Anomalies, symmetries, and asymmetries in the relaxation oscillation spectra of multimode standing-wave solid-state lasers

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When the laser medium only partially fills the cavity of a multimode solid-state laser, there are significant changes in the spectral profile of the emission, the intensity fluctuation power spectra, and modulation transfer functions in comparison with lasers which have media that entirely fill the cavity. These effects are computed according to several approximate models for the multimode interactions and compared with experimental measurements. Symmetries and asymmetries in the multimode optical spectrum have corresponding features in the relaxation oscillation spectra. Changes in the laser cavity detuning and laser excitation lead to relatively abrupt changes in some variables (modal intensities, low-frequency relaxation oscillations) while the total intensity and largest relaxation oscillation frequency vary as they would for a single-mode laser.

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I. INTRODUCTION

The dynamics of multimode solid-state lasers—including the transient evolution of the modal intensities, their response to modulation, and their noise spectra—has been the subject of recent investigations because of applications of diode-pumped Nd:YAG and microchip lasers in precision measurements, ranging from gravity-wave detection systems and ultrashort pulse generation to intracavity spectroscopy, and from communications systems to frequency doubling. Interest has been further extended to the operation of lasers with polarization dynamics of the electric field, as is commonly observed in some solid-state laser crystals and in many fiber lasers. Multimode operation of standing-wave solid-state lasers leads to more than one relaxation oscillation frequency. Typically, these frequencies are manifest as peaks in the power spectra of the intensity fluctuations and in the modulation transfer functions of both the total intensity and the intensities of the individual modes, except that in cases of high modal symmetry some peaks are missing in the spectra and in the modulation transfer function for the total intensity. The emission spectrum is narrowed by strong intermode competition, moderated by the longitudinal spatial hole burning, but when the lasing medium only partially fills the laser cavity, the spectrum may be narrower or broader or modulated since the intermode competition is limited to a shorter length.

The theories of the operation of these lasers have generally been derived with few changes from the early analyses by Tang, Statz, and de Mars [1] (see also Statz, Tang, and Lavine [2] extending the analysis to semiconductor lasers and including diffusion of the excited centers in the active medium) in the 1960's for flash lamp pumped ruby lasers. Their theory explained the persistence of several modes and the absence of many other modes at cavity frequencies for which the small signal gain exceeded the laser threshold. In modern laser-pumped devices, which have short high-gain media and short laser cavities, the number of interacting modes with small signal gain above threshold can be limited to a few, simplifying the results and facilitating detailed comparisons of theoretical, computational, and experimental results.

Single-mode solid-state lasers commonly exhibit damped relaxation oscillations after they are switched on or after they are perturbed from steady state. As the laser approaches steady state, the excess energy is shared alternately in the gain medium and in the laser intensity. The presence of weakly damped relaxation oscillations indicates that there is a narrow range of frequencies for enhanced (resonant) response to periodic perturbations that modulate the intensity, and this feature leads to peaks in the power spectrum of white-noise-driven intensity fluctuations [3].

In multimode solid-state lasers the oscillations in the total intensity are remarkably similar to those found in single-mode lasers (in their dependence on the degree of excitation above the lasing threshold and on the relaxation rates of the population inversion and the field in the cavity). In the multimode laser this primary modulation of the total intensity corresponds to in-phase modulation of the intensities of the individual modes. But in many multimode lasers there are also competing interactions of the different laser modes, which result in modulation in the intensities of the individual modes at frequencies lower than the characteristic relaxation oscillation frequency of the total intensity; but these modal oscillations combine to yield much weaker modulation of the total intensity because they are at least partly out of phase with each other. Selective feedback of detected intensity variations to modify the laser excitation can be used to en-

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hance the total intensity modulation or to suppress the individual mode oscillations at these lower frequencies [4,5].

In almost all of the analytical and computation work on models of these lasers in recent years, the emphasis has been on ''rate equation'' models for the intensities of the standing-wave longitudinal modes coupled to the population inversion. It is usually argued that the dynamics of the dipole moments of the material can be eliminated adiabatically, and this approximation seems satisfactory if the spectral range of the oscillating modes is much less than the homogeneous linewidth of the transition, a condition typically satisfied in multimode solid-state lasers because mode competition suppresses many modes for which the unsaturated gain is above threshold. The population inversion must necessarily be taken to be dependent on the longitudinal spatial coordinate in multimode lasers (whether they have ring or Fabry-Perot cavities) because of spatial hole burning. This causes the exact form of rate equation models to be integrodifferential equations for the dynamics of the spatially varying population inversion and the modal intensities. It has been shown that these equations have only one steady-state solution [6], regardless of the parameter values.

Because the integrodifferential equations are cumbersome and difficult to analyze, some sort of additional approximation has usually been applied. The simplest and most common approximation for lasers with Fabry-Perot cavities is to describe the longitudinal variation of the population inversion as the combination of a spatially uniform component and the amplitude of a grating at the fundamental spatial frequency of the standing-wave pattern formed by each oscillating mode. Such models are excellent approximations when the intermode frequency spacing is sufficiently larger than the cavity mode linewidths and when the medium completely fills the laser cavity. The most common dynamical phenomena are sensitive primarily to the coupling of the modal intensities and the real Fourier spatial amplitudes of the inversion, a condition that typically results in as many relaxation oscillation frequencies as there are modes, with the highest of these frequencies being similar to the relaxation oscillation of a single-mode solid-state laser with similar total output power.

By contrast, when the intermode spacing is of the same order or less than the cavity mode linewidth, it is clear that rate equation models are unlikely to provide all of the important physics and more complicated models are needed, sometimes referred to as ''phase-sensitive models'' or ''four-wave-mixing models,'' which include as well the more rapidly varying inversion gratings induced by interference of the optical fields of different modes [7–12]. These models allow for FM modulation of each of the cavity mode electric field amplitudes, and these modulations can play an important role in mode-switching phenomena, not only in standing-wave lasers, but also in unidirectional ring lasers and bidirectional ring lasers. Our interests lead us to focus on several particular phenomena, not previously well explored, that are restricted to the first type of models.

When the material has a gain spectrum that is symmetric about a central frequency, such as the Lorentzian line shape common for rare-earth transitions in solid-state lasers, there are situations in which the emitted optical spectrum is symmetric about the central emission frequency. The cavity detuning can be used to position the modes spectrally so that one mode is at the material resonance frequency and other modes are paired in their intensities or so that each mode is paired with another with symmetric detuning and equal intensity. An even higher symmetry of equal gain (and intensity) for all modes has been considered previously in some theoretical analyses of multimode ''antiphased dynamics,'' but the more limited symmetry in which pairs of modes in the spectrum have the same gain is the experimental limit. This degree of symmetry causes the relaxation oscillations to separate into those that are ''compensated'' (entirely absent in the total intensity) and those that are ''uncompensated'' (with some resultant modulation of the total intensity) [13,14]. When the symmetrical detuning of the modes is broken, each of the relaxation oscillation frequencies leads to some modulation of the total intensity. For experimental comparisons, the symmetrical situations have special significance as without expensive optical equipment one can tune one of the laser modes to the line center by observing the disappearance of the compensated relaxation oscillations in the total intensity. Thereby one can obtain spectral tuning information from the dynamics of the total intensity as a way to offset thermally induced drifts in the detuning caused by temperature changes of the cavity or heating of the medium from its absorption of the pump radiation. As an added complication, the symmetry of the spectrum is also affected by longitudinal nonuniformities in the pumping, including the partial filling of the laser cavity by the lasing medium [15].

In this paper we identify the normal modes of modal intensity modulation, which correspond to the different relaxation oscillation frequencies in order to understand how these modes are differently excited by a particular source of noise or by parameter modulation. This is connected with the question of how pump modulation can affect the total intensity or individual mode intensities by exciting competing relaxation oscillations of different type. These investigations are among the easiest to perform experimentally and their results offer a test of the suitability of different theoretical models.

There is also interest in an extension of recent studies [16,17] that examined conditions for the disappearance of some of the low-frequency relaxation oscillations. From work done heretofore, it seems that each new optical mode that enters the dynamics brings an additional relaxation oscillation frequency and that the square of each of the frequencies grows linearly with increasing excitation of the laser. We show in this paper that these features are not universal characteristics of solid-state lasers. In addition we show that the optical spectrum does not necessarily evolve smoothly or uniformly as the laser cavity is detuned, a reminder of the importance of intermode competition in determining the emission spectrum.

II. MODEL OF STANDING-WAVE SOLID-STATE LASERS

The dynamics of multimode lasers depends critically on the strengths of the intermode couplings that are governed by the extent of the overlap of the standing-wave gratings burned by the different modes in the population inversion. This coupling depends on the location of the laser medium inside the cavity. If a short laser rod is located close to one of the mirrors, the situation has been characterized by the filling factor l/L, where l is the length of the lasing medium and L is the cavity length [13,15,18–20]. An anomaly in recently reported experimental results is that lasers of different filling factors seem to be reasonably explained by comparison with theories for lasers with a filling factor of unity. In recent experiments, work on standing-wave systems has focused on laser-pumped microchip lasers [21,22], longitudinally diode-pumped Nd:YAG crystals filling about one-third of the laser cavity [4,23], or on fiber lasers; the latter two cases have filling factors much less than unity.

From careful investigations of the effect of the cavity filling factor, we have found that there are certain critical values which give behavior remarkably like that observed when the filling factor is unity. We have also found that there are other experimentally accessible values of the filling factor that lead to unusual, or unexpected, variations in modal intensities, relaxation oscillation frequencies, and modulation transfer functions. While formally understood for decades and included in some theoretical analyses [15,18,24–26], the effects of the cavity filling factor have been usually neglected in studies of laser dynamical oscillations, though Evdokimova and Kaptsov did note that a lower cavity filling factor tended to shift the relaxation oscillation frequencies lower as well.

Our specific experimental and theoretical investigations focus on a Fabry-Perot Nd:YAG laser. The choice of a mathematical model might seem to be relatively trivial from the literature, but recent investigations suggest that there are critically important (and somewhat inconsistent) approximation strategies that make the agreed-upon integrodifferential equations more tractable. The basic Tang, Statz, and deMars rate equations are

$$\frac{dI_k(\tau)}{d\tau}=GI_k(\tau)\left[g_k\int_0^1 n(z,\tau)\psi_k^2(z)dz-1-\beta_k\right], \quad (1)$$

$$\frac{\partial n(z,\tau)}{\partial\tau}=A(z)-n(z,\tau)\left[1+\sum g_j\psi_j^2(z)I_j(z)\right], \quad (2)$$

which can be obtained from the more general multimode semiclassical Maxwell-Bloch equations, assuming that (1) the polarization of the laser medium is adiabatically eliminated; (2) the lasing field is expanded in the cavity eigenfunctions; and (3) the inversion gratings resulting from mode interference, which oscillate at the intermode beat frequencies, are negligibly small.

The symbols in Eqs. (1) and (2) have the following meaning: $I_k(\tau)$ is the normalized and time-dependent modal intensity; $n(z,\tau)$ is the normalized spatial density of the population inversion; $\tau=t/T_1$ is the time measured in the units of the inverse of the inversion decay rate; *x, y, z* are Cartesian coordinates normalized to the cavity length L; $A(z)$ is the pumping parameter, which reflects the unsaturated inversion distribution along the cavity; $\psi_k(z)$ is the resonator eigenfunction, and $\psi_k(z)=\sqrt{2}\sin(\pi q_k z)$ for longitudinal modes; πq_k is the wave number (where q_k is a large integer corresponding to the number of half-wavelengths for that mode in the cavity), and $k=1,2,...,K$, where K is the full number of laser modes included in the truncated model; g_k is the kth mode gain coefficient $G=T_1/T_c$, where T_c is the photon lifetime in the cavity; and β_k represents additional losses in kth mode with respect to the loss rate of the reference level specified by $1/T_c$. The index of refraction of the active medium can be incorporated in the model without loss of generalization by taking a length for the medium that corresponds to its optical length and by adjusting the cavity length L to match the total optical length of the cavity.

The variables in Eqs. (1) and (2) are, therefore, the mode intensities, which depend solely on time, and the inversion density, which depends also on the spatial coordinates; for this reason a partial derivative $\partial n/\partial\tau$ is used in Eq. (2).

Equations such as these are the integrodifferential, but nonetheless they have steady-state solutions and those steady-state solutions can be analyzed for their stability. Though integrodifferential equations may, in principle, have a phase space of infinite dimension, it is not surprising if there is only a finite number of stability analysis eigenvalues, though that number may change as the parameters are varied. We will see from the truncated approximate models that there are strong suggestions about the number of eigenvalues of these integrodifferential equations.

Several different approximations have been made to obtain ordinary differential equations. If the inversion is represented by a constant term plus a Fourier expansion including all spatial harmonics of the length of the resonator, the expressions would be exact, but the number of equations would be infinite.

There are two spatial nonuniformities to consider in a laser that is partially filled by the active medium. At the very least, as in the Statz, Tang, and deMars model, one must include the spatial gratings created by the standing wave of each mode separately. These require terms proportional to $\cos(2\pi q_k z)$ for each mode that is operating. In addition some account must be taken of the finite length of the gain medium in the laser cavity and it has recently been proposed [27] that this can be adequately represented by taking the first few long wavelength harmonics proportional to $\cos(2\pi pz)$, where p is an integer from 1 to K, and K is the number of modes.

In this case the inversion can be written as follows:

$$\begin{aligned} n(z,\tau)=&D_0(\tau)+2\sum_{p=1}^{K-1}D_p(\tau)\cos(2\pi pz)\\ &-2\sum_{k=1}^{K}N_k(\tau)\cos(2\pi q_k z)\\ &-2\sum_{k=1}^{K}N_{2k}(\tau)\cos(4\pi q_k z)+\cdots. \end{aligned} \quad (3)$$

Here D_p indicates the amplitudes of the low spatial harmonics of the inversion caused by the finite length of the laser medium or the nonuniformity of the pumping intensity within the medium. More generally these terms describe large scale spatial inhomogeneities in the population inversion. Variables N_k represent the amplitudes of high spatial harmonic gratings with periods of one-half of the modal wavelengths, as induced by spatial hole burning by the lasing modes. In general the gratings of the both scales should be also represented in the expansion of the pumping parameter, but the typical absence of small scale structure in the intensity of the pumping light (unless it is also a standing wave) allows us to limit ourselves to the expression

$$A(z)=A_0+2\sum_{p=1}^{K-1} A_p \cos(2\pi pz). \tag{4}$$

Using expansions (3) and (4) Pieroux and Mandel [27] arrive at the following set of equations that are $3K$ in number:

$$\frac{dI_k}{d\tau}=GI_k[g_k(D_0+N_k)-1-\beta_k], \tag{5}$$

$$\frac{dN_k}{d\tau}=-\left(1+\sum_{m=1}^{K} g_mI_m\right)N_k-\frac{1}{2}\sum_{m=1}^{K} g_mI_mD_{k-m}, \tag{6}$$

$$\frac{dD_0}{d\tau}=A_0-D_0\left(1+\sum_{m=1}^{K} g_mI_m\right)-\sum_{m=1}^{K} g_mI_mN_m, \tag{7}$$

$$\frac{dD_p}{d\tau}=A_p-\left(1+\sum_{m=1}^{K} g_mI_m\right)D_p-\frac{1}{2}\sum_{m=1}^{K-p} g_mI_mN_{m+p} -\frac{1}{2}\sum_{m=1+p}^{K} g_mI_mN_{m-p}. \tag{8}$$

For a uniform pump distribution along the laser rod,

$$A(z)=\begin{cases} A, & z_1\leqslant z\leqslant z_2 \\ 0, & 0\leqslant z\leqslant z_1, z_2\leqslant z\leqslant 1, \end{cases}$$

the coefficients A_p are determined by the expressions:

$$A_0=A(z_2-z_1), \quad \text{and} \quad A_p=A\frac{\sin(2\pi pz_2)-\sin(2\pi pz_1)}{2\pi p(z_2-z_1)}, \tag{9}$$

where $z_2-z_1=l/L$ is the length of the lasing medium l normalized to the cavity length L. We will suppose that the laser medium is situated near one mirror of the cavity, as is the case when one surface of the Nd:YAG crystal is coated to be highly reflecting at 1.06 μm while the other surface of the crystal rod is antireflection coated. In this case Eq. (9) reduces to

$$A_p=A\frac{\sin(2\pi pl/L)}{2\pi pl/L}. \tag{10}$$

Considering a Lorentzian gain profile, the gain coefficients of the modes are

$$g_j=\left\{1+\left[\left(\frac{K+1}{2}-j\right)\delta-\delta_0\right]^2\right\}^{-1}, \tag{11}$$

where δ and δ_0 are the intermode frequency spacing and the detuning of the central cavity mode frequency from central frequency of the gain line, respectively, normalized to the half-width of the gain spectrum.

In making the transition from Eqs. (1) and (2) to Eqs. (5)–(8) all small scale gratings with $k>K$, as well as large scale ones with $p\geqslant K$, are ignored; and this is, apparently, sufficient to capture the important features of the dynamics. Analysis of the steady states and stability of those steady states reveals that there are typically $2K+1$ eigenvalues with modestly negative real parts (of order the population decay rate, which means of order unity in our normalized units of time) and $K-1$ eigenvalues with very much more negative eigenvalues. This suggests that the truncation of the integrodifferential equations has introduced additional (essentially spurious) eigenvalues to the exact dynamics.

An alternative approximation of the integrodifferential equations introduced by Evdokimova and Kaptsov retains the Fourier expansion and truncation of the original Tang, Statz, and deMars equations and deals quite differently with the finite length of the medium. The $2K+1$ equations [18] are

$$\frac{dI_k}{d\tau}=GI_k\left[g_k\left(D_0+\sum_{m=1}^{K} b_{km}N_m\right)-1-\beta_k\right], \tag{12}$$

$$\frac{dN_k}{d\tau}=-\left(1+\sum_{m=1}^{K} g_mI_m\right)N_k-\frac{1}{2}g_kI_kD_0, \tag{13}$$

$$\frac{dD_0}{d\tau}=A_0-D_0\left(1+\sum_{m=1}^{K} g_mI_m\right)-\sum_{m=1}^{K} g_mI_mN_m. \tag{14}$$

The coupling coefficients between the modes

$$b_{km}=\frac{\sin[2\pi(k-m)l/L]}{2\pi(k-m)l/L} \tag{15}$$

do not differ in form from the coefficients of the series (7). The dimension of the model (12)–(14) is lower than that of model (5)–(8), though the number of eigenvalues of a stability analysis now corresponds to the number of important eigenvalues in the analysis of the model (5)–(8). The models do not agree exactly except in the case of uniform filling of the cavity by the laser medium. Numerical computations show that the models lead to very similar results for a finite medium when it is uniformly excited along its longitudinal dimension. Model (5)–(8) seems to be a more accurate description when the excitation of the medium is not uniform along its length. Pieroux and Mandel have noted that their model allows for both bistability (multiple steady-state solutions) and a Hopf bifurcation to spontaneous dynamical pulsations at low frequencies, though these results are inconsistent with the analyses of the steady states and stability for the integrodifferential equations [6]. Nonetheless, for the range of problems addressed in this paper the results of the two approximate models are essentially equivalent.

III. NUMERICAL RESULTS

Investigations of the dynamical behavior of a particular model begin with finding steady states and then determining the behavior following small deviations from the stationary state. Linearizing the system near the steady states we can find both eigenvectors and real or complex eigenvalues, which characterize the modes of relaxation of the system. However, it is difficult to advance very far analytically. One of the few analyses was given by Khanin *et al.* [13,14] who, after making the simplifying assumption that there would be a large number of stationary lasing modes in a Fabry-Perot laser with (nearly) equal intensities, which allowed them to neglect the deviations of the saturated inversion distribution along the cavity length from a homogeneous distribution, showed that all relaxation modes are separated into compensated and uncompensated categories. They showed that the number of compensated relaxation oscillation modes is determined by the number of optical modes that have the same intensity.

The eigenvectors of rate equation model (1) for mode amplitudes and inversion gratings clearly demonstrate the character of the relaxation modes. The eigenvectors can be found by applying numerical methods to the linearized system. In a matrix form the problem on eigenvalues is formulated as follows:

$$[a_{jk}](V_k)=\alpha_j(V_j),$$

where a_{jk} is the matrix of the coefficients of the linearized system; V_j are the components of the eigenvector; α_j is the eigenvalue. The complex eigenvalues correspond to transient relaxation oscillations at a frequency given approximately by the imaginary part.

Figure 1 shows the result obtained for lasers with five lasing modes. The frequencies of the optical modes are symmetrically detuned from the center of the Lorentzian gain line, while the losses of all optical modes are equal. For a small number of optical modes the number of pairs of complex-conjugated characteristic roots and, consequently, the number of relaxation modes is maximal and equal to *K*. The length of an arrow indicates the amplitude of the variation of the intensity of an optical mode, while its direction reflects the relative phase of variations relative to the modulation of the other modes.

The figure shows that all relaxation oscillations are divided into two categories. The first one is represented by a single frequency, which appears in a single-mode model, and this frequency of modulation is the highest in the eigenfrequency spectrum of dynamical modes. It corresponds to in-phase oscillations of all mode intensities, and thus is always represented in the modulation of the total intensity. The $K-1$ remaining relaxation modes belong to at least partially compensated modulation due to mode competition. The amplitudes and phases of the components of the eigenvectors prove that these relaxation oscillations are either compensated or uncompensated. Figure 1 shows the compensated

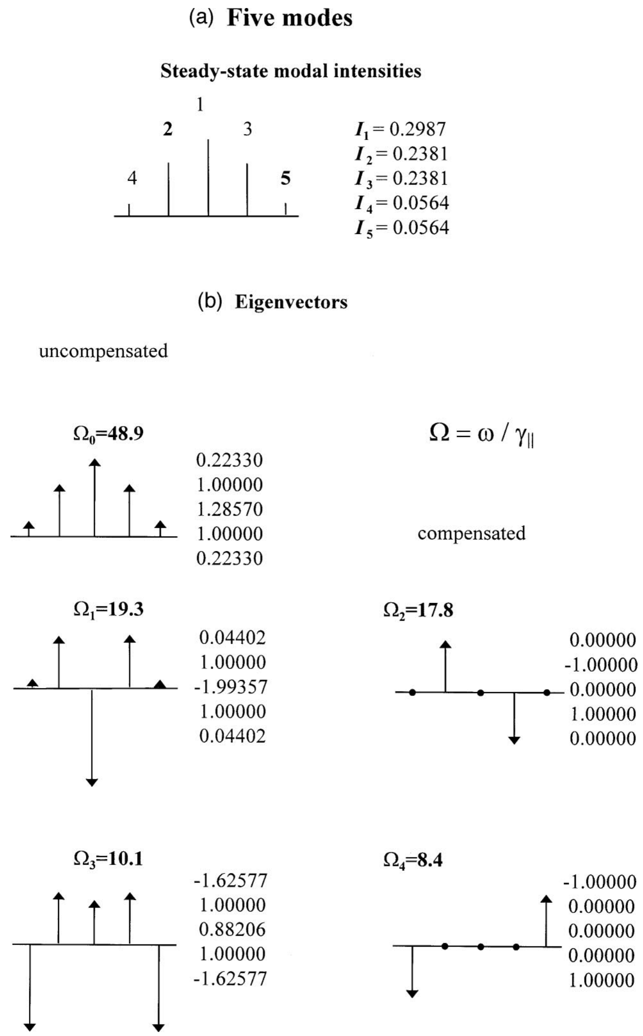

FIG. 1. (a) Schematic of the optical spectrum of a laser with five modes of nonzero intensity showing the mode enumeration used elsewhere. (b) Schematic of the five eigenvector perturbations of the modal intensities corresponding to the five relaxation oscillation eigenvalues that are numbered by subscripts, which increase in order of decreasing frequency.

relaxation oscillation components of the eigenvector, which belong to symmetrically detuned optical modes, are strictly equal in magnitude and opposite in sign. The definition of compensated relaxation oscillations relies exactly on this property. All relaxation oscillations that do not possess this property are regarded as uncompensated relaxation oscillations. Compensated modes of relaxation oscillation exist only when the linear coefficients of the gain of symmetrically detuned optical modes are equal, i.e., only for amplitude degeneracy in an ideal laser medium with a Lorentzian gain profile. Compensated modes of modulation do not cause variations in the total intensity, so their existence is revealed only if the dynamics of individual modes are examined.

Numerical methods allow us not only to find the eigenvectors of a system, but also to trace their dependence on the control parameters, in particular, the pump and the cavity filling factor. The latter case is most interesting. Figure 2 shows the steady-state intensities of five symmetrically de-

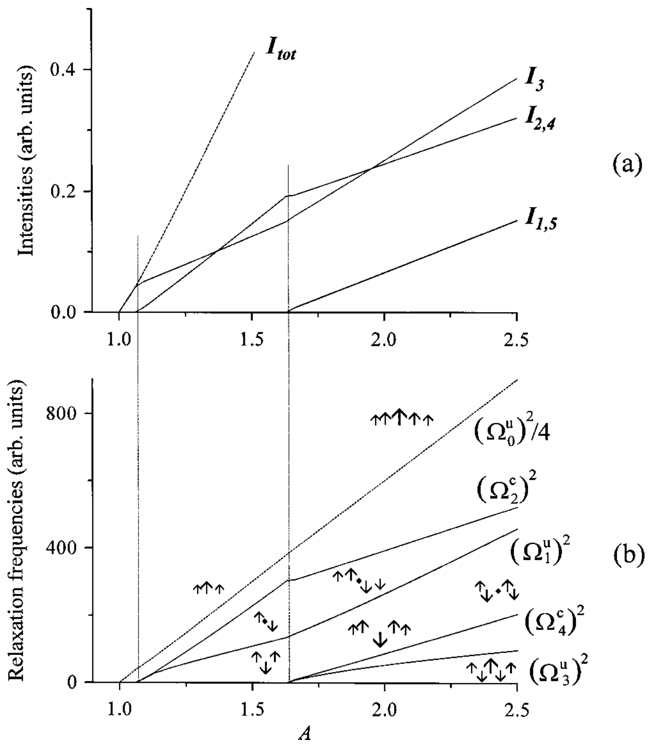

FIG. 2. (a) Dependencies of the total and modal intensities on the pump parameter A for $G=2500$, $\delta=0.132$, $\delta_0=0.0$, $1/L=0.4$. (b) Corresponding dependencies of the squares of the relaxation oscillation frequencies with indications of their eigenvectors. Superscripts in the frequencies indicate whether the modulation of the total intensity is compensated (c) or uncompensated (u). Kinks and nonlinear piecewise evolution are evident near the onset of new modes in the optical spectrum.

tuned modes ($K=5,\delta_0=0$) and their sum, along with relaxation oscillation frequencies, plotted vs the pump parameter, when the filling coefficient is fixed at $l/L=0.4$. In this case the distribution of the gain coefficients is as follows: $\gamma_{1,5}<\gamma_{2,4}<\gamma_3=1$. However, in the range between $A=1.3$ and $A=1.9$ the intensity of the central mode (with the highest small signal gain) is smaller here than the nearest side modes. This situation, which results from mode competition, is possible only for $l/L<1$. This effect was observed in recent experiments with Nd:YAG lasers [4,23]. The number of lasing modes also depends on the pumping level. In the case of perfect symmetry, new modes enter the lasing process in pairs, giving birth to a pair of additional relaxation oscillation frequencies. One of the newly born relaxation modes is compensated, while the other is uncompensated, and their frequencies are different.

The influence of the filling factor l/L on the intensities of lasing modes and the frequencies of relaxation oscillations at a fixed pump parameter is shown in Fig. 3. In the interval $0.1<l/L<0.4$ the central mode is fully suppressed, and the number of lasing modes is four. The complete suppression of a mode has a significant effect on the number and ordering of the relaxation mode frequencies. There are also values of filling factor and pump at which the spectrum of the relaxation frequencies is close to that observed in the case of uniform cavity filling, as shown in Fig. 3. This possibility

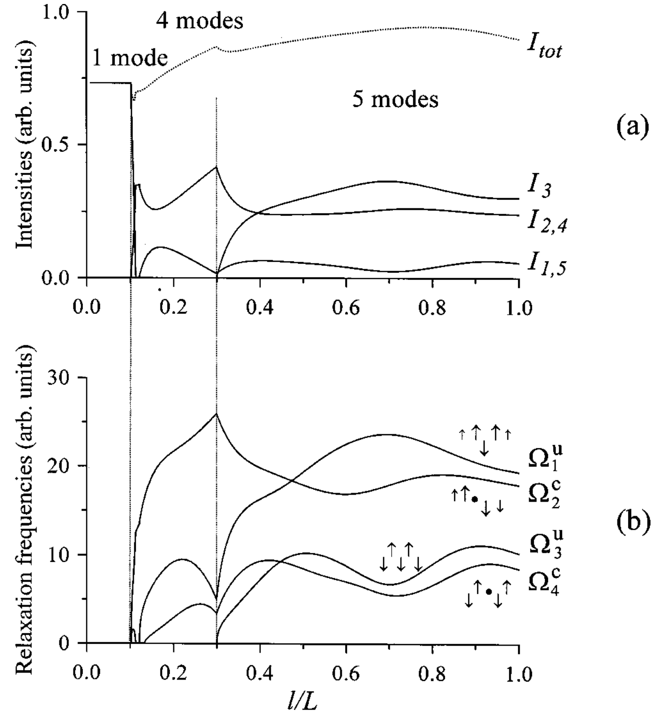

FIG. 3. Dependencies of (a) the total and modal intensities and (b) low relaxation oscillation frequencies on the filling factor $1/L$, for $A=2.0$, $G=2500$, $\delta=0.132$, $\delta_0=0.0$. Domains of one, four, and five modes with nonzero intensities are indicated.

should be taken into account while interpreting experimental results.

Nontrivial results have been also obtained in the study of the functions that describe the transfer of modulation from some parameter, for example, pumping, to intensity, either total or modal. The calculated transfer functions for the five-mode model, discussed above, is given in Fig. 4. Since com-

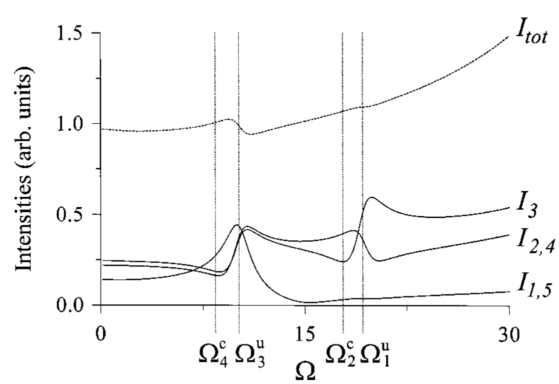

FIG. 4. Modulation transfer functions (amplitude of linear response at the modulation frequency of the total and modal intensities to modulation of the pump parameter) for $A=2.0$, $G=2500$, $\delta=0.132$, $\delta_0=0.0$, and $1/L=0.4$. Relaxation oscillation frequencies are indicated by vertical dashed lines. Compensated forms of modulation cannot be excited by pump modulation in the symmetric model. Some of the resonances in the response functions appear as kinks rather than peaks because of the combined responses with different phases and different strengths of response to the different normal modes of modulation.

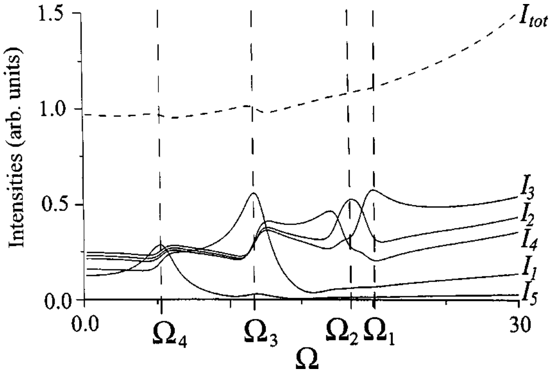

FIG. 5. Modulation transfer functions for $A=2.0$, $G=2500$, $\delta=0.132$, $\delta_0=0.002$, and $1/L=0.4$. Relaxation oscillation frequencies are indicated by vertical dashed lines. Given the broken symmetry caused by the detuning δ_0 resonant response to modulation appears for each of the relaxation oscillation frequencies as their eigenvectors cannot be classified according to the ''compensated'' and ''uncompensated'' criterion.

pensated relaxation oscillations are not excited by pump modulation, in transfer functions for individual modes there are resonances only at frequencies of uncompensated oscillations. These ''resonances'' more often appear as kinks rather than peaks, since modulation at a definite frequency excites all uncompensated oscillations, though with different amplitudes and phases that depend on the detuning of the modulation frequency from the corresponding resonance frequencies. This makes it hard to extract the relaxation oscillation frequencies from experimental spectra, as there is no single feature such as a peak that is reliably correlated with each resonance frequency except in special cases. However, this is not an absolute rule. The presence of the relaxation frequencies Ω_1^u and Ω_0^u does not affect the transfer functions of the weakest modes $I_{1,5}(\Omega)$. As a result we observe a peak, not a kink at frequency Ω_3^u for the spectra and transfer functions of the weakest modes. The vertical scale of Fig. 4 has been selected so that the peak at frequency Ω_0 does not appear (it is off scale to the right).

Disturbance of the detuning, which breaks the symmetry, slightly changes the situation. In this case there are no longer any exactly compensated oscillations. This symmetry breaking is thus accompanied by an increase in the number of resonance peculiarities that are observed in the transfer functions (Fig. 5) since all relaxation oscillation modes can now be excited by pump modulation.

The symmetry breaking due to a difference in mode losses, in particular, when they are randomly perturbed, leads to the same results. These kinds of perturbations often occur from parasitic resonances caused by back reflections from various components (mirrors, lenses, detectors) in the beam line. There may also be equivalent gain variations that affect the pumping of the spatial harmonics of the pump parameter [15]. Intensities of symmetrically detuned modes can be therefore differ even in the absence of the detuning δ_0. However, the effect on the modal intensities introduced by a difference in losses can be offset by a certain detuning. Our computations show that in this quasisymmetric case there are

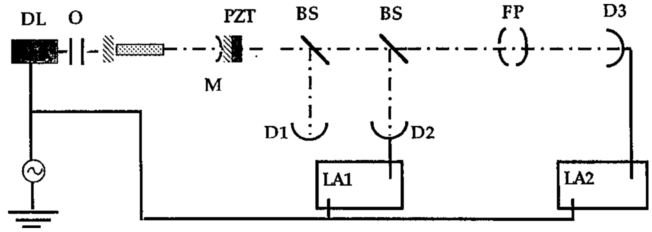

FIG. 6. Experimental setup. DL 808-nm-diode laser array; O, collimating and focusing optics; M, external spherical mirror mounted on a piezoelectric transducer PZT; BS, beam splitters; FP, Fabry-Perot optical spectrum analyzer, $D1$, detector for power readings; $D2$, detector for total intensity modulation; $D3$, detector for light transmitted by FP; LA1 and LA2, lockin amplifiers.

no fully compensated oscillations, and all relaxation oscillations are present to some degree in the transfer functions. Thus, parasitic selection of modes or selective absorption in intracavity media could be responsible for these phenomena in experimental situations.

IV. EXPERIMENTAL RESULTS

Our experiments were carried out on an Nd:YAG laser with diode laser pumping as shown schematically in Fig. 6. The lasing medium was a cylindrical crystal 1 cm in length and 2 mm in diameter with plane parallel faces and with 1% Nd doping. One face of the crystal had a dichroic coating for high reflectivity at the lasing wavelength of 1.06 μm and high transmission at the pump wavelength of 808 nm the other crystal surface had a broadband antireflection coating. The second laser mirror was a spherical mirror with 98.5% reflectivity at 1.06 μm and a radius of curvature of 10 cm, which could be finely positioned longitudinally by a piezoelectric transducer (PZT). The laser cavity length was varied

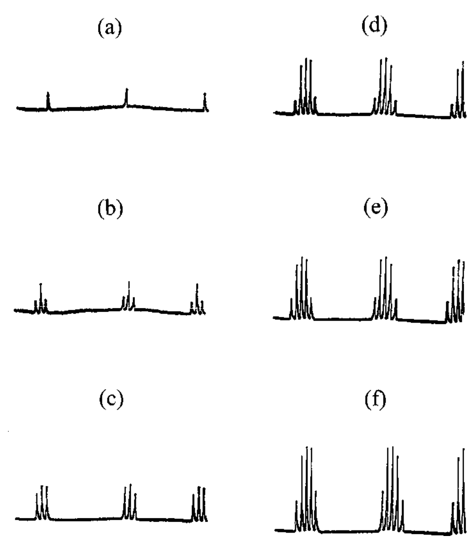

FIG. 7. Optical spectra for different powers of the diode pumping for a cavity length of 8.48 cm giving a mode spacing of 1.616 GHz. Nd:YAG laser threshold parameter A has the following values: (a) 1.02; (b) 1.06; (c) 1.16; (d) 1.30; (e) 1.38; (f) 1.43.

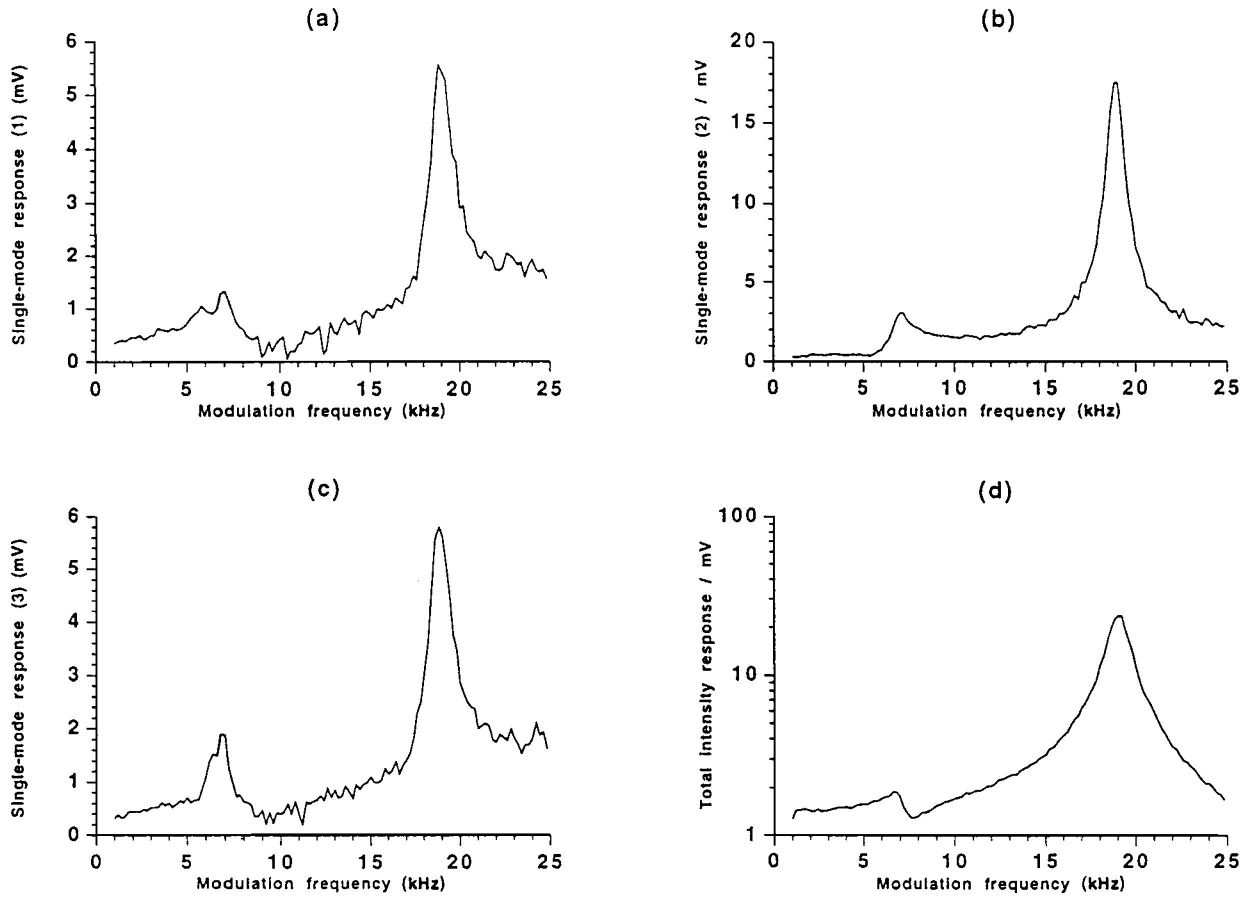

FIG. 8. Modulation transfer functions for cavity length of 8.48 cm and $A=1.06$ for (a) side mode, (b) central mode, (c) side mode, (d) total intensity.

from 5 cm to 10 cm, so that filling factors in the range of 0.45–0.15 could be explored.

For a cavity length of 8.48 cm (filling factor of 0.194) only a few percent above the lasing threshold we observed three modes, but at the maximum level of pumping, 2.5 times above threshold, only five modes were generated spanning less than 5 GHz, indicating a significant compression of the above-threshold portion of the gain spectrum resulting from mode competition. By contrast, for shorter cavity length of 3.81 cm (filling factor of 0.39) we observed both complete and partial suppression of the central mode or of alternate modes. In general, for the shorter cavity lengths (larger filling factor) we observed a broader spectrum, often in excess of 20 GHz for pumping two times above threshold.

Mode composition was monitored by a Fabry-Perot optical spectrum analyzer with a free spectral range of 7.5 GHz. Sample spectra for a cavity length of 8.48 cm (giving a mode spacing of 1.616 GHz) are shown in Fig. 7 for different levels of output power under detuning conditions that gave approximately symmetric spectra. Evidence for parasitic reflection, nonuniform pumping, or other symmetry breaking phenomena was given by the inequality of one pair of side modes when the other pair of side modes in a five-mode spectrum was balanced in intensity.

The noise level in laser emission over the whole low-frequency domain we observed (up to 100 kHz) was very low, being tenths of percent. So, in power spectra, the relaxation peaks were seen very poorly. The use of a locking amplifier in obtaining amplitude transfer functions allowed us to acquire necessary information on specific features of relaxation oscillations at minimum perturbation of pump parameter and, consequently, of laser operation. The ratio of the amplitude of modulating signal to the constant current level of pump laser was as small a fraction of percent. Such level of external perturbation secured the linear laser response to pump modulation.

Modulation transfer functions for total intensity and for three different single modes selected by the Febry-Perot spectrum analyzer are shown in Fig. 8 at low output power. Only two relaxation oscillation frequencies are evident for the central mode and the total power but the side mode transfer functions show two low frequencies at approximately 7 kHz and 6 kHz. The lower ''resonances'' are differently shaped kinks with different phasing of the high- and low-frequency sides for different modes, in agreement with the predictions of the model. Despite the logarithmic scale for the transfer function for the total power, there is no evidence of low-frequency peaks. The details were generally weakly sensitive to detuning, but we did find that there were some conditions for which symmetric spectra could not be maintained at all, leading to gaps in the plot of Fig. 9(a), which shows the total and modal intensities vs the pump parameter.

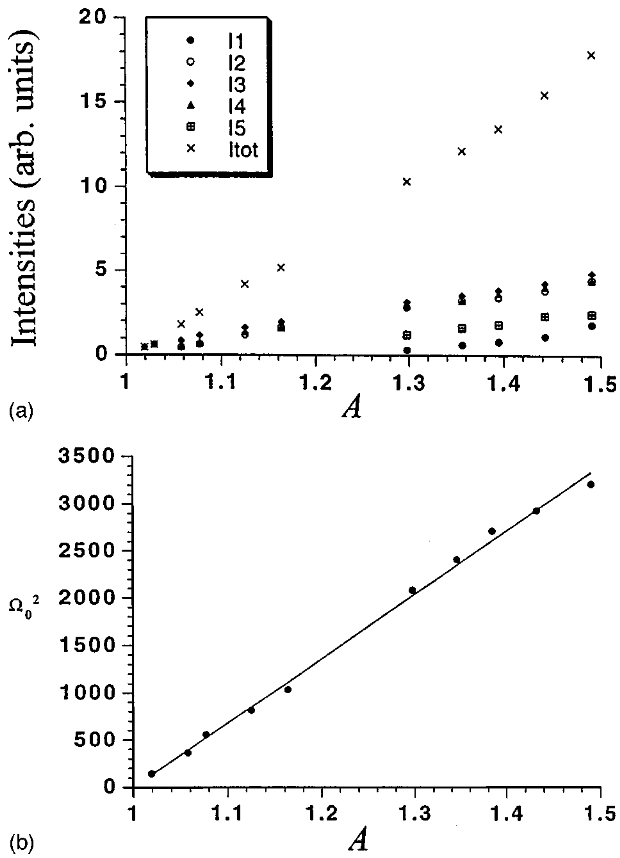

FIG. 9. For cavity length of 8.48 cm. (a) Total power and modal power (arb. units) vs pump parameter A. (b) Frequency of the largest relaxation oscillation peak (from total intensity transfer function) squared vs A, yielding $G \approx 15\,000$.

Figure 9(b) shows the highest relaxation oscillation frequency vs pump parameter from which we can extract the parameter G from the model ($G \approx 15\,000$). Figure 10 shows that the slopes of the modal intensities vs pump parameter decreased whenever the number of modes increased. Moreover there were regions where symmetric spectra with a strong central mode could not be maintained stably. In these regions the optical spectrum was very sensitive to slight amounts of detuning, with sharp jumps in the spectrum such as are shown in Fig. 11, which demonstrate both asymmetric spectra and an example of suppression of the central mode.

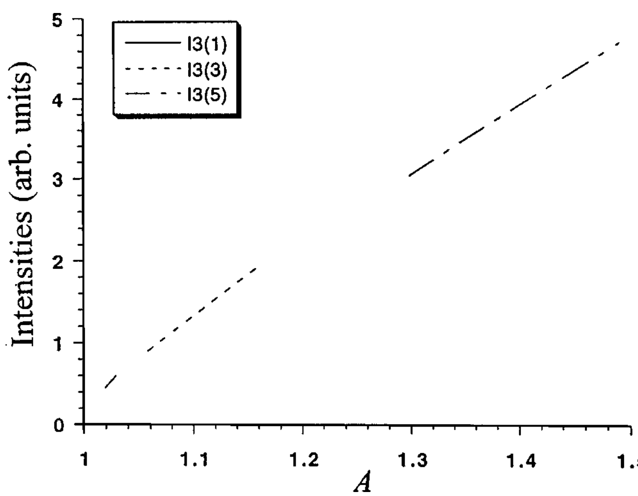

FIG. 10. For cavity length of 8.48 cm, power of the central vs A fitted in three regions of single-mode, three-mode, and five-mode emission with slopes 15.6, 10.1, and 8.7, respectively, with approximately 5% uncertainties.

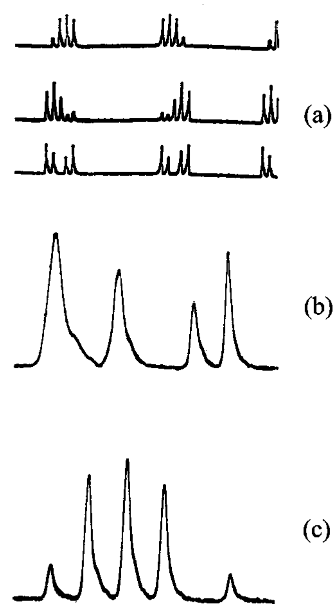

FIG. 11. Optical spectra for $A = 1.26$ and cavity length of 8.48 cm for small variations in the detuning (with essentially constant total power emission) showing various side-mode suppression, spectral asymmetries, and central-mode suppression. At this pump value a symmetric spectrum could not be obtained.

The abrupt shifts in mode intensities with detuning, mode suppression, and the consequences for the dependence of the relaxation oscillation frequencies on the pump and modulation transfer functions for the total intensity and for the modal intensities are in generally good agreement with the theoretical and numerical predictions. See, for example, Fig. 12, in which we present computed results from the model for $A = 1.2$ and a filling factor of 0.2, to show the nonequidistant and nonsymmetric spectra with a quick hop in the spectral profile with a small detuning change. One exception is that conditions of apparently symmetrically arranged modal spectra gave peaks in the total intensity transfer function at what were predicted to be compensated relaxation oscillation frequencies, and pump modulation strongly excites compensated relaxation oscillations as found in the modal intensity modulation transfer functions. We conclude that these results are most likely due to nonuniform pumping, as adjustments in the modal spectral symmetry from moving the focusing optics could be compensated by minor adjustments in detuning.

V. DISCUSSION

The most important results in this work concern the peculiarities of dynamic behavior of multimode class B lasers with nonuniform cavity filling by laser medium. These are

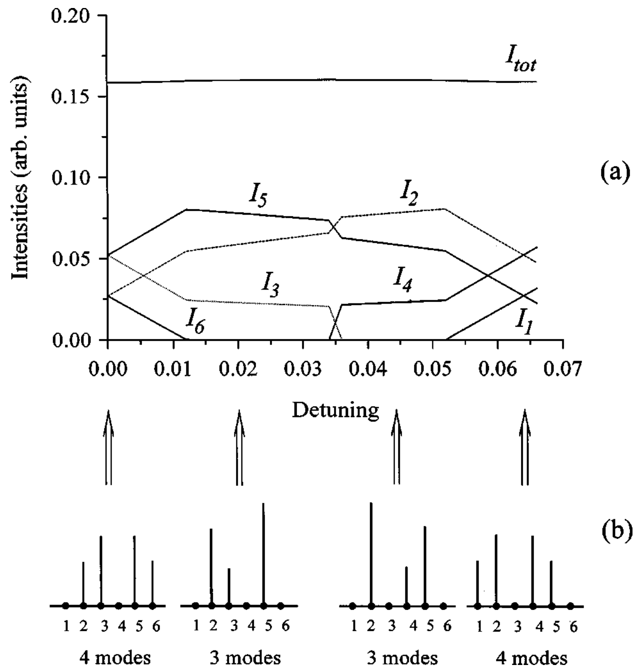

FIG. 12. (a) Dependencies of the total and modal intensities on the detuning δ_0 for $G=2500$, $\delta=0.064$, and $1/L=0.2$. In the vicinity of $\delta_0=0$ ($\delta_0=\delta$), the intensity of the central mode I_4 (I_3) is suppressed. (b) Optical spectra for different detunings δ_0.

the theoretical results obtained numerically, which are in good agreement with the experimental data.

(1) The eigenvectors for the eigenvalues of the stability of the steady-state solution of the rate equation model for relatively small number of lasing modes have been found. The structure of these vectors shows that the relaxation oscillations for situations of symmetric mode detunings can be separated into compensated and uncompensated modulations. Compensated modulations exist only when there is perfect symmetry in the detunings and pumpings of the optical modes, otherwise all modulations lead to some degree of modulation of the total intensity.

(2) The number of lasing modes depends on the pumping level. In case of perfect symmetry, new modes enter the lasing process in pairs, giving birth to a pair of relaxation oscillations. One of the newly born relaxation oscillations is compensated, while the other is uncompensated, and their frequencies are different.

(3) The profile of the unsaturated inversion as determined by the size and location of the laser medium inside the cavity (the filling factor) as well as by the spatial distribution of pumping along the laser medium significantly affects the mode competition. Changes in the filling factor can result in suppression of optical modes with potentially large gain (for example, the central mode) by weaker modes.

(4) The filling factor also has a considerable effect on the ordering of the relaxation mode frequencies. There are values of the filling factor, at which the spectrum of the relaxation frequencies is close to that observed in case of uniform cavity filling. This possibility should be taken into account when interpreting experimental results.

(5) Nontrivial results have been obtained in the study of transfer functions. These are, first of all, the manifestation of resonances in response to pump modulation at frequencies of compensated oscillations, where no salient features were anticipated. We explain the excitation of the compensated relaxation oscillations from pump modulation by a hidden symmetry breaking present in the system of optical modes. If factors, which lead to selective discrimination among the individual modes, act here, then we can restore the apparent symmetry of the optical spectrum, counterbalancing (neutralizing) the effect of, for example, weak parasitic mode selection of pumping nonuniformities by detuning the cavity. However, this compensation to equalize the intensities does not make the system really symmetric and thus the relaxation oscillations are not correspondingly compensated.

During the completion of this work we learned of studies by Stamatescu and Hamilton [28] of the effects of a partially filled cavity on the relaxation oscillation frequencies and on the relative phasing of the response of these different modes of oscillation to pump modulation. Mandel, Nguyen, and Otsuka paper [29] have also recently completed a study of the relaxation oscillation frequencies of a three-mode laser with arbitrary gains for the three modes. Of special note are their results that the unequal gains may lead to rational fractional relations among the relaxation oscillation frequencies, which may promote additional resonances in the modulation response, phenomena confirmed by experimental studies of a microchip laser (with filling factor equal to unity).

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[1] C. L. Tang, H. Statz, and G. de Mars, J. Appl. Phys. **34**, 2289 (1963).

[2] H. Statz, C. L. Tang, and J. M. Lavine, J. Appl. Phys. **35**, 2581 (1964).

[3] D. E. McCumber, Phys. Rev. **141**, 306 (1966).

[4] P. Khandokhin, Ya. I. Khanin, J.-C. Celet, D. Dangoisse, and P. Glorieux, Opt. Commun. **123**, 372 (1995).

[5] K. Otsuka, D. Pieroux, and P. Mandel, Opt. Commun. **108**, 265 (1994).

[6] A. V. Ghiner and K. G. Folin, *Dynamics of Free Lasing of Solid-State Lasers* (Nauka, Novosibirsk, 1979) in Russian; A. V. Ghiner, K. P. Komarov, and K. G. Folin, Opt. Commun. **19**, 350 (1975).

[7] P. Mandel, C. Etrich, and K. Otsuka, IEEE J. Quantum Electron. **29**, 836 (1993).

[8] P. A. Khandokhin, Ya. I. Khanin, I. V. Koryukin, and P. Mandel, IEEE J. Quantum Electron. **31**, 647 (1995).

[9] J. R. Tredicce, L. M. Narducci, N. B. Abraham, D. K. Bandy,

and L. A. Lugiato, Opt. Commun. **56**, 435 (1986).

[10] L. M. Narducci, J. R. Tredicce, L. A. Lugiato, N. B. Abraham, and D. K. Bandy, Phys. Rev. A **33**, 842 (1986).

[11] H. Zeghlache, P. Mandel, N. B. Abraham, L. M. Hoffer, G. L. Lippi, and T. Mello, Phys. Rev. A **37**, 470 (1988).

[12] I. V. Koryukin, P. Khandokhin, Ya. Khanin, and P. Mandel, Kvant. Elektron. (Moscow) **22**, 1081 (1995) [Quantum Electron. **25**, 1045 (1995)].

[13] Ya. I. Khanin, *Principles of Laser Dynamics* (North-Holland/Elsevier, Amsterdam, New York, 1995).

[14] A. S. Meller, P. A. Khandokhin, and Ya. I. Khanin, Kvant. Electron (Moscow) **13**, 2278 (1986) [Sov. J. Quantum Electron. **16**, 1502 (1986)].

[15] E. S. Kovalenko, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **10**, 1765 (1967) [Radiophys. Quantum Electron. **10**, 989 (1967)]; E. S. Kovalenko and L. I. Shangina, *ibid.* **12**, 846 (1969); [**12**, 674 (1969)]; A. A. Gusev, S. V. Kruzhalov, L. N. Pakhomov, and V. Yu. Petrun'kin, Pis'ma Zh. Tekh. Fiz. **4**, 1250 (1978) [Sov. Tech. Phys. Lett. **4**, 503 (1979)].

[16] P. A. Khandokhin, P. Mandel, I. V. Koryukin, B. A. Nguyen, and Ya. I. Khanin, Phys. Lett. A **235**, 248 (1997).

[17] P. A. Khandokhin, P. Mandel, I. V. Koryukin, B. A. Nguyen, and Ya. I. Khanin, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **40**, 161 (1997).

[18] O. Evdokimova and L. Kaptsov, Kvant. Elektron. **16**, 1557 (1989) [Sov. J. Quantum Electron. **19**, 1001 (1989)].

[19] V. R. Mironenko and V. I. Yudson, Opt. Commun. **34**, 397 (1980); Zh. Eksp. Teor. Fiz. **79**, 1174 (1980).

[20] V. R. Mironenko and V. I. Yudson, Opt. Commun. **41**, 126 (1982).

[21] K. Otsuka, in *Chaos in Optics*, edited by R. Roy, Proc. SPIE **2039**, 182 (1993).

[22] E. Lacot and F. Stoeckel, J. Opt. Soc. Am. B **9**, 2034 (1996).

[23] N. B. Abraham, L. L. Everett, C. Iwata, and M. B. Janicki, in *Laser Optics '93: Laser Physics*, edited by A. Mak, Proc. SPIE **2095**, 16 (1994).

[24] O. N. Evdokimova and L. Kaptsov, Kvant. Elektron. **17**, 901 (1990) [Sov. J. Quantum Electron. **20**, 824 (1990)].

[25] E. A. Viktorov, V. A. Sokolov, E. V. Tkachenko, and V. I. Ustyugov, Opt. Spektrosk. **68**, 920 (1990) [Opt. Spectrosc. **68**, 537 (1990)].

[26] K. N. Evtyukhov, L. N. Kaptsov, and I. V. Mitin, Zh. Prikl. Spektrosk. **32**, 18 (1980) [J. Appl. Spectrosc. **32**, 10 (1980)].

[27] D. Pieroux and P. Mandel, Quantum Semiclassic. Opt. **9**, L17 (1997).

[28] L. Stamatescu and M. W. Hamilton, Phys. Rev. E **55**, 2115 (1997).

[29] P. Mandel, B. A. Nguyen, and K. Otsuka, Quantum Semiclassic. Opt. **9**, 365 (1997).