General optical state truncation and its teleportation

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A setup is proposed in which the number-state expansion of a one-mode traveling-wave optical field is truncated so as to leave its vacuum, one-, and two-photon components. The process is a realization of quantum teleportation on a three-state basis. The possibility of generalization to the first *n* components is also considered.

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I. INTRODUCTION

Quantum teleportation, invented by Bennett *et al.* [1], is one of the essential primitives of quantum communication. It relies on quantum entanglement, one of the most peculiar fundamental features of quantum mechanics, especially in the nonlocal case. Not surprisingly several realizations of quantum teleportation have been proposed [2–6] and some of them have been realized experimentally [7–11].

Quantum teleportation exploits the von Neumann projection principle, which has also been applied in quantum state design: measurements can bring a physical system to a desired quantum state. In schemes of this kind, several outcomes of the measurements are possible; some of them signify that the desired state is prepared, while the procedure has to be repeated if the other outcomes are obtained. This is similar to discrete variable quantum teleportation: an efficiency of the method can be defined, which describes the probability of obtaining the proper state. Recently Dakna *et al.* have proposed a scheme in which an arbitrary quantum state of a traveling electromagnetic field can be generated by a method relying on quantum measurement [12]. Their apparatus, consisting of an array of many detectors and beam splitters, has an efficiency depending on the state itself. Though this arrangement is capable of generating arbitrary states, the efficiency can be quite low in some cases, so the set of states that can actually be produced with high efficiency is restricted.

In a paper of Pegg, Phillips, and Barnett [13] it is shown that a one-mode traveling-wave optical state can be truncated so as to leave only its vacuum and one-photon components. This is an approach to quantum state design similar to that of Dakna *et al.* The proposed arrangement, called ‘‘quantum scissors,’’ consists of two beam splitters and two photon-counting detectors. It exploits quantum measurement and nonlocality. As shown by Villas-Bôas, de Almeida, and Moussa [4], it is a realization of quantum teleportation of a two-state system, the basis states being the vacuum and the one-photon Fock state. The above authors analyze the operation of the arrangement in a noisy environment. In a very recent paper [14] it is shown that quantum scissors have good fidelity in the presence of imperfections. The quantum scissors device is interesting from at least two points of view: quantum state design and teleportation of states of a traveling-wave electromagnetic field.

Quantum scissors are capable of converting a classical state to a highly nonclassical one. For example, if the input is a low-intensity coherent state, the truncation yields a coherent superposition of vacuum and one-photon states. This state is known to possess squeezing properties [15], and can be used as a reference state of projection synthesis [16,17]. The quantum scissors work for other (even mixed) input states too; thus they are capable of generating several kinds of superpositions and mixtures of vacuum and one-photon states. The question arises naturally, as to whether the class of preparable states can be enlarged. One possibility is that of Dakna *et al.* [12], which applies more beam splitters and detectors. We follow a different method: we do not raise the number of components of the arrangement, but we examine the facilities introduced by the freedom of using beam splitters with appropriate parameters. It turns out that a truncation so as to leave vacuum, one-, and two-photon superpositions needs no significant extension of current experimental expertise. This generalized quantum scissors device can generate a larger class of nonclassical states. For example, by cutting a squeezed vacuum state, a coherent superposition of vacuum and two-photon states can be obtained, which may also be used as a reference state in projection synthesis, and its squeezing properties have been analyzed in Ref. [15]. The truncation of coherent states also makes an interesting class of nonclassical states feasible. The arrangement works for any pure and mixed input state.

The other aspect of the operation of the quantum scissors device is quantum teleportation. The suggested realizations of this phenomenon can be divided into two groups: teleportation of discrete and continuous quantum variables. Our argument is concerned with the discrete case, which usually means teleportation of the state of a two-state system (a qbit), though several authors address the question of generalization to discrete systems with more than two basis states. The latter question is discussed by Stenholm and Bardroff [18] in a general form. Our approach is different. The generalized quantum scissors creates a superposition of vacuum, one-, and two-photon Fock states of a one-mode traveling-wave field, and teleports it at the same time. If the input state of the generalized scissors is already a superposition of this kind, it is simply teleported. This is a teleportation on the three-dimensional Hilbert space spanned by $\{|0\rangle,|1\rangle,|2\rangle\}$.

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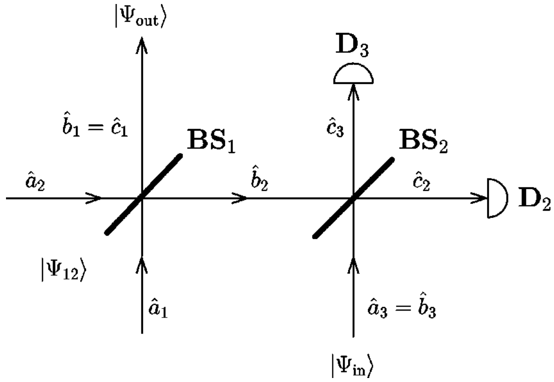

FIG. 1. The ''quantum scissors'' device, the subject of our analysis. It consists of the BS_1 and BS_2 beam splitters and D_2 and D_3 photon counters. The numbering of detectors is consistent with the indexing of spatial modes. $|\Psi_{in}\rangle$ is the incoming state, $|\Psi_{out}\rangle$ is the output state. The modes $\hat{a}_1$ and $\hat{a}_2$ are in the $|\Psi_{12}\rangle$ ancillary state necessary for the operation of the device. The annotation used in this paper for the annihilation operators of the spatial modes is shown.

We analyze this particular situation in detail. The discussion yields suggestive insight into the process of transporting quantum information in this case. We have also investigated the possibility of further generalization: truncating up to the nth Fock component.

The manipulation of entangled states of the electromagnetic field is carried out with passive linear multiports in most cases. Although the description of such systems has received extensive coverage in the literature, not all the possibilities involved in such devices have been exploited up to date. For generalized quantum scissors the optimization of beam splitter parameters with respect to all of the SU(2) parameters is required. On the other hand, several consequences can follow from the symmetry properties of a multiport. For example, even in the lossless and noiseless case, quantum teleportation arrangements have a nontrivial limitation: the no-go theorem for Bell-state detection [19]. In the setup discussed here, such limitations emerge in a particularly clear form as a consequence of the linearity and photon number conservation and thus mainly of the SU(2) symmetry of the beam splitters.

This paper is organized as follows. In Sec. II the generalized quantum scissors device is introduced. In Sec. III the possibility of truncation up to the n photon components is considered. In Sec. IV the teleportation aspect is analyzed. Section V summarizes the results.

II. STATE TRUNCATION UP TO TWO-PHOTON STATES

The quantum scissors device is depicted in Fig. 1. The notation used is also shown in the figure. We label the modes with their annihilation operators. We are given an arbitrary state in the input mode $\hat{a}_3$. For simplicity consider a pure state

$$|\Psi_{\rm in}\rangle=\sum_{k=0}^{\infty}\gamma_k|k\rangle \tag{1}$$

as input, but the generalization to mixed states is straightforward. Our aim is to obtain a truncated state

$$|\Psi_{\rm out}\rangle=\sqrt{N}(\gamma_0|0\rangle+\gamma_1|1\rangle+\gamma_2|2\rangle) \tag{2}$$

as output in mode $\hat{c}_1$, $N=1/\Sigma_{k=0}^{2}|\gamma_k|^2$ being a renormalization constant. The operation of the device consists of unitary evolution and a measurement process. The unitary evolution can be divided into two steps: action of the BS_1 and BS_2 beam splitters. Operators $\hat{a}$, $\hat{b}$, and $\hat{c}$ belong to the stages of unitary evolution, and their indices refer to the spatial modes. At the beginning, modes $\hat{a}_1$ and $\hat{a}_2$ are in a given state $|\Psi_{12}\rangle$. This is the ''reference state'' of the projection synthesis. The state of the whole system of three modes is $|\Psi\rangle=|\Psi_{12}\rangle\otimes|\Psi_{\rm in}\rangle$ initially.

We describe the unitary evolution in the Heisenberg picture. In the first step the effect of BS_1 can be described by the unitary transformation

$$\begin{pmatrix}\hat{b}_1^\dagger\\ \hat{b}_2^\dagger\end{pmatrix}=\begin{pmatrix}e^{-i\phi_t}\cos\tau_1 & e^{-i\phi_r}\sin\tau_1\\ -e^{i\phi_r}\sin\tau_1 & e^{i\phi_t}\cos\tau_1\end{pmatrix}\begin{pmatrix}\hat{a}_1^\dagger\\ \hat{a}_2^\dagger\end{pmatrix}, \tag{3}$$

while the input mode is not modified: $\hat{a}_3^\dagger=\hat{b}_3^\dagger$. In Eq. (3), ϕ_t and ϕ_r are the phase shifts imparted to the transmitted and reflected beams, and $(\cos\tau_1)^2$ and $(\sin\tau_1)^2$ are the transmittance and reflectance of the beam splitter, respectively. The next step is the action of BS_2 on modes $\hat{b}_2$ and $\hat{b}_3$ yielding $\hat{c}_2$ and $\hat{c}_3$,

$$\begin{pmatrix}\hat{c}_2^\dagger\\ \hat{c}_3^\dagger\end{pmatrix}=\begin{pmatrix}e^{-i\eta_t}\cos\tau_2 & e^{-i\eta_r}\sin\tau_2\\ -e^{i\eta_r}\sin\tau_2 & e^{i\eta_t}\cos\tau_2\end{pmatrix}\begin{pmatrix}\hat{b}_2^\dagger\\ \hat{b}_3^\dagger\end{pmatrix}, \tag{4}$$

while $\hat{c}_1=\hat{b}_1$. This is followed by a measurement, i.e., a projection to a photon-number eigenstate in modes $\hat{c}_2$ and $\hat{c}_3$. The detectors are assumed to be ideal photon counters. Given a reference state $|\Psi_{12}\rangle$ we will find suitable parameters for BS_1 and BS_2 to carry out the state truncation described above.

The ancillary state $|\Psi_{12}\rangle$ has to be experimentally available in order to make the idea of quantum scissors realistic. The desired output state of Eq. (2) contains a maximum of two photons. These two photons originate from $|\Psi_{12}\rangle$, since evidently no light reaches the output from the input. Thus $|\Psi_{12}\rangle$ has to contain at least two photons. We choose the state $|\Psi_{12}\rangle=|11\rangle$, consisting of a pair of temporally correlated photons, because it can be generated in the way described at the end of this section.

Suppose that from $|\Psi_{12}\rangle=|11\rangle$ the BS_1 general beam splitter produces the intermediate state

$$|\Psi'_{12}\rangle=\beta_0|20\rangle+\beta_1|11\rangle+\beta_2|02\rangle, \tag{5}$$

so the state of all three modes after the first step of unitary evolution is the product of this and the state in Eq. (1):

$$|\Psi'\rangle = |\Psi'_{12}\rangle \otimes |\Psi_{\rm in}\rangle = \sum_{n=0}^{\infty} \gamma_n(\beta_0|20n\rangle + \beta_1|11n\rangle + \beta_2|20n\rangle) = \sum_{n=0}^{\infty} \frac{\gamma_n}{\sqrt{n!}} \left(\frac{\beta_0}{\sqrt{2}} \hat{b}_1^{\dagger 2} \hat{b}_3^{\dagger n} + \beta_1 \hat{b}_1^{\dagger} \hat{b}_2^{\dagger} \hat{b}_3^{\dagger n} + \frac{\beta_2}{\sqrt{2}} \hat{b}_2^{\dagger 2} \hat{b}_3^{\dagger n} \right) |000\rangle. \tag{6}$$

As the result of the action of BS_2, $|\Psi'\rangle$ turns into

$$|\Psi''\rangle = \sum_{n=0}^{\infty} \frac{\gamma_n}{n!} \sum_{k=0}^{n} \binom{n}{k} (\sin\tau_2)^k (\cos\tau_2)^{n-k} e^{i(k\eta_r-(n-k)\eta_t)} \Bigg(\frac{\beta_0}{\sqrt{2}} \hat{c}_1^{\dagger 2} \hat{c}_2^{\dagger k} \hat{c}_3^{\dagger n-k} + \beta_1 \cos\tau_2 e^{i\eta_t} \hat{c}_1^{\dagger} \hat{c}_2^{\dagger k+1} \hat{c}_3^{\dagger n-k}$$
$$- \beta_1 \sin\tau_2 e^{-i\eta_r} \hat{c}_1^{\dagger} \hat{c}_2^{\dagger k} \hat{c}_3^{\dagger n-k+1} + \frac{\beta_2}{\sqrt{2}} (\cos\tau_2)^2 e^{2i\eta_t} \hat{c}_2^{\dagger k+2} \hat{c}_3^{\dagger n-k} + \frac{\beta_2}{\sqrt{2}} (\sin\tau_2)^2 e^{-2i\eta_r} \hat{c}_2^{\dagger k} \hat{c}_3^{\dagger n-k+2}$$
$$- 2\frac{\beta_2}{\sqrt{2}} \cos\tau_2 \sin\tau_2 e^{i(\eta_t-\eta_r)} \hat{c}_2^{\dagger k+1} \hat{c}_3^{\dagger n-k+1} \Bigg) |000\rangle. \tag{7}$$

The output state in the case of a given detection event can now be determined by projecting $|\Psi''\rangle$ to the number state corresponding to the result of the measurement carried out on modes $\hat{c}_2$ and $\hat{c}_3$. Due to considerations of photon-number conservation it suffices to examine the detection events in which the total number of detected photons is 2. Let us examine the case in which one photon on the D_2 and one on D_3 detectors are detected in coincidence. It will turn out that the other two possible detection events (two photons on one of the detectors and no photons on the other) are inadequate choices.

After the coincident detection of one photon on D_2 and one on D_3 the state of the system becomes the projection of $|\Psi''\rangle$ of Eq. (7) to $|11\rangle$ in modes 2 and 3. The state of the output mode obtained this way reads up to a normalization constant:

$$-\sqrt{2}\cos\tau_2 \sin\tau_2 e^{i\eta}\beta_2\gamma_0|0\rangle + \cos(2\tau_2)\beta_1\gamma_1|1\rangle$$
$$+ \sqrt{2}\cos\tau_2\sin\tau_2 e^{-i\eta}\beta_0\gamma_2|2\rangle, \tag{8}$$

where $\eta = \eta_t - \eta_r$. Comparing to the desired state in Eq. (2), we see that to achieve the truncation

$$-\sqrt{2}\cos\tau_2\sin\tau_2 e^{i\eta}\beta_2 = \sqrt{2}\cos\tau_2\sin\tau_2 e^{-i\eta}\beta_0$$
$$= \cos(2\tau_2)\beta_1 = K \tag{9}$$

must hold. This is the condition for the β coefficients of Eq. (5). The efficiency of the truncation is K^2/N, where N is the renormalization constant of Eq. (2). It is maximal if K is maximal (this depends on the device), and $N=1$ (this depends on the input state). When the beam-splitter parameters are chosen optimally, K has to be maximal.

Now, given the state $|\Psi_{12}\rangle = |11\rangle$ incident on BS_1 a set of parameters for BS_1 and BS_2 have to be found to make BS_1 capable of generating the state in Eq. (5) with the β coefficients fulfilling Eq. (9). The intermediate state $|\Psi'_{12}\rangle$ in Eq. (5), leaving the beam splitter, is a point of the vector space spanned by the vectors $\{|20\rangle, |11\rangle, |02\rangle\}$. On varying the parameters ϕ_t, ϕ_r, and τ_1 of BS_1, this point perambulates around a set of points in this vector space. [This is called the SU(2) orbit of the point $|11\rangle$.] The coordinates of the parametrized set of points read

$$\cos(2\tau_1)|11\rangle + \frac{\sqrt{2}}{2}\sin(2\tau_1)e^{i\phi}|20\rangle - \frac{\sqrt{2}}{2}\sin(2\tau_1)e^{-i\phi}|02\rangle, \tag{10}$$

where $\phi = \phi_t - \phi_r$. Equation (9) also defines a parametrized set of points in this vector space with coordinates β_1, β_2, and β_3, the parameters being η and τ_2. Each point in this set represents the appropriate state that is required for state truncation if the parameters of BS_2 are chosen to be η and τ_2. The required beam-splitter parameters are the coordinates of the intersection of the two point sets in Eqs. (9) and (10).

The solution is the following: for the phase shifts $\phi_t - \phi_r = \eta_t - \eta_r$ must hold. Otherwise the relative phase of the Fock components is modified. We may choose $\phi_t = \phi_r = \eta_t = \eta_r = 0$, which is convenient, because in this case the matrices in Eqs. (3) and (4) are real. The τ parameters have to satisfy

$$\tan(2\tau_1)\tan(2\tau_2) = 2. \tag{11}$$

The factor K in Eq. (9), and thus the maximum probability of detection of the coincidence under discussion, depends on the τ values. K itself also has a maximum at the optimal choice of τ parameters,

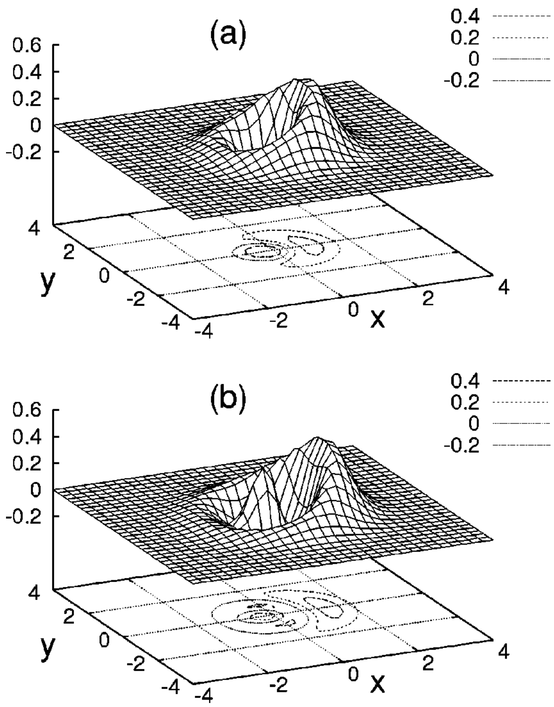

FIG. 2. Highly nonclassical states obtained by state truncation. The starting state is the coherent state $|\alpha=2\rangle$. (a) Shows the Wigner function of the state obtained using the quantum scissors device of Pegg, Phillips, and Barnett [13]. (b) Shows the Wigner function of the result of generalized state truncation up to the two-photon states. The realization and teleportation of the latter is the subject of this paper.

$$\tau_1=\tau_2=\tfrac{1}{2}\arctan(\pm\sqrt{2}). \tag{12}$$

Thus in the optimal case the two beam splitters have to be identical, with transmittance either 0.21 or 0.79. The optimum value of the renormalization factor is $K=1/3$, which means that the coincident detection of one photon on each detector occurs at most in 1/9 of the cases. In these cases the state truncation is successful. The probability is exactly 1/9 if the incoming state is already a superposition of vacuum, one-, and two-photon states. This is the case of teleportation and the teleportation efficiency is 1/9. Otherwise the probability is proportional to $\Sigma_{n=0}^{2}|\gamma_n|^2$. The device can be used with success if N has a low value, i.e., the incoming state contains the vacuum, one-, and two-photon states with large enough weights.

As mentioned in the Introduction, there are several interesting nonclassical states that could be prepared with this method. For example, Fig. 2 shows the Wigner functions of a weak coherent state truncated with the Pegg-Phillips-Barnett quantum scissors, and with our generalized quantum scissors. Both states are highly nonclassical. Truncation of squeezed vacuum states would yield superpositions of the vacuum and two-photon number states.

Let us return to the case of detecting two photons on one of the detectors and none on the other. These are interesting counterexamples, since the above discussed intersection of the point sets is empty in these cases; thus the state truncation cannot be carried out using any kind of beam splitter.

In the rest of this section we briefly outline some of the problems that would arise, in addition to detector imperfections and noise, if one had to put the quantum scissors scheme into practice. In order to obtain interference, one has to ensure the simultaneous arrival of the states at the proper beam splitters. In the case of interferometric Bell-state measurements with polarization states [6,20], the coincidence event indicating the result of measurement can occur only in case of proper timing. In our case the number of photons arriving at BS_2 in the input state is not limited, and therefore coincidence at the detectors D_2 and D_3 itself does not guarantee the simultaneous arrival of the states that have to interfere; we have to possess additional information on the timing of the states. The two-mode state $|11\rangle$ can be generated using nondegenerate parametric downconversion, which is a prevalent technique even in applied physics [21,22]. However, direct application of the photon pair that emerges in the nonlinear process is not easily applicable to our scheme, because in this case there is no indication of the presence of the photon couple. In the scheme of Pegg, Phillips, and Barnett, one of the photons belonging to a downconverted pair (the idler) is applicable for signifying the existence of the other one (the signal). In our case two downconversions are required (this may be achieved by reflecting the pump back to the crystal as in Refs. [7,10]), and the coincident detection of the idlers from both processes would ensure that the signals are in state $|11\rangle$ at the time of the coincidence. This would also be necessary for the scheme of Villas-Bôas *et al.* [4], who suggest the application of two quantum scissors one after one other. On the other hand, one has to ensure the simultaneous arrival of the state to be truncated in mode $\hat{b}_3$ and the state in mode $\hat{b}_2$ (the reference state) at the beam splitter BS_2. The input state for the scissors may be a weak coherent state obtained by attenuation of the pump beam that generated the reference beam. This makes the relative timing of the reference and input states possible, by changing the path length of the latter, similarly to Bouwmeester *et al.* [7]. The interference between weak coherent states and downconverted photons has been observed [23–25]. The proper arrival of the input state may be verified by splitting the coherent state with an additional beam splitter, and placing a detector at one of the branches. Alternatively, one may carry out a coincidence test by applying an additional detector on the output of the scissors device. The latter is applicable because once the timing of the reference state is guaranteed via the coincidence of the downconversion idlers, the only case when anything else than vacuum is truncated occurs if the input state arrives at BS_2 in time. Moreover, in an experiment the direct verification of the success of truncation ensures that the input state and the corresponding branch of the entangled state have interfered. The verification of the truncation process may even be carried out via a single homodyne measurement because of the very characteristic shape of the Wigner function of the output state.

TABLE I. The optimal transmittance ($\cos^2 \tau_2$) of the beam splitter BS_2 and the maximal efficiency (K^2) of the generalized quantum scissors device cutting up to the n-photon Fock components. Photon counters D_2 and D_3 count d_2 and d_3 photons, respectively. Many of the entries belong to hypothetical arrangements, which are not experimentally feasible at the present state of the art.

n	$d_2=n,\ d_3=0$		$d_2=n-1,\ d_3=1$	
	$\cos^2\tau_2$	K^2	$\cos^2\tau_2$	K^2
1	0.5	0.25	0.5	0.25
2	0.5	0.10	0.21 or 0.79	0.11
3	0.5	0.047	0.5	0.047
4	0.5	0.023	0.38 or 0.62	0.028
5	0.5	0.012	0.5	0.019
6	0.5	0.0062	0.42 or 0.58	0.012
7	0.5	0.0032	0.5	0.0093
8	0.5	0.0016	0.44 or 0.56	0.0056

III. FURTHER GENERALIZATION OF QUANTUM SCISSORS

In this section we present a theoretical investigation of the possibility of further generalization of the scheme discussed. Our aim is now to truncate the number-state expansion of an arbitrary incoming state up to the n-photon component. In this section we take a more general point of view: we omit the beam splitter BS_1, and suppose that an intermediate state

$$|\Psi'_{12}\rangle=\sum_{k=0}^{n}\beta_k|n-k,k\rangle \tag{13}$$

is already prepared by some method. Although it may be generated by BS_1 using some reference state $|\Psi_{12}\rangle$, unlike the case discussed in Sec. II, we are now not concerned with the question of preparing this state at the present state of the art. Our aim is the theoretical analysis of the possibilities that would emerge if this state could be prepared. Furthermore, suppose that D_2 and D_3 are ideal photon counters and they are counting d_2 and d_3 photons, respectively. The method is the same as in the case of the truncation up to two-photon components: the state of the system after the action of BS_2 has to be calculated, and the result has to be projected to the appropriate state determined by the measurement result. The difference is that since BS_1 is omitted, the result will be a set of β parameters of Eq. (13) and parameters of BS_2.

In general, the state obtained reads

$$|\Psi_{\mathrm{out}}\rangle=\frac{\sqrt{N}}{K}\sum_{j=0}^{\infty}\sum_{k=0}^{n}\beta_k\gamma_j D_{jk}|k\rangle=\sqrt{N}\sum_{k=0}^{n}\gamma_k|k\rangle, \tag{14}$$

where $N=1/\Sigma_{k=0}^{n}|\gamma_k|^2$ and K are normalization constants and D_{jk} describes both BS_2 and the measurement carried out. It is easy to show that Eq. (14) can be solved for the β coefficients if and only if $d_2+d_3=n$. That is, we detect a total photon number of n. (We consider both $\hat{c}_2$ and $\hat{c}_3$ as being in a photon-number eigenstate.)

Just as in the case of truncation up to the two-photon component, the desired measurement occurs with a probability of K^2/N. K^2 describes the efficiency of the truncation as a function of the parameter τ_2 of BS_2. An optimal quantum scissors device can be obtained by choosing τ_2 so that K^2 is maximal.

We discuss two detection events, as examples. If D_2 detects n photons and D_3 none, Eq. (14) has the solution

$$\beta_k=\frac{Ke^{-i[k\eta_t+(n-k)\eta_r]}}{\sqrt{\binom{n}{k}}(\cos\tau_2)^k(\sin\tau_2)^{n-k}}, \tag{15}$$

and

$$K^2=\left(\sum_{k=0}^{n}\frac{1}{\binom{n}{k}(\cos\tau_2)^{2k}(\sin\tau_2)^{2n-2k}}\right)^{-1}. \tag{16}$$

From Eq. (16) it can be seen that the optimal value of the $(\cos\tau_2)^2$ transmittance is 1/2; BS_2 has to be a 50-50 beam splitter. For $n=1$ it is the Pegg-Phillips-Barnett device. For $n=2$, the efficiency of the process would be approximately the same as that for the case discussed in Sec. II, but unlike in Sec. II, there are no beam-splitter parameters for BS_1 so that it could generate the required $|\Psi_{12}\rangle$ reference state of Eq. (15) from the state $|11\rangle$.

The other example is when D_2 detects $n-1$ photons and D_3 one photon at the same time. In this case, Eq. (14) gives

$$\beta_k=\frac{Ke^{-i[(k-1)\eta_t+(n-k-1)\eta_r]}}{\sqrt{\frac{1}{n}\binom{n}{k}}(\cos\tau_2)^{k-1}(\sin\tau_2)^{n-k-1}[n(\cos\tau_2)^2-k]}, \tag{17}$$

and

$$K^2=\left(\sum_{k=0}^{n}\frac{1}{\frac{1}{n}\binom{n}{k}(\cos\tau_2)^{2k-2}(\sin\tau_2)^{2n-2k-2}[n(\cos\tau_2)^2-k]^2}\right)^{-1}. \tag{18}$$

For $n=2$ this is the case discussed in Sec. II.

Table I contains the optimal transmittance ($\cos^2\tau_2$) of BS_2 and the maximal efficiency values for the two examples above. Note the significant decrease in efficiency for large photon numbers.

IV. QUANTUM SCISSORS AS A TELEPORTATION SCHEME

Let us direct our attention to the teleportation process involved in the operation of the device described above. Both the scheme suggested in Refs. [26,20] and realized in Bouwmeester *et al.* [7], and the teleportation scheme suggested by Villas-Bôas *et al.* [4], based on the quantum scissors of Pegg, Phillips, and Barnett are optical realizations of quantum teleportation with two-state systems. The entangled state in the latter scheme is generated by beam splitter BS_1 from the $|\Psi_{12}\rangle$ reference state, and the analysis in Ref. [4] shows that a Bell-state analysis is implemented by the BS_2 beam splitter and D_2, D_3 detectors.

The generalized quantum scissors described in Sec. II do this operation on three-state systems, namely, the basis states are $\{|0\rangle,|1\rangle,|2\rangle\}$. If the input of the quantum scissors is the superposition of these states, it is simply teleported. If the state is an arbitrary state, only its vacuum, one-, and two-photon components are teleported, and thereby the state becomes truncated. The role of the entangled state and Bell-state analysis is not as comprehensible as in the above case. We would like to provide insight into the process. This can be achieved by considering the properties of the beam splitter transformation.

The description of a lossless beam splitter with Schwinger angular momenta is well known [27]. The input and output operators can be transformed to angular momentum operators, which are more suitable for examining the action of a beam splitter. Using the notation used for BS_2 in Fig. 1, consider the Schwinger operators

$$\hat{l}=\tfrac{1}{2}(\hat{b}_1^\dagger\hat{b}_1+\hat{b}_2^\dagger\hat{b}_2),$$
$$\hat{m}=\tfrac{1}{2}(\hat{b}_1^\dagger\hat{b}_1-\hat{b}_2^\dagger\hat{b}_2). \tag{19}$$

Here $\hat{l}$ measures one-half of the total number of photons, proportional to the energy of the incoming state. $\hat{m}$ is the photon-number difference. The two-mode Fock states are common eigenstates of these operators. Denoting the eigenvalues by l and m, the standard inequality $-l\leq m\leq l$ holds: eigenstates with given l (given energy) can correspond to $2l+1$ different values of m, forming an SU(2) multiplet. These multiplets are labeled by the eigenvalue l, e.g., for $l=1$, the states in the multiplet are $|20\rangle$, $|11\rangle$, and $|02\rangle$, corresponding to $m=-1,0,1$. Similar operators and multiplet structure can be defined for the output states.

Since the beam splitter is passive and linear [essentially an SU(2) device], the multiplets span invariant subspaces of the beam-splitter action. This is a consequence of photon-number conservation. The beam splitter does not mix the states corresponding to different l values.

A measurement outcome, detection of given numbers of photons on D_2 and D_3 in coincidence, is a projection to a two-mode Fock state in modes $\hat{c}_2$, $\hat{c}_3$. Therefore measurement outcomes can also be grouped into multiplets. We shall see that this multiplet structure explains the teleportation process and its limitations.

Consider now

$$|\Psi_{\rm in}\rangle=\gamma_0|0\rangle+\gamma_1|1\rangle+\gamma_2|2\rangle \tag{20}$$

as input state in mode $\hat{a}_3$. The output state in mode $\hat{c}_1$ is determined by the outcome of the measurement, and the state of the whole system before the measurement, $|\Psi_m\rangle$. The operator $\hat{A}^\dagger$ creating $|\Psi_m\rangle$ from the vacuum is a polynomial of the creation operators $\hat{c}_1^\dagger$, $\hat{c}_2^\dagger$, and $\hat{c}_3^\dagger$. An outcome of a measurement means detection of n photons on D_2 and m photons on D_3. The state after measurement of such an outcome is created from the vacuum by an operator which is a sum of all the summands of $\hat{A}^\dagger$ containing $\hat{c}_2^{\dagger n}\hat{c}_3^{\dagger m}$, renormalized. We shall call the operators $\hat{c}_2^{\dagger n}\hat{c}_3^{\dagger m}$ "outcome operators," since they correspond to a given measurement outcome. In order to determine the possible output states, one has to examine the structure of $\hat{A}^\dagger$.

As mentioned before, since the operators $\hat{c}_2^\dagger$ and $\hat{c}_3^\dagger$ are obtained from the transformation of Eq. (4), it is worth grouping the outcome operators into multiplets indexed with the eigenvalues l of $\hat{l}$. The outcomes in which the total number of detected photons is $2l$ correspond to the same multiplet. Let us introduce the notation

$${}^{2l}\hat{M}_m=\hat{c}_2^{\dagger\, l+m}\hat{c}_3^{\dagger\, l-m}, \quad m=-l,\ldots,l, \tag{21}$$

for the outcome operators. Furthermore, given a set of arbitrary coefficients ${}^{2l}\mathcal{A}_m$, $m=-l,\ldots,l$, let there be

$${}^{2l}\hat{\mathcal{M}}_{\mathcal{A}}=\sum_{m=-l}^{l}{}^{2l}\mathcal{A}_m\,{}^{2l}\hat{M}_m, \tag{22}$$

a linear combination of outcome operators in the lth multiplet, with coefficients ${}^{2l}\mathcal{A}$ depending on the beam splitter parameters. Different script letters in the index will mean a different set of parameters in this notation.

Notice that since the γ_n coefficients in the $\hat{A}^\dagger$ polynomial originate from Eq. (20) and there are always two photons initially incident in modes $\hat{a}_1$ and $\hat{a}_2$, the summands in $\hat{A}^\dagger$ that contain a given γ_n have to create a three-mode Fock state with total number of photons $n+2$. This is a consequence of the linearity of the system. On the other hand, $\hat{c}_1^\dagger$ can appear at maximum as a second power in $\hat{A}^\dagger$ because no photons from mode $\hat{a}_3$ get into mode $\hat{c}_1$ and in modes $\hat{a}_1$ and $\hat{a}_2$ only two photons are incident. Consequently, $\hat{A}^\dagger$ has the following structure:

$$\hat{A}^{\dagger}=\gamma_0({}^2\hat{\mathcal{M}}_{\mathcal{A}}+{}^1\hat{\mathcal{M}}_{\mathcal{B}}\hat{c}_1^{\dagger}+{}^0\hat{\mathcal{M}}_{\mathcal{C}}\hat{c}_1^{\dagger 2})+\gamma_1({}^3\hat{\mathcal{M}}_{\mathcal{D}}+{}^2\hat{\mathcal{M}}_{\mathcal{E}}\hat{c}_1^{\dagger}+{}^1\hat{\mathcal{M}}_{\mathcal{F}}\hat{c}_1^{\dagger 2})+\frac{\gamma_2}{\sqrt{2}}({}^4\hat{\mathcal{M}}_{\mathcal{G}}+{}^3\hat{\mathcal{M}}_{\mathcal{H}}\hat{c}_1^{\dagger}+{}^2\hat{\mathcal{M}}_{\mathcal{I}}\hat{c}_1^{\dagger 2}). \quad (23)$$

It can be seen that the multiplet structure suggested by the nature of the beam-splitter transformation is reflected in the structure of the operator creating the output state. Only the outcomes in the ${}^2\hat{\mathcal{M}}_{\mathcal{A}}$, ${}^2\hat{\mathcal{M}}_{\mathcal{E}}$, and ${}^2\hat{\mathcal{M}}_{\mathcal{I}}$ multiplets appear with all three γ coefficients. Only the outcomes in these multiplets can provide teleportation, since the state obtained after the measurement on mode $\hat{c}_1$ depends on all three γ coefficients. In the case of a measurement outcome corresponding to another multiplet some of the information is lost. The whole information is transferred if the total number of detected photons is 2.

The ${}^{2l}\mathcal{A}$, ${}^{2l}\mathcal{E}$, and ${}^{2l}\mathcal{I}$ coefficients depend on the beam-splitter parameters. With the parameters determined in Sec. II, ${}^2\mathcal{A}_0={}^2\mathcal{E}_0={}^2\mathcal{I}_0=1/3$. In the case of a measurement outcome described by ${}^2\hat{M}_0$, i.e., detection of one photon on D_2 and one on D_3 in coincidence, the output in mode $\hat{c}_1$ becomes the same as the input state in Eq. (20). This is a case of successful teleportation, which happens in 1/9 of the cases, regardless of the input state in Eq. (20).

In the case of detecting two photons on either D_2 or D_3, described by ${}^2\hat{M}_{-1}$ and ${}^2\hat{M}_1$, the teleportation is successful in the sense that information involved in $|\Psi_{\text{in}}\rangle$ is transferred, but the Fock coefficients of the state obtained are multiplied by different constants. This is the case analogous with the detection of other than the singlet Bell state in the original scheme of Bennett *et al.*: the output has to undergo a given unitary transformation in order to obtain the teleported state. Since the coefficients of these summands are not equal, they cannot be factored out. The probability of these outcomes, and thus the efficiency of this ''distorted teleportation,'' depend on the state $|\Psi_{\text{in}}\rangle$ also.

Finally, the outcomes corresponding to the other multiplets yield unsuccessful teleportation because some of the information (some of the coefficients describing the input state) is irrecoverably lost. This information cannot be regained using passive elements, so the ''no-go theorem'' prevails.

V. CONCLUSION

We have shown how an arbitrary one-mode traveling-wave field can be truncated to its first three Fock components. The method, a generalization of a result of Pegg, Phillips, and Barnett [13], employs the projection principle. We have also examined the possibility of further generalization. This quantum scissors device can be a useful tool for traveling-wave quantum state engineering. The states that can be prepared with the application of the quantum scissors device are highly nonclassical.

The operation of the discussed device involves quantum nonlocality, namely, it is quantum teleportation on a finite basis set of the first three Fock states. Thus our scheme provides a possible realization of a discrete basis quantum teleportation with three basis states. We have analyzed the properties of this teleportation process in the ideal case. It seems to us that the argument presented leads closer to an understanding of the operation and limitations of quantum teleportation.

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