Radiative properties of an atom in the vicinity of a mirror

Reza Matloob
Department of Physics, University of Kerman, Kerman, Iran
(Received 4 January 2000; published 19 July 2000)

The decay rate of an excited atom is described using Fermi's golden rule. Welton's interpretation of the Lamb shift is extended by introducing a damping term in the Heisenberg equation of motion associated with the fluctuation in the position of the electron. These expressions are related to the imaginary part of the vector potential Green function through the fluctuation dissipation theorem and Kubo's formula. The results are applied to the calculation of the radiative properties of an atom in the vicinity of a perfect mirror.

PACS number(s): 12.20.Ds, 42.50.−p, 03.65.−w

I. INTRODUCTION

The ground state of the quantized field in general and that of the electromagnetic field in particular motivates a new picture for the vacuum. The undeniable indications of the reality of the zero-point field are seen both on macroscopic objects, as the Casimir force [1–4], and on microscopic systems, as the radiative properties of atoms and molecules [4]. The Casimir effect can be discussed as the response of the vacuum against the presence of macroscopic objects immersed in it [5–12]. The characteristic radiative behaviors of microscopic systems are explained as the reaction of these systems against the existence of the zero-point field [4]. In other words, the theoretical interpretations of these effects are achieved by considering the presence of the vacuum field in the theory.

The conventional theory of quantum electrodynamics deals with isolated atomic systems in free space. This is of course an idealization of realistic situations in which atoms are always at finite distances of metallic or dielectric surfaces in a real environment. This apparently bears an error made in the context of high precision experiments aiming at measuring the fundamental atomic constants.

The presence of a boundary surface or surfaces gives rise to alterations of the electromagnetic field operators and subsequently to a variation of the structure of the fluctuating field of the vacuum. The effect is displayed in all the phenomena that originate basically in one way or another from the vacuum field. The level shifts and the decay rate of microscopic systems are both examined either for the different ways of thinking about the role of vacuum in these phenomena [13–18], or on the basis of the modification of the effects due to the alteration of the structure of the fluctuating field of the vacuum. The variation of the level shifts as well as the change in the decay rate of an excited atom in front of a mirror, inside a Fabry-Perot cavity, in the vicinity of a dielectric surface, and some other configurations are well known in the literature [19–23].

It seems that the existing variety of ways of dealing with these effects gives rise to simplifications of the theory and softens out the unnecessary complexity as far as possible. The medley of different approaches may also slightly improve the current status of our understanding of the effects. The present paper is in fact the extension of previous work [24] to the case in which a boundary surface is involved. We present the base formulation of the effects in Sec. II. The evaluation of the appropriate Green function needed for the later calculations is provided briefly in Sec. III. In Sec. IV we use the general expressions of Sec. II to calculate the level shifts and the decay rate of an atom in the vicinity of a perfect mirror. Finally, in the concluding section, Sec. V, the main points of the present formulation are summarized.

II. THE BASE CALCULATION

The full quantum electrodynamics description of the radiative properties of an atom requires the explicit form of the field operators. A difficulty usually arises when the modifications of these effects are to be examined inside a cavity or adjacent to a boundary surface. This is due to the intricate structure of the field expressions in the presence of a boundary surface or surfaces. The presentation of an alternative approach that does not involve the explicit form of the field operators is consequently of special concern.

A. The decay rate of an excited atom

The decay rate of an initially excited atom in the dipole approximation is given by Fermi's golden rule

$$\Gamma=\frac{2\pi}{\hbar^2}\sum_f |\langle f|\boldsymbol{\mu}\cdot\hat{\mathbf{E}}(\mathbf{r}_0,t)|0\rangle|^2\delta(\omega_f-\omega_0), \qquad (2.1)$$

where $\mathbf{r}_0$, ω_0, and $\boldsymbol{\mu}$ are the position, transition frequency, and dipole moment of the atom, respectively. The kets $|0\rangle$ and $|f\rangle$ show the vacuum and final states of the electromagnetic field. The electric field operator can be written in the form of

$$\hat{\mathbf{E}}(\mathbf{r},t)=\frac{1}{\sqrt{2\pi}}\int_0^{+\infty} d\omega[\hat{\mathbf{E}}^+(\mathbf{r},\omega)e^{-i\omega t}+\hat{\mathbf{E}}^-(\mathbf{r},\omega)e^{+i\omega t}], \qquad (2.2)$$

where the positive and negative frequency parts involve only the photon annihilation and creation operators. Substitution of Eq. (2.2) into Eq. (2.1) yields

$$\Gamma=\frac{1}{\hbar^2}\sum_f \int_0^{+\infty} d\omega \int_0^{+\infty} d\omega' \langle 0|\boldsymbol{\mu}\cdot\hat{\mathbf{E}}^+(\mathbf{r}_0,\omega)|f\rangle \times\langle f|\boldsymbol{\mu}\cdot\hat{\mathbf{E}}^-(\mathbf{r}_0,\omega')|0\rangle e^{-i(\omega-\omega')t}\delta(\omega_f-\omega_0). \quad (2.3)$$

The electric field operator includes sums over the annihilation and creation operators of all modes of the electromagnetic field, whose frequencies ω and ω' must equal ω_f. The summation over final states in Eq. (2.3) is therefore redundant and its removal gives

$$\Gamma=\frac{1}{\hbar^2}\int_0^{+\infty} d\omega \int_0^{+\infty} d\omega' \mu_\alpha \langle 0|\hat{E}^+_\alpha(\mathbf{r}_0,\omega)\hat{E}^-_\beta(\mathbf{r}_0,\omega')|0\rangle \times\mu_\beta e^{-i(\omega-\omega')t}\delta(\omega-\omega_0), \quad (2.4)$$

where repeated subscript indices are summed over and represent Cartesian coordinates $\alpha,\beta=x,y,z$.

Taking advantage of the gauge in which the scalar potential vanishes, that is

$$\hat{\mathbf{E}}^+(\mathbf{r},\omega)=i\omega\hat{\mathbf{A}}^+(\mathbf{r},\omega),$$

$$\hat{\mathbf{B}}^+(\mathbf{r},\omega)=\boldsymbol{\nabla}\times\hat{\mathbf{A}}^+(\mathbf{r},\omega), \quad (2.5)$$

the electric field correlation function is related to the imaginary part of the vector potential Green function by using the fluctuation dissipation theorem and Kubo's formula [25]

$$\langle 0|\hat{E}^+_\alpha(\mathbf{r},\omega)\hat{E}^-_\beta(\mathbf{r}',\omega')|0\rangle = 2\hbar\omega^2\,\mathrm{Im}\,G_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega)\,\delta(\omega-\omega'). \quad (2.6)$$

Employing Eq. (2.6), expression (2.4) can be simplified as

$$\Gamma=\frac{2}{\hbar}\omega^2\,\mathrm{Im}[\boldsymbol{\mu}\cdot\mathbf{G}(\mathbf{r}_0,\mathbf{r}_0,\omega_0)\cdot\boldsymbol{\mu}]. \quad (2.7)$$

It is more convenient to use the dimensionless vector potential Green function $\mathcal{G}_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega)$, defined as

$$G_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega)=\frac{\omega}{4\pi\epsilon_0 c^3}\mathcal{G}_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega), \quad (2.8)$$

where ϵ_0 is the permittivity of free space and c is the velocity of light. Using Eq. (2.8), the decay-rate (2.7) can be rewritten in the form of

$$\Gamma_\alpha=\frac{3}{2}\Gamma_0\,\mathrm{Im}\,\mathcal{G}_{\alpha\alpha}(\mathbf{r}_0,\mathbf{r}_0,\omega_0), \quad (2.9)$$

where

$$\Gamma_0=\frac{\mu^2\omega_0^3}{3\pi\epsilon_0 c^3\hbar} \quad (2.10)$$

is the decay rate of the excited atom in free space. Notice that the subscript index ''α'' in Eq. (2.9) refers to the different orientations of the dipole moment of the atom and a summation over the repeated indices is not applied.

B. The level shifts of an atom

The position of an atomic electron fluctuates around a mean value due to the existence of the fluctuating field of the vacuum, that is

$$\hat{\mathbf{R}}(\mathbf{r},t)=\hat{\mathbf{R}}_0(\mathbf{r},t)+\Delta\hat{\mathbf{R}}(\mathbf{r},t), \quad (2.11)$$

where $\hat{\mathbf{R}}_0(\mathbf{r},t)$ and $\Delta\hat{\mathbf{R}}(\mathbf{r},t)$ are the mean position of the electron and its deviation from the mean value, respectively. This gives rise to a modification on the instantaneous potential energy of the electron which is given by

$$V(\hat{\mathbf{R}}_0+\Delta\hat{\mathbf{R}})=V(\hat{\mathbf{R}}_0)+[(\Delta\hat{\mathbf{R}}\cdot\boldsymbol{\nabla})]V(\hat{\mathbf{R}}) +\frac{1}{2}[(\Delta\hat{\mathbf{R}}\cdot\boldsymbol{\nabla})]^2V(\hat{\mathbf{R}})+\cdots, \quad (2.12)$$

where the argument $(\mathbf{r},t)$ has been omitted for simplicity. The first term on the right-hand side is the instantaneous potential energy in the absence of the vacuum field which is

$$V(\hat{\mathbf{R}})=-\frac{Ze^2}{4\pi\epsilon_0 R}, \quad (2.13)$$

where $\hat{\mathbf{R}}=\hat{\mathbf{R}}_0$ and Ze and $-e$ are the electric charge of the nucleus and electron, respectively. The other terms on the right-hand side of Eq. (2.12) display the correction due to the presence of the fluctuating field of the vacuum. The energy-level shift of a given state is therefore obtained by evaluating the expectation value of the correction terms of the potential, whose leading term is of the form

$$\Delta E_n=\frac{1}{2}\langle[(\Delta\hat{\mathbf{R}}\cdot\boldsymbol{\nabla})]^2V(\hat{\mathbf{R}})\rangle =\frac{Ze^2}{8\pi\epsilon_0}Q_{\alpha\beta}\langle[\Delta\hat{\mathbf{R}}(\mathbf{r}_0,t)]_\alpha[\Delta\hat{\mathbf{R}}(\mathbf{r}_0,t)]_\beta\rangle, \quad (2.14)$$

where the repeated indices are summed over the three Cartesian coordinates $\alpha,\beta=x,y,z$, and

$$Q_{\alpha\beta}=-\langle n|\frac{\partial^2}{\partial X_\alpha\partial X_\beta}\left(\frac{1}{R}\right)|n\rangle. \quad (2.15)$$

In writing Eq. (2.14), account has been taken of $\langle\Delta\hat{\mathbf{R}}(\mathbf{r},t)\rangle=0$. Note that the state of the system is a product state of the electromagnetic field $|0\rangle$ and the atomic state $|n\rangle$. The integer n indicates the principal quantum number.

In the dipole approximation $\Delta\hat{\mathbf{R}}(\mathbf{r},t)$ satisfies the Heisenberg equation

$$m\frac{d^2}{dt^2}\Delta\hat{\mathbf{R}}(\mathbf{r},t)+m\Gamma\frac{d}{dt}\Delta\hat{\mathbf{R}}(\mathbf{r},t)=-e\hat{\mathbf{E}}(\mathbf{r},t), \quad (2.16)$$

where m is the observed mass of the electron. This equation resembles Welton's interpretation of the Lamb shift when

$\Gamma \to 0$ [17]. The presence of this term and the absence of a restoring force term in Eq. (2.16) denote that the motion of $\Delta\hat{\mathbf{R}}(\mathbf{r},t)$ is assumed to be overdamped with the damping constant Γ much greater than the natural frequency of the undamped motion. The fluctuation $\Delta\hat{\mathbf{R}}(\mathbf{r},t)$ may be decomposed into positive and negative frequency parts having the Fourier transform

$$\Delta\hat{\mathbf{R}}(\mathbf{r},t)=\frac{1}{\sqrt{2\pi}}\int_0^{+\infty} d\omega[\Delta\hat{\mathbf{R}}^+(\mathbf{r},\omega)e^{-i\omega t} + \Delta\hat{\mathbf{R}}^-(\mathbf{r},\omega)e^{+i\omega t}]. \tag{2.17}$$

Substitution of Eqs. (2.2) and (2.17) into Eq. (2.16) yields

$$\Delta\hat{\mathbf{R}}^+(\mathbf{r},\omega)=\frac{e}{m\omega(\omega+i\Gamma)}\hat{\mathbf{E}}^+(\mathbf{r},\omega) \tag{2.18}$$

for the positive frequency parts. The Hermitian conjugate of Eq. (2.18) gives $\Delta\hat{\mathbf{R}}^-(\mathbf{r},\omega)$. The explicit form of $\Delta\hat{\mathbf{R}}(\mathbf{r},t)$ in terms of $\hat{\mathbf{E}}^{\pm}(\mathbf{r},\omega)$ is obtained by inserting Eq. (2.18) and its negative frequency counterpart into Eq. (2.17). We find that

$$\Delta\hat{\mathbf{R}}(\mathbf{r},t)=\frac{1}{\sqrt{2\pi}}\left(\frac{e}{m}\right)\int_0^{+\infty}\frac{d\omega}{\omega}\left[\frac{\hat{\mathbf{E}}^+(\mathbf{r},\omega)}{(\omega+i\Gamma)}e^{-i\omega t} + \frac{\hat{\mathbf{E}}^-(\mathbf{r},\omega)}{(\omega-i\Gamma)}e^{+i\omega t}\right]. \tag{2.19}$$

Therefore

$$\langle[\Delta\hat{\mathbf{R}}(\mathbf{r},t)]_\alpha[\Delta\hat{\mathbf{R}}(\mathbf{r},t)]_\beta\rangle = \frac{e^2}{2\pi m^2}\int_0^{+\infty}\frac{d\omega}{\omega(\omega+i\Gamma)}\int_0^{+\infty}\frac{d\omega'}{\omega'(\omega'-i\Gamma)} \times\langle 0|\hat{E}_\alpha^+(\mathbf{r},\omega)\hat{E}_\beta^-(\mathbf{r},\omega')|0\rangle e^{-i(\omega-\omega')t}. \tag{2.20}$$

Using Eq. (2.6), we find that

$$\langle[\Delta\hat{\mathbf{R}}(\mathbf{r},t)]_\alpha[\Delta\hat{\mathbf{R}}(\mathbf{r},t)]_\beta\rangle=\frac{e^2\hbar}{\pi m^2}\int_0^{+\infty}d\omega\frac{\mathrm{Im}\,G_{\alpha\beta}(\mathbf{r},\mathbf{r},\omega)}{(\omega^2+\Gamma^2)}. \tag{2.21}$$

Substitution of Eq. (2.21) into Eq. (2.14) provides the level shifts of an atom

$$\Delta E_n=\frac{Ze^4\hbar}{8\pi^2\epsilon_0 m^2}\int_0^{+\infty}d\omega Q_{\alpha\beta}\frac{\mathrm{Im}\,G_{\alpha\beta}(\mathbf{r}_0,\mathbf{r}_0,\omega)}{(\omega^2+\Gamma^2)}. \tag{2.22}$$

The range of validity of the dipole approximation, which has been used in Eq. (2.16), necessitates the introduction of a cutoff frequency for the upper limit of integration in Eq. (2.22). It is customary to choose $mc/\hbar$ corresponding to the Compton wavelength of the electron. As in the decay rate, it is advantageous to use the dimensionless vector potential Green function, defined by Eq. (2.8), to express Eq. (2.22) as follows:

$$\Delta E_n=\frac{Ze^4\hbar}{32\pi^3\epsilon_0^2c^3m^2}\int_0^{mc/\hbar}dq\frac{q}{(q^2+\gamma^2)} \times Q_{\alpha\beta}\,\mathrm{Im}\,\mathcal{G}_{\alpha\beta}(\mathbf{r}_0,\mathbf{r}_0,\omega), \tag{2.23}$$

where $q=\omega/c$ and $\gamma=\Gamma/c$. This expression can be used for the evaluation of quantum electrodynamics level shifts provided that the damping coefficient Γ and the different components of the tensor $\mathcal{G}_{\alpha\beta}(\mathbf{r},\mathbf{r},\omega)$ are given.

III. THE GREEN FUNCTION

Consider a perfectly conducting medium with a single plane interface at $z=0$ which fills the half space $z\geqslant 0$. The x and y axes lie within the interface. To recover the notion of an ideal mirror, we assume a frequency-independent reflection coefficient of unity for this interface.

The electromagnetic field operators are governed by the Maxwell's equations which in the frequency domain in free space are of the form

$$\boldsymbol{\nabla}\times\hat{\mathbf{E}}^+(\mathbf{r},\omega)=i\omega\hat{\mathbf{B}}^+(\mathbf{r},\omega), \tag{3.1}$$

$$\boldsymbol{\nabla}\times\hat{\mathbf{B}}^+(\mathbf{r},\omega)=-i\frac{\omega}{c^2}\hat{\mathbf{E}}^+(\mathbf{r},\omega)+\frac{1}{\epsilon_0c^2}\hat{\mathbf{J}}^+(\mathbf{r},\omega). \tag{3.2}$$

Combining Eqs. (2.5), (3.1), and (3.2), and taking into account the definition of the Fourier time transformed vector potential Green function, that is

$$\hat{A}_\alpha^+(\mathbf{r},\omega)=\sum_\beta\int d\mathbf{r}'G_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega)\hat{J}_\beta^+(\mathbf{r}',\omega), \tag{3.3}$$

one can easily show that

$$\sum_\mu\left(q^2\delta_{\lambda\mu}-\frac{\partial^2}{\partial x_\lambda\partial x_\mu}+\delta_{\lambda\mu}\nabla^2\right)G_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega) = -\frac{1}{\epsilon_0c^2}\delta_{\lambda\nu}\delta(\mathbf{r}-\mathbf{r}'). \tag{3.4}$$

The different components of $G_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)$ are obtained from Eq. (3.4) with the appropriate boundary conditions. The boundary conditions at the plane interface of the conductor are governed by the boundary conditions on the different components of the electromagnetic fields. The details of the calculations are omitted here for the sake of brevity, and the complete description is given elsewhere [12].

The explicit form of the coordinate space Green function in this configuration can be written as

$$G_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega)=\mathsf{G}_{\alpha\beta}(\mathbf{r},\mathbf{r}_0',\omega)\pm\mathsf{G}_{\alpha\beta}(\mathbf{r},\mathbf{r}_1',\omega) - \frac{1}{3\epsilon_0\omega^2}\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}'), \tag{3.5}$$

where the plus sign holds for $\alpha\beta=xz,yz,zz$ and the minus sign holds for the other components. The tensor $\mathsf{G}_{\alpha\beta}(\mathbf{r},\mathbf{r}_i',\omega)$ is given by

$$\mathsf{G}_{\alpha\beta}(\mathbf{r},\mathbf{r}_i',\omega)=\frac{q^3}{4\pi\epsilon_0\omega^2}\left\{\left(\frac{1}{(qr_{rel})}+\frac{i}{(qr_{rel})^2}-\frac{1}{(qr_{rel})^3}\right)\delta_{\alpha\beta}-\left(\frac{1}{(qr_{rel})}+\frac{3i}{(qr_{rel})^2}-\frac{3}{(qr_{rel})^3}\right)\frac{(\mathbf{r}_{rel}\mathbf{r}_{rel})_{\alpha\beta}}{(r_{rel})^2}\right\}e^{iqr_{rel}},\tag{3.6}$$

where $\mathbf{r}_{rel}\mathbf{r}_{rel}$ is the normal Cartesian dyadic and

$$\mathbf{r}_{rel}=\mathbf{r}-\mathbf{r}_i',\quad i=0,1.\tag{3.7}$$

The vectors $\mathbf{r}_i'$ are defined as

$$\mathbf{r}_0'=\mathbf{r}',\quad \mathbf{r}_1'=\mathbf{r}'-2z'\hat{k}.\tag{3.8}$$

The typical structure of a semi-infinite response function is seen in Eq. (3.5). The bulk part, which is the first term together with the δ function term, is associated with the direct communication between the two points $\mathbf{r}$ and $\mathbf{r}'$ and resembles the free space Green function. The second term displays the communication between the points via a reflection in the perfect conducting interface. This term corresponds to the so-called image source.

IV. THE RADIATIVE PROPERTIES OF AN ATOM IN THE VICINITY OF AN IDEAL MIRROR

Though in a practical situation we deal with a good conductor instead of a perfect conductor having reflection coefficient of unity for all frequencies, it is customary to use the ideal mirror approximation for simplicity. The modification of the radiative properties of an atom in the vicinity of a perfect reflecting mirror has a simplicity which allows us to work it out easily in almost any formulation. The typical behavior of this configuration illuminates some points which may be useful in more complicated situations.

A. The decay rate of an excited atom

Regarding the symmetry of the present problem there are two different orientations for the dipole moment of the excited atom which are known as perpendicular and parallel orientations. Let us first consider the case in which the dipole moment of the excited atom is perpendicular to the surface of the mirror at a distance $z=z_0\leqslant 0$. This case is represented by Γ_z in our notation. The zz component of the dimensionless vector potential Green function needed for substitution in Eq. (2.9) is given by Eq. (3.5) with the use of Eqs. (2.8) and (3.6)–(3.8). We find that

$$\mathcal{G}_{zz}(\mathbf{r}_0,\mathbf{r}_0,\omega_0)=\left\{\frac{2}{3}i-2\left[\frac{i}{(2q_0z_0)^2}-\frac{1}{(2q_0z_0)^3}\right]e^{2iq_0z_0}\right\},\tag{4.1}$$

where $q_0=\omega_0/c$. Substitution of Eq. (4.1) into Eq. (2.9) yields

$$\Gamma_z=\Gamma_0\left\{1-3\left[\frac{\cos(2q_0z_0)}{(2q_0z_0)^2}-\frac{\sin(2q_0z_0)}{(2q_0z_0)^3}\right]\right\}.\tag{4.2}$$

It is seen that in the limit $z_0\to 0$ the decay rate (4.2) tends to $\Gamma_z=2\Gamma_0$. This is due to the fact that the image dipole of the atom is in phase with the atomic dipole moment.

A similar consideration is applicable to the parallel orientation. The symmetry of the configuration in the xy plane indicates that this can be represented by either Γ_x or Γ_y in the present notation. The xx component of the tensor $\mathcal{G}_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega)$ needed for insertion in Eq. (2.9) is given by

$$\mathcal{G}_{xx}(\mathbf{r}_0,\mathbf{r}_0,\omega_0)=\left\{\frac{2}{3}i-\left[\frac{1}{(2q_0z_0)}+\frac{i}{(2q_0z_0)^2}-\frac{1}{(2q_0z_0)^3}\right]e^{2iq_0z_0}\right\}.\tag{4.3}$$

Substitution of Eq. (4.3) into Eq. (2.9) leads to the following expression for the decay rate

$$\Gamma_x=\Gamma_0\left\{1-\frac{3}{2}\left[\frac{\sin(2q_0z_0)}{(2q_0z_0)}+\frac{\cos(2q_0z_0)}{(2q_0z_0)^2}-\frac{\sin(2q_0z_0)}{(2q_0z_0)^3}\right]\right\}.\tag{4.4}$$

We see that this expression tends to zero when $z_0\to 0$. This denotes the fact that the image dipole of the atom is out of phase with the atom dipole moment in this case.

The typical behaviors of these two decay rates in terms of the distance of the atom from the mirror are depicted in Fig. 1. Note that the wavelength of the electromagnetic field radiated by the atom is the only scale of length involved in this phenomenon. Therefore, the distance from the mirror is measured in units of $\lambda_0=2\pi c/\omega_0$. The decay rate is measured in the unit of decay rate in free space. We see that for both orientations of the dipole moment of the atom, when the atom is many wavelengths away from the mirror, the reflected field is weak and therefore the decay rate tends to the free-space decay rate. If the distance to the mirror is decreased, the mirror effect becomes appreciable. In the case of parallel (perpendicular) orientation the decay rate tends to zero (twice the free space value), when the dipole is very close to the mirror.

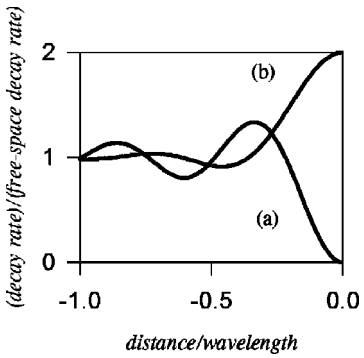

FIG. 1. Variation of the relative decay rate of an atomic system in the vicinity of a perfect mirror (a) for parallel orientation and (b) for perpendicular orientation.

The average decay rate of an excited atom is defined as

$$\Gamma_{av}=\frac{1}{3}(\Gamma_z+2\Gamma_x)$$
$$=\Gamma_0\left\{1-\left[\frac{\sin(2q_0z_0)}{(2q_0z_0)}+2\frac{\cos(2q_0z_0)}{(2q_0z_0)^2}-2\frac{\sin(2q_0z_0)}{(2q_0z_0)^3}\right]\right\}. \tag{4.5}$$

The variation of Γ_{av}/Γ_0 in terms of the distance of the atom from the mirror is shown in Fig. 2. As one expects, it is seen that

$$\lim_{z_0\to 0}\Gamma_{av}=\frac{2}{3}\Gamma_0 \tag{4.6}$$

and retains the free-space value when $z_0\to\infty$.

B. The level shifts of an atom

The symmetry of the present case allows us to simplify the general expression of the quantum electrodynamics level shifts given by Eq. (2.23). The explicit forms of $\mathcal{G}_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega)$, obtained from Eqs. (3.5) and (3.6) along with the use of Eq. (2.8), show that the off-diagonal elements of $\mathcal{G}_{\alpha\beta}(\mathbf{r}_0,\mathbf{r}_0,\omega)$ are zero. Therefore, Eq. (2.23) can be rewritten as

$$\Delta E_n=\frac{Ze^4\hbar}{32\pi^3\epsilon_0^2c^3m^2}\int_0^{mc/\hbar}dq\,\frac{q}{(q^2+\gamma^2)}$$
$$\times\sum_\alpha Q_{\alpha\alpha}\,\mathrm{Im}\,\mathcal{G}_{\alpha\alpha}(\mathbf{r}_0,\mathbf{r}_0,\omega), \tag{4.7}$$

in which $\alpha=x,y,z$. Using Eq. (2.15), the diagonal elements of tensor $Q_{\alpha\beta}$ are

$$Q_{\alpha\alpha}=-\langle n|\frac{\partial^2}{\partial X_\alpha^2}\left(\frac{1}{R}\right)|n\rangle=\frac{4\pi}{3}|\psi_n(0)|^2, \tag{4.8}$$

where in the last step the symmetry of the potential is taken into account. Substitution of Eq. (4.8) into Eq. (4.7) leads to the following expression for the atomic level shifts:

$$\Delta E_n=\frac{Ze^4\hbar}{24\pi^2\epsilon_0^2c^3m^2}|\psi_n(0)|^2$$
$$\times\int_0^{mc/\hbar}dq\,\frac{q}{(q^2+\gamma^2)}\mathrm{Im}\sum_\alpha\mathcal{G}_{\alpha\alpha}(\mathbf{r}_0,\mathbf{r}_0,\omega). \tag{4.9}$$

Employing the explicit forms of the diagonal elements of tensor $\mathcal{G}_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega)$, it is easy to show that

$$\mathrm{Im}\sum_\alpha\mathcal{G}_{\alpha\alpha}(\mathbf{r}_0,\mathbf{r}_0,\omega)$$
$$=2\left\{1-\left[\frac{\sin(2qz_0)}{(2qz_0)}+2\frac{\cos(2qz_0)}{(2qz_0)^2}-2\frac{\sin(2qz_0)}{(2qz_0)^3}\right]\right\}. \tag{4.10}$$

Substitution of Eq. (4.10) into Eq. (4.9) yields

$$\Delta E_n=\frac{Ze^4\hbar}{12\pi^2\epsilon_0^2c^3m^2}|\psi_n(0)|^2\int_0^{mc/\hbar}dq\,\frac{q}{(q^2+\gamma^2)}$$
$$\times\left\{1-\left[\frac{\sin(2qz_0)}{(2qz_0)}+2\frac{\cos(2qz_0)}{(2qz_0)^2}-2\frac{\sin(2qz_0)}{(2qz_0)^3}\right]\right\}. \tag{4.11}$$

The integration in Eq. (4.11) is all that needs to be done, numerically if necessary, provided that a numerical value has been assigned for γ. This can be accomplished easily if we examine the latter equation in the limit $z_0\to\infty$. Imposing the limit, we find that

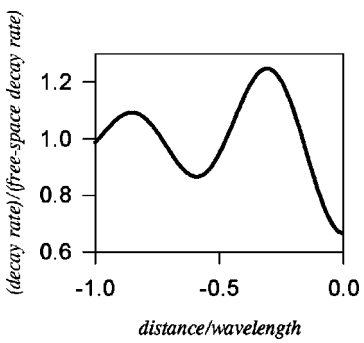

FIG. 2. Variation of the average relative decay rate of an atomic system in the vicinity of a perfect mirror.

$$\Delta E_n^0=\frac{Ze^4\hbar}{12\pi^2\epsilon_0^2c^3m^2}|\psi_n(0)|^2\int_0^{mc/\hbar}dq\,\frac{q}{(q^2+\gamma^2)}$$
$$=\frac{1}{12\pi^2\epsilon_0^2}\left(\frac{Ze^4\hbar}{m^2c^3}\right)|\psi_n(0)|^2\ln\left(\frac{mc}{\gamma\hbar}\right), \qquad (4.12)$$

where $mc/\hbar\gg\gamma$ is assumed in order to neglect γ^2 in the numerator of the argument of the logarithm. The superscript index ''0'' denotes that the latter quantity is evaluated for free space. Comparison of Eq. (4.12) with the traditional approaches to the level shifts of an atom in free space [14] allows us to assign a numerical estimate to γ. We find that

$$\gamma=|E_n-E_m|_{av}/\hbar c=17.8R_\infty/\hbar c, \qquad (4.13)$$

where R_∞ is the Rydberg unit of energy and $|E_n-E_m|_{av}$ is Bethe's average excitation energy associated with the two states of the atom involved in the emission and absorption of *virtual photons*. We can use this value in the theory of level shifts whenever it is needed.

The other extreme case of special interest is the limit $z_0\to 0$. The imposition of this limit on Eq. (4.11) yields

$$\lim_{z_0\to 0}\Delta E_n=\frac{2}{3}\Delta E_n^0, \qquad (4.14)$$

which is in harmony with Eq. (4.6) obtained for the decay rate of an excited atom. These two expressions denote the fact that both the decay rate and the level shifts of an atomic system experience the constructive and destructive nature of the interference of the vacuum field in the vicinity of the mirror in the same way.

Using Eq. (4.12) one may express Eq. (4.11) in the following form:

$$\Delta E_n=\Delta E_n^0\left\{1-\left[\ln\left(\frac{mc}{\gamma\hbar}\right)\right]^{-1}\int_0^{mc/\hbar}dq\,\frac{q}{(q^2+\gamma^2)}\right.$$
$$\left.\times\left[\frac{\sin(2qz_0)}{(2qz_0)}+2\frac{\cos(2qz_0)}{(2qz_0)^2}-2\frac{\sin(2qz_0)}{(2qz_0)^3}\right]\right\}. \qquad (4.15)$$

No matter how far the calculation may be proceeded analytically, the integration in Eq. (4.15) is all that needs to be done numerically. As far as the typical behavior of Eq. (4.15) is concerned, we may in most cases be content with the variation of ΔE_n in terms of the distance of the atom from the mirror. The difficulty, however, arises because of the lack of a clearly evident scale of length in this problem. As it was seen in the case of decay rate, an appropriate scale of length is needed to show the characteristic behavior of the phenomenon properly.

The integration in Eq. (4.15) can be simplified with some algebra, so that

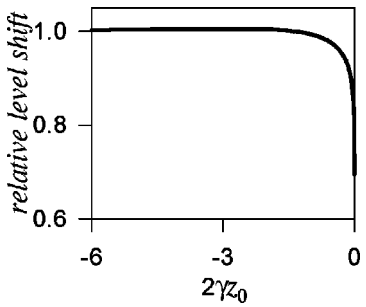

FIG. 3. Variation of the relative level shifts of an atom in the vicinity of a perfect mirror.

$$\frac{\Delta E_n}{\Delta E_n^0}=1-(\ln\xi)^{-1}\left\{\frac{2}{(2\gamma z_0)^2}\left[\frac{\sin[(2\gamma z_0)\xi]}{(2\gamma z_0)\xi}-1\right]\right.$$
$$+\left[\frac{1}{(2\gamma z_0)}\left(1+\frac{2}{(2\gamma z_0)^2}\right)\right.$$
$$\left.\left.-\frac{2}{(2\gamma z_0)^2}\frac{d}{d(2\gamma z_0)}\right]f(2\gamma z_0)\right\}, \qquad (4.16)$$

where

$$f(2\gamma z_0)=\int_0^\xi\frac{dx}{(x^2+1)}\sin[(2\gamma z_0)x] \qquad (4.17)$$

and $\xi=(mc/\gamma\hbar)$. We see that $l=(2\gamma)^{-1}\simeq 4$ Å is the proper scale of length for this equation where γ is given by Eq. (4.13).

The typical behavior of the energy level shifts versus the distance of the atom from the mirror is depicted in Fig. 3. The vertical axis is in the units of the shifts in free space and the distance on the horizontal axis is in the units of $1/(2\gamma)$. As one expects, it is seen that the relative atomic level shifts take the value 2/3 when $z_0\to 0$ which is consistent with Eq. (4.14). Increasing the distance of the atom from the mirror, the energy level shifts increase rapidly and achieve the free-space limit.

V. CONCLUSION

We developed previous work on the radiative properties of an atom [24] to the effects of the presence of a boundary surface. The general expressions (2.9) and (2.23) describe the decay rate of an atom or molecules as well as the energy-level shifts of an atom in terms of the imaginary part of the dimensionless vector potential Green function.

The results obtained by the application of these two expressions for the atom in the vicinity of a perfect mirror is in harmony with those appearing in the literature using different methods [19–23] . However, the mechanism of the level shifts is mixed with the modification of the effect due to the presence of a boundary surface or surfaces in these calculations. This usually affects the distance dependence of the effect which could be important when a more complicated geometry is involved. What is advantageous in our formal-

ism is that these two problems have been separated to clarify the effect of the cavity properly. As was seen in Sec. IV, the present approach reveals the similarities between the decay rate and level shifts of atomic systems in the presence of a mirror.

The formalism of the image method of the dipole decay rate was explained in both the classical and quantum domains [13]. What is not well appreciated is that the energy-level shifts of an atom may also be treated by the help of the image method. This is seen clearly in the present formulation. What is new is that, as for the relative decay rate, the relative level shifts tend to 2/3 for $z_0 \rightarrow 0$. This shows that the decay rate and the level shifts both experience the presence of the mirror identically when $z_0 \rightarrow 0$.

Experiments testing the change in the decay rate of an atom in the vicinity of a mirror and some other configurations are numerous in literature [13]. The results are in agreement with the theoretical predictions. It is seen that the variation of induced energy-level shifts in the vicinity of a mirror are small to be detected easily, unless the distance between the atom and the mirror is small. In practical situations, however, other interactions come into play in this limit.

The questions which may naturally arise in the application of this formalism are first on the accuracy of the numerical estimated value of γ, as given by Eq. (4.13), and second on the inconsistency of the calculations with the causality considerations. As for the first question we must note that the value of the damping constant $\Gamma = c\gamma$ does not apparently affect the typical behavior of the level shifts given by Fig. 3. This merely slightly changes the scale of length on the horizontal axis. Regarding the second question, one should necessarily improve on the ideal notion of a perfect conductor having a frequency-independent reflection coefficient of unity to remedy the present ambiguity. The work on this problem is under way.

ACKNOWLEDGMENT

We are grateful to R. Loudon for comments and suggestions that helped us to improve the present work.

[1] H. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
[2] H. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).
[3] E. Lifshitz, Zh. Éksp. Teor. Fiz. **29**, 94 (1956) [Sov. Phys. JETP **2**, 73 (1956)].
[4] P. W. Milonni, *The Quantum Vacuum* (Academic Press, New York, 1994).
[5] G. Plunien, B. Muller, and W. Greiner, Phys. Rep. **134**, 87 (1986), and references therein.
[6] E. Elizalde and A. Romeo, Am. J. Phys. **59**, 711 (1991).
[7] P. W. Milonni, R.J. Cook, and M.E. Goggin, Phys. Rev. A **38**, 1621 (1988).
[8] D. Kupiszewska and J. Mostowski, Phys. Rev. A **41**, 4636 (1990).
[9] D. Kupiszewska, Phys. Rev. A **46**, 2286 (1992).
[10] D. Kupiszewska, J. Mod. Opt. **40**, 517 (1993).
[11] R. Matloob, A. Keshavarz, and D. Sedighi, Phys. Rev. A **60**, 3410 (1999).
[12] R. Matloob, Phys. Rev. A **60**, 3421 (1999).
[13] E. Hinds, in *Cavity Quantum Electrodynamics*, Supplement to Advances in Atomic, Molecular, and Optical Physics, edited by P. R. Berman (Academic Press, New York, 1994).
[14] H. A. Bethe, Phys. Rev. **72**, 339 (1947).
[15] N. M. Kroll and W. E. Lamb, Phys. Rev. **75**, 388 (1949).
[16] J. B. French and V. F. Weisskopf, Phys. Rev. **75**, 1240 (1949).
[17] T. A. Welton, Phys. Rev. **74**, 1157 (1948).
[18] E. A. Power, Am. J. Phys. **34**, 516 (1966).
[19] M. Babiker and G. Barton, Proc. R. Soc. London, Ser. A **326**, 255 (1972); **326**, 277 (1972).
[20] G. Barton, J. Phys. B **7**, 2134 (1974).
[21] G. Barton, Proc. R. Soc. London, Ser. A **410**, 141 (1987); **410**, 175 (1987).
[22] D. Meschede, W. Jhe, and E. A. Hinds, Phys. Rev. A **41**, 1587 (1990).
[23] M. R. Philpott, Chem. Phys. Lett. **19**, 435 (1973).
[24] R. Matloob, Phys. Rev. A **61**, 062103 (2000).
[25] L. Landau and E. Lifshits, *Statistical Physics*, (Pergamon, Oxford, 1980), Part 2.