

## Quantum Zeno effect in a probed down-conversion process

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The distortion of a spontaneous down-conversion process caused by an auxiliary mode coupled to the idler wave is analyzed. In general, a strong coupling with the auxiliary mode tends to hinder the down-conversion in a nonlinear medium. On the other hand, provided that the evolution is disturbed by the presence of a phase mismatch, the coupling may increase the speed of down-conversion. These effects are interpreted as being manifestations of quantum Zeno and anti-Zeno effects, respectively, and they are understood by using the dressed mode picture of the device. The possibility of using the coupling as a nontrivial phase-matching technique is pointed out.

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### I. INTRODUCTION

In quantum optics a down-conversion process may be visualized as the decay of a pump photon into a pair of signal and idler photons of lower frequency. Provided the pumping remains undepleted and phase matching takes place, the energy of the spontaneously down-converted light monotonically increases and that of the pump beam monotonically decreases. From this point of view the down-conversion process may be looked at as a decay process of an unstable system. It is well known that frequent monitoring of a quantum system leads to inhibition of its evolution. This phenomenon is called the quantum Zeno effect [1,2]. Recently, a thought experiment has been suggested [3], in which it is possible to determine the place where the conversion of the pump photon took place inside a nonlinear crystal. The idea goes as follows. The nonlinear crystal is transversely cut into  $N$  pieces, which are then carefully aligned so that the signal and pump photons leaving, say, the  $k$ th slice becomes the input signal and pump photons to the  $(k+1)$ th slice of the crystal. The idler photons, on the other hand, are removed after each slice, thus allowing for a future measurement to be performed on them. If, for example, an ideal detector placed into the path of the idler mode after the  $k$ th slice clicks, it is then obvious that the decay of a pump photon took place somewhere inside the  $k$ th slice. By increasing the number of slices, the actual position of the creation of the signal and idler photons becomes more certain. It has been shown in [3], in accordance with the Misra-Sudarshan theorem [2], that the probability of emission of the down-converted pair decreases with increasing  $N$  and for very large number of crystal slices (continuous observation) the decay of the pump photon never occurs. It has also been shown [4,5] that provided the phase-matching condition is not fulfilled in the process of down-conversion, the observa-

tion may, on the contrary, *enhance* the emission for a properly chosen  $N$  (anti-Zeno or inverse Zeno effect). This Zeno/anti-Zeno interplay has a simple explanation in terms of destructive and constructive interference of subsequent emissions inside the nonlinear crystal [3–5]. Here we shall demonstrate that a Zeno-like behavior occurs also when, instead of cutting the crystal, we couple one of the down-converted beams with an auxiliary mode. Although, strictly speaking, such a linear coupling cannot be interpreted as being the realization of a measurement according to the tenets of von Neumann, the dynamics of the nonlinear coupler mimics very well the Zeno behavior of the arrangement in [3]. It is worth noting, in this context, that the idea of considering continuous interaction with an external agent as a sort of “steady gaze” at the system goes back to Kraus [6] and has recently been revived in relation with the quantum Zeno effect [7]. Schulman [8], in particular, has even provided a quantitative relation between the Zeno effect produced by pulsed measurements (in the sense of [2]) and continuous observation (in the sense discussed above) performed by an external system.

The paper is organized as follows. In the second section a theoretical model of the nonlinear coupler is introduced. In the third section the Zeno-like behavior of the nonlinear coupler is demonstrated. In the fourth section the dressed mode picture of the device under investigation is developed and a formal analogy between a phase mismatch and the coupling of the down-conversion process to an auxiliary mode is explored. Finally, the observed Zeno and anti-Zeno effects are thoroughly discussed in the fifth section, by using the results obtained.

### II. MODEL

Consider a nonlinear coupler made up of two waveguides, through which four modes, pump  $p$ , signal  $s$ , idler  $i$ , and auxiliary mode  $b$ , propagate in the same direction (see Fig. 1). The nonlinear waveguide is filled with a second-order nonlinear medium in which ultraviolet pump photons are down converted to signal and idler photons of lower fre-

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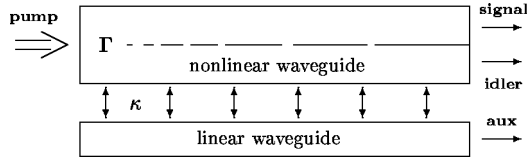


FIG. 1. Outline of the nonlinear coupler.

quency. In addition, the idler mode is allowed to exchange energy, e.g., by means of evanescent waves, with the auxiliary mode  $b$  propagating through a linear medium. In the following we will assume that all four modes are monochromatic and their frequencies are fixed, e.g., by placing narrow interference filters in front of the detectors. Provided the amplitudes of the fields inside the coupler vary little during an optical period (slowly varying envelope approximation), and provided the linear coupling is sufficiently weak so that it can be described by coupled mode theory (Born approximation) [9], the effective Hamiltonian of our device reads ( $\hbar = c1$ )

$$H = \omega_p a_p^\dagger a_p + \omega_s a_s^\dagger a_s + \omega_i a_i^\dagger a_i + \omega_b b^\dagger b + (\Gamma a_p a_s^\dagger a_i^\dagger e^{i\Delta t} + \kappa a_i^\dagger b + \text{H.c.}). \quad (1)$$

Here  $\omega_\alpha$  is the frequency of mode  $\alpha$ ,  $\Delta = (\mathbf{k}_p - \mathbf{k}_s - \mathbf{k}_i)_z$  is the nonlinear phase mismatch,  $\Gamma$  and  $\kappa$  are the nonlinear and linear coupling constants, respectively, and the propagation variable  $z$  has been replaced by the evolution parameter  $t$ . Usually,  $\kappa$  is proportional to the overlap between the idler and auxiliary modes [9], whereas the nonlinear coupling constant  $\Gamma$  is proportional to the second-order nonlinear susceptibility  $\chi^{(2)}$  [10]. It is convenient to split the Hamiltonian (1) into free and interaction parts,

$$H = H_0 + H_I. \quad (2)$$

In order to get rid of the free evolution in the Heisenberg equations of motion,

$$\dot{a} = -i[a, H_0 + H_I], \quad (3)$$

where  $a$  is the annihilation operator of a particular mode, we introduce the new field operators

$$a'_\alpha = e^{i\omega_\alpha t} a_\alpha \quad (\alpha = p, s, i), \quad (4)$$

and analogously for  $b$ . Substituting these new variables together with the Hamiltonian (2) into Eq. (3), we arrive at the equations of motion

$$\dot{a}' = -i[a', H'_I], \quad (5)$$

where

$$H'_I = \Gamma a'_p a'_s a'_i e^{i\Delta t} + \kappa a'_i b' e^{i(\omega_i - \omega_b)t} + \text{H.c.} \quad (6)$$

Because the Hamiltonian (1) contains products of three operators, the equations of motion (3) and (5) are nonlinear. The nonlinearity accounts mainly for saturation effects and must be taken into account whenever the pump beam be-

comes depleted (e.g., medium in a cavity). On the other hand, if the pumping is sufficiently strong and if the nonlinear interaction is weak so that only a small fraction of the pump photons is removed from the input beam, we can simplify our problem by describing the strong pump wave in classical terms, i.e., we let  $a_p = \xi \exp(i\omega_p t)$ , where  $\xi$  and  $\omega_p$  denote the complex amplitude and the frequency of the classical pump wave, respectively. With the help of the strong pump wave approximation, the interaction Hamiltonian of our problem, Eq. (6), is simplified as follows:

$$H_I = \Gamma a_s^\dagger a_i^\dagger e^{i\Delta t} + \kappa a_i^\dagger b + \text{H.c.}, \quad (7)$$

where we assumed that the frequency-matching conditions hold:  $\omega_p - \omega_s - \omega_i = 0$  and  $\omega_b = \omega_i$ . The amplitude  $\xi$  has been absorbed in the coupling constant  $\Gamma$  and all operators are written without primes, for simplicity. The dynamics of the nonlinear coupler (7) reduces to the dynamics of the phase-matched spontaneous down-conversion process provided that  $\kappa = \Delta = 0$  and the initial state is taken as  $|\Psi_0\rangle = |\text{vac}\rangle_s \otimes |\text{vac}\rangle_i$ . As we already mentioned in the Introduction, the average number of signal and idler photons originating in the crystal of length  $L$ ,

$$\langle a_{s,i}^\dagger a_{s,i} \rangle_{\text{vac}} = \sinh^2 \Gamma L \quad (\kappa = \Delta = 0), \quad (8)$$

is then an (exponentially) increasing function of  $L$ . We stress that the model exhibits an essentially irreversible behavior: there is an exponential energy production. Formally, from a mathematical viewpoint, this is due to the unboundedness of the Hamiltonian (6) and (7) from below, when  $\kappa = 0$  (actually, when  $\kappa < \Gamma$ ). Physically, the model is sensible as long as the pumping is strong enough and the classical approximation is valid for the pump wave.

### III. LINEAR COUPLING TURNED ON

The behavior of the down-conversion process dramatically changes when one of the two down-converted modes (e.g., the idler mode) is coupled to an auxiliary mode via a linear interaction. The Hamiltonian (7) yields, when  $\Delta = 0$  (phase matching),

$$\dot{a}_s = -i\Gamma a_i^\dagger,$$

$$\dot{a}_i = -i\Gamma a_s^\dagger - i\kappa b \quad (\Delta = 0), \quad (9)$$

$$\dot{b} = -i\kappa a_i,$$

and we are interested in the regime of weak nonlinearity, expressed by the condition  $\kappa > \Gamma$ . Notice that two opposite tendencies compete in Eqs. (9): an elliptic structure, leading to oscillatory behavior, governed by the coupling parameter  $\kappa$ ,

$$\ddot{a}_i = -\kappa^2 a_i, \quad \ddot{b} = -\kappa^2 b, \quad (10)$$

and a hyperbolic structure, yielding exponential behavior, governed by the nonlinear parameter  $\Gamma$ ,

$$\ddot{a}_s = \Gamma^2 a_s, \quad \ddot{a}_i = \Gamma^2 a_i. \quad (11)$$

The threshold between these two regimes occurs for  $\Gamma = \kappa$ .

The system of equations (9) is easily solved and the number of output signal photons, which is the same as the number of pump photons decayed, reads

$$\langle a_s^\dagger a_s \rangle_{\text{vac}} = \frac{\Gamma^2}{\chi^2} \sin^2 \chi L + \frac{\kappa^2 \Gamma^2}{\chi^4} (1 - \cos \chi L)^2, \quad (12)$$

where  $\chi = \sqrt{\kappa^2 - \Gamma^2}$ . Hereafter, the symbol  $\langle \dots \rangle_{\text{vac}}$  denotes averaging with respect to the initial vacuum state  $|\Psi_0\rangle = |\text{vac}\rangle_s \otimes |\text{vac}\rangle_i \otimes |\text{vac}\rangle_b$  [11]. Unlike the case of phase-matched down conversion (8), the exchange of energy between all modes now becomes periodic when  $\kappa > \Gamma$ . As the linear coupling becomes stronger, the period of the oscillations gets shorter and the amplitude of the oscillations decreases as  $\kappa^{-2}$ , namely,

$$\langle a_s^\dagger a_s \rangle_{\text{vac}} \sim \frac{\Gamma^2}{\kappa^2} \sin^2 \kappa L + \frac{\Gamma^2}{\kappa^2} (1 - \cos \kappa L)^2 = \frac{4\Gamma^2}{\kappa^2} \sin^2 \frac{\kappa L}{2} \quad (\kappa \gg \Gamma). \quad (13)$$

For very strong coupling [12] the down-conversion process is completely frozen, the medium becomes effectively linear, and the pump photons propagate through it without ‘‘decay.’’ Notice that in this situation, even if  $L$  is increased, the number of down-converted photons is bounded [compare with the opposite case, Eq. (8)]. This can be interpreted as a manifestation of the quantum Zeno effect in the following sense: by increasing the coupling with the auxiliary mode, one performs a better ‘‘observation’’ of the idler mode and therefore of the ‘‘decay’’ of the pump. The hindering of the evolution results. There is an intuitive explanation of this behavior: since the linear coupling changes the phases of the amplitudes of the interacting modes, the constructive interference yielding exponential increase of the converted energy (8) is destroyed, and down conversion becomes frozen. We shall come back to this point and corroborate this intuitive picture in the next section.

The proposed interpretation in terms of the quantum Zeno effect is readily understandable and rather appealing. On the other hand, one should remark that since only the output fields are accessible to measurement in the experimental setup in Fig. 1, no relevant information is readily available about the place where the signal and idler photon are created [13]. In this sense, no bona fide measurement is being performed on the fields. The situation would be different if we provided the auxiliary waveguide with some photodetection device like an array of highly efficient photodetectors. For sufficiently strong linear coupling, the decay product (the idler photon) would enter the auxiliary mode soon after emission, it could then be detected by a pixel of the photodetection array, and we could thereafter infer the place where the emission had taken place. As there is no such detection device present in the setup in question, the *coherent* superposition of the two possibilities ‘‘the idler photon is in the

idler mode’’ and ‘‘the idler photon is in the auxiliary mode’’ is maintained through the evolution and no decomposition of the wave function occurs. Nevertheless, it is still possible (and useful) to speak about the quantum Zeno effect in the more general sense given above. A discussion of this point is given in [14] in connection with the experiment performed by Itano *et al.* [15].

#### IV. DRESSED MODES

We now look for the modes dressed by the interaction  $\kappa$ . This will provide an alternative interpretation and a more rigorous explanation of the result obtained above. Let us diagonalize the Hamiltonian (1) with respect to the linear coupling. By setting  $\omega_i = \omega_b$  and  $\kappa$  real, it is easy to see that in terms of the dressed modes

$$\begin{aligned} c &= (a_i + b)/\sqrt{2}, \\ d &= (a_i - b)/\sqrt{2}, \end{aligned} \quad (14)$$

the Hamiltonian (1) reads

$$\begin{aligned} H &= \omega_p a_p^\dagger a_p + \omega_s a_s^\dagger a_s + \omega_c c^\dagger c + \omega_d d^\dagger d + \frac{\Gamma}{\sqrt{2}} a_p a_s^\dagger c^\dagger e^{i\Delta t} \\ &+ \frac{\Gamma}{\sqrt{2}} a_p a_s^\dagger d^\dagger e^{i\Delta t} + \text{H.c.}, \end{aligned} \quad (15)$$

where the dressed energies are

$$\begin{aligned} \omega_c &= \omega_i + \kappa, \\ \omega_d &= \omega_i - \kappa. \end{aligned} \quad (16)$$

If  $\Delta = 0$ , in the strong pump limit, by following the procedure of Sec. II, instead of Eq. (7), we get the following interaction Hamiltonian:

$$H_I = \frac{\Gamma}{\sqrt{2}} a_s^\dagger c^\dagger e^{i\kappa t} + \frac{\Gamma}{\sqrt{2}} a_s^\dagger d^\dagger e^{-i\kappa t} + \text{H.c.} \quad (\Delta = 0), \quad (17)$$

where we assumed as before that the frequency-matching conditions holds:  $\omega_p - \omega_s - \omega_i = 0$ . By comparing the Hamiltonian (17) with the Hamiltonian (7) when  $\kappa = 0$ ,

$$H_I = \Gamma a_s^\dagger a_i^\dagger e^{i\Delta t} + \text{H.c.} \quad (\kappa = 0), \quad (18)$$

describing down-conversion with phase mismatch  $\Delta$ , it is apparent that the coupling and the phase mismatch influence the down-conversion process in the same way. In fact, for large values of the phase mismatch  $\Delta$  it is easy to find that

$$\langle a_s^\dagger a_s \rangle_{\text{vac}} \sim \frac{4\Gamma^2}{\Delta^2} \sin^2 \frac{\Delta L}{2} \quad (\Delta \gg \Gamma), \quad (19)$$

which is to be compared with Eq. (13). The coupling of the idler mode  $a_i$  with the auxiliary mode  $b$  yields two dressed modes  $c$  and  $d$  to which the pump photon can decay. They

are completely decoupled and due to their energy shift (16) exhibit a phase mismatch  $\pm\kappa$ . Since the phase mismatch effectively shortens the time during which a fixed phase relation holds between the interacting beams, the amount of converted energy is smaller than in the ideal case of perfectly phase-matched interaction. This explains the results of Sec. III. A strong linear coupling then makes the subsequent emissions of converted photons interfere destructively and the nonlinear interaction is frozen. In this respect the disturbances caused by the coupling and by frequently repeated measurements are similar and we can interpret the phenomenon as a quantum Zeno effect.

### V. COMPETITION BETWEEN THE COUPLING AND THE MISMATCH

In the previous section we saw that the nonlinear interaction was affected by both linear coupling and phase mismatch in the same way, namely, the effectiveness of the nonlinear process was reduced under their action. In this section we show that when both disturbing elements are present in the dynamics of the down-conversion process, the linear coupling can, rather surprisingly, compensate for the phase mismatch and vice versa, so that the probability of emission of the signal and idler photons can almost return back to its undisturbed value.

We start from the equations of motion generated by the full interaction Hamiltonian (7),

$$\begin{aligned}\dot{a}_s &= -i\Gamma a_i^\dagger e^{i\Delta t}, \\ \dot{a}_i &= -i\Gamma a_s^\dagger e^{i\Delta t} - i\kappa b \quad (\Delta \neq 0, \kappa \neq 0), \\ \dot{b} &= -i\kappa a_i.\end{aligned}\quad (20)$$

Although it is easy to write down the explicit solution of the system (20), we shall here provide only a qualitative discussion of the solution. The main features are then best demonstrated with the help of a few figures. Eliminating idler and auxiliary mode variables from Eq. (20), we get a differential equation of the third order for the annihilation operator of the signal mode. Its characteristic polynomial [upon the substitution  $a_s(t) = a_s(0)\exp(i\lambda t)$ ]

$$\lambda^3 + 2\Delta\lambda^2 + (\Delta^2 - \kappa^2 + \Gamma^2)\lambda + \Delta\Gamma^2, \quad \kappa \neq 0, \quad (21)$$

is recognized as a cubic polynomial in  $\lambda$  with real coefficients. An oscillatory behavior of the signal mode occurs only provided the polynomial (21) has three real roots (*causus irreducibilis*), i.e., its determinant  $D$  must obey the condition  $D < 0$ . Expanding the determinant in the small nonlinear coupling parameter  $\Gamma$  and keeping terms up to the second order in  $\Gamma$  we obtain

$$D = -\frac{\kappa^2}{27} [(\kappa^2 - \Delta^2)^2 - (5\Delta^2 + 3\kappa^2)\Gamma^2], \quad \Gamma \ll \Delta, \kappa. \quad (22)$$

It is seen that a mismatched down conversion behaves in either an oscillatory or a hyperbolic way, depending on the

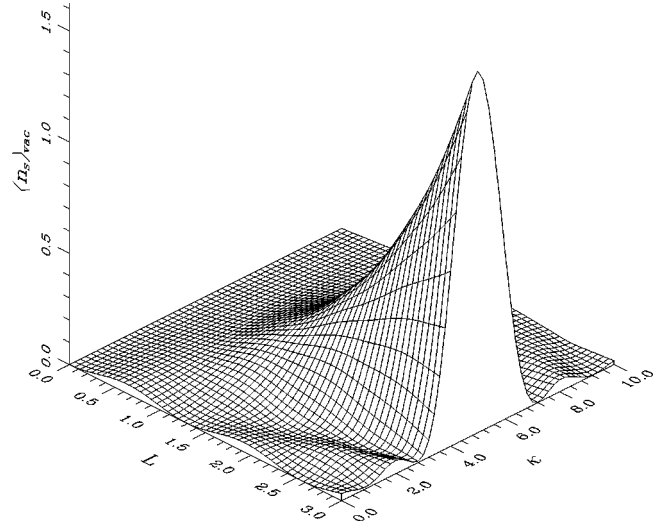


FIG. 2. Mean number of output signal photons  $\langle n_s \rangle$  behind the nonlinear medium as a function of interaction length  $L$  and strength  $\kappa$  of linear coupling. The nonlinear mismatch and nonlinear coupling parameter are  $\Delta = 5$  and  $\Gamma = 0.5$ , respectively; all quantities are in arbitrary units.

strength of the coupling with the auxiliary mode. The values of  $\kappa$  lying at the boundary between these two types of dynamics are determined by solving the equation  $D = 0$ . The only two nontrivial solutions are

$$\kappa_{1,2} = \sqrt{\Delta^2 + \frac{3}{2}\Gamma^2 \pm \sqrt{8\Delta\Gamma}}. \quad (23)$$

The case  $\Delta \gg \Gamma$  is of main interest in this section (otherwise we have the situation already described in Sec. III). Hence we can, eventually, drop  $\Gamma^2$  in Eq. (23). The resulting intervals are

$$\kappa \in \begin{cases} \langle \Delta - \sqrt{2}\Gamma, \Delta + \sqrt{2}\Gamma \rangle, & \text{hyperbolic behavior} \\ \langle 0, \Delta - \sqrt{2}\Gamma \rangle \cup (\Delta + \sqrt{2}\Gamma, \infty) & \text{oscillatory behavior.} \end{cases} \quad (24)$$

The behavior of the mismatched down-conversion process is shown in Fig. 2 for a particular choice of  $\Delta$ . In the absence of linear coupling the down-converted light shows oscillations and the overall effectiveness of the nonlinear process is small due to the presence of phase mismatch  $\Delta$ . However, as we switch on the coupling between the idler and auxiliary modes, the situation changes. On increasing the strength of the coupling the period of the oscillations gets longer and its amplitude gets larger. When  $\kappa$  becomes larger than  $\Delta - \sqrt{2}\Gamma$  the oscillations are no longer seen and the intensity of the signal beam starts to grow monotonically. We can say that in this regime the initial nonlinear mismatch has been compensated by the coupling.

The interplay between nonlinear mismatch and linear coupling is illustrated in Fig. 3. A significant production of signal photons is a clear manifestation of an anti-Zeno effect. In correspondence with the observations in [4,5], such an anti-Zeno effect occurs only provided a substantial phase mismatch is introduced in the process of down conversion. It is worthwhile to compare the interesting behavior seen in Fig.



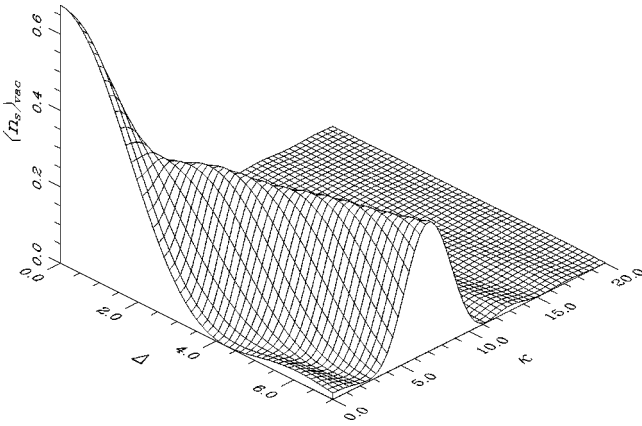


FIG. 3. Interplay between linear coupling and phase mismatch. The mean number of output signal photons  $\langle n_s \rangle$  leaving the nonlinear medium of length  $L=1.5$  is shown vs strength  $\kappa$  of linear coupling and nonlinear mismatch  $\Delta$ . The nonlinear coupling parameter is fixed at  $\Gamma=0.5$ . All quantities are in arbitrary units.

3 with the Zeno and anti-Zeno effects observed in a sliced nonlinear crystal (Fig. 1 in [5]). It can be seen that the coupling parameter  $\kappa$  here plays a role similar to the number of slices  $N$  into which the crystal is cut in the latter scheme. Moreover, the sharpness of the “observation” ( $\kappa$  or  $N$ ) at which a maximum output intensity occurs is approximately a linear function of the introduced phase mismatch in both schemes. There are, however, also some points of difference. For example, the maximum output intensity obtainable for a given  $\Delta$  by slicing the crystal decreases with increasing phase mismatch  $\Delta$  [5]. On the other hand, no matter how strong the mismatch is, it can always be removed with the help of a suitable linear coupling (and vice versa). This difference is due to the  $1/N$  scaling of the intensities of output light generated by a process under observation [3–5]. An analogous factor is missing here, in Eq. (12).

Several intuitive explanations of the anti-Zeno-like behavior seen in Fig. 3 are at hand. From the point of view of constructive and destructive interference, one can say that since the linear coupling effectively changes the phase relations among interacting modes, the destructive interference of subsequent pump photon decays caused by phase mismatch is suppressed in the same way as the constructive interference has been suppressed in the case of a perfectly matched interaction.

Figure 3 can also be interpreted in analogy with the dressed state description of interaction of atoms with intense light [16]. In terms of the dressed modes  $c$  and  $d$  of Eq. (14), if  $\Delta \neq 0$ , in place of the Hamiltonian (17) one gets

$$H_I = \frac{\Gamma}{\sqrt{2}} a_s^\dagger c^\dagger e^{i(\Delta+\kappa)t} + \frac{\Gamma}{\sqrt{2}} a_s^\dagger d^\dagger e^{i(\Delta-\kappa)t} + \text{H.c.}, \quad (25)$$

which yields the equations of motion

$$\dot{a}_s = -i \frac{\Gamma}{\sqrt{2}} c^\dagger e^{i(\Delta+\kappa)t} - i \frac{\Gamma}{\sqrt{2}} d^\dagger e^{i(\Delta-\kappa)t},$$

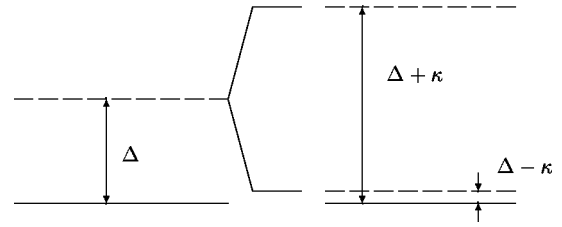


FIG. 4. Energy scheme of a mismatched down-conversion process subject to linear coupling. The bottom solid lines denote a resonant process.

$$\begin{aligned} \dot{c} &= -i \frac{\Gamma}{\sqrt{2}} a_s^\dagger e^{i(\Delta+\kappa)t}, \\ \dot{d} &= -i \frac{\Gamma}{\sqrt{2}} a_s^\dagger e^{i(\Delta-\kappa)t}. \end{aligned} \quad (26)$$

The energy scheme implied by Eq. (26) is shown in Fig. 4. Under the influence of the coupling with the auxiliary mode  $b$  the mismatched down conversion splits into two dressed energy-shifted interactions. It is apparent that when  $\kappa = \pm \Delta$  one of the two interactions becomes resonant. The other one is “counter-rotating” and acquires a phase mismatch  $2\Delta$ , yielding oscillations. Also, the amplitude of such oscillations decreases as  $\Delta^{-2}$  and the mode output becomes negligible compared to the other interaction. The use of the rotating wave approximation in Eq. (26) is fully justified in this case and the system is easily solved. The output signal intensity reads

$$\langle a_s^\dagger a_s \rangle_{\text{vac}} = \sinh^2 \left( \frac{\Gamma}{\sqrt{2}} L \right) \quad (\kappa = \pm \Delta) \quad (\Delta \gg 1/L) \quad (27)$$

[compare with Eq. (8)]. The linear coupling to an auxiliary mode compensates for the phase mismatch up to a change in the effective nonlinear coupling strength  $\Gamma \rightarrow \Gamma/\sqrt{2}$ .

As a matter of fact, the condition  $\kappa = \pm \Delta$  can be interpreted also as a condition for achieving the so-called quasi-phase-matching in the nonlinear process. A quasi-phase-matched regime of generation [17] is usually forced by creating an artificial lattice inside a nonlinear medium, e.g., by periodic modulation of the nonlinear coupling coefficient. Periodic change of sign of  $\Gamma$  (rectangular modulation) yields the effective coupling strength  $\Gamma \rightarrow 2\Gamma/\pi$  [17], where, as before,  $\Gamma$  is the coupling strength of the phase-matched interaction. Thus the continuous “observation” of the idler mode even gives a slightly better enhancement of the decay rate than the most common quasi-phase-matching technique.

To summarize, the statement “the down-conversion process is mismatched” means that the nonlinear process is out of resonance in the sense that the momentum of the decay products (signal and idler photons) differs from the momentum carried by the pump photon before the decay took place. When the linear interaction is switched on the system gets dressed and the energy spectrum changes. A careful adjustment of the coupling strength  $\kappa$  makes it then possible to

tune the nonlinear interaction back to resonance. In this way the probability of pump photon decay can be greatly enhanced. This occurs when  $\kappa \approx \pm \Delta$  and explains why the anti-Zeno effect takes place along the line  $\kappa = \Delta$  in Fig. 3.

## VI. CONCLUSION

In this article a down-conversion process disturbed by the presence of a linear coupling between the idler and some auxiliary mode has been discussed. Although, strictly speaking, such a coupling is not a measurement in von Neumann's sense, we found a striking similarity between the dynamics of our system and the dynamics of the down-conversion processes taking place in a sliced nonlinear crystal, where a Zeno interpretation is feasible and appealing.

In some sense, the Zeno effect is a consequence of the new dynamical features introduced by the coupling with an external agent that (through its interaction) “looks closely”

at the system. When this interaction can be effectively described as a projection operator in the sense of von Neumann, we obtain the usual formulation of the quantum Zeno effect in the limit of very frequent measurements. In general, the description in terms of projection operators may not apply, but the dynamics can be modified in a way that is strongly reminiscent of the Zeno effect. Examples of the type analyzed in this paper call for a broader definition of the “quantum Zeno effect.”

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