Tunneling times in the Copenhagen interpretation of quantum mechanics

M. Abolhasani[1,2,*] and M. Golshani[1,3,†]
[1]Institute for Studies in Theoretical Physics and Mathematics (IPM), P.O. Box 19395-5531, Tehran, Iran
[2]Department of Physics, Khajeh Nassir-Al-Deen Toosi University of Technology, P.O. Box 15418, Tehran, Iran
[3]Department of Physics, Sharif University of Technology, P.O. Box 11365-9161, Tehran, Iran
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Recently, people have calculated tunneling's characteristic times within Bohmian mechanics. Contrary to some characteristic times defined within the framework of the standard interpretation of quantum mechanics, these have reasonable values. Here, we introduce one of the available definitions for tunneling's characteristic times within the standard interpretation as the best definition that can be accepted for the tunneling times. We show that, due to experimental limitations, Bohmian mechanics leads to the same tunneling times.

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I. INTRODUCTION

A problem that does not have a clearcut answer in quantum mechanics is the time that it takes for an electron to pass through a potential barrier. This is a problem that is important from both a theoretical perspective [1,2] and a technological view [3,4].

In quantum mechanics, time enters as a parameter rather than an observable (to which an operator can be assigned). Thus, there is no direct way to calculate tunneling times. People have tried to introduce quantities that have the dimension of time and can somehow be associated with the passage of the particle through the barrier. These efforts have led to the introduction of several times, some of which are completely unrelated to the others [5–17]. Some people have used the Larmor precession as a clock [5] to measure the duration of tunneling for a steady state [6,7] or for a wave packet [8]. Others have used Feynman paths like real paths to calculate an average tunneling time with the weighting function $\exp\{iS[x(t)]/\hbar\}$, where S is the action associated with the path $x(t)$—where $x(t)$'s are Feynman paths initiated from a point on the left of the barrier and ending at another point on the right of it [9]. On the other hand, a group of people have used some features of an incident wave packet and the comparable features of the transmitted packet to introduce a delay as tunneling time [10,18]. There are many other approaches, some of which are mentioned in Refs. [10–17]. But, there is no general consensus among physicists about the meaning of them and about which, if any, of them is the proper tunneling time. In Bohmian mechanics [19], however, there is a unique way of identifying the time of passage through a barrier. This time has a reasonable behavior with respect to the width of the barrier and the energy of the particle [20,21].

It is expected that with the availability of reliable experimental results in the near future, an appropriate definition can be selected from the available ones, or that the ground would be prepared for a more appropriate definition of the transmission time. But now, we want to use the definition of tunneling time in the framework of Bohmian mechanics to select one of available definitions for quantum tunneling times (QTT) within the standard interpretation as the best definition.

Our paper is organized as follows: after introducing the Olkhovsky-Recami and Muga-Brouard-Sala QTT, by using a heuristic argument in Sec. II, we introduce, in Sec. III, the Bohmian QTT. Then, in Sec. IV we give a critical discussion about Cushing's thought experiment and about what it really measures.

II. TUNNELING'S CHARACTERISTIC TIMES IN THE COPENHAGEN FRAMEWORK

To begin with, we consider the time at which a particle passes through a definite point in space. We describe the particle by a Gaussian wave packet which is incident from the left. The most natural way to estimate this time of passage is to find the time at which the peak of the wave packet passes through that point. But this is not the right criterion for finding the time of passage of the particle (even if the wave packet is symmetrical). To clarify the matter, we divide the packet, in the middle, into two parts. The probability of finding the particle in the front section is $\frac{1}{2}$ and the same is true for the back section. We represent the transit time of the center of gravity of the front section by t_1 and that of the back section by t_2. The average time for the particle's passage through that point is $T=\frac{1}{2}(t_1+t_2)$. If the transit time for the peak of the wave is denoted by t, we have

$$t_1=t-\frac{x_1}{v_g}, \tag{1a}$$

$$t_2=t+\frac{x_2}{v_g}, \tag{1b}$$

where v_g is the group velocity of the wave packet and x_1 and x_2 are, respectively, the distances of the centers of gravity of the front and the back sections of the packet from its peak position, when these centers pass the point under consideration. Thus, we have

$$T=\frac{1}{2}(t_1+t_2)=t+\frac{x_2-x_1}{2v_g}. \tag{2}$$

*Electronic address: majida@theory.ipm.ac.ir
†Electronic address: golshani@ihcs.ac.ir

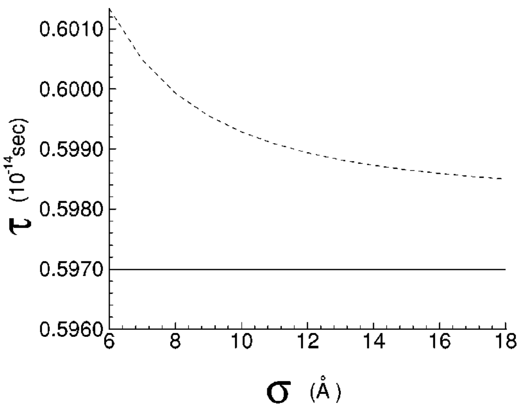

FIG. 1. Average time for the transmission of the probability flux (. . .) and the passage of the peak (—), for packets having different widths (σ).

If the wave packet did not spread, x_1 and x_2 would remain equal and T would be equal to t. But, since the wave packet spreads, $T \neq t$. In fact, the average transit time for the particle is later than that of the wave's peak. Because the spreading of the packet decreases the transit time of the center of gravity of the wave's front section, and increases that of the back section. But the change is not symmetrical (i.e., $x_1 \neq x_2$), as the back section of the wave experiences the spreading for a longer time.

Now, consider a wave packet $\psi(x,t)$, which is incident from the left and approaches a far point x. The best time that we can attribute to the particle's passage through x is

$$\tau(x) = \frac{\int_0^\infty t|\psi(x,t)|^2 v(x,t)dt}{\int_0^\infty |\psi(x,t)|^2 v(x,t)dt}, \tag{3}$$

where $v(x,t) = j(x,t)/|\psi(x,t)|^2$, $j(x,t)$ being the probability current density. In fact, we have divided the wave packet into infinitesimal elements. The transit time when the particle is in one of these elements is weighted by the probability of finding the particle there (i.e., $|\psi(x,t)|^2 dx = |\psi(x,t)|^2 \times v(x,t)dt$). Figure 1 illustrates the difference between this time and the time that the peak passes that point. For narrow wave packets, for which the rate of spreading is large, this difference is large. From Eq. (3), one can define a distribution for the transit time through x,

$$P(x,t) = \frac{|\psi(x,t)|^2 v(x,t)}{\int_0^\infty |\psi(x,t)|^2 v(x,t)dt} = \frac{j(x,t)}{|T|^2}, \tag{4}$$

where $|T|^2$ is the transition probability for passing through x_0. Dumont and Marchioro introduced this definition for the distribution of the time at which a particle passes through the far side of a potential barrier [22]. They did not find it possible to define the time spent by the particle in the barrier. Leavens showed that this is also the distribution for the same time in Bohmian mechanics [23].

By looking at Eq. (3), one notices that $\tau(x)$ is in fact the average time for the passage of the probability density $|\psi|^2$ through x. Since the probability density represents the probability of the presence of the particle, it is natural to take the average time for the passage of probability density through a point as a measure of the average time for particle's passage through that point. But, while part of the probability flux passes through the barrier, the particle itself might not be detected on the other side of the barrier. We do not, however, expect to get a definite prediction for an individual system, and in the laboratory we usually consider an ensemble of systems. Thus, it is natural to take the average time for the passage of the probability density as a measure of the average time for the particle's passage. From now on, we talk about the particle's average time of transit. Consider, a particle incident on a barrier from the left. Then, one can easily extend Eq. (3) to define average times for the particle's entrance into the barrier (τ_{in}), the particle's exit from the right side of the barrier (τ_{out}^T), and the particle's exit from the left side of the barrier (τ_{out}^R). To simplify the matter we use the following notations:

$$(\cdots\Theta)_x^\pm = \int_0^\infty dt \cdots (\pm) j(x,t)\Theta[\pm j(x,t)], \tag{5}$$

where $j(x,t)$ represents the probability current density at the point x at time t, and Θ is the usual step function. Using this definition and Eq. (3), we define τ_{in}, τ_{out}^R, and τ_{out}^T as

$$\tau_{in} = \frac{\int_0^\infty dt\, t j(a,t)\Theta[+j(a,t)]}{\int_0^\infty dt\, j(a,t)\Theta[+j(a,t)]} = \frac{(t\Theta)_a^+}{(\Theta)_a^+}, \tag{6a}$$

$$\tau_{out}^T = \frac{\int_0^\infty dt\, t j(b,t)\Theta[+j(b,t)]}{\int_0^\infty dt\, j(b,t)\Theta[+j(b,t)]} = \frac{(t\Theta)_b^+}{(\Theta)_b^+}, \tag{6b}$$

$$\tau_{out}^R = \frac{\int_0^\infty dt\, t(-)j(a,t)\Theta[-j(a,t)]}{\int_0^\infty dt(-)j(a,t)\Theta[-j(a,t)]} = \frac{(t\Theta)_a^-}{(\Theta)_a^-}, \tag{6c}$$

where a and b represent the coordinates of the left and right side of the barrier, respectively. Using these times, one can write the times that the particle spends in the barrier before the transmission (τ_T^{OM}) or reflection (τ_R^{OM}) as

$$\tau_T^{OM} = \tau_{out}^T - \tau_{in}, \tag{7a}$$

$$\tau_R^{OM} = \tau_{out}^R - \tau_{in}. \tag{7b}$$

We shall call them OM times[1] (referring to Olkhovsky and Recami [24] and Muga, Brouard, and Sala [25]). The average time spent by the particle in the barrier, irrespective of being transmitted or reflected, the so-called dwelling time, is thus given by

$$\tau_d^{OM}=(\Theta)_b^+\,\tau_T^{OM}+(\Theta)_a^-\,\tau_R^{OM}, \tag{8}$$

where $(\Theta)_b^+$ and $(\Theta)_a^-$ represent the probability of the particle's exit from the right and left sides of the barrier, respectively. Now, the probability of the particle's exit from the right, $(\Theta)_b^+$, is equal to the probability of the particle's transmission through the barrier, $|T|^2$. But the probability of the particle's exit from the left, $(\Theta)_a^-$, is not equal to the probability of reflection from the barrier, $|R|^2$, because the particle could be reflected without entering the barrier. Using Eq. (7), one can write Eq. (8) in the form

$$\tau_d^{OM}=(\Theta)_b^+\,\tau_{out}^T+(\Theta)_a^-\,\tau_{out}^R-(\Theta)_a^+\,\tau_{in}, \tag{9}$$

where we have made use of the fact that $(\Theta)_b^+ +(\Theta)_a^- =(\Theta)_a^+$, which follows from the conservation of probability. The first two terms in Eq. (9) represent the average of a particle's exit time from the barrier, irrespective of the direction of the exit. Using Eq. (6) we can write the right-hand side of Eq. (9) in the form

$$\tau_d^{OM}=\int_0^\infty dt\, t[j(b,t)-j(a,t)]. \tag{10}$$

Using the continuty equation, one can easily show that Eq. (10) coincides with the standard dwelling time defined by

$$\tau_D=\int_0^\infty dt\int_a^b|\psi(x,t)|^2dx. \tag{11}$$

III. TUNNELING'S CHARACTERISTIC TIMES IN BOHMIAN FRAMEWORK

In the causal interpretation of quantum mechanics, proposed by Bohm [19], a particle has a well-defined position and velocity at each instant, where the latter is obtained from a field $\psi(x,t)$ satisfying the Schrödinger equation. If the particle is at x at the time t, its velocity is given by

$$v(x,t)=\frac{j(x,t)}{|\psi(x,t)|^2}. \tag{12}$$

For a particle that is prepared in the state $\psi(x,0)$ at $t=0$, any uncertainty in its dynamical variables is a result of our ignorance about its initial position x_0. Our information about the particle's initial position is given by a probability distribution $|\psi(x_0,0)|^2$. If we know the initial position x_0 of the particle, we can find its position at a later time, $x(x_0;t)$, from Eq. (12). Then, when a particle encounters a barrier, it is determined whether the particle passes through the barrier or not, and one can determine when the particle enters the barrier and when it leaves the barrier. Thus the time spent by the particle within the barrier is easily calculated. But, since we do not know the particle's initial position, we consider an ensemble of initial positions, given by the distribution $|\psi(x_0,t)|^2$. Then we calculate the average time spent by the particle within the barrier. To compare the time of reflection or transmission in this framework with OM characteristic times, we first consider the time of arrival at x_1, for a particle that was at x_0 at $t=0$

$$t(x_1;x_0)=\int_{C_{x_0}}dx\; t(x;x_0)\,\delta(x_1-x), \tag{13}$$

where the integral is defined along the Bohmian path C_{x_0} which starts at x_0. This relation can also be written in the form

$$t(x_1;x_0)=\int_0^\infty dt|v(x(x_0;t),t)|\,t\,\delta(x_1-x(x_0;t)), \tag{14}$$

where

$$\delta(x_1-x(x_0;t))=\frac{\delta(t(x_1)-t)}{|v(x(x_0;t),t)|}. \tag{15}$$

Since it is possible for the particle to pass the point x_1 twice (due to reflection from the barrier), we define $t^{\pm}(x_0;x_1)$ in the following manner:

$$t^{\pm}(x_0;x_1)=\int_0^\infty dt|v(x(x_0;t),t)|\,t\,\delta(x_1-x(x_0;t))\times\Theta(\pm v(x(x_0;t),t)), \tag{16}$$

where t^+ and t^- correspond to the cases where the particle passes x_1 from left to right and from right to left, respectively. Since for long periods of time, a particle either passes or is reflected (depending on its x_0), we define Θ_R and Θ_T in the following way [20,21]:

$$\Theta_T(x_0)=1,\quad \Theta_R(x_0)=0\quad (\text{for transmission}), \tag{17a}$$

$$\Theta_T(x_0)=0,\quad \Theta_R(x_0)=1\quad (\text{for reflection}). \tag{17b}$$

Thus, we have $\Theta_T(x_0)+\Theta_R(x_0)=1$. Using these functions, the average times spent by the transmitted and the reflected particles, τ_T^B and τ_R^B, respectively, are given by

$$\tau_T^B=\frac{\langle t^+(x_0;b)\Theta_T(x_0)\rangle-\langle t^+(x_0;a)\Theta_T(x_0)\rangle}{\langle\Theta_T(x_0)\rangle}, \tag{18a}$$

$$\tau_R^B=\frac{\langle t^-(x_0;a)\Theta_R(x_0)\rangle-\langle t^+(x_0;a)\Theta_R(x_0)\rangle}{\langle\Theta_R(x_0)\rangle}, \tag{18b}$$

where

[1] Note that in the orginal definition of Olkhovsky and Recami, temporal integrations run from $-\infty$ to $+\infty$. In Ref. [26] they discussed that the substitution of integrals of the type $\int_0^\infty$ for integrals $\int_{-\infty}^{+\infty}$ have physical significance. Any way, we shall use relations (6).

$$\langle \cdots \rangle = \int_{-\infty}^{+\infty} dx_0 \cdots |\psi(x_0,t)|^2, \qquad (19)$$

but $\langle \Theta_R(x_0)\rangle = |T|^2$ and $\langle \Theta_R(x_0)\rangle = |R|^2$ [20]. Thus, we have for the dwelling time

$$\tau_d^B = |T|^2 \tau_T^B + |R|^2 \tau_R^B$$
$$= \langle t^+(x_0;b)\Theta_T(x_0)\rangle + \langle t^-(x_0;a)\Theta_R(x_0)\rangle$$
$$- \langle t^+(x_0;a)\rangle, \qquad (20)$$

where we have made use of the fact that $\Theta_T + \Theta_R = 1$. Using the fact that $\int_{-\infty}^{+\infty} dx_0 f(x(x_0,t),t)|\psi(x_0,0)|^2 \delta(x - x(x_0,t)) = f(x,t)|\psi(x,0)|^2$, one can easily show that

$$\langle t^+(x_0;b)\Theta_T(x_0)\rangle = (t\Theta)_b^+, \qquad (21a)$$

$$\langle t^-(x_0;a)\Theta_R(x_0)\rangle = (t\Theta)_a^-, \qquad (21b)$$

$$\langle t^+(x_0;a)\rangle = (t\Theta)_a^+. \qquad (21c)$$

Thus, τ_d^B is equal to τ_d^{OM} and therefore equal to τ_D. Of course, the equality of τ_d^B and τ_D was shown earlier by Leavens [20,21]. But the relations (21) are new and they are important because they show the relation between OM characteristic times and those defined in Bohmian mechanics. Notice that in the causal interpretation of Bohm, one defines two average entrance times τ_{in}^T and τ_{in}^R, depending on whether the particle is reflected or transmitted:

$$\tau_{in}^T = \frac{\langle t^+(x_0,a)\Theta_T(x_0)\rangle}{\langle \Theta_T(x_0)\rangle}, \qquad (22a)$$

$$\tau_{in}^R = \frac{\langle t^+(x_0,a)\Theta_R(x_0)\rangle}{\langle \Theta_R(x_0)\rangle}, \qquad (22b)$$

whereas OM have defined only one average time. This is because in the standard interpretation of quantum mechanics, it is not definite whether a particle that has entered a barrier, is transmitted or reflected. It is natural to have the average time for particle's entrance, irrespective of whether it is reflected or transmitted, to be equal to $|T|^2\tau_{in}^T + |R|^2\tau_{in}^R = \langle t^+(x_0;a)\rangle$ in the Bohmian framework. Then, we must have $(t\Theta)_a^+ = |T|^2\tau_{in}^T + |R|^2\tau_{in}^R$, which is easy to prove.

It is natural to expect that the average time for particle's transmision through a potential barrier to be a function of the width of the barrier. This time should generally increase with the width of barrier. However, due to quantum effects, one does not expect it to be a linear function of this width. Most of the times defined within the framework of the standard interpretation of quantum mechanics, do not have this property, and some of them even yield negative times! On the other hand, we expect the transition time to decrease with the increase in the energy of the incident particle. The transition time in OM approach has both of these properties . The diagrams in Figs. 2 and 3 represent the trasmission time as a function of the width of the barrier and as a function of the particle's energy, respectively, for both Bohmian and OM times. The numerical method used to solve the time-dependent Schrödinger equation was the fourth order (in time steps δt) symmetrized product formula method, developed by De Readt [29]. We chose $\delta x = \pi/30k_0$, where $k_0 = \sqrt{2mE_0/\hbar^2}$ and $\delta t = \delta x^2/25$ in all calulations (E_0 is the energy of the incident Gaussian wave packet). One notices that OM transmission time coincides with that of the Bohm-

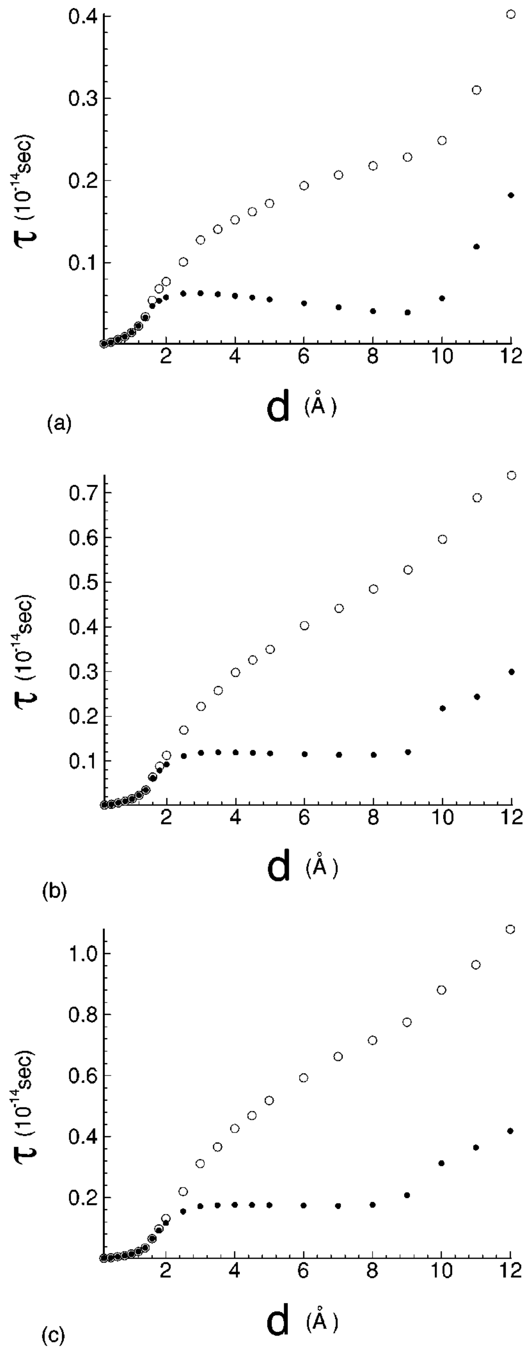

FIG. 2. Diagrams (a), (b), and (c) show the dependence of the transmission time τ_T in terms of the width of a square barrier with the height $V_0 = 10$ eV and the incident energy $E_0 = \hbar^2 k_0^2/2m = 5$ eV. These diagrams represent, respectively, Gaussian wave packets having the width $\sigma = 6$, 12, and 18 Å. We have shown the Bohmian results by hollow spheres and those of the standard interpretation by solid circles.

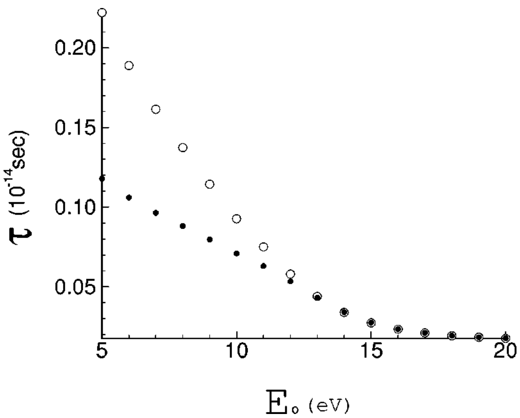

FIG. 3. This diagram shows the transit time for a square barrier of width $b-a=3$ Å and of height $V_0=10$ eV, for Gaussian wave packets having $\sigma=12$ Å and differents energies. We have shown Bohmian results by hollow spheres and those of the standard interpretation by solid circles.

ian case for large $|T|^2$ (i.e., $d<2$ in the diagrams of Fig. 2 and $E_0>V_0$ in the diagram of Fig. 3). This is natural, because while the average time for the particle's exit from the right side of the barrier is always the same in both approaches, in the limit of $|T|^2\rightarrow 1$, the average entrance time for the tansmitted particle is the same in the OM approach and in the Bohmian approach ($|T|^2\rightarrow 1 \Rightarrow \tau_{in}^T \rightarrow \tau_{in}$). Thus in this limit we have $\tau_T^B=\tau_T^{OM}$. On other hand, as we said earlier, the average time for the particle's exit from the left, in the OM approach, is generally different from that of the Bohmian case. But, if we choose a to be a point far (relative to the width of the wave packet) from the left side of the barrier, then the time for the particle's exit from the left side is the same in both approaches, since, in this case for $|R|^2 \rightarrow 1$, the average time of entrance for reflected particles in the causal approach becomes equal to the average time of entrance in the OM approach ($|R|^2\rightarrow 1 \Rightarrow \tau_{in}^R \rightarrow \tau_{in}$). Thus we have $\tau_R^B=\tau_R^{OM}$. It appears that the OM approach gives the most natural definition for a positive definite transmission time, within the framework of the standard interpretation of quantum mechanics.[2]

IV. EXPERIMENTAL TEST

It is generally believed that the standard quantum mechanics and the Bohmian mechanics have identical predictions for physical observables. On the other hand, there is no Hermitian operator associated with time. Is it possible to consider a phenomenon involving time, e.g., tunneling, to differentiate between these two theories? By considering a thought experiment, Cushing gave a positive response to this question. His argument was the following [1,2].

(1) There is presently no satisfactory account of a quantum tunneling time (QTT) in the standard quantum mechanics.

(2) There is a well-defined account of QTT in Bohmian interpretation. If it can be measured, then such a measurement would constitute a test of the interpretation .

(3) It might be possible to measure the Bohmian QTT with an experiment of a certain type.

(4) Therefore, from points (2) and (3), if an experiment of that type is possible, such an experiment could serve as a test of Bohm's interpretation.

(5) Because of point (1), the outcome of an experiment of that type would not support or refute the Copenhagen interpretation.

In a recent paper, Bedard [2], by referring to points (1), (2), and (5) questioned Cushing's conclusion. Her argument was based on the fact that the two theories have different microontologies. Therefore, the QTT obtainable from Bohmian mechanics has no counterpart in the standard quantum mechanics. Thus, the measurement of such a time cannot be considered a test between the two theories. Here, we shall question point (3), i.e., the claim that Cushing's thought experiment can be used to measure Bohmian times.

Cushing's experiment consists of a potential barrier between x_1 and x_2 with width d ($d=b-a$). A detector D_T is located at x_2 on the right of the barrier and a detector D_R is located at x_1 on the left of the barrier ($x_1<a<b<x_2$). Electrons are incident from the left. D_T records the arrival times of the transmitted electrons at x_2 and D_R records the times of the reflected electron at x_1. The distance from x_1 to the left side of the barrier (a) is much more than the width of the wave packet. The same holds for x_2. The recording of the arrival time of the incident electron at x_1 will collapse the wave function. In that case, any subsequent tunneling time prediction on the basic of the known incident wave packet would be quite useless. To resolve this problem, Cushing considers the preparation of the state of the incident particle at x_1, rather than its detection. Thus, the time recorded at x_1 is the preparation time for the transit of the particle, if D_T would detect it, and the preparation time for the reflected paticle, if D_R would detect it. To provide this condition, we prepare a source of electrons in front of which there is a shutter. The shutter starts to open little before $t_0=0$ and closes little after t_0. Thus t_0 is the most probable time for the passage of the electron from x_1. In other words, t_0 is the time when the peak of the wave packet passes x_1. By choosing a weak source, we can be sure to have at most one electron emerging from the shutter's opening. The time of passage for the particle through x_1 is $t_1=t_0\pm\Delta x/v_o$, where Δx is the width of the packet and v_0 is the speed of the particle. Cushing claims that ''in principle, this error could be made as small as we like (for large enough v_0)'' [1]. In our opinion, the error must be compared with τ_T, not with t_1. In fact, we want to obtain τ_T which is the difference between the two times (t_2-t_1), where t_2 is the time that the electron is detected at x_2 by D_T). The error could be small if we compare it with t_1 and t_2 but not if we compare it with their difference. Thus, we must have

[2]Of course, in relation (6b) if we chose b, a point in the interior of barrier width, τ_T^{OM} may become negative, but small in absolute value [28,27].

$$\frac{\Delta x}{v_0} \ll \tau_T . \quad (23)$$

By referring to Fig. 3, one can see that τ_T decreases quicker than $1/v_0$. Thus, the increase in v_0 decreases the right-hand side of Eq. (23) more than its left-hand side and we are not able to decrease the relative error in this way. One may hope to obtain condition (23) by decreasing Δx. But decreasing Δx is not useful. Because, by referring to Figs. 2(a)–2(c) one can see that τ_T decreases almost linearly with Δx.

Experimental limitations dictate that the arrival time of particles to the barrier be measured independent of whether they shall be reflected or transmitted (i.e., state preparation time). Thus, although Bohm's theory considers τ_{in}^T and τ_{in}^R, it must pay attention only to the measurements of $\tau_{in} = |T|^2 \tau_{in}^T + |R|^2 \tau_{in}^R$, to avoid experimental limitations. In this way the precise time that it attributes to the particle's transmission through the barrier or reflection is τ_T^{OM} and τ_R^{OM}, respectively. Thus, at the experimental level, even in the case of tunneling times we have the same predictions in the two theories. In fact, here we encounter a problem like the case of the celebrated two-slit experiment. In the framework of Bohmian mechanics, all particles observed on the lower (upper) half of the screen must come from the lower (upper) slit. But, any effort to know which particle came from which slit destroys the interference pattern. Thus, in the two-slit experiment, the two theories come to the same result due to experimental limitations. It appears that, from various definitions given for QTT in the framework of the standard quantum mechanics, our choice of OM's is the best because, in our opinion, it is the best time that can be related to the tunneling phenomena in the framework of the standard interpretation of quantum mechanics. We can justify our claim in the following way.

(1) There is a unique and well-defined account of QTT in Bohm's interpretation.

(2) There are several accounts of QTT in standard interpretation.

(3) These two theories have the same prediction for observables.

(4) The Bohmian prediction for QTT coincides with one of the Copenhagen's QTT (OM's).

In fact, OM's is the only definition that gives the same result, at the experimental level, as the Bohmian mechanics, although it does not associate an operator with τ_T (at least up to now). In this way, we have used a theory with additional microontology (Bohmian mechanics) to give the best definition for a quantity in a theory with less microontology. Bohm's theory may also shed light on other definitions of QTT in the standard quantum mechanics.

V. CONCLUSION

Considering the fact that the microontology of the Copenhagen theory includes the wave function (probability amplitude), and not pointlike particles, the best time one could attribute to the passage of a particle from a point of space is the average time of the passage of probability flux [Eq .(3)]. The generalization of this time to QTT leads one to OM's times. On the other hand, the microontology of Bohmian mechanics includes pointlike particles in addition to the wave function, and it leads uniquely to the Bohmian QTT [Eq. (18)]. We have compared them for different widths and energies of the wave packets in Figs. 2 and 3 by use of numerical calculations.

Now, the Bohmian QTT could not be measured due to experimental limitations. The best times that could be obtained in Bohmian mechanics are the same as OM's. The agreement of one of the several[3] available definitions of the QTT in the Copenhagen quantum mechanics with the unique definition of the Bohmian mechanics separates it from the others, because it is reasonable to expect the same prediction for the two theories, even in the case of the QTT.

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[3] In fact, only the systematic projector approach of Brouard, Sala, and Muga leads to an infinite hierarchy of possible mean transmission and reflection times [16].

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