Disappearance of the dressed bound states in photodetachment from a short-range potential by an intense high-frequency laser field

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It is proved that in three dimensions, and contrary to what is usually found in one dimension, the number of bound Kramers-Hennebrger states always reduces to zero in strong fields if the range of the potential is short and its depth finite. Numerical results showing the disappearance of the dressed ground state of an exponential potential in an intense high-frequency field are also presented.

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I. INTRODUCTION

Much progress has been made in the theory of photodetachment in very strong laser fields through the study of simple one-dimensional (1D) models in which a single active electron is initially bound by a short-range potential. As first noted by Bhatt, Piraux, and Burnett more than twelve years ago [1], the number of dressed bound states supported by such one-dimensional systems tends to increase with the intensity if the frequency is sufficiently high. The appearance in strong fields of additional, ''light-induced'' dressed states has been described for a variety of potentials, e.g., for the $1D$ polarization potential [1,2], the $1D$ zero-range potential [3], the $1D$ Gaussian potential [4,5], and the $1D$ square well [5]. Their counterparts in real systems have recently been found in three-dimensional, correlated multielectron calculations taking the long-range Coulomb interaction into account, in the form of new nonautoionizing bound states of H^- and of multiply charged negative hydrogen ions appearing in ultraintense laser fields [6–8].

The present article is concerned with photodetachment from a short-range *three*-dimensional potential by a high-frequency field. This case has received less attention than those mentioned above. It is generally assumed that it is broadly similar with regard to the appearance of light-induced states. However, it is shown below that there is no proliferation of such states in three dimensions if the potential is of finite depth and has no Coulomb tail. To the contrary, the number of dressed bound states tends to *decrease* when the intensity increases, and always reduces to zero in the strong field limit: increasing the intensity leads to the disappearance of the dressed bound states that the potential may support in weak field, rather than to the appearance of new ones.

Under the effect of an incident field described in dipole approximation by the vector potential $\boldsymbol{\mathcal{A}}(t)$, the wave function of an electron initially bound by a potential $V_{\rm at}(\boldsymbol{r})$ varies in time according to the equation

$$i\hbar\frac{\partial}{\partial t}\Phi(\boldsymbol{r},t)=\left(-\frac{\hbar^2}{2m}\nabla^2-\frac{ie\hbar}{m}\boldsymbol{\mathcal{A}}(t)\cdot\boldsymbol{\nabla}+V_{\rm at}(\boldsymbol{r})\right)\Phi(\boldsymbol{r},t). \tag{1}$$

Some insight into the strong field dynamics of the system can be gained by passing to the Kramers-Henneberger (KH) frame, whose origin is located at the position

$$\boldsymbol{\alpha}(t)=\frac{e}{m}\int_{-\infty}^{t}\boldsymbol{\mathcal{A}}(t')dt' \tag{2}$$

in the laboratory frame. Changing $\boldsymbol{r}$ into $\boldsymbol{r}+\boldsymbol{\alpha}(t)$ in Eq. (1) yields the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Phi_{\rm KH}(\boldsymbol{r},t)=\left(-\frac{\hbar^2}{2m}\nabla^2+V_{\rm at}[\boldsymbol{r}+\boldsymbol{\alpha}(t)]\right)\Phi_{\rm KH}(\boldsymbol{r},t), \tag{3}$$

where $\Phi_{\rm KH}(\boldsymbol{r},t)\equiv\Phi[\boldsymbol{r}+\boldsymbol{\alpha}(t),t]$. In the laboratory frame, the origin of the Kramers-Henneberger frame follows the classical trajectory of an electron freely quivering in the field. If the field is stationary and the quiver motion is too fast for the wave function to respond immediately to the variation of $V_{\rm at}[\boldsymbol{r}+\boldsymbol{\alpha}(t)]$, which happens if the frequency and the intensity are sufficiently high, the electron behaves, in first approximation, as if it were submitted to the effective potential [9]

$$V_{\rm dr}(\alpha_0,\boldsymbol{r})=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}V_{\rm at}[\boldsymbol{r}+\boldsymbol{\alpha}(t)]dt. \tag{4}$$

The average potential $V_{\rm dr}(\alpha_0,\boldsymbol{r})$ is often called the ''dressed potential.'' It depends on ω and on the electric field amplitude only through the excursion amplitude $\alpha_0=\max|\boldsymbol{\alpha}(t)|$. This parameter increases with the intensity, and, while $\alpha_0=0$ and $V_{\rm dr}(\alpha_0,\boldsymbol{r})\equiv V_{\rm at}(\boldsymbol{r})$ in zero field, $V_{\rm dr}(\alpha_0,\boldsymbol{r})$ greatly differs from $V_{\rm at}(\boldsymbol{r})$ at high intensity. Typically, $V_{\rm dr}(\alpha_0,\boldsymbol{r})$ has a larger range and a smaller depth than $V_{\rm at}(\boldsymbol{r})$. The bound states supported by the former, the so-called KH states, are found by solving the equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2+V_{\rm dr}(\alpha_0,\boldsymbol{r})\right)u_n(\alpha_0,\boldsymbol{r})=w_n(\alpha_0)u_n(\alpha_0,\boldsymbol{r}). \tag{5}$$

If the intensity is high enough, the KH states whose binding energy is somewhat smaller than the photon energy are good approximations of exact dressed bound states supported by $V_{\rm at}(\boldsymbol{r})$ in a stationary field [4,5]. The latter correspond to quasistationary solutions of Eq. (1) or (3) satisfying bound-

ary conditions appropriate to bound states decaying by multiphoton ionization [9,10]. Conversely, one can expect that any dressed bound state reduces to a KH state in the high-frequency limit.

The one-dimensional case is examined in Sec. II, for the sake of completeness. It is proved that the number of KH states always increases without bound when α_0 increases, if the field-free potential is both short range and nonpositive everywhere. Additional states appear in this case because the dressed potential becomes more and more elongated as α_0 increases, and the increase of its range is sufficiently fast to compensate the concomitant decrease of its depth.

The range of any non-Coulombic, nonpositive three-dimensional dressed potential increases with α_0, too, *but not in every direction*. Hence, in three dimensions the decrease in the depth of $V_{\rm dr}(\alpha_0,\boldsymbol{r})$ may be accompanied by a reduction in the number of KH states that it supports. That is, some of the eigenenergies $w_n(\alpha_0)$ may become zero, at which point the corresponding states cease to be bound and become resonances. In fact, and this is proved in Sec. III A, all KH states disappear as bound states in the high-intensity limit when $V_{\rm at}(\boldsymbol{r})$ has a finite depth and goes to zero faster than $1/r^3$ for $|\boldsymbol{r}|\to\infty$. The proof does not apply to the 3D δ potential. It does not apply either to potentials decreasing as slowly as a Coulomb potential at large distances, and therefore there is no contradiction with the persistence of the KH states in atomic hydrogen [9] and with the appearance of new ones in negative ions [6–8].

The disappearance of the KH states for short-range potentials implies that of the corresponding exact dressed states in intense high-frequency fields. Numerical results showing the disappearance of the dressed ground state of an exponential potential are given in Sec. III B in illustration of this point. Not all discrete, physical Floquet states vanish, however, since those emanating from field-free resonances remain.

II. THE 1D CASE

We consider only potentials that are nonpositive everywhere, i.e., such that $V_{\rm at}(x)\leqslant 0$ for all x. We also assume that there exist two positive constants V_0 and a such that

$$V_{\rm at}(x)\leqslant -V_0<0 \tag{6}$$

for $|x|\leqslant a$. Let $W(x)$ be the square well potential

$$W(x)=\begin{cases} -V_0, & |x|\leqslant a,\\ 0, & |x|>a. \end{cases} \tag{7}$$

The corresponding dressed potential is

$$W_{\rm dr}(\alpha_0,x)=\frac{\omega}{2\pi}\int_0^{2\pi/\omega} W(x+\alpha_0\sin\omega t)\,dt. \tag{8}$$

Clearly, we have

$$V_{\rm dr}(\alpha_0,x)\leqslant W_{\rm dr}(\alpha_0,x)\leqslant 0 \tag{9}$$

for all x. A straightforward calculation shows that the minimum of $|W_{\rm dr}(\alpha_0,x)|$ on the interval $|x|\leqslant\alpha_0-a$, for $\alpha_0>a$, occurs at $x=0$. At this point,

$$W_{\rm dr}(\alpha_0,x=0)=-\frac{2}{\pi}V_0\arcsin\frac{a}{\alpha_0}. \tag{10}$$

Therefore, we also have, for all values of x,

$$W_{\rm dr}(\alpha_0,x)\leqslant U(\alpha_0,x)\leqslant 0, \tag{11}$$

where $U(\alpha_0,x)$ is the square well potential

$$U(\alpha_0,x)=\begin{cases} -U_0, & |x|\leqslant R,\\ 0, & |x|>R, \end{cases} \tag{12}$$

with $U_0=(2aV_0)/(\pi\alpha_0)$ and $R=\alpha_0-a$. The number of bound states supported by $U(\alpha_0,x)$ is roughly proportional to $(U_0R^2)^{1/2}$ [11], and thus increases without limit when α_0 increases. Because of the relations (9) and (11), the same is true for the dressed potential $V_{\rm dr}(\alpha_0,x)$.

There is also proliferation of KH states for the one-dimensional zero-range potential [3]

$$V_{\rm at}(x)=-B\delta(x),\quad B>0, \tag{13}$$

although it does not satisfy the condition (6), because the corresponding dressed potential never exceeds $U(\alpha_0,x)$ for $U_0=B/(\pi\alpha_0)$ and $R=\alpha_0$. On the other hand, there is no proliferation for potentials with an attractive Coulomb tail; the infinite accumulation of bound states at the continuum threshold indeed prevents additional ones from appearing at high intensity. There may not be any additional KH states either if the potential is not attractive everywhere. An example is the Morse potential

$$V_{\rm at}(x)=D(e^{-2bx}-2e^{-bx}),\quad D,b>0, \tag{14}$$

which is repulsive for $x\ll 0$: it can be shown analytically that the dressed Morse potential supports fewer bound states at large α_0 than at $\alpha_0=0$ [12].

Incidentally, a similar reasoning can be used, with the help of Ref. [13], for showing that in two dimensions $V_{\rm dr}(\alpha_0,\boldsymbol{r})$ always supports at least one bound state if $V_{\rm at}(\boldsymbol{r})$ is attractive everywhere.

III. THE 3D CASE

A. Disappearance of the KH states

For simplicity, we assume that $V_{\rm at}(\boldsymbol{r})\leqslant 0$ everywhere. This condition is not essential here, as the proof developed below can be immediately extended to the case where $V_{\rm at}(\boldsymbol{r})>0$ in some region of space by considering the potential

$$V_{\rm at}^{(-)}(\boldsymbol{r})=\begin{cases} V_{\rm at}(\boldsymbol{r}), & V_{\rm at}(\boldsymbol{r})\leqslant 0,\\ 0, & V_{\rm at}(\boldsymbol{r})>0, \end{cases} \tag{15}$$

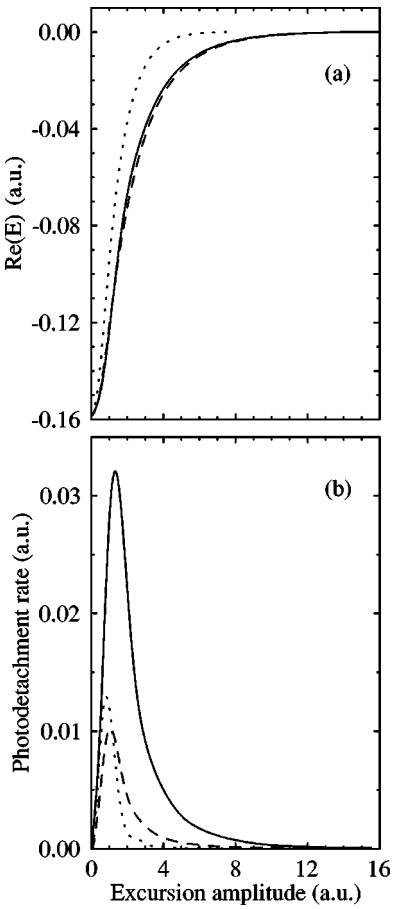

FIG. 1. Variation with the excursion amplitude α_0 of the ground state quasienergy for photodetachment from the potential (30). (a) the real part of the quasienergy; (b) the photodetachment rate. All quantities are expressed in atomic units. Solid curve: $\omega=1$ a.u., linear polarization; dashed curve: $\omega=2$ a.u., linear polarization; dotted curve: $\omega=2$ a.u., circular polarization.

instead of $V_{\rm at}(\boldsymbol{r})$ itself. We also assume (1) that $V_{\rm at}(\boldsymbol{r})$ is of finite depth, i.e., that $\max|V_{\rm at}(\boldsymbol{r})|$ is finite, and (2), that $|V_{\rm at}(\boldsymbol{r})|$ decreases sufficiently fast at large distances for the integral

$$I[V_{\rm at}]=\int |V_{\rm at}(\boldsymbol{r})|d\boldsymbol{r} \tag{16}$$

to exist. It follows from the definition of the dressed potential that $V_{\rm dr}(\alpha_0,\boldsymbol{r})\leqslant 0$ everywhere and that $|V_{\rm dr}(\alpha_0,\boldsymbol{r})|$ is bounded:

$$|V_{\rm dr}(\alpha_0,\boldsymbol{r})|\leqslant V_0(\alpha_0), \tag{17}$$

and $V_0(\alpha_0)=\max|V_{\rm dr}(\alpha_0,\boldsymbol{r})|$ is finite. Moreover, the integral

$$I[V_{\rm dr}]=\int |V_{\rm dr}(\alpha_0,\boldsymbol{r})|d\boldsymbol{r} \tag{18}$$

does not depend on α_0:

$$I[V_{\rm dr}]=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left(\int |V_{\rm at}[\boldsymbol{r}+\boldsymbol{\alpha}(t)]|d\boldsymbol{r}\right)dt$$

$$=\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left(\int |V_{\rm at}(\boldsymbol{r}')|d\boldsymbol{r}'\right)dt$$

$$=I[V_{\rm at}]. \tag{19}$$

The depth of the dressed potential $V_0(\alpha_0)$ is a decreasing function of α_0 (at least for sufficiently large values of this variable). This can be seen from the following argument. In view of the finiteness of $I[V_{\rm at}]$ and of $V_{\rm at}(\boldsymbol{r})$, one can always find two strictly positive constants U_0 and a such that

$$|V_{\rm at}(\boldsymbol{r})|\leqslant U(\boldsymbol{r})\equiv\frac{U_0}{a^2+|\boldsymbol{r}|^2} \tag{20}$$

for any $\boldsymbol{r}$. Hence, $\max|V_{\rm dr}(\alpha_0,\boldsymbol{r})|\leqslant\max U_{\rm dr}(\alpha_0,\boldsymbol{r})$, where

$$U_{\rm dr}(\alpha_0,\boldsymbol{r})=U_0\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{dt}{a^2+|\boldsymbol{r}+\boldsymbol{\alpha}(t)|^2}. \tag{21}$$

Assuming that the incident field is monochromatic and propagates in the z direction, the vector $\boldsymbol{\alpha}(t)$ has the general form

$$\boldsymbol{\alpha}(t)=\alpha_0\sin(\omega t+\phi)\hat{\boldsymbol{x}}+\alpha_0\tan\chi\cos(\omega t+\phi)\hat{\boldsymbol{y}}, \tag{22}$$

with $-\pi/4\leqslant\chi\leqslant\pi/4$. The phase ϕ is unimportant and can be set to zero. Since

$$a^2+|\boldsymbol{r}+\boldsymbol{\alpha}(t)|^2\geqslant a^2+(x+\alpha_0\sin\omega t)^2, \tag{23}$$

we have

$$U_{\rm dr}(\alpha_0,\boldsymbol{r})\leqslant U_0\frac{\omega}{2\pi}\int_0^{2\pi/\omega}[a^2+(x+\alpha_0\sin\omega t)^2]^{-1}dt$$

$$\leqslant(U_0/a)\,{\rm Im}\{[(x-ia)^2-\alpha_0^2]^{-1/2}\}$$

$$\leqslant(U_0/a)|(x-ia)^2-\alpha_0^2|^{-1/2}. \tag{24}$$

It follows from the last relation that

$$U_{\rm dr}(\alpha_0,\boldsymbol{r})\leqslant U_0(2a^3\alpha_0)^{-1/2} \tag{25}$$

if $\alpha_0>a$. Therefore $V_0(\alpha_0)$ decreases as least as fast as $1/\alpha_0^{1/2}$ for $\alpha_0\to\infty$.

Now, in three dimensions, the number of bound states supported by the dressed potential, $N[V_{\rm dr}]$, cannot exceed the limit set by the Cwikel-Lieb-Rozenbljum bound [14,15]. In the present case, the latter implies that

$$N[V_{\rm dr}]\leqslant\eta\int |V_{\rm dr}(\alpha_0,\boldsymbol{r})|^{3/2}d\boldsymbol{r}, \tag{26}$$

where η is a constant that does not depend on α_0. This relation holds provided the integral appearing in the right-hand side exists. But it follows from Eqs. (16)–(19) that

$$\int |V_{\rm dr}(\alpha_0,\boldsymbol{r})|^{3/2}d\boldsymbol{r}\leqslant V_0^{1/2}(\alpha_0)I[V_{\rm at}]. \tag{27}$$

Therefore the right-hand side of Eq. (26) exists and $N[V_{\rm dr}]$ tends to zero at least as fast as $1/\alpha_0^{1/4}$ for $\alpha_0 \to \infty$.

B. Disappearance of a dressed bound state

The KH states are approximations of the exact dressed bound states of the system. Those can be obtained by looking for solutions of Eq. (1) in the Floquet form [10],

$$\Phi(\boldsymbol{r},t) = e^{-iEt} \sum_N e^{-iN\omega t} \phi_N(\boldsymbol{r}), \tag{28}$$

for a stationary incident field—e.g., for a vector potential

$$\boldsymbol{\mathcal{A}}(t) = \mathcal{A}_0 \mathrm{Re}[\hat{\boldsymbol{\epsilon}} \exp(-i\omega t)]. \tag{29}$$

In order to represent a decaying bound state, the wave function must reduce for $|\boldsymbol{r}| \to \infty$ to a superposition of outgoing waves in the open channels and exponentially damped waves in the closed channels. These Gamow-Siegert boundary conditions make the quasienergy E complex. The rate of photodetachment is $\Gamma = -2\,\mathrm{Im}\,E/\hbar$.

The variation with α_0 of the ground state quasienergy for the potential

$$V_{\rm at}(\boldsymbol{r}) = -2\exp(-r) \tag{30}$$

is presented in Fig. 1. (Atomic units are used throughout this section.) This potential supports only one bound state. Its binding energy, 0.158 a.u., is much smaller than the photon energies for which results are given in the figure, $\hbar\omega = 1$ or 2 a.u. The high-frequency approximation can thus be expected to be reliable. The quasienergy was calculated *ab initio*, by complex scaling [16], using the method and program described in Ref. [17].

The results show that the real part of the quasienergy slowly approaches 0 when α_0 increases, both for linear and for circular polarization. It eventually passes the threshold (i.e., it becomes positive), after which the dressed ground state has unphysical features and is a shadow state [18]. For a given α_0, $\mathrm{Re}\,E$ for linear polarization is about the same at $\omega = 2$ a.u. as at $\omega = 1$ a.u. This strongly suggests that the trajectory of the quasienergy follows the variation of the eigenenergy of the corresponding KH state, as is expected at these high frequencies. Note also the sharp and sustained drop in the photodetachment rate visible in Fig. 1(b), which is diagnostic of adiabatic stabilization. Each curve peaks at a value of α_0 close to where the rate of ionization from the ground state of atomic hydrogen is a maximum for the same frequency, i.e., close to the intensity for which $(\hbar\omega/2P)^{1/2} \approx 1$, where P is the ponderomotive energy [19].

Numerical results similar to those of Figs. 1(a) and 1(b) have been obtained by Day for photodetachment from a spherical square well in the high-frequency regime [20]. However, there is neither disappearance of the dressed bound state nor adiabatic stabilization in photodetachment from the three-dimensional zero-range potential by a circularly polarized field [21]. Again, there is no contradiction since this potential does not satisfy the assumptions under which the disappearance of KH states is proved in Sec. III A.

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