N$) are with the receiver. A $2N$-bit piece of classical information transmitted from the sending station to the receiving station enables the receiver to reconstruct the original state.

III. TELEPORTATION OF THE N-QUBIT FIELD STATE

After giving a scheme to teleport the two-qubit state, we would like to generalize this scheme for the N-qubit state as shown in Fig. 2. Let us consider a N-qubit entangled field state in N high-Q cavities as

$$|\psi(A_1\ldots A_N)\rangle = \sum_{n_1,\ldots,n_N=0}^{1} C_{n_1,\ldots,n_N}|n_1,\ldots,n_N\rangle. \quad (31)$$

We want to teleport this entangled state in A_i ($i=1,2,\cdots,N$) high-Q cavities to C_i ($i=1,2,\ldots,N$) high-Q cavities.

In the first step of the teleportation of state (31), we need N pairs of entangled cavities

$$|\psi(B_iC_i)\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{B_i}|1\rangle_{C_i} + |1\rangle_{B_i}|0\rangle_{C_i}), \quad (32)$$

where $i=1,2,\ldots,N$. These N entangled pairs of cavities can be prepared as mentioned earlier by passing two-level atoms initially in the excited state through the two resonant cavities and by setting a $\pi/2$ pulse and a π pulse, respectively, in the two cavities. As before, cavities B_i ($i=1,2,\ldots,N$) are with the sender and cavities C_i ($i=1,2,\ldots,N$) belong to receiver. We now define 2^{2N} basis states in cavities $A_1A_2\ldots A_NB_1B_2\ldots B_N$ as

$$|\psi_{j_1,\ldots,j_N,k_2,\ldots,k_N}(A_1\ldots A_NB_1\ldots B_N)\rangle = \sum_{p_1,\ldots,p_N=0}^{1} \exp[i\pi(j_1p_1+j_2p_2+\cdots+j_Np_N)] \times |p_1\rangle_{A_1}|p_2\rangle_{A_2}\cdots|p_N\rangle_{A_N}|(1-p_1-k_1)\text{mod}2\rangle_{B_1} \times|(1-p_2-k_2)\text{mod}2\rangle_{B_2}\times\cdots\times|(1-p_N-k_N)\text{mod}2\rangle_{B_N}$$

$$= \sum_{p_1,\ldots,p_N=0}^{1} \prod_{m=1}^{N} [e^{i\pi j_m p_m}|p_m\rangle_{A_m} \times |(1-p_m-k_m)\text{mod}2\rangle_{B_m}]. \tag{33}$$

The combined state in the cavities $A_1\ldots A_N B_1\ldots B_N C_1\ldots C_N$ in terms of basis states can be written as

$$|\psi(A_1\ldots A_N B_1\ldots B_N C_1\ldots C_N)\rangle = \sum_{j_1,\ldots,j_N=0}^{1} \sum_{k_1,\ldots,k_N=0}^{1} \sum_{p_1,\ldots,p_N=0}^{1} \times C_{p_1,\ldots,p_N}|\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1\ldots A_N B_1\ldots B_N)\rangle \times \prod_{m=1}^{N} e^{i\pi j_m p_m}|(p_m+k_m)\text{mod}2\rangle_{C_m}. \tag{34}$$

We now make measurement of the 2^{2N} basis states of the $A_1\ldots A_N B_1\ldots B_N$ system. It has $2N$ parameters; N parameters correspond to the phase, while the remaining N parameters correspond to the photon numbers inside the cavities $A_1,\ldots A_N, A_N, B_1,\ldots,B_N$. Thus the state $|\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1\ldots A_N B_1\ldots B_N)\rangle$ can be determined in two sets of measurements, the first determining $k_1,k_2,\ldots,k_N$ via the total number of photons inside the cavities, and the second determining $j_1,j_2,\ldots,j_N$ via the relative phase. For the determination of photon numbers we use Ramsey interferometry. We send an atom in ground state $|b\rangle$ through two cavities A_1 and B_1 and two Ramsey zones R_1 and R_2 with interaction time $\theta=\pi$ in each cavity. The atom is resonant with the two Ramsey zones and off-resonant with the cavities. Detection of the atom in either the excited state $|a\rangle$ or the ground state $|b\rangle$ makes the probable outcomes of $|\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1\ldots A_N B_1\ldots B_N)\rangle$ to $N/2$ of total N values. We then send a second atom in the ground state through A_2 and B_2 with the same interaction time, which reduces the probable outcomes by half. Similarly, we continue the procedure and send the last atom through A_N and B_N. A detection of the atom in either the excited state $|a\rangle$ or the ground state $|b\rangle$ completely determines the values of $k_1,k_2,\ldots,k_N$ according to the following outcomes. For example, k_n is equal to 1 if the outcome of the nth atom is $|a\rangle$, and k_n is zero if the outcome of the nth atom is $|b\rangle$. For example, if the outcome of each atom is $|b\rangle$, except the last outcome, i.e., $|a\rangle$, then the Bell state is $|\psi_{j_1,\ldots,j_N,0,0,\ldots,1}(A_1\ldots A_N B_1\ldots B_N)\rangle$.

For the determination of phase factors $j_1,j_2,\ldots,j_N$ we make a measurement in the cavities $A_1,A_2,\ldots,A_N$ only after evacuating the cavities $B_1,B_2,\ldots,B_N$. For this purpose we follow the same procedure used earlier for the two-qubit state. We send N two-level atoms initially in ground state $|b\rangle$ one by one through the cavities $B_1,B_2,\ldots,B_N$. After the passage through the cavity, the atomic internal states $|a\rangle$ and $|b\rangle$ are mixed by a classical field such that $|a\rangle\rightarrow(|a\rangle+|b\rangle)/\sqrt{2}$ and $|b\rangle\rightarrow(|a\rangle-|b\rangle)/\sqrt{2}$. A subsequent detection of these atoms in state $|a\rangle$ or $|b\rangle$ introduces phase factors yielding 2^N possible outcomes of atomic states. Next we make measurements in $A_1,A_2,\ldots,A_N$ in order to determine $j_1,j_2,\ldots,j_N$. We remove one photon from A_1 by sending a two-level atom in its ground state $|b\rangle$ followed again by a classical field that mixes the levels such that $|a\rangle\rightarrow(|a\rangle+|b\rangle)/\sqrt{2}$ and $|b\rangle\rightarrow(|a\rangle-|b\rangle)/\sqrt{2}$. Detection of the atom in $|a\rangle$ or $|b\rangle$ determines the value of j_1. It is zero if the atom is detected in $|a\rangle$ and one if the atom is detected in $|b\rangle$. We then repeat the process with other cavities. Finally we send the Nth atom in $|b\rangle$ from cavity A_N, after mixing and detection of the atom in $|a\rangle$ or $|b\rangle$ determines j_N. For each combination of the first step, while evacuating $B_1,B_2,\ldots,B_N$, we get 2^N combinations in the second step. Finally, we have a total of 2^{2N} different combinations. Each combination has $2N$ outcomes of atomic states—N outcomes each for evacuation of B_n and A_n. We compare the first N outcomes of any combination among the total of 2^{2N} with the last N outcomes of the same combination. When these are the same we get $j=0$ and when they are reversed with each other we have $j=1$. For example, if all the first N outcomes of a combination among 2^{2N} combinations are similar to the last N outcomes of the same combination then we have all j equal to 0. However, if any nth outcome of the first N outcomes is reversed with respect to the nth outcome of the last N then that $j_n=1$. If all the outcomes of the first N are reversed with all the outcomes of the last N of that combination then we have all j equal to 1. This completes the procedure of measuring the Bell states $|\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1\ldots A_N B_1\ldots B_N)\rangle$. A determination of Bell state $|\psi_{j_1,\ldots,j_N,k_1,\ldots,k_N}(A_1\ldots A_N B_1\ldots B_N)\rangle$ projects the state of the field in cavities $C_1,C_2,\ldots,C_N$ into the entangled state $|\psi(C_1\ldots C_N)\rangle$ as

$$|\psi(C_1\ldots C_N)\rangle = \sum_{p_1,\ldots,p_N=0}^{1} C_{p_1,\ldots,p_N} \times \prod_{m=1}^{N} e^{i\pi j_m p_m}|(p_m+k_m)\text{mod}2\rangle_{C_m}. \tag{35}$$

In the third and final step of the quantum teleportation, a manipulation of the cavities $C_1,C_2,\ldots,C_N$ needs to be done to bring state $|\psi(C_1C_2\ldots C_N)\rangle$ to form $|\psi(A_1A_2\ldots A_N)\rangle$. This transformation of state involves two steps. One is the removal of phases and the other is the appropriate transformation of photon numbers.

First we consider the transformation of phase only. It depends upon the value of j. If all j are 0, then we have to do nothing and the original state is recovered. However, if any j_n among N values of j is 1 then it has an additional phase with it. For the removal of this phase we send a two-level atom in a coherent superposition of states $|a\rangle$ and $|b\rangle$ through the cavity C_n in such a way that ground state $|b\rangle$ picks the phase. If the atom is detected in $|b\rangle$ then the original state is recovered, otherwise we have to repeat the process until it is detected in $|b\rangle$. If there are m values of j that are equal to 1 out of N values of j then we pass m atoms in coherent superposition of states $|a\rangle$ and $|b\rangle$ one by one from those m cavities and detect the atoms in ground state $|b\rangle$. If all the j are 1 then we pass N atoms in $(|a\rangle+|b\rangle)/\sqrt{2}$ from all N cavities and detect atoms in ground state $|b\rangle$.

Next we consider the transformation of photon numbers in the cavities. This transformation depends upon the values of k. If all the k are 0 then we have to do nothing and the original state is recovered. However, if any k_n among N values of k is 1, then we have to change 0 and 1 photon from cavity C_n. For this purpose we pass a two level-atom in its ground state $|b\rangle$ through cavity C_n with a π pulse followed by its passage through a classical field again with a π pulse. Finally the atom passes through an empty cavity C_n' such that the atom in excited state $|a\rangle$ leaves the cavity in ground state $|b\rangle$ while leaving one photon inside the cavity and the atom in ground state $|b\rangle$ leaves the cavity in ground state with no photon inside the cavity. This leads the field states in the cavities $C_1, C_2, \ldots, C_N$ in the entangled state (31) and the teleportation is complete. If there are m values of k that are equal to 1 out of N values then we repeat the same process as above by sending m two-level atoms one by one in ground state $|b\rangle$ from each m cavity and proceed further as mentioned earlier until the completion of the process. If all the k are 1 then we pass N atoms in the ground state from all N cavities followed by a classical field that mixes $|a\rangle$ and $|b\rangle$ as $|a\rangle\rightarrow|b\rangle$ and $|b\rangle\rightarrow|a\rangle$ and finally through N empty cavities. The field state in the cavities $C_1, C_2, \ldots, C_N$ have thus been projected to a state that has all the information about the amplitudes $C_{n_1,n_2,\ldots,n_N}$. This completes the transformation process and hence the teleportation of the N-qubit state.

IV. CONCLUSION

We have presented a scheme for the quantum teleportation of a two-qubit entangled state of the form (1) from a pair of cavities at the sender's end to another pair of cavities at the receiver's end. The scheme employs atomic interaction with high-Q cavities. We need two entangled states of two particles each for the teleportation of a two-particle entangled state. Sending one particle of each entangled state to the sender and the other particle to the receiver is sufficient to teleport the entangled state of two qubits. This scheme is then generalized for the teleportation of the N-qubit entangled state in N high-Q cavities of the form (31). For this purpose we need N entangled states of two qubits each. Sending one particle of each entangled state to the sending station and the other particle of that state to the receiving station is enough for the teleportation process.

The proposed scheme of teleportation consists of three steps. The first step involves preparation of quantum entangled states of type (2) and (3) between two high-Q cavities. The second and third steps involve optical Ramsey interferometry and single-photon transfer. All these require controlled interaction times between atoms and cavities, negligible cavity loss, and no spontaneous decay during the whole teleportation process. Controlling the interaction time in the cavities can easily be achieved by properly setting, through Stark field adjustment, the times during which atom is resonant with each cavity [6]. About the spontaneous decay we propose the Rydberg atom in circular states with principle quantum number $\approx$50. They have a long radiative lifetime (30 ms) and a very strong coupling to radiation [28]. A negligible cavity loss is also required during the whole process of teleportation. Cavity lifetimes for high-Q cavities should be long enough as all the interactions of atom with cavities should be completed before the cavity dissipation. High-quality factors of such cavities and control of atomic beams during the whole teleportation process may pose limitations on the suggested scheme.

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