PHYSICAL REVIEW A, VOLUME 62, 013809

Three-photon Hong-Ou-Mandel interference at a multiport mixer

Richard A. Campos*
Hudson Valley Research Park, East Fishkill, New York 12533
(Received 15 December 1999; published 14 June 2000)

We consider a six-port triangular arrangement of beam splitters designed to couple three electromagnetic fields through U(3) unitary transformations. We present conditions for Hong-Ou-Mandel destructive interference of output triple coincidences when single photons are presented, one at each of the three input ports. Unlike the corresponding four-port effect, three-photon Hong-Ou-Mandel interference is sensitive to mixer phases and input-output port reversal.

PACS number(s): 42.50.Ar, 42.25.Hz, 03.65.Fd

I. INTRODUCTION

Over a decade ago, realizations of double-quantum interference effects using correlated photon pairs from parametric down-conversion unequivocally confirmed the fundamental nature of boson statistics. In a notable 1987 experiment [1], Hong, Ou, and Mandel (HOM) demonstrated a cancellation of interbeam coincidences behind a 50:50 optical mixer when the photons entered one into each of the two input ports of the device with adjustable simultaneity. Said differently, the photons exit the symmetric mixer in a superposition state containing an equal potentiality for the registration of the photon pair at either of the two output ports. The HOM experiment therefore confirms the enhanced tendency of photons to cluster together due to boson statistics [2]. The physical basis for this two-photon effect is quantum-mechanical destructive interference between probability amplitudes for double transmission and double reflection at the optical mixer, when experimental conditions favor their indistinguishability.

As larger equal numbers of photons are brought into each input port of the 50:50 two-beam mixer, theoretical calculations show that the boson clustering effect generalizes into an arcsine distribution of photon pairs, with strictly zero probability for the registration of odd photon numbers [3]. Recent experiments confirm these higher-order multiphoton effects at a single optical mixer [4].

The striking beauty and simplicity of this phenomenon presumes wider application toward more complex optical systems, particularly to those mixing arrangements that contain two-beam HOM modules. The purpose of this paper is to determine the necessary conditions for a three-photon HOM effect [5] in the next-order generalization of a simple optical mixer, called a tritter [6]. This is a six-port device that can be assembled from a triangle of two-beam mixers [7]. Below we derive the criteria for three-photon HOM interference, starting from general unitary considerations for the tritter. In this fashion we explore how the effect depends on tritter optical parameters, as well as identify a simpler mixing configuration to illustrate the essential nature of three-photon HOM interference.

*Electronic address: ravarcampos@compuserve.com

II. THREE-PHOTON HOM EFFECT

A. Unitary considerations

We begin with a collection of N quantum harmonic oscillators whose creation operators $\mathbf{\hat{a}}=\{\hat{a}_n\}$ for $n=1,...,N$ obey the boson commutator relations $[\hat{a}_n,\hat{a}_m^\dagger]=\delta_{nm}\hat{I}$. It is well established that the N-dimensional harmonic oscillator is governed by U(N) unitary symmetry [8]. This means that the Hamiltonian form is preserved under the unitary transformations

$$\begin{aligned}\hat{b}_n&=\hat{U}(\gamma)\hat{a}_n\hat{U}^\dagger(\gamma)=\sum_{m=1}^{N}u_{nm}(\gamma)\hat{a}_m,\\ \hat{b}_n^\dagger&=\hat{U}(\gamma)\hat{a}_n^\dagger\hat{U}^\dagger(\gamma)=\sum_{m=1}^{N}u_{nm}^*(\gamma)\hat{a}_m^\dagger.\end{aligned}\tag{1}$$

The unitary operator is often written in the product form,

$$\hat{U}(\gamma)=\prod_{k=1}^{N^2}\exp(is_k\hat{R}_k),\tag{2}$$

where $\gamma=\{s_k\}$ are the free parameters of the transformation, and the operators $\hat{R}_k$ are selected from among the N^2 Hermitian operators $\mathbf{\hat{F}}=\{\hat{F}_k\}$ obeying the U(N) commutator relations

$$[\hat{F}_n,\hat{F}_m]=i\sum_k g_{nmk}\hat{F}_k\tag{3}$$

with group structure constants g_{nmk}.

For a general pure state that is created with the boson operators according to

$$|\Psi\rangle=f(\mathbf{\hat{a}}^\dagger)|0\rangle,\tag{4}$$

we reexpress the creation operators through the inverse unitary transformations

$$\hat{a}_n^\dagger=\hat{U}^\dagger(\gamma)\hat{b}_n^\dagger\hat{U}(\gamma)=\sum_{m=1}^{N}u_{mn}(\gamma)\hat{b}_m^\dagger,\tag{5}$$

and collect the terms in the expansion to generate the joint probability distribution at the N output ports of the U(N) optical mixer. In the reverse process, the output ports of the

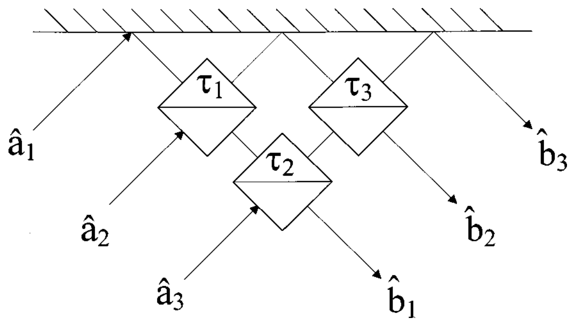

FIG. 1. Three-photon interference effects are considered for a triangular arrangement of optical mixers, called a tritter. The mixers are labeled by their transmittances τ together with the input and output electromagnetic field modes $\hat{a}$ and $\hat{b}$, respectively. For clarity, mixer phase shifts are not labeled but are discussed in the text.

mixer are loaded according to $|\Psi\rangle = f(\hat{\mathbf{b}}^\dagger)|0\rangle$, and Eq. (1) generates the joint probability distribution at the input ports.

B. The tritter

An optical realization of U(3) unitary transformations is shown in Fig. 1 [7]. This triangular arrangement of two-beam mixers couples three harmonic-oscillator modes $\hat{a}_n$ $(n=1,2,3)$ that represent equipolarized, single-mode optical fields [9]. If the first and third optical mixers have 50:50 partition ratios ($\tau_1=\tau_3=\frac{1}{2}$) while the remaining mixer has a 67:33 partition ratio ($\tau_2=\frac{2}{3}$), then for certain choices of phases an input photon distributes with equal probability into all output ports. This has been called a *symmetric multiport device* [10]. Our analysis below preserves the general formalism, as it leads to results that are not available for the symmetric multiport arrangement.

In analogy with the HOM experiment, we look for a solution that provides three-photon coincidence cancellation at all three output ports of the tritter when one photon fills each of the input ports. We begin with the most general SU(2) unitary transformation of a two-beam mixer [3,11],

$$\begin{pmatrix} \sqrt{\tau}\exp(i\alpha) & \sqrt{\rho}\exp(i\beta) \\ -\sqrt{\rho}\exp(-i\beta) & \sqrt{\tau}\exp(-i\alpha) \end{pmatrix}, \tag{6}$$

where $\tau=1-\rho$ is the transmittance, while α and β are associated phase factors for the device. For example, the common choice $\alpha=0$ and $\beta=\pi/2$ yields a phase factor i at both reflection arms [12]. The matrix representation of Fig. 1 is constructed by sequential cascading of two-beam mixer representations. The unitary generators for these matrices are

$$\begin{aligned} \hat{B}_1 &:= \exp(-i\phi_1\hat{F}_3)\exp(-i\theta_1\hat{F}_2)\exp(-i\psi_1\hat{F}_3), \\ \hat{B}_2 &:= \exp[-i\phi_2(\hat{F}_4+\tfrac{1}{2}\hat{F}_3)]\exp(-i\theta_2\hat{F}_6) \\ &\quad\times\exp[-i\psi_2(\hat{F}_4+\tfrac{1}{2}\hat{F}_3)], \\ \hat{B}_3 &:= \exp[-i\phi_3(\hat{F}_4-\tfrac{1}{2}\hat{F}_3)]\exp(-i\theta_3\hat{F}_8) \\ &\quad\times\exp[-i\psi_3(\hat{F}_4-\tfrac{1}{2}\hat{F}_3)], \end{aligned} \tag{7}$$

such that by the unitary shifting properties $\hat{b}_n=\hat{B}_m\hat{a}_n\hat{B}_m^\dagger$,

$$B_1 = \begin{pmatrix} \sqrt{\tau_1}e^{i\alpha_1} & \sqrt{\rho_1}e^{i\beta_1} & 0 \\ -\sqrt{\rho_1}e^{-i\beta_1} & \sqrt{\tau_1}e^{-i\alpha_1} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} \sqrt{\tau_2}e^{i\alpha_2} & 0 & \sqrt{\rho_2}e^{i\beta_2} \\ 0 & 1 & 0 \\ -\sqrt{\rho_2}e^{-i\beta_2} & 0 & \sqrt{\tau_2}e^{-i\alpha_2} \end{pmatrix}, \tag{8}$$

$$B_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\tau_3}e^{i\alpha_3} & \sqrt{\rho_3}e^{i\beta_3} \\ 0 & -\sqrt{\rho_3}e^{-i\beta_3} & \sqrt{\tau_3}e^{-i\alpha_3} \end{pmatrix},$$

respectively, provided $\tau_m := \cos^2(\theta_m/2)$ and $(\alpha_m,\beta_m) := \frac{1}{2}(\psi_m \pm \phi_m)$, where ϕ_m and ψ_m are Euler angles in the unitary representations of Eq. (7). The Hermitian operators $\hat{\mathbf{F}}$ are Jordan-Schwinger maps [13] of Gell-Mann SU(3) matrices [14],

$$\begin{aligned} &\hat{F}_1 := \tfrac{1}{2}(\hat{a}_1^\dagger\hat{a}_2+\hat{a}_2^\dagger\hat{a}_1), \quad \hat{F}_5 := \tfrac{1}{2}(\hat{a}_1^\dagger\hat{a}_3+\hat{a}_3^\dagger\hat{a}_1), \\ &\hat{F}_2 := \frac{1}{2i}(\hat{a}_1^\dagger\hat{a}_2-\hat{a}_2^\dagger\hat{a}_1), \quad \hat{F}_6 := \frac{1}{2i}(\hat{a}_1^\dagger\hat{a}_3-\hat{a}_3^\dagger\hat{a}_1), \\ &\hat{F}_3 := \tfrac{1}{2}(\hat{a}_1^\dagger\hat{a}_1-\hat{a}_2^\dagger\hat{a}_2), \quad \hat{F}_7 := \tfrac{1}{2}(\hat{a}_2^\dagger\hat{a}_3+\hat{a}_3^\dagger\hat{a}_2), \\ &\hat{F}_4 := \tfrac{1}{4}(\hat{a}_1^\dagger\hat{a}_1+\hat{a}_2^\dagger\hat{a}_2-2\hat{a}_3^\dagger\hat{a}_3), \quad \hat{F}_8 := \frac{1}{2i}(\hat{a}_2^\dagger\hat{a}_3-\hat{a}_3^\dagger\hat{a}_2). \end{aligned} \tag{9}$$

These operators satisfy Eq. (3) with the antisymmetric structure constants $g_{123}=g_{456}=g_{478}=1$, $g_{158}=g_{176}=g_{257}=g_{268}=g_{356}=g_{387}=\frac{1}{2}$. We also utilize the mirror matrix representation

$$M_m := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + (e^{i\zeta}-1)\begin{pmatrix} \delta_{m1} & 0 & 0 \\ 0 & \delta_{m2} & 0 \\ 0 & 0 & \delta_{m3} \end{pmatrix}, \tag{10}$$

where ζ is the mirror reflection phase factor. By stacking these unitary matrices in the order $M_3B_3M_2B_2B_1M_1$, we arrive at the general transformation matrix $u_{nm}(\gamma)$ linking the output operators $\hat{\mathbf{b}}$ to the input operators $\hat{\mathbf{a}}$ [Eq. (1)],

$$\begin{pmatrix} \sqrt{\tau_1\tau_2}e^{i(\zeta+\alpha_1+\alpha_2)} & \sqrt{\rho_1\tau_2}e^{i(\beta_1+\alpha_2)} & \sqrt{\rho_2}e^{i(\beta_2)} \\ -\sqrt{\rho_1\tau_3}e^{i(2\zeta-\beta_1+\alpha_3)}-\sqrt{\tau_1\rho_2\rho_3}e^{i(\zeta+\alpha_1-\beta_2+\beta_3)} & \sqrt{\tau_1\tau_3}e^{i(\zeta-\alpha_1+\alpha_3)}-\sqrt{\rho_1\rho_2\rho_3}e^{i(\beta_1-\beta_2+\beta_3)} & \sqrt{\tau_2\rho_3}e^{i(\beta_3-\alpha_2)} \\ \sqrt{\rho_1\rho_3}e^{i(3\zeta-\beta_1-\beta_3)}-\sqrt{\tau_1\rho_2\tau_3}e^{i(2\zeta+\alpha_1-\beta_2-\alpha_3)} & -\sqrt{\tau_1\rho_3}e^{i(2\zeta-\alpha_1-\beta_3)}-\sqrt{\rho_1\rho_2\tau_3}e^{i(\zeta+\beta_1-\beta_2-\alpha_3)} & \sqrt{\tau_2\tau_3}e^{i(\zeta-\alpha_2-\alpha_3)} \end{pmatrix}. \tag{11}$$

For a U(3) HOM experiment, single photons are presented at each input port of the tritter, according to the state creation function $f(\hat{\mathbf{a}})=\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_3^\dagger$ described in Eq. (4). There are six paths to coincidence at the output ports. For example, the amplitude for the ordered transfer of single photons from input ports (1,2,3) to output ports (1,2,3) is obtained from the product of the right diagonal matrix elements of the transposed matrix [Eq. (5)], namely, $\tau_1\tau_2\tau_3\exp(i3\zeta)+\tau_2\sqrt{\tau_1\tau_3\rho_1\rho_2\rho_3}\exp[i(2\zeta+\Phi)]$, where we define

$$\Phi:=\alpha_1-\alpha_3+\beta_1-\beta_2+\beta_3=\psi_1-\phi_3+\tfrac{1}{2}(\phi_2-\psi_2). \tag{12}$$

For sixfold path indistinguishability, the sum of the six possible amplitudes must be nulled, leading to the condition

$$(\tau_1-\rho_1)(\tau_2-\rho_2)(\tau_3-\rho_3)+2\sqrt{\tau_1\tau_3\rho_1\rho_2\rho_3}\times[(1-3\tau_2)e^{i(\Phi-\zeta)}+e^{-i(\Phi-\zeta)}]=0. \tag{13a}$$

A notable feature of this key result is the sensitivity to mirror and mixer phase shifts ζ and Φ, respectively. Since the reversal of input and output ports leads to the complex conjugate of Eq. (13a), uncanceled output triple coincidences will also inherit this feature and will therefore generally not be symmetric to input-output port reversal. This is not the case for HOM interference at a single, uncoupled mixer. The equivalent amplitude in the latter case is obtained from Eq. (6),

$$(\tau-\rho)=0. \tag{13b}$$

This is completely phase insensitive, and requires only a mixer with 50:50 partition ratio.

Another interesting feature of Eq. (13a) is the manifestation of one, rather than three, mirror bounces (see Fig. 1). Careful tracking of terms reveals that the mirror reflection between the first and third mixers is the only contributing phase factor. This may lend flexibility to the experimental design.

The factorized form of Eq. (13a) facilitates a solution by simple inspection. Note that the first term in the expression vanishes if we select 50:50 partition ratios for any of the mixers. Choosing $\tau_1=\frac{1}{2}$ for convenience, it follows that the vanishing of the term $(1-3\tau_2)\exp[i(\Phi-\zeta)]+\exp[-i(\Phi-\zeta)]$ leads to the additional solutions

$$\tau_2=\tfrac{2}{3},\qquad \Phi=\zeta\pm\pi, \tag{14}$$

without a specific requirement for τ_3. We will explore this degree of freedom further below. From the phase condition of Eq. (14), a standard choice of mirror reflection $\zeta=\pi$ provides the additional requirement $\Phi=0$. A simple way to meet this is to balance the pathlength difference between the arms of the tritter, and to verify that all three mixers satisfy the Stokes relations $(\alpha_m=\beta_m=0)$ [12].

For Stokes mixers, the probability of triple coincidence at the output ports of the tritter generally obtains,

$$P_{\text{out}}(1,1,1)=[2\sqrt{\tau_1\tau_3\rho_1\rho_2\rho_3}(3\tau_2-2)+(\tau_1-\rho_1)(\tau_2-\rho_2)\times(\tau_3-\rho_3)]^2. \tag{15}$$

This contrasts sharply with the equivalent expression derived from classical probability for distinguishable particles,

$$P_{\text{out}}^{\text{classical}}(1,1,1)=[(\tau_1\tau_3\rho_1\rho_2\rho_3)(4\tau_2^2+2\rho_2^2+2)+(\tau_1^2+\rho_1^2)\times(\tau_2^2+\rho_2^2)(\tau_3^2+\rho_3^2)]. \tag{16}$$

These expressions are compared in Fig. 2 for $\tau_1=\tau_3=\frac{1}{2}$ as a function of the transmittance τ_2. In the quantum theory, the probability of triple coincidence vanishes at $\tau_2=\frac{2}{3}$ and thereafter remains near a value of 0.01, which is approximately 25 times lower than the classical prediction. In the limiting case $\tau_2\rightarrow 0$, the second mixer transforms into a mirror, and the tritter becomes a balanced Mach-Zehnder interferometer for ports 1 and 2. For this configuration, the tritter translates the input state at the upper input ports to the two upper output

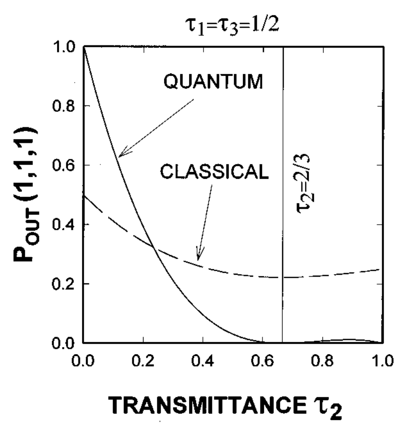

FIG. 2. The probability of triple photon coincidence at the output ports of the tritter (Fig. 1) as a function of the second mixer's transmittance is compared to the equivalent expression obtained from classical considerations for distinguishable particles. Nonclassical cancellation of triple coincidence obtains for the transmittance values $\tau_1=1/2$ and $\tau_2=2/3$, with no further requirement for τ_3. For the purposes of this figure, we chose $\tau_3=1/2$. In the limiting case $\tau_2\rightarrow 0$ the top portion of the optical circuit forms a Mach-Zehnder interferometer, which translates the input state to the output ports, hence $P_{\text{out}}(1,1,1)=1$.

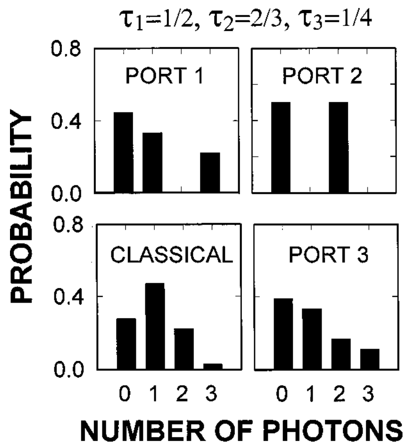

FIG. 3. The additional specifications $\tau_3=1/4$ or $\tau_3=3/4$ supplement three-photon HOM interference by even-odd photon sorting at the output ports of the tritter. The resulting marginal photon-number probability distributions for $\tau_3=1/4$ differ from classical predictions. The selection $\tau_3=3/4$ exchanges the photon-number distributions observed at output ports 2 and 3.

ports, hence $P_{\text{out}}(1,1,1)=1$. This does not occur for classical particles as they independently partition into binomial statistics [2].

C. Even-odd photon sorting

The degree of freedom (τ_3) available for three-photon HOM interference has the additional consequence of allowing even-odd photon sorting at the output ports of the tritter. This follows from the remaining probabilities for exit photon arrangements,

$$\begin{aligned}
P_{\text{out}}(0,0,3)&=6[\sqrt{\tau_3\tau_2\tau_1\rho_1}(\rho_3-\tau_3\rho_2)\\
&\quad+\tau_3(\tau_1-\rho_1)\sqrt{\tau_2\rho_3\rho_2}]^2,\\
P_{\text{out}}(0,3,0)&=6[\sqrt{\tau_2\tau_1\rho_3\rho_1}(\tau_3-\rho_3\rho_2)\\
&\quad-\rho_3(\tau_1-\rho_1)\sqrt{\tau_3\tau_2\rho_2}]^2,\\
P_{\text{out}}(3,0,0)&=6\tau_2^2\tau_1\rho_2\rho_1,
\end{aligned}\tag{17}$$

and also

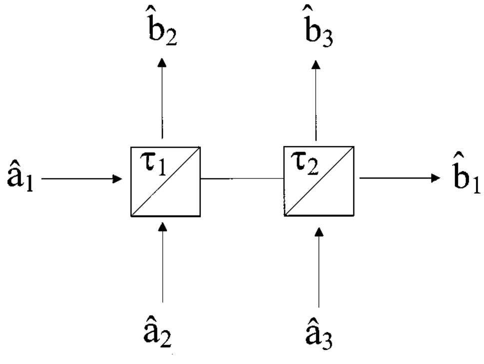

FIG. 4. Elimination of the third optical mixer reduces the tritter to this optical circuit.

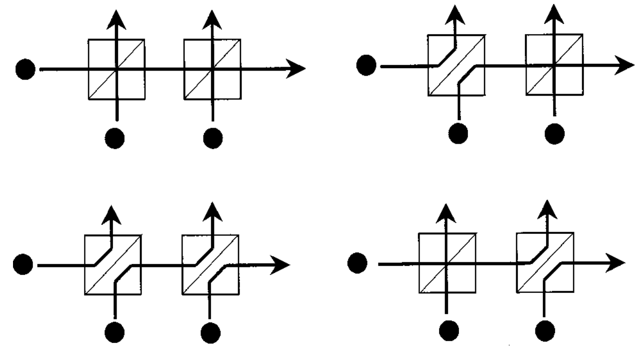

FIG. 5. Three-photon HOM interference for the setup of Fig. 4 results from the cancellation of four probability amplitudes, in a manner analogous to two-photon HOM interference.

$$\begin{aligned}
P_{\text{out}}(0,1,2)&=2[\sqrt{\tau_2\tau_1\rho_3\rho_1}(1-3\tau_3-3\tau_3\rho_2)-(\tau_3-2\rho_3)\\
&\quad\times(\tau_1-\rho_1)\sqrt{\tau_3\tau_2\rho_2}]^2,\\
P_{\text{out}}(1,0,2)&=2[\sqrt{\tau_1\rho_2\rho_1}(3\tau_3\tau_2+\rho_3-\tau_3)\\
&\quad-(\tau_2-\rho_2)(\tau_1-\rho_1)\sqrt{\tau_3\rho_3}]^2,\\
P_{\text{out}}(0,2,1)&=2[\sqrt{\tau_3\tau_2\tau_1\rho_1}(3\rho_3\rho_2+3\rho_3-1)+(2\tau_3-\rho_3)\\
&\quad\times(\tau_1-\rho_1)\sqrt{\tau_2\rho_3\rho_2}]^2,\\
P_{\text{out}}(1,2,0)&=2[\sqrt{\tau_1\rho_2\rho_1}(3\rho_3\tau_2+\tau_3-\rho_3)\\
&\quad+(\tau_2-\rho_2)(\tau_1-\rho_1)\sqrt{\tau_3\rho_3}]^2,\\
P_{\text{out}}(2,0,1)&=2[\sqrt{\tau_2\rho_3\rho_2}(\tau_1-\rho_1)\\
&\quad+(\tau_2-2\rho_2)\sqrt{\tau_3\tau_2\tau_1\rho_1}]^2,\\
P_{\text{out}}(2,1,0)&=2[\sqrt{\tau_2\tau_1\rho_3\rho_1}(\tau_2-2\rho_2)\\
&\quad-(\tau_1-\rho_1)\sqrt{\tau_3\tau_2\rho_2}]^2,
\end{aligned}$$

when $\tau_3=\frac{1}{4}$ or $\frac{3}{4}$, in addition to the specifications $\tau_1=\frac{1}{2}$ and $\tau_2=\frac{2}{3}$. Marginal photon-number distributions at the output ports of the tritter (Fig. 3) highlight the separation of even and odd photons at the first two output ports, for $\tau_3=\frac{1}{4}$. The figure also contains the classical prediction for distinguishable particles. Careful inspection of Eq. (17) pinpoints the root cause of this effect to the *additional cancellation of exit probabilities for both single- and three-photon clusters* at output port 2 ($\tau_3=\frac{1}{4}$) or 3 ($\tau_3=\frac{3}{4}$). In U(2) HOM interference, the two photons always exit the optical mixer in pairs for a 50:50 mixer. Because $\tau_3\neq\tau_1$, the cancellation of triple clusters is an exclusive feature of the *asymmetric* multiport device [10].

D. Special case

The essential character of three-photon HOM interference emerges by removal of the third optical mixer ($\tau_3=1$). The output probability of triple coincidence for the resulting optical arrangement (Fig. 4) is readily obtained from Eq. (15),

$$\lim_{\tau_3 \to 1} P_{\text{out}}(1,1,1) = (\tau_1 - \rho_1)^2 (\tau_2 - \rho_2)^2. \tag{18}$$

U(3) HOM interference now results when both optical mixers have 50:50 partition ratios. This special case brings out a close resemblance to the corresponding U(2) effect. Figure 5 outlines the four interfering probability amplitudes that cancel exit triple coincidences. They represent all possible permutations of probability amplitudes for double transmission and double reflection of photon pairs at both mixers.

III. CONCLUSION

Three-photon HOM interference at a tritter exhibits a wider range of phenomena as a consequence of the larger dimensionality of its unitary representation. Unlike the corresponding double-beam effect, destructive interference cannot occur if all three mixers share identical partition ratios. To achieve cancellation of output triple coincidences, only two mixer transmittances require specification. The third mixer's transmittance represents an additional degree of freedom that permits even-odd photon sorting in the marginal photon-number distributions at the output ports of the tritter. The development of triply correlated photon sources would enable verification of these predictions [15].

ACKNOWLEDGMENT

I thank C. C. Gerry for valuable discussions.

[1] C. K. Hong, Z. Y. Ou, and L. Mandel, Phys. Rev. Lett. **59**, 2044 (1987). For a recent topical review see L. Mandel, Rev. Mod. Phys. **71**, S274 (1999).

[2] When the two photons arrive at only one input port of the symmetric mixer as a preclustered pair, no boson statistical effects are observed. The photons register a binomial output photon-number distribution consistent with the independent behavior of classical particles. Experimental confirmation is provided by J. Brendel, S. Schütrumpf, R. Lange, W. Martienssen, and M. O. Scully, Europhys. Lett. **5**, 223 (1988); R. Lange, J. Brendel, E. Mohler, and W. Martienssen, *ibid.* **5**, 619 (1988).

[3] R. A. Campos, B. E. A. Saleh, and M. C. Teich, Phys. Rev. A **40**, 1371 (1989).

[4] Z. Y. Ou, J.-K. Rhee, and L. J. Wang, Phys. Rev. Lett. **83**, 959 (1999); Phys. Rev. A **60**, 593 (1999).

[5] Three-photon effects in classic Young interference are considered by Th. Richter, Phys. Rev. A **42**, 1817 (1990); Quantum Opt. **3**, 115 (1991).

[6] D. M. Greenberger, M. A. Horne, and A. Zeilinger, Phys. Today **46** (8), 22 (1993).

[7] M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani, Phys. Rev. Lett. **73**, 58 (1994).

[8] G. A. Baker, Jr., Phys. Rev. **103**, 1119 (1956).

[9] For a discussion of SU(3) transformations in polarization space, see A. V. Burlakov, M. V. Chekhova, O. A. Karabutova, D. N. Klyshko, and S. P. Kulik, Phys. Rev. A **60**, R4209 (1999).

[10] I. Jex, S. Stenholm, and A. Zeilinger, Opt. Commun. **117**, 95 (1995); M. Zukowski, A. Zeilinger, and M. A. Horne, Phys. Rev. A **55**, 2564 (1997).

[11] Various additional aspects of the lossless two-beam mixer are discussed by A. Zeilinger, Am. J. Phys. **49**, 882 (1981); B. Yurke, S. L. McCall, and J. R. Klauder, Phys. Rev. A **33**, 4033 (1986); S. Prasad, M. O. Scully, and W. Martienssen, Opt. Commun. **62**, 139 (1987); Z. Y. Ou, C. K. Hong, and L. Mandel, *ibid.* **63**, 118 (1987); H. Fearn and R. Loudon, *ibid.* **64**, 485 (1987); B. Huttner and Y. Ben-Aryeh, Phys. Rev. A **38**, 204 (1988); Z. Y. Ou and L. Mandel, Am. J. Phys. **57**, 66 (1989); H. Fearn and R. L. Loudon, J. Opt. Soc. Am. B **6**, 917 (1989); W. K. Lai, V. Bužek, and P. L. Knight, Phys. Rev. A **43**, 6323 (1991); U. Leonhardt, *ibid.* **48**, 3265 (1993); A. Luis and L. L. Sánchez-Soto, Quantum Semiclassic. Opt. **7**, 153 (1995); K. Jacobs and P. L. Knight, Phys. Rev. A **54**, R3738 (1996).

[12] The internal phase shifts of an optical mixer relate to its material composition and manner of construction, as discussed by M. W. Hamilton, Am. J. Phys. **68**, 186 (2000). Mach-Zehnder interferometry is sensitive to these phase shifts. Note that the tritter converts into a Mach-Zehnder interferometer if the second mixer is replaced by a mirror.

[13] L. C. Biedenharn and J. D. Louck, *Angular Momentum in Quantum Physics, Theory and Application* (Addison-Wesley, Reading, MA, 1981), p. 212.

[14] T. J. Nelson, Am. J. Phys. **36**, 791 (1968). Nelson's notation is $T_1 = F_1 = \lambda_1/2$, $T_2 = F_2 = \lambda_2/2$, $T_3 = F_3 = \lambda_3/2$, $U = 4F_4/3 = \lambda_8/\sqrt{3}$, $N = 2F_5 = \lambda_4$, $N' = 2F_6 = \lambda_5$, $M = 2F_7 = \lambda_6$, and $M' = 2F_8 = \lambda_7$, where λ_m are the Gell-Mann SU(3) matrices. See M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[15] A. Zeilinger, M. A. Horne, H. Weinfurter, and M. Zukowski, Phys. Rev. Lett. **78**, 3031 (1997); J. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter, and A. Zeilinger, Nature (London) **403**, 515 (2000).