Decoherence and coherent population transfer between two coupled systems

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We show that an arbitrary system described by two dipole moments exhibits coherent superpositions of internal states that can be completely decoupled from the dissipative interactions (responsible for decoherence) and an external driving laser field. These superpositions, known as dark or trapping states, can be completely stable or can coherently interact with the remaining states. We examine the master equation describing the dissipative evolution of the system and identify conditions for population trapping and also classify processes that can transfer the population to these undriven and nondecaying states. It is shown that coherent transfers are possible only if the two systems are nonidentical, that is the transitions have different frequencies and/or decay rates. In particular, we find that the trapping conditions can involve both coherent and dissipative interactions, and depending on the energy level structure of the system, the population can be trapped in a linear superposition of two or more bare states, a dressed state corresponding to an eigenstate of the system plus external fields or, in some cases, in one of the excited states of the system. A comprehensive analysis is presented of the different processes that are responsible for population trapping, and we illustrate these ideas with three examples of two coupled systems: single V- and Λ-type three-level atoms and two nonidentical two-level atoms, which are known to exhibit dark states. We show that the effect of population trapping does not necessarily require decoupling of the antisymmetric superposition from the dissipative interactions. We also find that the vacuum-induced coherent coupling between the systems could be easily observed in Λ-type atoms. Our analysis of the population trapping in two nonidentical atoms shows that the atoms can be driven into a maximally entangled state which is completely decoupled from the dissipative interaction.

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I. INTRODUCTION

The control and manipulation of information transfer processes is a topic of much current interest because of the many possible applications in quantum computation, teleportation, and quantum information theory. Ways of controlling decoherence, and of producing maximum entanglement, are of particular importance.

Information can be transferred between two systems by coherent or incoherent interactions. The coherent interactions can be stimulated by an external field such as a laser that can induce coherent oscillations of the dipole moments of the systems, or can produce coherent superpositions of their internal states. The incoherent interactions occur as spontaneous emission from one system to the other resulting from the coupling of the systems to the same modes of the vacuum field. Coherent interactions lead to nondissipative (reversible) transfers of population between systems, whereas transfers induced by spontaneous emission are dissipative (irreversible). Whilst coherent processes are easy to control, the spontaneous emission from two interacting systems leads to losses (decoherence) as only a small part of the radiation emitted by one system can be absorbed by the other. Moreover, these two processes are not complementary to each other, since any coherent interaction is accompanied by spontaneous emission. Therefore, the problem of transferring information in modified environments which suppress or reduce spontaneous emission has attracted considerable interest in recent years. It has been shown that an effective method to modify spontaneous emission is to place a radiating system in a frequency-dependent reservoir such as an electromagnetic cavity [1], an optical waveguide [2], or a photonic band-gap material [3], which changes the density of modes of the vacuum field into which the system can emit.

Another process that can modify spontaneous emission is quantum interference. It was predicted by Agarwal [4] in a degenerate three-level atom and is now a well-known phenomenon that can lead to many interesting effects, such as electromagnetically induced transparency [5], lasing without inversion [6], and the narrowing of optical transitions [7,8]. The essential feature of quantum interference is the existence of quantum superposition states, which can be decoupled from the coherent and incoherent interactions. These states, known as dark or trapped states, were also predicted in other configurations of three-level and multilevel atoms, as well as in multiatom systems [9,10], and many practical applications have been suggested, for example, in high-resolution laser spectroscopy [11], laser cooling [12], and quantum computing [13–15].

Although the trapping states have the common property that the population will stay in such a state for an extremely long time, they can, however, be implemented in different ways. In a multilevel system the population can be trapped in a linear superposition of two or more bare states, a dressed state corresponding to an eigenstate of the atoms plus external fields, or in some cases, in one of the excited states of the system. The starting point of the standard analysis of the origin of population trapping in a specific multilevel system is a numerical or analytical solution for the populations, the coherences, or the fluorescence or absorption spectra [16–20]. The results are then analyzed in terms of the parameters of the system such as the damping rates, detunings, and Rabi frequencies of the driving fields, and the origin of population trapping is usually explored in terms of dressed states of the system.

In this paper we demonstrate a qualitatively different approach to the problem of population trapping. We show how the master equation of two coupled systems enables us to identify conditions for population trapping and to classify the coherent and incoherent processes responsible for the transfer of population to a trapping state. Specifically, we examine the dynamics of two arbitrary systems coupled through the three-dimensional electromagnetic vacuum field and driven by a single-mode coherent laser field. The systems are represented by transition dipole moments which refer either to the two transitions in a single multilevel atom, or to the individual transitions in two separate two-level atoms. The master equation for two interacting systems can, of course, be solved directly in many cases and, as we have mentioned above, the conditions for population trapping can be found from the final results. Nevertheless, in some cases such a direct method can be laborious and uninformative. Our approach provides a simple picture of the processes responsible for the population trapping which enables us to obtain a better understanding of the physics of this effect. Using an unitary transformation of the dipole moments of the systems, we rewrite the master equation in the representation of superposition systems that are not coupled to each other through the vacuum field, but can be coupled through coherent interactions. We find the general condition for the complete decoupling of one of the superpositions from the dissipative interactions and identify coherent processes that can transfer the population between the superpositions. To our knowledge, the analysis of the conditions and processes responsible for the transfer of the population between two coupled systems has not been previously presented in the literature.

In Sec. II we give a general description of the master equation for two arbitrary systems and then, in Sec. III, we introduce a unitary transformation to the superposition systems. Finally, in Sec. IV we illustrate our approach for three specific examples of two coupled systems, and obtain a number of interesting results. For instance, our study of population trapping in the system of two nonidentical atoms shows that the atoms can be driven into a maximally entangled state which exhibits zero decoherence.

II. MASTER EQUATION OF TWO INTERACTING SYSTEMS

We start with a quite general description of two interacting systems, driven by a single-mode coherent laser field of amplitude E and phase ϕ. The systems, which we will call bare systems, are represented by induced dipole moments

$$\begin{aligned}\tilde{\boldsymbol{\mu}}_1 &= \boldsymbol{\mu}_1 S_1^+ + \boldsymbol{\mu}_1^* S_1^- ,\\ \tilde{\boldsymbol{\mu}}_2 &= \boldsymbol{\mu}_2 S_2^+ + \boldsymbol{\mu}_2^* S_2^- ,\end{aligned} \tag{1}$$

where $\boldsymbol{\mu}_i$ is the dipole matrix element of the ith system, and S_i^+ and S_i^- $(i=1,2)$ are dipole raising and lowering operators, respectively. The dipole moments are assumed to oscillate with different frequencies ω_1 and ω_2, and are coupled to the three-dimensional multimode electromagnetic field whose modes are in a vacuum state. The total Hamiltonian describing the energies of the systems, electromagnetic field and interactions, in the electric-dipole and RWA approximations, is composed of four terms

$$H = H_s + H_v + H_{sL} + H_{sv} , \tag{2}$$

where

$$H_s = \hbar\omega_1 S_1^+ S_1^- + \hbar\omega_2 S_2^+ S_2^- \tag{3}$$

is the Hamiltonian of the two bare systems,

$$H_v = \sum_{\mathbf{k}s} \hbar\omega_{\mathbf{k}s}\left(\hat{a}_{\mathbf{k}s}^{\dagger}\hat{a}_{\mathbf{k}s} + \frac{1}{2}\right) \tag{4}$$

is the Hamiltonian of the three-dimensional multimode electromagnetic field,

$$H_{sL} = -\frac{1}{2}\hbar[(\Omega_1 S_1^+ + \Omega_2 S_2^+)e^{i\omega_L t} + \text{H.c.}] \tag{5}$$

is the interaction of the systems with the coherent laser field, and

$$H_{sv} = \sum_{\mathbf{k}s} \{[\boldsymbol{\mu}_1\cdot\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_1)S_1^+ + \boldsymbol{\mu}_2\cdot\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_2)S_2^+]\hat{a}_{\mathbf{k}s} + \text{H.c.}\} \tag{6}$$

is the interaction of the bare systems with the multimode vacuum field. Here, ω_L is the frequency of the driving laser field, $\hat{a}_{\mathbf{k}s}^{\dagger}$ and $\hat{a}_{\mathbf{k}s}$ are the creation and annihilation operators of a photon in the mode $(\mathbf{k},s)$ with wave vector $\mathbf{k}$ and polarization s, the coefficient $\boldsymbol{\mu}_i\cdot\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)$ is the coupling constant of the dipole moment $\boldsymbol{\mu}_i$ with the mode function $\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)$ of the three-dimensional multimode vacuum field, evaluated at the position $\mathbf{r}_i$ of the ith dipole, and

$$\Omega_i = \boldsymbol{\mu}_i\cdot\mathbf{E}e^{i\mathbf{k}_L\cdot\mathbf{r}_i}/\hbar \tag{7}$$

is the Rabi frequency of the ith system located at a point $\mathbf{r}_i$ and $\mathbf{k}_L$ is the wave vector of the driving laser field. For a single laser coupled to both systems the Rabi frequencies Ω_1 and Ω_2 are related by

$$\Omega_2 = \Omega_1\frac{\mu_2\cos\theta_1}{\mu_1\cos\theta_2}e^{i\mathbf{k}_L\cdot(\mathbf{r}_2-\mathbf{r}_1)}, \tag{8}$$

where θ_i is the angle between $\boldsymbol{\mu}_i$ and the polarization vector of the laser field, $\mu_i=|\boldsymbol{\mu}_i|$ is the magnitude of the ith dipole moment, and $\exp[i\mathbf{k}_L\cdot(\mathbf{r}_2-\mathbf{r}_1)]$ is the phase difference arising from different positions of the dipoles.

A standard procedure employing the Born and Markoff approximations leads to a description of the dynamics of the systems in terms of the master equation for the reduced density operator ρ. For two general systems, the master equation can be written in the Lindblad form as

$$\dot{\rho} = \mathcal{L}_{nd}\rho + \mathcal{L}_d\rho, \tag{9}$$

where the Liouville operator $\mathcal{L}_{nd}$ describes the nondissipative part of the evolution

$$\mathcal{L}_{nd}\rho=-\frac{i}{\hbar}[H',\rho], \tag{10}$$

and $\mathcal{L}_d$ the dissipative part

$$\begin{aligned}\mathcal{L}_d\rho=&-\frac{1}{2}\Gamma_1(S_1^+S_1^-\rho+\rho S_1^+S_1^--2S_1^-\rho S_1^+)\\&-\frac{1}{2}\Gamma_{12}(S_1^+S_2^-\rho+\rho S_1^+S_2^--2S_2^-\rho S_1^+)\\&-\frac{1}{2}\Gamma_{12}(S_2^+S_1^-\rho+\rho S_2^+S_1^--2S_1^-\rho S_2^+)\\&-\frac{1}{2}\Gamma_2(S_2^+S_2^-\rho+\rho S_2^+S_2^--2S_2^-\rho S_2^+),\end{aligned} \tag{11}$$

with

$$\begin{aligned}H'=&\hbar(\omega_1+\delta_1^{(-)})S_1^+S_1^-+\hbar(\omega_2+\delta_2^{(-)})S_2^+S_2^-+\hbar\delta_1^{(+)}S_1^-S_1^+\\&+\hbar\delta_2^{(+)}S_2^-S_2^++\hbar\delta_{12}^{(-)}(S_1^+S_2^-+S_2^+S_1^-)\\&+\hbar\delta_{12}^{(+)}(S_1^-S_2^++S_2^-S_1^+)+H_{sL},\end{aligned} \tag{12}$$

and H_{sL} is given in Eq. (5). The coefficient

$$\Gamma_i=\pi\sum_{\mathbf{k}s}|\boldsymbol{\mu}_i\cdot\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)|^2\delta(k-k_i)\qquad(i=1,2) \tag{13}$$

is the spontaneous damping rate of the ith system resulting from the coupling of the system to the vacuum field, and

$$\Gamma_{12}=\Gamma_{21}=\pi\sum_{\mathbf{k}s}[\boldsymbol{\mu}_1\cdot\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_1)][\boldsymbol{\mu}_2^*\cdot\mathbf{g}_{\mathbf{k}s}^*(\mathbf{r}_2)]\delta(k-k_0), \tag{14}$$

is a generalized (cross-) damping rate arising from the coupling of the bare systems through the vacuum field. The terms proportional to Γ_{12} represent an incoherent exchange of the excitation between the systems such that one of the systems spontaneously emits photons which are then absorbed by the other system.

The remaining parameters

$$\delta_i^{(\pm)}=\mathrm{P}\frac{1}{c}\sum_{\mathbf{k}s}|\boldsymbol{\mu}_i\cdot\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)|^2\frac{1}{k\pm k_i} \tag{15}$$

represent a part of the Lamb shift, induced by the first-order coupling in the Hamiltonian H_{sv}, of the ground and excited states of the systems, while

$$\delta_{12}^{(\pm)}=\delta_{21}^{(\pm)}=\mathrm{P}\frac{1}{c}\sum_{\mathbf{k}s}[\boldsymbol{\mu}_1\cdot\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_1)][\boldsymbol{\mu}_2^*\cdot\mathbf{g}_{\mathbf{k}s}^*(\mathbf{r}_2)]\frac{1}{k\pm k_0} \tag{16}$$

is the vacuum-induced coupling between the systems. In Eqs. (13)–(16), $k=|\mathbf{k}|$, $k_i=\omega_i/c$, $k_0=(\omega_1+\omega_2)/2$, and P refers to the Cauchy principal value.

According to Eq. (12), the parameters $\delta_i^{(\pm)}$ can be considered as a part of the frequencies ω_1 and ω_2, and thus they can be omitted or, in general, can be included into the dynamics by redefining the frequencies to $\tilde{\omega}_i=\omega_i+\delta_i^{(-)}$. Therefore, we will not present calculations of the Lamb shift. However, we are interested in the qualitative effects of the interactions between the systems, and the role played by $\delta_{12}^{(\pm)}$ in their dynamics. It is evident from Eq. (12) that the parameter $\delta_{12}^{(\pm)}$ does not appear as a shift of the energies, but contributes to the coherent coupling between the bare systems [21]. Thus, the interaction with the vacuum field not only produces dissipative spontaneous emission but also leads to a coherent coupling between the systems.

We may find the explicit form of the damping rates and the coherent coupling coefficient by evaluating the sums in Eqs. (13), (14), and (16). In the plane-wave representation of the three-dimensional multimode field in free space, $\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)$ is defined as

$$\mathbf{g}_{\mathbf{k}s}(\mathbf{r}_i)=\left(\frac{ck}{2\pi\epsilon_0\hbar(2\pi)^3}\right)^{1/2}\hat{\mathbf{e}}_{\mathbf{k}s}e^{i\mathbf{k}\cdot\mathbf{r}_i}, \tag{17}$$

where $\hat{\mathbf{e}}_{\mathbf{k}s}$ is the unit polarization vector of the field mode $(\mathbf{k},\mathbf{s})$. In the spherical representation the unit orthogonal polarization vectors $\hat{\mathbf{e}}_{\mathbf{k}1}$ and $\hat{\mathbf{e}}_{\mathbf{k}2}$ may be taken as [22]

$$\begin{aligned}\hat{\mathbf{e}}_{\mathbf{k}1}&=(-\cos\theta\cos\phi,-\cos\theta\sin\phi,\sin\theta),\\\hat{\mathbf{e}}_{\mathbf{k}2}&=(\sin\phi,-\cos\phi,0),\end{aligned} \tag{18}$$

and the sum over $\mathbf{k}$ can be changed into an integral

$$\sum_{\mathbf{k}s}\to\frac{1}{c}\sum_{s=1}^{2}\int_0^\infty k^2\,dk\int_0^\pi\sin\theta\,d\theta\int_0^{2\pi}d\phi, \tag{19}$$

where (k,θ,ϕ) denote spherical coordinates. Substituting Eq. (19) into Eqs. (13) and (14), and assuming that the dipole moments are both linearly or both circularly polarized, we obtain the following explicit expressions for the spontaneous damping rates:

$$\Gamma_i=\frac{k_i^3\mu_i^2}{6\pi\epsilon_0\hbar}\qquad(i=1,2), \tag{20}$$

and the cross-damping rate

$$\begin{aligned}\Gamma_{12}=\frac{3}{4}\sqrt{\Gamma_1\Gamma_2}\Bigg\{&[(\hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2)-(\hat{\boldsymbol{\mu}}_1\cdot\hat{\mathbf{r}}_{12})(\hat{\boldsymbol{\mu}}_2\cdot\hat{\mathbf{r}}_{12})]\frac{\sin(k_0r_{12})}{k_0r_{12}}\\&+[(\hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2)-3(\hat{\boldsymbol{\mu}}_1\cdot\hat{\mathbf{r}}_{12})(\hat{\boldsymbol{\mu}}_2\cdot\hat{\mathbf{r}}_{12})]\\&\times\left[\frac{\cos(k_0r_{12})}{(k_0r_{12})^2}-\frac{\sin(k_0r_{12})}{(k_0r_{12})^3}\right]\Bigg\},\end{aligned} \tag{21}$$

where $r_{12}=|\mathbf{r}_2-\mathbf{r}_1|$, and $\hat{\boldsymbol{\mu}}_i$ and $\hat{\mathbf{r}}_{12}$ are unit vectors along the ith dipole moment and the line connecting the two systems, respectively.

Using Eq. (19) and the explicit expressions for Γ_i and Γ_{12}, the coherent coupling $\delta_{12}=\delta_{12}^{(+)}+\delta_{12}^{(-)}$ can be written as

$$\delta_{12}=\frac{\sqrt{\Gamma_1\Gamma_2}}{\pi}P\int_{-\infty}^{\infty}dk\,F(kr_{12})\left(\frac{1}{k-k_0}+\frac{1}{k+k_0}\right), \quad (22)$$

where $F(kr_{12})=\Gamma_{12}/\sqrt{\Gamma_1\Gamma_2}$, and Γ_{12} is given in Eq. (21).

The parameter δ_{12} depends on r_{12} and can have significantly different values depending on whether $kr_{12}=0$ or $kr_{12}\neq 0$. For $kr_{12}=0$ which, for example, occurs for two dipole moments in the same atom, $F(kr_{12})=1$ and then δ_{12} reduces to a form similar to the Lamb shift. When $kr_{12}\neq 0$, we can evaluate the integral by contour methods, and obtain

$$\begin{aligned}\delta_{12}=\frac{3}{4}\sqrt{\Gamma_1\Gamma_2}\bigg\{&-[(\hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2)-(\hat{\boldsymbol{\mu}}_1\cdot\hat{\mathbf{r}}_{12})(\hat{\boldsymbol{\mu}}_2\cdot\hat{\mathbf{r}}_{12})]\frac{\cos(k_0r_{12})}{k_0r_{12}}\\&+[(\hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2)-3(\hat{\boldsymbol{\mu}}_1\cdot\hat{\mathbf{r}}_{12})(\hat{\boldsymbol{\mu}}_2\cdot\hat{\mathbf{r}}_{12})]\\&\times\left[\frac{\sin(k_0r_{12})}{(k_0r_{12})^2}+\frac{\cos(k_0r_{12})}{(k_0r_{12})^3}\right]\bigg\}, \end{aligned}\quad (23)$$

which is the familiar retarded dipole-dipole interaction between the systems [23–27]. The parameters (21) and (23) depend on the mutual orientation of the dipole moments of the systems and their separation r_{12}. For large separations k_0r_{12} goes to infinity, and then $\Gamma_{12}=\delta_{12}=0$, independent of the mutual orientation of the dipole moments. By contrast, for very small separations (much smaller than the optical wavelength), k_0r_{12} goes to zero, and then Γ_{12} and δ_{12} reduce to

$$\Gamma_{12}=\sqrt{\Gamma_1\Gamma_2}(\hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2), \quad (24)$$

and

$$\delta_{12}=\frac{3\sqrt{\Gamma_1\Gamma_2}}{4(k_0r_{12})^3}[(\hat{\boldsymbol{\mu}}_1\cdot\hat{\boldsymbol{\mu}}_2)-3(\hat{\boldsymbol{\mu}}_1\cdot\hat{\mathbf{r}}_{12})(\hat{\boldsymbol{\mu}}_2\cdot\hat{\mathbf{r}}_{12})]. \quad (25)$$

For this case δ_{12} corresponds to the static dipole-dipole interaction potential. The magnitude of the parameters (24) and (25) depends on the mutual orientation of the dipole moments and vanishes when they are perpendicular. For parallel dipole moments the parameters attain their maximal values.

III. SUPERPOSITION SYSTEMS

Equations (11) and (12) illustrate the significance of the generalized damping rate Γ_{12} and the interaction energy δ_{12}. These two parameters are not associated with individual systems but appear as coupling terms between the two systems. The parameter Γ_{12} introduces a coupling through the dissipative process of spontaneous emission, while δ_{12} introduces a coherent coupling through the nondissipative process. These two couplings introduce off-diagonal terms into the master equation.

The traditional method of solving the master equation is to calculate equations of motion for the density-matrix elements and solve them by direct integration, or by a transformation to easily solvable algebraic equations. Another method is to diagonalize the Hamiltonian H', which leads to the dressed states of the system, and next to represent the dissipative part as spontaneous emission among these dressed states [28]. Here, we propose an alternative method where we introduce a unitary transformation of the dipole moments of the bare systems which diagonalizes the dissipative part of the master equation. In this approach the two coupled systems are represented by linear superpositions, which decay independently with significantly different rates, but which can be coupled through coherent interactions. This will allow us to identify the coherent processes which can transfer population between the two systems in the absence of the dissipative interaction.

We introduce new dipole operators S_s^+ and S_a^+ that are linear combinations of the S_1^+ and S_2^+ operators

$$\begin{aligned}S_s^+&=uS_1^++vS_2^+,\\S_a^+&=vS_1^+-uS_2^+,\end{aligned}\quad (26)$$

where u and v will be related through the condition

$$|u|^2+|v|^2=1, \quad (27)$$

which ensures that the transition to the superposition operators is a unitary transformation. The operators S_s^+ and S_a^+ represent, respectively, symmetric and antisymmetric superpositions of the dipole moments of the two bare systems. In terms of the operators (26), and with a proper choice of u and v, we can rewrite the dissipative part (11) of the master equation in a form

$$\begin{aligned}\mathcal{L}_d\rho=&-C_{ss}(S_s^+S_s^-\rho+\rho S_s^+S_s^--2S_s^-\rho S_s^+)\\&-C_{aa}(S_a^+S_a^-\rho+\rho S_a^+S_a^--2S_a^-\rho S_a^+)\\&-C_{sa}(S_s^+S_a^-\rho+\rho S_s^+S_a^--2S_a^-\rho S_s^+)\\&-C_{as}(S_a^+S_s^-\rho+\rho S_a^+S_s^--2S_s^-\rho S_a^+).\end{aligned}\quad (28)$$

By simple comparison of coefficients in Eqs. (11) and (28), we can express C_{mn} $(m,n=s,a)$, u and v in terms of Γ_i $(i=1,2)$ and Γ_{12} as

$$C_{ss}=\frac{1}{2}\,\frac{(\Gamma_1^2+\Gamma_2^2+2\Gamma_{12}\sqrt{\Gamma_1\Gamma_2})}{\Gamma_1+\Gamma_2}, \quad (29)$$

$$C_{aa}=\frac{(\sqrt{\Gamma_1\Gamma_2}-\Gamma_{12})\sqrt{\Gamma_1\Gamma_2}}{\Gamma_1+\Gamma_2}, \quad (30)$$

$$C_{sa}=C_{as}=\frac{1}{2}\,\frac{(\Gamma_1-\Gamma_2)(\sqrt{\Gamma_1\Gamma_2}-\Gamma_{12})}{\Gamma_1+\Gamma_2}, \quad (31)$$

and

$$u=\frac{\sqrt{\Gamma_1}}{\sqrt{\Gamma_1+\Gamma_2}}, \tag{32}$$

$$v=\frac{\sqrt{\Gamma_2}}{\sqrt{\Gamma_1+\Gamma_2}}. \tag{33}$$

The transformed dissipative part (28) of the master equation has a form similar to Eq. (11). Of course, the real advantage of any unitary transformation of Eq. (11) appears only if the transformed part is less complicated than the initial one. Although in general the two forms (11) and (28) look similar, the advantage of the transformed form (28) over (11) is obtained when $\Gamma_{12}=\sqrt{\Gamma_1\Gamma_2}$ and/or the damping rates of the original systems are equal ($\Gamma_1=\Gamma_2=\Gamma$). According to Eq. (21), $\Gamma_{12}=\sqrt{\Gamma_1\Gamma_2}$ when the two dipole moments are parallel and separated by distances smaller than the optical wavelength. When the damping rates are equal, $C_{sa}=C_{as}=0$, and then the symmetric and antisymmetric superpositions decay independently with the decay rates $\frac{1}{2}(\Gamma+\Gamma_{12})$ and $\frac{1}{2}(\Gamma-\Gamma_{12})$, respectively. In other words, for $\Gamma_1=\Gamma_2$ the transformation (26) diagonalizes the dispersive part of the master equation. Furthermore, if $\Gamma_{12}=\sqrt{\Gamma_1\Gamma_2}$ then $C_{aa}=C_{sa}=C_{as}=0$ regardless of the ratio between Γ_1 and Γ_2. In this case the antisymmetric superposition *decouples* from the dissipative interactions and consequently does not decay. This implies that spontaneous emission can be controlled and even suppressed by appropriately engineering the dissipative interaction Γ_{12} between the systems.

The above discussion shows that the basic feature of the two coupled systems is the existence of an antisymmetric superposition which can be decoupled from the dissipative interactions. The modification of the dissipative interactions is an example of quantum interference between two coupled systems, in that the spontaneous emission from one of them modifies the spontaneous emission from the other. This phenomenon leads to symmetric and antisymmetric superpositions which may decay independently with significantly modified rates. The decay rate of the antisymmetric superposition may be greatly reduced or even completely suppressed. An interesting question arises as to whether the nondecaying antisymmetric superposition can still be coupled to the coherent interactions. These interactions can coherently transfer population between the superpositions. In order to check this, we rewrite the Hamiltonian H' in terms of the S_s^+ and S_a^+ operators as

$$\begin{aligned}H'=&-\hbar\left[\left(\Delta_L+\frac{(\Gamma_1-\Gamma_2)}{\Gamma_1+\Gamma_2}\Delta\right)S_s^+S_s^-+\left(\Delta_L-\frac{(\Gamma_1-\Gamma_2)}{\Gamma_1+\Gamma_2}\Delta\right)S_a^+S_a^-+2\Delta\frac{\sqrt{\Gamma_1\Gamma_2}}{\Gamma_1+\Gamma_2}(S_s^+S_a^-+S_a^+S_s^-)\right]\\&+\hbar\delta_{12}\left[\frac{2\sqrt{\Gamma_1\Gamma_2}}{\Gamma_1+\Gamma_2}(S_s^+S_s^--S_a^+S_a^-)+\frac{(\Gamma_1-\Gamma_2)}{\Gamma_1+\Gamma_2}(S_s^+S_a^-+S_a^+S_s^-)\right]-\frac{\hbar}{2\sqrt{\Gamma_1+\Gamma_2}}[(\sqrt{\Gamma_1}\Omega_1+\sqrt{\Gamma_2}\Omega_2)(S_s^++S_s^-)\\&+(\sqrt{\Gamma_2}\Omega_1-\sqrt{\Gamma_1}\Omega_2)(S_a^++S_a^-)],\end{aligned} \tag{34}$$

where $\Delta=\frac{1}{2}(\tilde{\omega}_2-\tilde{\omega}_1)$ and $\Delta_L=\omega_L-\frac{1}{2}(\tilde{\omega}_1+\tilde{\omega}_2)$ is the detuning of the laser field from the average frequency of the two dipole moments.

The first term in Eq. (34) arises from the Hamiltonian H_s and shows that the energies of the symmetric and antisymmetric superpositions depend on the energy difference Δ between the bare systems and the damping rates Γ_i. Moreover, the energy difference Δ introduces a coherent coupling between the superpositions. If the bare systems are identical ($\Delta=0$ and $\Gamma_1=\Gamma_2$) then the superpositions have the same energies and there is no contribution to the coherent interaction from the Hamiltonian H_s.

In the transformed representation the interaction δ_{12} between the two bare systems, given by the second term in Eq. (34), has two effects on the coherent dynamics of the symmetric and antisymmetric superpositions. The first is a shift of the energies and the second is the coherent interaction between the superpositions. It is seen from Eq. (34) that the coherent interaction between the superpositions vanishes for identical atoms with $\Gamma_1=\Gamma_2$ and then the effect of δ_{12} is only the shift of the energies from their unperturbed values. It is interesting that the interaction δ_{12} shifts the energies in the opposite directions.

The third term in Eq. (34) represents the interaction of the superpositions with the driving laser field. We see that the symmetric superposition strongly couples to the laser field with an effective Rabi frequency proportional to $\Omega_1+\Omega_2$, whereas the Rabi frequency of the antisymmetric superposition is proportional to $\Omega_1-\Omega_2$ and vanishes for $\Omega_1=\Omega_2$. In the latter case, the laser field couples only to the symmetric superposition. According to Eq. (8), this takes place only if the dipole moments experience the same phase of the driving field.

We can rewrite the Hamiltonian (34) in a more compact form,

$$\begin{aligned}H'=&-\hbar[(\Delta_L+\Delta')S_s^+S_s^-+(\Delta_L-\Delta')S_a^+S_a^-\\&+\Delta_c(S_s^+S_a^-+S_a^+S_s^-)]\\&-\frac{\hbar}{2\sqrt{\Gamma_1+\Gamma_2}}[(\sqrt{\Gamma_1}\Omega_1+\sqrt{\Gamma_2}\Omega_2)(S_s^++S_s^-)\\&+(\sqrt{\Gamma_2}\Omega_1-\sqrt{\Gamma_1}\Omega_2)(S_a^++S_a^-)],\end{aligned} \tag{35}$$

where

$$\Delta' = \frac{1}{\Gamma_1+\Gamma_2}[(\Gamma_1-\Gamma_2)\Delta - 2\delta_{12}\sqrt{\Gamma_1\Gamma_2}] \tag{36}$$

and

$$\Delta_c = \frac{1}{\Gamma_1+\Gamma_2}[2\Delta\sqrt{\Gamma_1\Gamma_2} + \delta_{12}(\Gamma_1-\Gamma_2)]. \tag{37}$$

The physical interpretation of Eq. (35) is straightforward: Δ' is a shift of the energies of the superposition systems and Δ_c is the magnitude of the coherent coupling between the superpositions. The parameters depend on the vacuum-induced coherent coupling δ_{12}, which can strongly affect the coherent evolution of the systems. For $\delta_{12} \neq 0$ and identical bare systems the shift $\Delta' \neq 0$, but can vanish for nonidentical bare systems. This occurs for

$$\delta_{12} = \frac{1}{2}\frac{(\Gamma_1-\Gamma_2)\Delta}{\sqrt{\Gamma_1\Gamma_2}}. \tag{38}$$

In contrast to the shift Δ', which is different from zero for identical systems, the coherent coupling Δ_c can be different from zero only for nonidentical bare systems. However, even in this case the coupling can vanish, which happens for

$$\Delta = -\frac{\delta_{12}(\Gamma_1-\Gamma_2)}{2\sqrt{\Gamma_1\Gamma_2}}. \tag{39}$$

Thus, with the condition (39) and $\Gamma_{12} = \sqrt{\Gamma_1\Gamma_2}$ the antisymmetrical superposition of two nonidentical bare systems completely decouples from the interactions.

The master equation with the dissipative part (28) and the Hamiltonian (35) gives an elegant description of the physics involved in the existence of coherent superpositions in the interaction of two dipole systems, their dissipative interactions with the vacuum field (environment), and the coupling to the coherent interactions. An important point is that the master equation is quite general and can be applied to an arbitrary system composed of two dipole moments. The condition $\Gamma_{12} = \sqrt{\Gamma_1\Gamma_2}$ for the decoupling of the antisymmetric superposition from the dissipative interaction is valid for arbitrary dipole systems, whereas the presence of the coherent interaction between the superpositions depends on specific examples of the dipole systems and appears only if the bare systems are nonidentical with different energies and/or spontaneous damping rates. In the next section we will consider specific examples of two systems and discuss the conditions of their couplings to coherent interactions.

IV. EXAMPLES

Let us illustrate our considerations with three examples of a quantum system which is composed of two interacting subsystems. The three particular quantum systems we consider are a single V-type three-level atom, a single Λ-type three-level atom, and two nonidentical two-level atoms. Each of the three systems is represented by two dipole moments $\boldsymbol{\mu}_1$ (system 1) and $\boldsymbol{\mu}_2$ (system 2) coupled to the same vacuum field and driven by a coherent laser field. These systems are known to exhibit the population trapping phenomenon, that is, the system can be driven into a dark state from which the population is unable to leave.

A. Three-level *V* system

We consider a three-level atom in the V configuration composed of two nondegenerate excited levels $|1\rangle$ and $|2\rangle$ and a single ground level $|3\rangle$. The levels $|1\rangle$ and $|2\rangle$ can decay to the ground level by spontaneous emission with decay rates Γ_1 and Γ_2, respectively, whereas transitions between the excited levels are forbidden in the electric dipole approximation. The two interacting systems have dipole moments $\boldsymbol{\mu}_{13}$ and $\boldsymbol{\mu}_{23}$ sharing the same atomic ground level $|3\rangle$ and represented by the operators $S_1^+ = (S_1^-)^\dagger = |1\rangle\langle 3|$ and $S_2^+ = (S_2^-)^\dagger = |2\rangle\langle 3|$. In this three-level atom the superposition systems correspond to the symmetric and antisymmetric superpositions of the atomic excited states

$$|s\rangle = \frac{1}{\sqrt{\Gamma_1+\Gamma_2}}(\sqrt{\Gamma_1}|1\rangle + \sqrt{\Gamma_2}|2\rangle), \tag{40}$$

$$|a\rangle = \frac{1}{\sqrt{\Gamma_1+\Gamma_2}}(\sqrt{\Gamma_2}|1\rangle - \sqrt{\Gamma_1}|2\rangle). \tag{41}$$

The evolution of the system is described by a master equation of the same type as Eq. (9) with the specific form of the Hamiltonian H'. Since we have a single atom, the dipole moments are at the same point $\mathbf{r}_1 = \mathbf{r}_2$, the Rabi frequencies are related by

$$\Omega_2 = \Omega_1\sqrt{\frac{\Gamma_2}{\Gamma_1}}\frac{\cos\theta_1}{\cos\theta_2}, \tag{42}$$

and the cross-damping term is given by [7]

$$\Gamma_{12} = p\sqrt{\Gamma_1\Gamma_2}, \tag{43}$$

where $p = (\hat{\boldsymbol{\mu}}_1 \cdot \hat{\boldsymbol{\mu}}_2)$ determines the mutual polarization of the dipole moments of the two atomic transitions. For parallel dipole moments $p=1$, whereas $p=0$ for perpendicular polarizations. In the former case, the antisymmetric state decouples from the dissipative interaction and consequently does not decay. However, the population of this state can still evolve in time due to the coherent coupling to the symmetric state. In order to show this in more detail, we derive the equation of motion for the population ρ_{aa} of the antisymmetric state, which for $\Gamma_1 = \Gamma_2 = \Gamma$ is given by

$$\dot{\rho}_{aa} = -(1-p)\Gamma\rho_{aa} + i\Delta_c(\rho_{as} - \rho_{sa}). \tag{44}$$

In the derivation of Eq. (44), we have assumed equal Rabi frequencies, $\Omega_1 = \Omega_2$, and hence the antisymmetric state is not driven by the laser field. This will allow us to identify excitation channels different from the laser field. The first term on the right-hand side of Eq. (44) arises from the dissipative interaction of the antisymmetric state with the vacuum, while the second term arises from the coherent in-

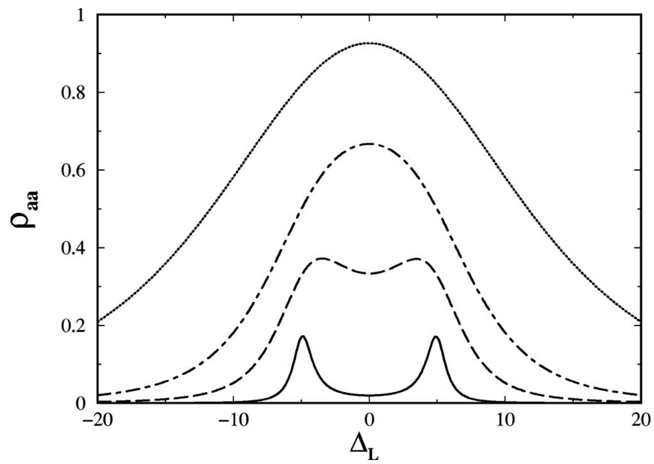

FIG. 1. The stationary population of the antisymmetric state $|a\rangle$ as a function of Δ_L for $\Gamma_2=1$, $\delta_{12}=0.1$, $\Delta=5$, $p=1$, and different Ω:$\Omega=1$ (solid line), $\Omega=5$ (dashed line), $\Omega=10$ (dashed-dotted line), $\Omega=25$ (dotted line). All parameters are scaled to Γ_1 throughout the figures and, for simplicity, we take $\Gamma_1=1$.

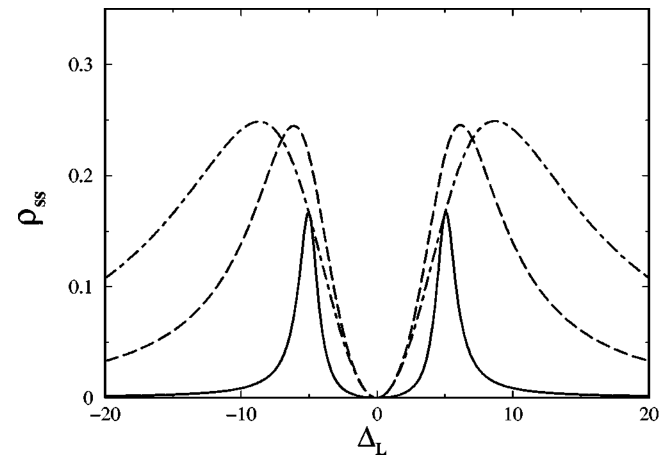

FIG. 2. The stationary population of the symmetric state $|s\rangle$ as a function of Δ_L for $\Gamma_2=1$, $\delta_{12}=0.1$, $\Delta=5$, $p=1$, and different Ω: $\Omega=1$ (solid line), $\Omega=5$ (dashed line), $\Omega=10$ (dashed-dotted line).

teraction with the symmetric state. Note that the interaction between the superpositions does not involve the ground state, and therefore is not accompanied by spontaneous emission. If $\Delta_c=0$ the steady-state population $\rho_{aa}=0$, unless $p=1$ and then ρ_{aa} retains its initial value. This is the population trapping effect, predicted by Agarwal [4], that a degenerate three-level atom excited initially into the antisymmetric superposition of the excited levels will stay in this state for all times. For $\Delta_c\neq 0$ and in the absence of the driving field, the steady-state population $\rho_{aa}=0$ regardless of the initial value. This implies that the coherent interaction destroys the population trapping in the state $|a\rangle$.

The role of the coherent coupling can reverse in the presence of the driving field. In this case the coherent coupling Δ_c can transfer the population from the driven $|s\rangle$ state to the undriven and nondecaying $|a\rangle$ state. This is shown in Fig. 1, where we plot the steady-state population ρ_{aa} as a function of Δ_L for $\Delta=5,\delta_{12}=0.1$ and different Ω. It is seen that the antisymmetric state is populated by the presence of the coherent coupling to the symmetric state. The amount of population in $|a\rangle$ increases with increasing Ω and attains the maximum value $\rho_{aa}\approx 1$ for $\Delta_L=0$ and very strong driving fields.

The coherent transfer of the population between the superpositions can leave $|s\rangle$ unpopulated despite that the state is continuously driven by the laser field. We illustrate this in Fig. 2, where we plot the population ρ_{ss} as a function of Δ_L for $\Delta=5,\delta_{12}=0.1$ and different Ω. For $\Delta_L=0$ the population $\rho_{ss}=0$ regardless of the value of Ω. The coherent interaction between the superpositions transfers the population to the state $|a\rangle$ leaving the state $|s\rangle$ unpopulated.

The appearance of the zero in the population ρ_{ss} results from the presence of the coherent coupling Δ_c, but the quantity which determines the position of the zero is the detuning Δ'. According to Eq. (36), for $\Gamma_1=\Gamma_2$ the detuning Δ' depends solely on the vacuum induced coherent coupling δ_{12}. Therefore, an experimental observation of a shift of the zero from the $\Delta_L=0$ position would provide evidence of the vacuum induced coherent interaction in the V system. In Fig. 2 we have chosen $\delta_{12}=0.1$, and even with such a large value of δ_{12} no shift of the zero is visible. Cardimona and Stroud [21] have shown that the effect of δ_{12} on the dynamics of the V system could be observed as a change in the fluorescence intensity profile. However, the predicted changes are also very small and could be difficult to observe. In Sec. IV B, however, we show that δ_{12} can have an experimentally significant effect on the dynamics of a Λ system.

The lack of population in the state $|s\rangle$, see Fig. 2, can be interpreted as a population trapping induced by the laser field and the coherent interaction. However, the induced trapping state is not entirely the $|a\rangle$ state but rather a linear superposition of the ground $|0\rangle$ and $|a\rangle$ states. Only in the limit of a strong driving field does the induced trapping state reduces to $|a\rangle$. An alternative way of viewing the process of transferring population from the state $|s\rangle$ to $|a\rangle$ is to employ the dressed-atom model of the system [28]. The dressed atom approach provides a transparent picture of the physical processes responsible for population transfer and trapping phenomena. In this model we use a fully quantum-mechanical description of the Hamiltonian H', which for the three-level system discussed here takes the form

$$H'=H_0+V_L, \tag{45}$$

where

$$H_0=-\hbar\Delta_L(S_s^+S_s^-+S_a^+S_a^-)+\hbar\omega_L a_L^\dagger a_L \tag{46}$$

is the Hamiltonian of the uncoupled system and the laser field, and

$$V_L=-\hbar\Delta(S_s^+S_a^-+S_a^+S_s^-)-\frac{\hbar}{\sqrt{2}}g(a_L^\dagger S_s^-+S_s^+a_L) \tag{47}$$

is the interaction Hamiltonian which includes the coherent coupling between the superposition states and the coupling of the symmetric state to the laser field. In Eq. (47), g is the system-field coupling constant, and a_L ($a_L^\dagger$) is the annihilation (creation) operator for the driving field mode.

For $\Delta_L=0$ the Hamiltonian H_0 has three degenerate eigenstates $|3,N\rangle$, $|a,N-1\rangle$, and $|s,N-1\rangle$, where $|i,N\rangle$ is the state with the atom in state $|i\rangle$ and N photons present in the driving laser mode. When we include the interaction V_L the degeneracy is lifted, resulting in triplets (dressed states)

$$|+,N\rangle=\frac{1}{\sqrt{2}}[-\alpha|a,N-1\rangle+|s,N-1\rangle-\sqrt{2}\beta|3,N\rangle],$$
$$|0,N\rangle=-\sqrt{2}\beta|a,N-1\rangle+\alpha|3,N\rangle,$$
$$|-,N\rangle=\frac{1}{\sqrt{2}}[-\alpha|a,N-1\rangle-|s,N-1\rangle-\sqrt{2}\beta|3,N\rangle], \tag{48}$$

with energies

$$E_{N,+}=N\omega_L+\Omega',$$
$$E_{N,0}=N\omega_L, \tag{49}$$
$$E_{N,-}=N\omega_L-\Omega',$$

where $\Omega'=\sqrt{\Delta^2+\frac{1}{2}\Omega^2}$, $\alpha=\Delta/\Omega'$, and $\beta=\Omega/2\Omega'$.

The dressed states (48) group into manifolds, each containing three states. Neighboring manifolds are separated by ω_L, while the states inside each manifold are separated by $\Omega'/2$. The dressed states are connected by transition dipole moments. It is easily verified that nonzero dipole moments occur only between states within neighboring manifolds. Using Eq. (48) and assuming that $\boldsymbol{\mu}_{13}=\boldsymbol{\mu}_{23}=\boldsymbol{\mu}$, we find that the transition dipole moments between $|i,N+1\rangle$ ($i=0,-,+$) and $|0,N\rangle$ are

$$\langle N+1,+|\boldsymbol{\mu}|0,N\rangle=\alpha\boldsymbol{\mu},$$
$$\langle N+1,0|\boldsymbol{\mu}|0,N\rangle=0, \tag{50}$$
$$\langle N+1,-|\boldsymbol{\mu}|0,N\rangle=-\alpha\boldsymbol{\mu},$$

whereas the transition dipole moments $\langle N,0|\boldsymbol{\mu}|i,N-1\rangle$ between $|0,N\rangle$ and the dressed states $|i,N-1\rangle$ of the manifold below are equal to zero. It is apparent from Eq. (50) that transitions to the state $|0,N\rangle$ are allowed from the states of the manifold above, but are forbidden to the states of the manifold below. Therefore, the state $|0,N\rangle$ is a trapping state such that the population can be transferred into this state, but cannot leave it. The transfers are allowed only when $\Delta\neq 0$, i.e., in the presence of the coherent coupling between the symmetric and antisymmetric superpositions. Otherwise, for $\Delta=0$, the state $|0,N\rangle$ is completely decoupled from the remaining dressed states. In this case the three-level system reduces to that equivalent to a two-level atom. We see from Eq. (48) that the dressed state $|0,N\rangle$ is a linear superposition of the $|a\rangle$ and $|3\rangle$ states, and reduces to the state $|a\rangle$ for a very strong driving field ($\Omega\gg\Delta$).

Thus, the coherent interaction between the superpositions can have a constructive or destructive effect on the population trapping in a V-type three-level atom. In the absence of the driving field the coupling has a destructive effect on the population trapping in that it depopulates the state $|a\rangle$. On the other hand, in the presence of the driving field the coupling has a constructive effect on the population trapping since it creates a trapping superposition state of the ground and the nondecaying antisymmetric states.

B. Three-level Λ system

Here, we consider a three-level Λ-type atom composed of a single upper level $|3\rangle$ and two ground levels $|1\rangle$ and $|2\rangle$. The two interacting systems have dipole moments $\boldsymbol{\mu}_{31}$ and $\boldsymbol{\mu}_{32}$ sharing the same atomic upper level $|3\rangle$. After introducing superposition operators $S_s^+=(S_s^-)^\dagger=|3\rangle\langle s|$ and $S_a^+=(S_a^-)^\dagger=|3\rangle\langle a|$, where $|s\rangle$ and $|a\rangle$ are the superposition states of the same form as Eqs. (40) and (41), we obtain the master equation of the same type as Eq. (9) with the dissipative part (28) and the Hamiltonian (35) given by

$$H'=-\hbar\Bigg\{(\Delta_L+\Delta')S_s^-S_s^++(\Delta_L-\Delta')S_a^-S_a^+ +\Delta_c(S_s^-S_a^++S_a^-S_s^+)+\frac{1}{2}\frac{\sqrt{\Gamma_1}\Omega}{\sqrt{\Gamma_1+\Gamma_2}}(S_s^++S_s^-)\Bigg\}, \tag{51}$$

with $\Gamma_{12}=p\sqrt{\Gamma_1\Gamma_2}$, and we have assumed that $\Omega_1=\Omega_2=\Omega$. Note that the ordering of the superposition operators in Eq. (51) is the reverse of that for the V system.

Following our procedure, we analyze conditions for population trapping using the equation of motion for the population ρ_{aa} of the antisymmetric state. For the Λ system, the equation of motion is of the following form:

$$\dot{\rho}_{aa}=\frac{2\Gamma_1\Gamma_2}{\Gamma_1+\Gamma_2}(1-p)\rho_{33}-i\Delta_c(\rho_{as}-\rho_{sa}). \tag{52}$$

In the steady state ($\dot{\rho}_{aa}=0$) with $p\neq 1$ and $\Delta_c=0$ the population in the upper state $\rho_{33}=0$. Thus the state $|3\rangle$ is not populated despite that is continuously driven by the laser. In this case the population is entirely trapped in the antisymmetric state [29]. This is the well-known coherent population trapping effect predicted by Alzetta *et al.* [30], and experimentally observed by Orriols [31] (see also [32]). However, for $p=1$ and $\Delta_c=0$ the antisymmetrical state decouples from the interactions, and then the steady-state population ρ_{33} is different from zero [33]. This shows that coherent population trapping is possible only in the presence of dissipative spontaneous emission from the upper level to the antisymmetric superposition state. Moreover, coherent population trapping does not appear even if $p\neq 1$. According to Eq. (52) this happens when $\Delta_c\neq 0$. We see that, similar to the V system, the coherent coupling destroys population trapping. This is shown in Fig. 3, where we plot the steady-state population ρ_{33} as a function of Δ for $\Delta_L=0$, $\Omega=5$, $\delta_{12}=0.1$,

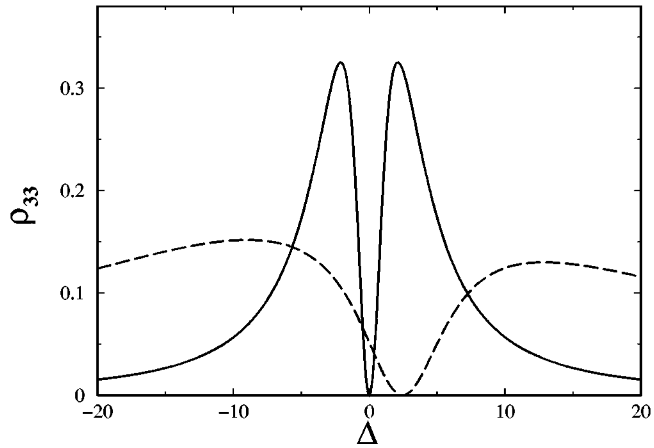

FIG. 3. The stationary population of the upper state $|3\rangle$ of a Λ-type atom as a function of the splitting Δ for $\Delta_L=0$, $\Omega=5$, $\delta_{12}=0.1$, $p=0.5$, and different Γ_2: $\Gamma_2=1$ (solid line), $\Gamma_2=50$ (dashed line).

$p=0.5$, and two different values of Γ_2. It is evident that the cancellation of the population ρ_{33} appears only at $\Delta_c=0$, i.e., in the absence of the coherent coupling between the antisymmetric and symmetric states. For $\Gamma_1=\Gamma_2$ the cancellation appears at $\Delta=0$, while for $\Gamma_1\neq\Gamma_2$ the effect appears at

$$\Delta=-\frac{1}{2}\frac{\Gamma_1-\Gamma_2}{\sqrt{\Gamma_1\Gamma_2}}\delta_{12}. \tag{53}$$

Thus, for Γ_1 significantly different from Γ_2, the shift can be large despite that δ_{12} is very small. Therefore, the vacuum-induced coherent coupling can be experimentally observed in the Λ system as a shift of the zero of the population ρ_{33}. Note that in contrast to the V system, where the effect of δ_{12} could be important for nearly degenerate transitions [21], in the Λ system the effect could be observed with nondegenerate transitions.

It is important to note that, in contrast to the V system, there is no laser-induced population trapping in the Λ system. We can show this by calculating the transition dipole moments between the dressed states of the system. The procedure of calculating the dressed states of the Λ system is the same as for the V system. The only difference is that now the eigenvalues of the unperturbed Hamiltonian H_0 are $|3,N-1\rangle$, $|a,N\rangle$, $|s,N\rangle$, and the dressed states, with $p=1$ and $\Gamma_1=\Gamma_2$, are given by

$$|+,N\rangle=\frac{1}{\sqrt{2}}[-\alpha|a,N\rangle+|s,N\rangle-\sqrt{2}\beta|3,N-1\rangle],$$

$$|0,N\rangle=-\sqrt{2}\beta|a,N\rangle+\alpha|3,N-1\rangle, \tag{54}$$

$$|-,N\rangle=\frac{1}{\sqrt{2}}[-\alpha|a,N\rangle-|s,N\rangle-\sqrt{2}\beta|3,N-1\rangle].$$

Although the dressed states (54) are similar to that of the V system [Eq. (48)], there is a crucial difference in that the transition dipole moments between $|i,N+1\rangle$ and $|0,N\rangle$ are all zero, but there are nonzero transition dipole moments between $|0,N\rangle$ and the dressed states $|i,N-1\rangle$ of the manifold below, since

$$\langle N,0|\boldsymbol{\mu}|\pm,N-1\rangle=\pm\alpha\boldsymbol{\mu},$$
$$\langle N,0|\boldsymbol{\mu}|0,N-1\rangle=0. \tag{55}$$

Therefore, population is unable to flow into the state $|0,N\rangle$, but can flow away from it. If $\Delta=0$ then $\alpha=0$, and the state $|0,N\rangle$ completely decouples from the remaining states. For $\Delta\neq0$ the state $|0,N\rangle$ is coupled to the remaining states, but does not participate in the dynamics of the system because it cannot be populated by transitions from the other states. There is no trapping state among the dressed states (54) as each state of a given manifold has nonzero transition dipole moments to the dressed states of the manifold below.

We conclude that the process responsible for the population trapping in the Λ system is different from that in the V system. In the former the trapping results from the dissipative decay of the population into the antisymmetric state, whereas in the latter the trapping appears only if the antisymmetric state is completely decoupled from the dissipative interaction. Moreover, in the presence of the coherent coupling between the superposition states no population is trapped in a specific state of the Λ system.

C. Two nonidentical two-level atoms

In this section we consider two nonidentical atoms separated by $\mathbf{r}_{12}$, coupled to each other via a retarded dipole-dipole interaction and to the three-dimensional electromagnetic vacuum field, leading to dissipative spontaneous decay. Each atom is modeled as a two-level system with ground state $|g_i\rangle$ ($i=1,2$) and excited state $|e_i\rangle$, connected by a transition dipole moment $\boldsymbol{\mu}_i$. The atoms are assumed to have the transition frequencies ω_1 and ω_2 respectively, and the corresponding decay rates Γ_1 and Γ_2. The master equation for this system involves all parameters appearing in Eqs. (28) and (35) with δ_{12} being the retarded dipole-dipole interaction (23). As we have mentioned in Sec. III, the dipole-dipole interaction has two effects on the dynamics of the system. The interaction shifts the energies of the superposition systems in opposite directions, and contributes to the coherent coupling between them. The latter happens only if $\Gamma_1\neq\Gamma_2$.

It is convenient to represent the superposition systems in terms of the so-called collective states of the two-atom system, which correspond to the symmetric and antisymmetric superpositions of the atoms [23,24]. In this representation, the two-atom system is equivalent to a single four-level system with a single ground state $|0\rangle=|g_1\rangle|g_2\rangle$, two intermediate (entangled) states

$$|+\rangle=\frac{1}{\sqrt{\Gamma_1+\Gamma_2}}(\sqrt{\Gamma_1}|e_1\rangle|g_2\rangle+\sqrt{\Gamma_2}|e_2\rangle|g_1\rangle), \tag{56}$$

$$|-\rangle=\frac{1}{\sqrt{\Gamma_1+\Gamma_2}}(\sqrt{\Gamma_2}|e_1\rangle|g_2\rangle-\sqrt{\Gamma_1}|e_2\rangle|g_1\rangle), \tag{57}$$

and a single upper state $|1\rangle = |e_1\rangle|e_2\rangle$. The entangled states (56) and (57) are independent of Δ, but depend on the damping rates Γ_1 and Γ_2. For $\Gamma_1 = \Gamma_2$ the states are maximally entangled, whereas for either $\Gamma_1 \ll \Gamma_2$ or $\Gamma_1 \gg \Gamma_2$, the entangled states reduce to the product states $|e_i\rangle|g_j\rangle$ $(i \neq j)$. In the basis of the collective states the superposition operators S_s^+ and S_a^+ are of the following form:

$$S_s^+ = \frac{1}{\Gamma_1+\Gamma_2}[2\sqrt{\Gamma_1\Gamma_2}|1\rangle\langle +| - (\Gamma_1-\Gamma_2)|1\rangle\langle -|] + |+\rangle\langle 0|, \tag{58}$$

$$S_a^+ = \frac{-1}{\Gamma_1+\Gamma_2}[2\sqrt{\Gamma_1\Gamma_2}|1\rangle\langle -| + (\Gamma_1-\Gamma_2)|1\rangle\langle +|] + |-\rangle\langle 0|. \tag{59}$$

Before proceeding further, it is worth pointing out the physical significance of various terms in Eqs. (58) and (59) to gain insight into the underlying dynamics of the system. We see that there are two channels of excitation in the two-atom system: The symmetrical channel $|0\rangle \to |+\rangle \to |1\rangle$, and the antisymmetrical channel $|0\rangle \to |-\rangle \to |1\rangle$. The channels are independent for identical atoms, but become correlated when $\Gamma_1 \neq \Gamma_2$. It is interesting to note that unequal damping rates correlate transitions only from the upper to the intermediate states, while the transitions from the intermediate states to the ground state remain independent.

Now, let us consider population trapping conditions in the two-atom system and the mechanism of population transfer between the superpositions, especially between the entangled states (56) and (57). As before, for the V and Λ systems, we assume that $\Gamma_{12} = \sqrt{\Gamma_1\Gamma_2}$ and derive the equation of motion for the population of the antisymmetric state $|-\rangle$, which is of the following form:

$$\dot{\rho}_{--} = \frac{(\Gamma_1-\Gamma_2)^2}{\Gamma_1+\Gamma_2}\rho_{11} + i\Delta_c(\rho_{+-}-\rho_{-+}) - \frac{1}{2}i\Omega\frac{(\Gamma_1-\Gamma_2)}{\sqrt{\Gamma_1(\Gamma_1+\Gamma_2)}}(\rho_{1-}-\rho_{-1}). \tag{60}$$

We see immediately that the antisymmetric state $|-\rangle$ does not decay, but can be populated by spontaneous emission from the upper state $|1\rangle$ and also by the nondissipative interaction with the state $|+\rangle$. The first condition is satisfied only when $\Gamma_1 \neq \Gamma_2$. The last condition is satisfied only when $\Delta_c = 0$. Thus, the transfer of population to the state $|-\rangle$ does not appear when the atoms are identical, but is possible for nonidentical atoms. In this case the upper state decays to a superposition of the intermediate states, but then only a part of the population, that part in the symmetric state $|+\rangle$, can decay to the ground state $|0\rangle$.

In Fig. 4 we plot the steady-state population of the state $|-\rangle$ as a function of Δ_L for two different types of nonidentical atoms. In the first case the atoms have the same damping rates $(\Gamma_1 = \Gamma_2)$ but different transition frequencies $(\Delta \neq 0)$, while in the second case the atoms have the same frequencies $(\Delta = 0)$ but different damping rates $(\Gamma_1 \neq \Gamma_2)$. It is seen from Fig. 4 that in both cases the antisymmetric state can be populated even if it is decoupled from the dissipative interaction and the driving field. The population is transferred to $|-\rangle$ through the coherent coupling Δ_c which, similar to the V-type atom, leaves the other excited states completely unpopulated. This is shown in Fig. 5, where we plot the steady-state populations ρ_{++} and ρ_{11} for $\Gamma_1 = \Gamma_2$, $\Delta = 1$, $\delta_{12} = 10$, and $\Omega = 5$. It is evident from Fig. 5 that for $\Delta_L = -\delta_{12}$ the states are not populated. In a similar way to the V system, the population is trapped in a linear superposition of the $|0\rangle$ and $|-\rangle$ states, and for a very strong field can be completely transferred to the state $|-\rangle$. This is shown in Fig. 6, where we plot the steady-state population ρ_{--} for the same parameters as in Fig. 5, but different Ω. Clearly, for a strong driving field the population is completely transferred to the state $|-\rangle$.

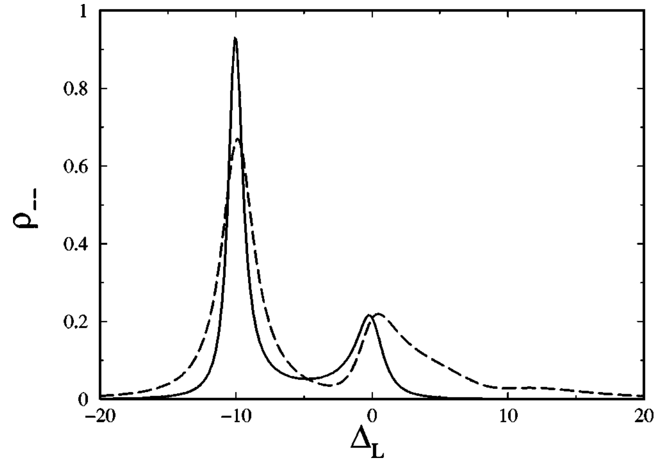

FIG. 4. The stationary population of the entangled state $|-\rangle$ of two nonidentical atoms for $\Omega = 5$, $\delta_{12} = 10$, $p = 1$, and $\Gamma_2 = 1$, $\Delta = 1$ (solid line), $\Gamma_2 = 2$, $\Delta = 0$ (dashed line).

This result shows that we can relatively easily prepare two atoms, with different transition frequencies, in a maximally entangled state. The closeness of the prepared state to the ideal one is measured by the fidelity F. Here F is equal to the obtained maximum population in the state $|-\rangle$. For Ω

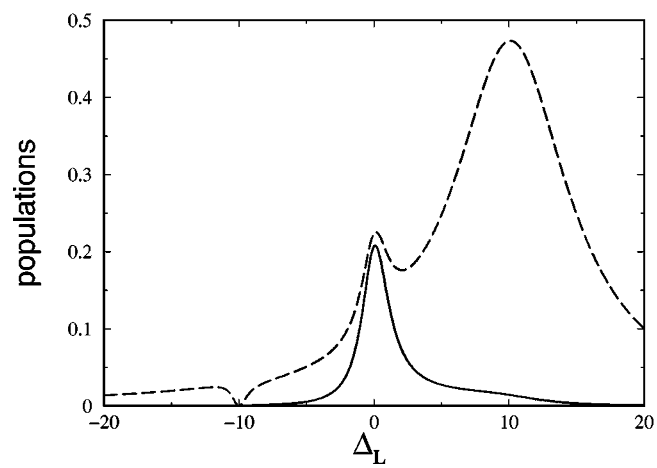

FIG. 5. The stationary population of the entangled state $|+\rangle$ and the upper state $|1\rangle$ of two nonidentical atoms for $\Gamma_2 = 1$, $\Omega = 5$, $\Delta = 1$, $\delta_{12} = 10$, and $p = 1$: ρ_{11} (solid line), ρ_{++} (dashed line).

$\gg\Gamma_1$ the fidelity of the prepared state is maximal, equal to 1. The system has the advantage that the maximally entangled state $|-\rangle$ is completely decoupled from the dissipative interaction, i.e., is a decoherence-free state.

V. SUMMARY

In this paper we have examined the dynamics of two systems, coupled through the three-dimensional vacuum field and driven by a single-mode laser field. The systems have been described in terms of the transition dipole moments, which refer either to two transitions in a single multilevel atom, or to the two transitions in two separate two-level atoms. We have shown that in each case the systems can be represented by coherent symmetric and antisymmetric superpositions whose dynamics depend solely on the frequencies of the dipole moments, their mutual polarizations, and the phase difference arising from possible different positions of the dipoles. For identical systems confined in a region much smaller than the resonant wavelength, so that the dipole moments experience the same phase, the antisymmetric superposition totally decouples from the dynamics and remains unaccessible by any interactions. A small frequency difference between the dipole moments introduces a coherent coupling between the superpositions, which can have a constructive or destructive effect on the population trapping. In the absence of the driving field the coupling destroys the population trapping that depopulates the dark superposition, while one can drive the population into the dark superposition in the presence of driving. We have also shown that the effect of population trapping does not necessarily require decoupling of the antisymmetric superposition from the dissipative interactions. For example, coherent population trapping, predicted in a Λ-type three-level atom, appears only in the presence of the dissipative coupling of the antisymmetric state to the atomic upper state. A similar feature occurs in the system of two nonidentical atoms. However, in this system, we also show that the atoms can be driven into a maximally entangled state which exhibits zero decoherence.

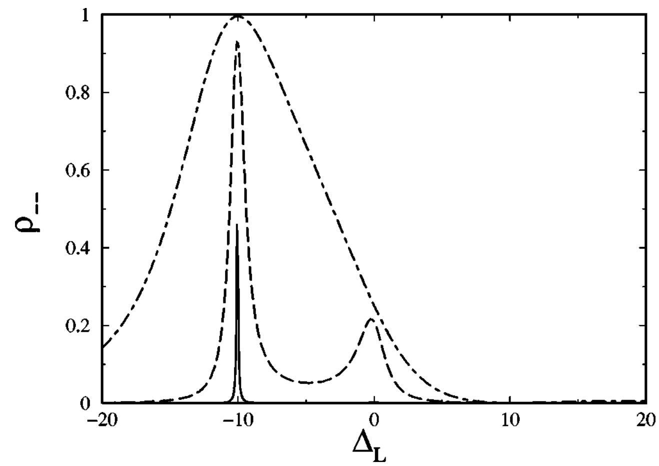

FIG. 6. The stationary population of the entangled state $|a\rangle$ as a function of Δ_L for $\Gamma_2=1$, $\delta_{12}=10$, $\Delta=1$, $p=1$, and different Ω: $\Omega=1$ (solid line), $\Omega=5$ (dashed line), $\Omega=20$ (dashed-dotted line).

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