Entanglement and generation of superpositions of atomic coherent states

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After making a general description of the properties of macroscopic superpositions of collective atomic states, we present a detailed analysis of a method for their experimental realization recently proposed by C.C. Gerry and R. Grobe [Phys. Rev. A **56**, 2390 (1997)] which is based on cavity QED techniques involving the manipulation of electromagnetic and matter fields. Specific conditions and limitations of the procedure are discussed.

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I. INTRODUCTION

A realization of a Schrödinger cat [1] is considered to be a quantum superposition of two coherent states which individually would evolve in different regions of the phase space associated to the physical problem under consideration [2]. These states have their historical origin in the Schrödinger cat paradox formulated in 1935, whose main motivation was to illustrate the bewildering paradoxes arising from the seemingly incomplete character of quantum theory [1]. They are considered to be essential for the analysis of the borderline region between the classical and quantum worlds. According to Zurek *et al.*, superpositions of states are not observed in classical physics because the density matrix associated to macroscopic objects becomes diagonal, that is, the off diagonal terms tend to zero almost instantaneously, i.e., decoherence time is extremely short [3]. Recently, however, methods have been proposed to diminish the dissipative effects of the environment by imposing couplings with a certain symmetry condition [4].

Lately new experiments have lead to the generation of mesoscopic Schrödinger cats for the quantized electromagnetic field [5] and for the quantized vibrational motion of the center of mass of a trapped ion [6]. An alternative and quite attractive realization would correspond to superpositions of the coherent states of a system of spins or equivalently, within the Dicke model [7], a system of N two-level atoms [8–10]. In the latter case, experimentalists are interested in studying the practical limits of quantum control methods in trapped atoms for several reasons besides the generation and analysis of nonclassical states, attempting to implement quantum logic and quantum computation [11] and trying to improve the signal-to-noise ratio in spectroscopy using maximally entangled states [12] which turn out to be examples of atomic Schrödinger cats.

In the last years several proposals have been made for the realization of atomic Schrödinger cat states. One of them, introduced by Agarwal, Puri, and Singh, considers an effective Hamiltonian quadratic in the population inversion operator which is analogous to the Hamiltonian of a radiation field in a Kerr medium [8]. In addition, a spin model was studied in [13] that leads to the generation of atomic Schrödinger cat states on the sphere. Under certain conditions and in the limit when the number of atoms tends to infinity it reduces exactly to the Kerr Hamiltonian. On the other hand, Gerry and Grobe have recently proposed two alternative methods for the generation of these states, where the essential ingredients are the use of state reduction techniques and of dispersive interactions between the atoms and the field [9,10]. Other proposals to yield special cases of macroscopic atomic states are related to methods that produce maximally entangled states in a system of N trapped ions. The construction of such entangled states have been also important for the elucidation of fundamental questions in the interpretation of quantum mechanics [14,15]. So-called ''crystallized'' Schrödinger cat states, related to point group representations, were introduced as an extension of the symmetry construction for the even and odd coherent states of the electromagnetic field oscillator [16].

In this work, we analyze in detail one of these proposals [9] which is particularly illustrative of quantum engineering techniques. Our objective is twofold. On the one hand we reconstruct and extend the steps of Ref. [9] in a simple and transparent fashion which can be followed by nonspecialists. On the other, we contribute to the analysis of feasibility of the proposed experiment by exploiting the analytic expressions we find at each step. The procedure of Gerry and Grobe involves a sequence of interactions which suitably modify the state of the N two-level atoms confined in a cavity and the quantum electromagnetic field within it. The dimensions of the cavity are chosen to be much smaller than the wavelength associated to the atomic transition in order to have control of the interaction of the atoms with the cavity electromagnetic field. This interaction becomes relevant only when an external driving field produces a Stark shift of the atomic levels. At some stages of the procedure, the cavity electromagnetic field is entangled with an additional atom passing through the cavity. For each step in the procedure, it is possible to obtain analytic expressions for the states of the system making the subsequent physical analysis more transparent. The Q function of the system of N atoms is explicitly presented. Partial characterizations of the atoms-light system illustrate clearly the consequences of quantum entanglement and nonlocality. Some restrictions on the transit times and interactions are pointed out.

This work is organized as follows. In the next section, the

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basic properties of coherent atomic states are described. Subsequent steps of the procedure are then treated in individual sections. Finally, a brief discussion is given at the end of the article.

II. ATOMIC COHERENT AND SCHRÖDINGER CAT STATES

Atomic coherent states have properties which can be associated to the angular momentum algebra [17,18]. Given an assembly of N two-level atoms, the corresponding Hilbert space is spanned by the set of 2^N product states,

$$|\phi_{i_1,i_2,\ldots,i_N}\rangle=\prod_{k=1}^{N}|\zeta_{i_k}\rangle_k \quad (i_k=1,2). \tag{2.1}$$

Defining collective operators in terms of the Pauli matrices which connect the two internal states of each atom,

$$J_\mu=\frac{1}{2}\sum_n \sigma_\mu^n \quad (\mu=x,y,z)$$

$$J_\pm=\frac{1}{2}\sum_n \sigma_\pm^n, \tag{2.2}$$

$$J^2=J_x^2+J_y^2+J_z^2,$$

one arrives to the su(2) or angular momentum algebra

$$[J_+,J_-]=2J_z, \quad [J_z,J_\pm]=\pm J_\pm. \tag{2.3}$$

A natural alternative basis set to that defined by (2.1) would be the set of eigenstates of the operator J_z which is proportional to the energy operator for a free evolution of the system

$$H_a=\hbar\omega_a J_z, \tag{2.4}$$

where $\hbar\omega_a$ is the energy difference between the two atomic levels. The elements of this basis are usually written in the form $|J\ M\rangle$, $2J=N$, and are called Dicke states [7].

Similarly to harmonic oscillator coherent states, the atomic coherent states are obtained by applying a unitary transformation $\exp(\alpha J_+-\alpha^* J_-)$ to the collective ground state $|J-J\rangle$ for which the N atoms are in the ground state:

$$|\alpha;J\rangle\equiv e^{\alpha J_+-\alpha^* J_-}|J,-J\rangle. \tag{2.5}$$

By using a matrix representation of J_+ and J_- it can be shown that

$$e^{\alpha J_+-\alpha^* J_-}=e^{\tau J_+}e^{ln(1+|\tau|^2)J_z}e^{\tau^* J_-}, \tag{2.6}$$

with $\alpha\equiv\theta/2\ e^{-i\phi}$, and $\tau=\tan(\theta/2)\ e^{-i\phi}$. The latter representation leads directly to the result

$$|\alpha;J\rangle=(1+|\tau|^2)^{-J}\sum_{M=-J}^{J}\sqrt{\frac{2J!}{(J-M)!(J+M)!}}\tau^{J+M}|J,M\rangle,$$
$$=\sum_{M=-J}^{J}\mathcal{D}^J_{M-J}(\phi,-\theta,-\phi)|J,M\rangle \tag{2.7}$$

where $\mathcal{D}^J_{MJ}$ is the Wigner rotation matrix. That is, as expected from the nature of the **J** operators, this transformation is equivalent to performing a rotation through an angle θ about an axis $\hat{n}=(\sin\phi,-\cos\phi,0)$ in the Bloch sphere:

$$R_{\theta,\phi}=e^{\alpha J_+-\alpha^* J_-}. \tag{2.8}$$

Thus the Bloch states (2.7) can be labeled by either α or by $(\theta,\ \phi)$

$$|\alpha;J\rangle\equiv|\theta,\phi\rangle=R_{\theta,\phi}|J,-J\rangle. \tag{2.9}$$

Notice that, by construction, two-level atoms in a coherent state are not entangled since (2.7) can be written as a product state for individual atoms. In fact, each atom is represented by a $J=1/2$ coherent state

$$|\alpha;J\rangle=\prod_{k=1}^{N}|\alpha;1/2\rangle_k. \tag{2.10}$$

Atomic coherent states have many properties analogous to those of harmonic coherent states. They span an overcomplete set

$$(2J+1)\int\frac{d\Omega}{4\pi}|\theta,\phi\rangle\langle\theta,\phi|=1, \tag{2.11}$$

and they are not orthogonal

$$\langle\theta,\phi|\theta',\phi'\rangle=\left(\frac{(1+\tau^*\tau')^2}{(1+|\tau|^2)(1+|\tau'|^2)}\right)^J,$$
$$=e^{iJ(\phi-\phi')}[\cos\tfrac{1}{2}(\theta-\theta')\cos\tfrac{1}{2}(\phi-\phi')$$
$$-i\cos\tfrac{1}{2}(\theta+\theta')\sin\tfrac{1}{2}(\phi-\phi')]^{2J}. \tag{2.12}$$

They are also minimum uncertainty states for the set of rotated operators $(J_\eta,J_\upsilon,J_\nu)=R_{\theta\phi}(J_x,J_y,J_z)R^{-1}_{\theta\phi}$,

$$\langle J_\eta^2\rangle\langle J_\upsilon^2\rangle=\tfrac{1}{4}\langle J_\nu\rangle^2, \tag{2.13}$$

where it was used that $\langle J_\eta\rangle=\langle J_\upsilon\rangle=0$.

The atomic coherent states can be produced by classical driving fields of constant amplitude (Sec. IV) and behave almost as classical dipoles. As shown by Arecchi *et al.*, the similarities are not fortuitous, but are related to the group contraction method of the angular momentum generators to the harmonic oscillator operators [18].

Also in analogy with vibrational cat states [2,16], atomic cat states can be defined as a superposition of a finite number of atomic coherent states. For instance, even and odd cat states correspond to

$$|\Psi_{\pm}(\gamma,\delta)\rangle = N_{\pm}[|\gamma,\delta\rangle \pm |\gamma,[\delta+\pi]_{\mathrm{mod}\,2\pi}\rangle], \quad (2.14)$$

where (γ,δ) correspond to the angles in (2.9) and the normalization factor is given by

$$N_{\pm} = \frac{1}{\sqrt{2}}\left[1 \pm \left(\frac{1-|\tau|^2}{1+|\tau|^2}\right)^{2J}\right]^{-1/2}$$

$$= \frac{1}{\sqrt{2 \pm 2\cos^{2J}\gamma}}. \quad (2.15)$$

The states $|\Psi_+\rangle$ and $|\Psi_-\rangle$ are orthogonal since they carry the symmetric and antisymmetric irreducible representations of the point group C_2, respectively [16]. Contrary to coherent states, atoms in a Schrödinger cat state are in general entangled as their wavefunctions cannot be written as a product of individual functions.

The maximally entangled state

$$|\Psi_{\mathrm{max}}\rangle = 1/\sqrt{2}[|J,-J\rangle \pm |J,J\rangle] \quad (2.16)$$

is a particularly relevant example of a Schrödinger cat. It cannot be written as a direct product of functions comprising any subset of the N two-level atoms. This state maximizes the signal-to-noise ratio in spectroscopic measurements [12].

As usual, the phase space description for a collection of atoms involves the introduction of a c-number function with a one-to-one correspondence to the quantum mechanical state of the system. Similarly to the harmonic oscillator case, coherent states provide a natural scheme for the definition of quasiprobability distribution functions. Given a density matrix ρ its expression in a diagonal Bloch representation [18]

$$\rho = \int P(\theta,\phi)|\theta,\phi\rangle\langle\theta,\phi|d\Omega \quad (2.17)$$

defines the P function. Another useful function is the Q function defined by [8]

$$Q(\theta,\phi) = \frac{2J+1}{4\pi}\langle\theta,\phi|\rho|\theta,\phi\rangle. \quad (2.18)$$

The $Q_{J;M}$ function of a Dicke state $|JM\rangle$ is proportional to the probability $F_{\theta;\phi}$ of finding $[J=N/2; M=(N_e-N_g)/2]$ the distribution of N atoms in the ground N_g or excited N_e states of a system described by a Bloch function $|\theta,\phi\rangle$,

$$\frac{4\pi}{2J+1}Q_{J;M}(\theta,\phi) = |\mathcal{D}^J_{M-J}(\phi,-\theta,-\phi)|^2$$

$$\equiv F_{\theta;\phi}(J,M). \quad (2.19)$$

The $Q_{\gamma;\delta}$ function of a coherent state $|\gamma;\delta\rangle$ can be directly obtained from Eq. (2.12) yielding

$$Q_{\gamma;\delta}(\theta,\phi) = \frac{2J+1}{4\pi}\left(\frac{1+\cos\zeta}{2}\right)^{2J}, \quad (2.20)$$

with ζ the angle between (θ,ϕ) and (γ,δ) given by

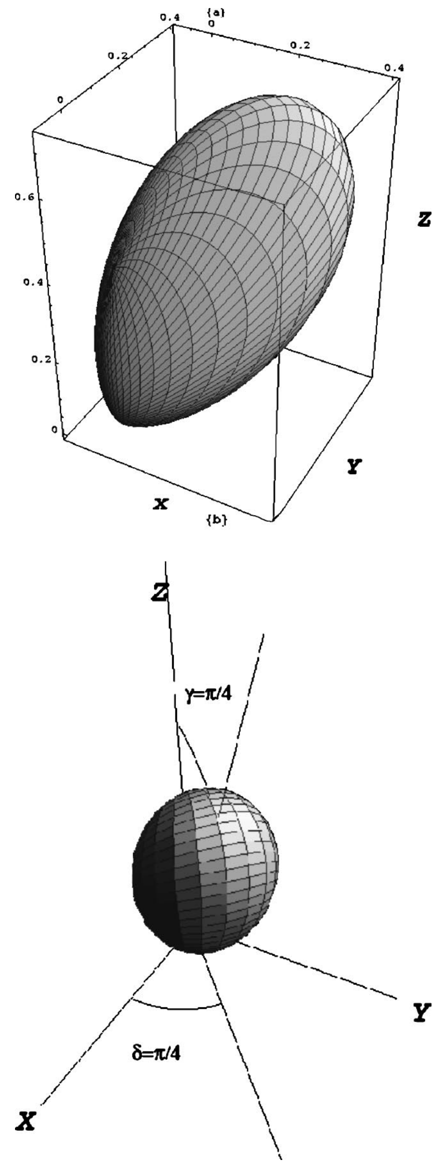

FIG. 1. (a) Plot of the $Q_{\gamma,\delta}(\theta,\phi)$ function for five two-level atoms in a $|\gamma=\pi/4, \delta=\pi/4\rangle$ Bloch state. (b) The (γ,δ) values of the Bloch state are shown. This and the following Q plots are made in spherical coordinates $[Q(\theta,\phi),\theta,\phi]$.

$$\cos\zeta = \cos\theta\cos\gamma + \sin\theta\sin\gamma\cos(\phi-\delta). \quad (2.21)$$

Notice that the Q function takes the maximum value in the direction (γ,δ), whereas it is a minimum in the direction $(\pi-\gamma, \pi+\delta)$, as can be seen in Figs. 1. In addition, it is symmetric around the (γ,δ) axis.

Finally, the $Q^{\pm}_{\gamma\delta}$ function of the even and odd cat states can be written in the form

$$Q^{\pm}_{\gamma;\delta}(\theta,\phi) = N^2_{\pm}\Bigg(Q_{\gamma;\delta}(\theta,\phi) + Q_{\gamma;\delta+\pi}(\theta,\phi)$$

$$\pm \frac{2J+1}{4\pi}\frac{(-1)^J}{2^{2J}}\{[\sin\theta\sin\gamma\sin(\phi-\delta)$$

$$+ i(\cos\theta+\cos\gamma)]^{2J} + [\sin\theta\sin\gamma\sin(\phi-\delta)$$

$$-i(\cos\theta+\cos\gamma)]^{2J}\}\Bigg). \tag{2.22}$$

In particular,

$$Q^{\pm}_{\pi/2;0}(\theta,\phi)=\frac{2J+1}{4\pi}\frac{1}{2^{2J+1}}\{(1+\sin\theta\cos\phi)^{2J} + (1-\sin\theta\cos\phi)^{2J} \pm(-1)^{J}[(\sin\theta\sin\phi+i\cos\theta)^{2J} + (\sin\theta\sin\phi-i\cos\theta)^{2J}]\}. \tag{2.23}$$

We observe that the interference terms are zero when the direct terms are maximal, that is, for $\theta=\pi/2,\phi=0$ or π, while in the orthogonal plane $\phi=\pi/2$, the direct terms take their minimum value and the interference terms reach their maximum absolute value $(2J+1/4\pi)2^{-2J}$ at $\theta=0$ or π which, however, diminish exponentially with J.

The simplest Schrödinger cat atomic states are superpositions of two distinguishable coherent atomic states which have a clear classical counterpart (dead and alive cat in the language of Schrödinger). To have a complete analogy to the well-known paradox, these superpositions must be entangled with another quantum system (the analog of the radioactive atom). In the procedure for constructing a realization of a Schrödinger cat analyzed here, the N-atom system is entangled to the quantum electromagnetic field. In the next section, a technique for the manipulation of the quantum state of a cavity quantum electromagnetic field is described. A similar procedure was used by Brune *et al.* [5,19] to create electromagnetic cat states and it also constitutes the first step in Gerry and Grobe's proposal.

III. FIRST ATOM PASSING THROUGH THE CAVITY

We consider N identical two-level atoms of frequency ω_a inside a cavity whose characteristic frequency nearest to ω_a is ω_c. The dimensions of the cavity should be much smaller than the wavelength $\lambda_a=2\pi c/\omega_a$ corresponding to the atomic transition. Then the Hamiltonian of the system is approximately given by two terms, the first determining the free behavior of the N two-level atoms and the second representing the free electromagnetic field

$$H_0=\hbar\omega_a J_z+\hbar\omega_c a^\dagger a. \tag{3.1}$$

A different type of two-level atom (from here on called atom 1) is prepared with classical microwave fields in the superposition of states [5]

$$|\chi\rangle_1\equiv\frac{1}{\sqrt{2}}(|g_1\rangle+|e_1\rangle), \tag{3.2}$$

with $|g_1\rangle$ and $|e_1\rangle$ indicating its ground and excited states, respectively. This atom is sent through the cavity where the initial state of the system is a combination of the vacuum state of the electromagnetic field and the N atoms in their ground state. That is, the initial state of the system is,

$$|\psi(0)\rangle=|0\rangle_{\rm em}|J-J\rangle|\chi\rangle_1. \tag{3.3}$$

The atom 1 transition frequency is chosen so that $\omega_1\simeq\omega_c$. Thus, there is an interaction between the atom and the cavity field that, in the rotating wave approximation, valid for weak coupling, can be described by the Hamiltonian

$$H=H_0+\left\{\frac{\hbar\omega_1}{2}\sigma_z^{(1)}+\hbar\Omega(\sigma_+^{(1)}a+\sigma_-^{(1)}a^\dagger)\right\} \times\{\theta(t)-\theta(t-\tau_1)\}, \tag{3.4}$$

where $\theta(t-\tau_1)$ is the step function and τ_1 is the time of flight of the atom passing through the cavity. The coupling frequency Ω is determined by the magnitude of the dipole moment of the atomic transition $\langle e_1|\vec{x}|g_1\rangle$ and is a measure of the strength of the coupling of the atom with the electromagnetic field [20].

In our case the problem reduces to finding the action of the evolution operator on the state (3.3), or equivalently to solving the corresponding time-dependent Schrödinger equation. We follow the procedure used by Jaynes and Cummings [20,21], in which the Hamiltonian is rewritten in terms of a pair of commuting operators. The evolution operator of the system then takes the form

$$U_0=\exp\{-i\omega_a\hat{J}_z\tau_1\}\exp\{-i(\hat{C}_1+\hat{C}_2)\tau_1\}, \tag{3.5}$$

where the first exponential corresponds to the N two-level atoms, while the second characterizes the interaction between the passing atom and the cavity field, that is

$$\hat{C}_1=\omega_c\left(a^\dagger a+\frac{\sigma_z^{(1)}}{2}\right),$$

$$\hat{C}_2=\Omega(\sigma_+^{(1)}a+\sigma_-^{(1)}a^\dagger)-\frac{\Delta\omega}{2}\sigma_z^{(1)}, \tag{3.6}$$

with the definition $\Delta\omega\equiv\omega_c-\omega_1$.

These operators commute, implying that from the set of Fock states for the radiation field and spin eigenstates for the two-level atoms we can construct a representation in which they are both diagonal [20]. The states that diagonalize the two-level atom interacting with a single mode of the radiation field are known as dressed states. In general, their structure can be considered a clear manifestation of the entanglement between the atom and the electromagnetic field.

Here we only need to find the expansion of the states $|0,g_1\rangle$ and $|0,e_1\rangle$, defining the states of the field and the atom, in terms of the eigenstates of the operators (3.6), $|\phi(n,k)\rangle$, with $k=1,2$, i.e.,

$$\hat{C}_1|\phi(0,k)\rangle=\frac{\omega_c}{2}|\phi(0,k)\rangle,$$

$$\hat{C}_2|\phi(0,k)\rangle=(-1)^{k+1}\lambda_0|\phi(0,k)\rangle, \tag{3.7}$$

with $\lambda_0=\sqrt{(\Delta\omega/2)^2+\Omega^2}$.

It is then easy to show that

$$\hat{C}_1|0,g_1\rangle = -\frac{\omega_c}{2}|0,g_1\rangle, \tag{3.8}$$

$$\hat{C}_2|0,g_1\rangle = \frac{\Delta\omega}{2}|0,g_1\rangle, \tag{3.9}$$

and as a consequence the evolution of the state $|0,g_1\rangle$ is determined by the expression

$$\exp\{-i(\hat{C}_1+\hat{C}_2)\tau_1\}|0,g_1\rangle = \exp\left\{-i\left(\frac{\Delta\omega}{2}-\frac{\omega_c}{2}\right)\tau_1\right\}|0,g_1\rangle. \tag{3.10}$$

The state $|0,e_1\rangle$ can be expanded in terms of the kets $|\phi(n,k)\rangle$ as

$$|0,e_1\rangle = \sin\theta_0|\phi(0,1)\rangle + \cos\theta_0|\phi(0,2)\rangle, \tag{3.11}$$

where $\tan 2\theta_0 = 2\Omega/\Delta\omega$. By means of (3.7) and (3.8) we also obtain the result

$$\begin{aligned}\exp\{-i(\hat{C}_1+\hat{C}_2)\tau_1\}|0,e_1\rangle\\ = e^{\{-i\,\omega_c/2\,\tau_1\}}\{(\cos\lambda_0\tau_1 + i\cos 2\theta_0\sin\lambda_0\tau_1)|0,e_1\rangle\\ - i\sin 2\theta_0\sin\lambda_0\tau_1|1,g_1\rangle\}.\end{aligned} \tag{3.12}$$

Therefore, evaluating $|\psi(\tau_1)\rangle = U_0|\psi(0)\rangle$, we get

$$|\psi(\tau_1)\rangle = e^{i\omega_a J\tau_1}|J,-J\rangle|\Theta(\Omega,\omega_c,\theta_0,\tau_1)\rangle, \tag{3.13}$$

where

$$\begin{aligned}|\Theta(\Omega,\omega_c,\theta_0,\tau_1)\rangle\\ = \frac{1}{\sqrt{2}}\{e^{i(\omega_c-\Delta\omega)\tau_1/2}|0,g_1\rangle\\ - ie^{-i\omega_c\tau_1/2}\sin 2\theta_0\sin\lambda_0\tau_1|1,g_1\rangle\\ + e^{-i\omega_c\tau_1/2}(\cos\lambda_0\tau_1 + i\sin\lambda_0\tau_1\cos 2\theta_0)|0,e_1\rangle\}.\end{aligned} \tag{3.14}$$

In resonance, we have $\Delta\omega=0$, $\lambda_0=\Omega$, and $\theta_0=\pi/4$, which simplifies the expression (3.14) for the state of the system to

$$\begin{aligned}|\Theta_1\rangle \equiv |\Theta(\Omega,\omega_c,\pi/4,\tau_1)\rangle\\ = \frac{e^{i\omega_c\,\tau_1/2}}{\sqrt{2}}\{|0,g_1\rangle - ie^{-i\omega_c\tau_1}\sin\Omega\tau_1|1,g_1\rangle\\ + e^{-i\omega_c\tau_1}\cos\Omega\tau_1|0,e_1\rangle\}.\end{aligned} \tag{3.15}$$

The superposition character of the original atom1 state has been now transferred to the cavity's electromagnetic field. Notice that the presence of a photon within the cavity yields an interaction with the N atoms in the cavity which, however, changes the energy spacing between the levels in a negligible way because of the large detuning with the cavity mode.

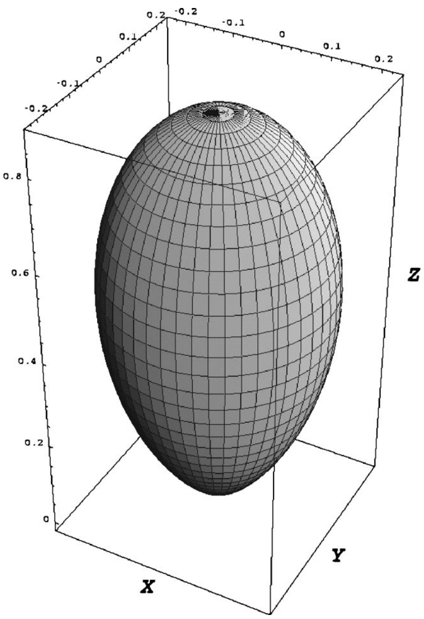

FIG. 2. $Q(\theta,\phi)$ function for five two-level atoms in their ground state. As the number of atoms is increased the spheroid becomes longer and narrower.

The Q function for the N two-level atoms during this step of the procedure

$$Q_{J-J}(\theta,\phi) = \frac{2J+1}{4\pi}\cos^{4J}\theta/2, \tag{3.16}$$

is illustrated in Fig. 2.

IV. GENERATION OF BLOCH STATES

Consider now a classical driving field of constant amplitude ϵ, produced by a single mode laser, applied to the atoms inside the cavity for an interval $(\tau_2-\tau_1)$. This driving field is described, in the rotating wave approximation, by the interaction Hamiltonian

$$H_I = \frac{ik\epsilon}{2}(J_+ - J_-), \tag{4.1}$$

where k is the dipole moment coupling the N atoms to the driving field. The state of the system (3.15) will evolve according to the expression

$$\begin{aligned}|\psi(\tau_1,\tau_2)\rangle\\ = e^{-i\omega_a J_z(\tau_2-\tau_1)+k\epsilon/2\hbar(J_+-J_-)(\tau_2-\tau_1)}e^{i\omega_a J\tau_1}|J,-J\rangle\\ \times e^{-i\omega_c a^\dagger a(\tau_2-\tau_1)}|\Theta_1\rangle,\end{aligned} \tag{4.2}$$

where the evolution operator has been separated into two pieces. The first part has effect only on the state $|J,-J\rangle$. It is clearly related to the rotation (2.8) and will generate an SU(2) coherent or Bloch state. The second part of the evolution operator yields the free evolution of the state (3.14).

We now use a faithful representation for the SU(2) generators (spin operators) to evaluate the Baker-Campbell-Hausdorff formula

$$\exp\left\{-i\omega_a \hat{J}_z(\tau_2-\tau_1)+\frac{k\epsilon}{2\hbar}(\hat{J}_+-\hat{J}_-)(\tau_2-\tau_1)\right\}$$
$$=e^{\alpha_+\hat{J}_+}e^{\alpha_-\hat{J}_-}e^{\alpha_0\hat{J}_z}, \tag{4.3}$$

finding that the parameters are related by the expressions

$$\alpha_0=\ln\left(\cos w_0(\tau_2-\tau_1)+i\frac{\omega_a}{2\omega_0}\sin\omega_0(\tau_2-\tau_1)\right)^{-2}, \tag{4.4}$$

$$\alpha_-=\frac{k\epsilon}{2\hbar\omega_0}\sin\omega_0(\tau_2-\tau_1)\left(\cos\omega_0(\tau_2-\tau_1)\right.$$
$$\left.+i\frac{\omega_a}{2\omega_0}\sin\omega_0(\tau_2-\tau_1)\right), \tag{4.5}$$

$$\alpha_+=\frac{k\epsilon}{2\hbar\omega_0}\frac{\sin\omega_0(\tau_2-\tau_1)}{\left[\cos\omega_0(\tau_2-\tau_1)+i\frac{\omega_a}{2\omega_0}\sin\omega_0(\tau_2-\tau_1)\right]}, \tag{4.6}$$

with

$$\omega_0=\sqrt{\frac{k^2\epsilon^2}{4\hbar^2}+\frac{\omega_a^2}{4}}.$$

The second exponential operator appearing in (4.2) can be evaluated directly.

The state of the system is then given by

$$|\psi(\tau_1,\tau_2)\rangle=e^{i2\zeta_1 J}|\alpha_+;J\rangle|\Theta_{12}\rangle, \tag{4.7}$$

where

$$|\Theta_{12}\rangle=\frac{1}{\sqrt{2}}\{e^{i\omega_c\tau_1/2}|0,g_1\rangle+e^{-i\omega_c\tau_1 2}\cos\Omega\tau_1|0,e_1\rangle$$
$$-ie^{i\omega_c\tau_1/2}e^{-i\omega_c\tau_2}\sin\Omega\tau_1|1,g_1\rangle, \tag{4.8}$$

and the phase takes the value

$$\zeta_1=\arctan\left\{\frac{\omega_a}{2\omega_0}\tan\omega_0(\tau_2-\tau_1)\right\}. \tag{4.9}$$

By introducing the strength parameter $x=(k\epsilon/\hbar\omega_a)$, the parameters θ_+ and ϕ_+ ($\alpha_+=\theta_+/2e^{-i\phi_+}$) of the Bloch state can be written in the form

$$\theta_+=\frac{2x}{\sqrt{(1+x^2)\mathrm{ctan}^2\omega_0(\tau_2-\tau_1)+1}},$$

$$\phi_+=\arctan\left(\frac{1}{\sqrt{1+x^2}}\tan\omega_0(\tau_2-\tau_1)\right). \tag{4.10}$$

For a given value of x, θ_+ and ϕ_+ are periodic functions of time with frequency ω_0. The maximum value $\theta_{max}=2x$ corresponds to $(\tau_2-\tau_1)=(2n+1)(\pi/2\omega_0)$, with $n=0,1,2,\ldots$. In Fig. 3, the direction (θ_+,ϕ_+) as a function of time is shown for interactions with $0\leqslant x\leqslant 1$. It can be observed that the precession of the (θ_+,ϕ_+) axis on the Bloch sphere depends strongly on the application time of the classical fields, particularly for $\omega_0(\tau_2-\tau_1)\sim n\pi$.

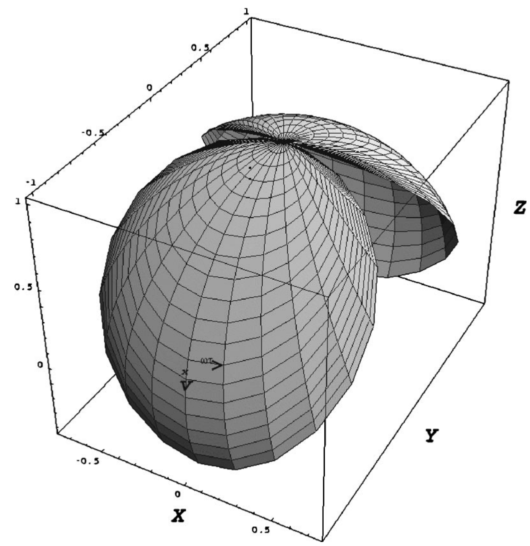

FIG. 3. Precession of the (θ_+,ϕ_+) axis on the Bloch sphere, characterizing the atomic coherent state generated by the classical fields, as a function of $\omega_0(\tau_2-\tau_1)$ and the strength parameter x. For a given value of x, the (θ_+,ϕ_+) axis starts in the z direction for $\omega_0(\tau_2-\tau_1)=0$ and evolves counterclockwise in $\pi/10$ steps. The x parameter is varied from zero to one in steps of 0.05 along the meridians.

Once the atomic Bloch state has been generated, the next step has the purpose of entangling the N-atom system with the cavity electromagnetic field.

V. STARK EFFECT

If an additional electric field is properly applied, the N atoms experience a modification on their energy levels leading to a new transition frequency $\omega_a{}'$ which is again assumed to be different from the characteristic frequency ω_c of the cavity, but near enough to induce a dispersive interaction [9]

$$H=\hbar\omega_a{}'J_z+\hbar\omega_c a^\dagger a+\hbar\chi a^\dagger a J_z. \tag{5.1}$$

The parameter χ is proportional to the square of the dipole atomic moment and inversely proportional to the detuning. Notice that the interaction Hamiltonian does not change either the population of the Dicke atomic system or the number of photons within the cavity.

This Hamiltonian yields an evolution operator which can be factorized as a product of three terms, that is

$$U=e^{-i\chi a^\dagger a J_z(\tau_3-\tau_2)}e^{-i\omega_a{}'J_z(\tau_3-\tau_2)}e^{-i\omega_c a^\dagger a(\tau_3-\tau_2)}. \tag{5.2}$$

The action of the last exponential on the state $|\Theta_{12}\rangle$ can be easily calculated and yields

$$|\Theta_{123}\rangle = \frac{1}{\sqrt{2}}\{e^{i\omega_c\tau_1/2}|0,g_1\rangle + e^{-i\omega_c\tau_1/2}\cos\Omega\tau_1|0,e_1\rangle - ie^{i\omega_c\tau_1/2}e^{-i\omega_c\tau_3}\sin\Omega\tau_1|1,g_1\rangle\}. \tag{5.3}$$

The second exponential operator of expression (5.2) can now be applied to the state $|\alpha_+;J\rangle$. If the result

$$\exp\{-i\omega_a{}'J_z\tau\}\exp\{\alpha_+J_+\}\exp\{i\omega_a{}'J_z\tau\} = \exp\{\alpha_+e^{-i\omega_a{}'\tau}J_+\} \tag{5.4}$$

is used, one obtains

$$\exp\{-i\omega_a{}'J_z(\tau_3-\tau_2)\}|\alpha_+;J\rangle = e^{i\omega_a{}'J(\tau_3-\tau_2)}|\alpha_+{}';J\rangle, \tag{5.5}$$

where $\alpha_+{}' = \alpha_+e^{-i\omega_a{}'(\tau_3-\tau_2)}$. This expression implies that

$$\theta'_+ = \theta_+, \tag{5.6}$$

$$\phi'_+ = \phi_+ + \omega_a{}'(\tau_3-\tau_2). \tag{5.7}$$

Finally, by means of expressions (5.2) to (5.5) the evolution of the state (4.7) is found to be

$$\begin{aligned}|\psi(\tau_1,\tau_2,\tau_3)\rangle &= U|\psi(\tau_1,\tau_2)\rangle\\ &= e^{i2\zeta_1J}e^{i\omega_a{}'J(\tau_3-\tau_2)}e^{-i\chi a^\dagger aJ_z(\tau_3-\tau_2)}|\alpha_+{}';J\rangle|\Theta_{123}\rangle\\ &= e^{i2\zeta_1J}e^{i\omega_a{}'J(\tau_3-\tau_2)}\frac{1}{\sqrt{2}}[e^{i\omega_c\tau_1/2}|0,g_1\rangle|\alpha_+{}';J\rangle\\ &\quad + e^{-i\omega_c\tau_1/2}\cos\Omega\tau_1|0,e_1\rangle|\alpha_+{}';J\rangle\\ &\quad - ie^{i\omega_c\tau_1/2}e^{-i\omega_c\tau_3}e^{i\chi J(\tau_3-\tau_2)}\sin\Omega\tau_1|1,g_1\rangle\\ &\quad \times|\alpha_+{}'';J\rangle],\end{aligned} \tag{5.8}$$

with $\alpha_+{}'' = \alpha_+{}'e^{-i\chi(\tau_3-\tau_2)}$. We thus find that

$$\theta''_+ = \theta_+, \tag{5.9}$$

$$\phi''_+ = \phi_+ + (\omega_a{}' + \chi)(\tau_3-\tau_2). \tag{5.10}$$

The main consequence of this step is that the cavity electromagnetic field and the N two-level atoms are now entangled, that is, the collective atomic state will be affected by the presence or absence of a photon within the cavity. Notice that at this stage the two-level atoms' Q function can be obtained from the reduced density matrix

$$\rho_{\rm red} = \langle 0g_1|\rho|0g_1\rangle + \langle 1g_1|\rho|1g_1\rangle + \langle 0e_1|\rho|0e_1\rangle, \tag{5.11}$$

resulting in

$$Q_{\rm red}(\theta,\phi) = \tfrac{1}{2}[(1+\cos^2\Omega\tau_1)Q_{\theta'_+,\phi'_+}(\theta,\phi) + \sin^2\Omega\tau_1 Q_{\theta''_+,\phi''_+}(\theta,\phi)]. \tag{5.12}$$

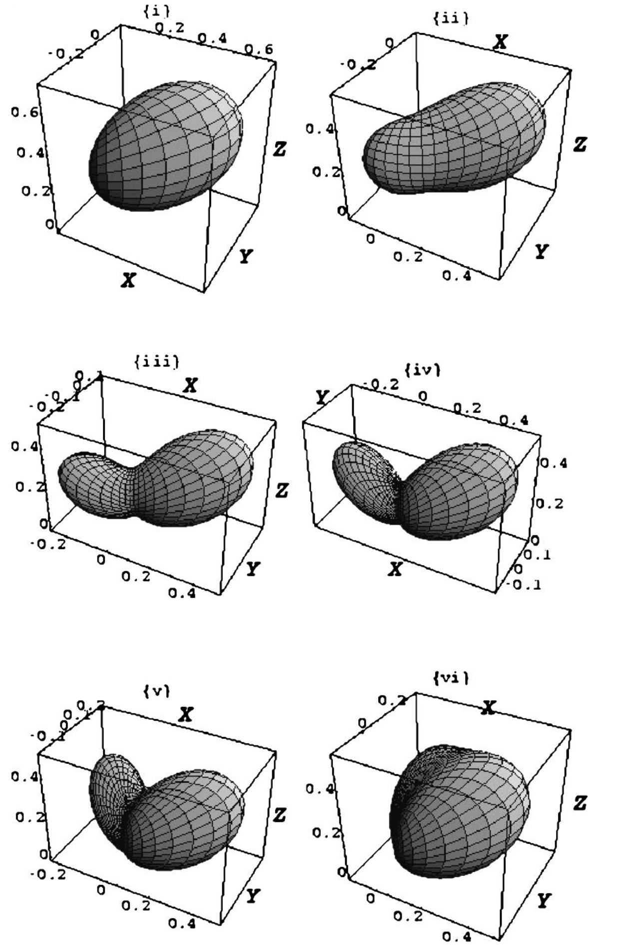

FIG. 4. The evolution of the $Q_{\rm red}(\theta,\phi)$ function, Eq. (5.12), for five two level atoms is displayed, as a function of the values $\chi(\tau_3-\tau_2)=k\pi/4$ with $k=1,2,\ldots,6$. The function is considered to be in a frame rotating around z axis with frequency ω_a and with the parameters $\theta'_+=\pi/4$, $\phi'_+=0$ and $\Omega\tau_1=\pi/4$.

In this case $Q_{\rm red}$ corresponds to the Q function of a mixture of coherent atomic states not exhibiting the characteristic interference terms of an atomic Schrödinger cat state. The behavior of $Q_{\rm red}$ as a function of $\chi(\tau_3-\tau_2)$ is illustrated in Figs. 4. The purpose of the next step is to create Schrödinger cat states of the system.

VI. SECOND ATOM PASSING THROUGH THE CAVITY

A detector can be connected to measure the state of the first atom when it leaves the cavity. Then, depending on wether the atom 1 is in its ground or excited state, the system can be either

$$|\phi_{g_1}\rangle = N_{g_1}\langle g_1|\psi(\tau_1,\tau_2,\tau_3)\rangle \tag{6.1}$$

or

$$|\phi_{e_1}\rangle = N_{e_1}\langle e_1|\psi(\tau_1,\tau_2,\tau_3)\rangle. \tag{6.2}$$

The parameters N_{e_1} and N_{g_1} denote the corresponding normalization factors.

A second two-level atom, prepared to be in its ground state $|g_2\rangle$, is sent through the cavity. Similarly to the first atom, this atom has a transition frequency $\omega_2=\omega_1$ close to the one of the cavity and thus the interaction can be modeled, within the same rotating wave approximation, by a Hamiltonian analogous to that given by Eq. (3.4). By a similar procedure to that outlined in the third section one can find the evolution of these states, leading to the following resonance results

$$|\phi_{g_1}(\tau_1,\tau_2,\tau_3,\tau_4)\rangle = \frac{1}{\sqrt{1+\sin^2\Omega\tau_1}}\{e^{i\omega_c(\tau_4-\tau_3)/2}|0,g_2\rangle|\overline{\alpha'_+};J\rangle - ie^{-i\omega_c(\tau_4+\tau_3)/2}e^{i\chi J(\tau_3-\tau_2)}\sin\Omega\tau_1\{\cos\Omega(\tau_4-\tau_3) \times|1,g_2\rangle - i\sin\Omega(\tau_4-\tau_3)|0,e_2\rangle\}|\overline{\alpha''_+};J\rangle\}, \quad (6.3)$$

where $\overline{\alpha'_+}=\alpha_+{}'e^{-i\omega_a(\tau_4-\tau_3)}$ and $\overline{\alpha''_+}=\alpha_+{}''e^{-i\omega_a(\tau_4-\tau_3)}$, and

$$|\phi_{e_1}(\tau_1,\tau_2,\tau_3,\tau_4)\rangle = |0,g_2\rangle|\overline{\alpha'_+};J\rangle\}. \quad (6.4)$$

We observe that the structure of the wave function has a mixture of Bloch states if, after leaving the cavity, atom 1 is determined to be in the ground state. From now on, we restrict the discussion to this case.

When the second atom is outside the cavity, it can still be manipulated by means of a $\pi/2$ pulse with a resonant field that yields the transformation [9]

$$|e_2\rangle \mapsto \frac{1}{\sqrt{2}}\{|g_2\rangle+|e_2\rangle\}, \quad (6.5)$$

$$|g_2\rangle \mapsto \frac{1}{\sqrt{2}}\{|g_2\rangle-|e_2\rangle\}. \quad (6.6)$$

We denote by $|\tilde{\phi}_{g_1}\rangle$ the state function resulting from applying this transformation to $|\phi_{g_1}\rangle$ Finally, a selective device discriminating the state of the second atom is activated. As a result the system within the cavity will be described by one of the normalized wave functions,

$$|\tilde{\psi}_{g_1,g_2}\rangle = N_{g_2}\langle g_2|\tilde{\phi}_{g_1}\rangle = N_-\{[|\overline{\alpha'_+};J\rangle - \lambda_0|\overline{\alpha''_+};J\rangle]|0\rangle + \lambda_1|\overline{\alpha''_+};J\rangle|1\rangle\}, \quad (6.7)$$

$$|\tilde{\psi}_{g_1,e_2}\rangle = N_{e_2}\langle e_2|\tilde{\phi}_{g_1}\rangle = N_+\{[|\overline{\alpha'_+};J\rangle + \lambda_0|\overline{\alpha''_+};J\rangle]|0\rangle - \lambda_1|\overline{\alpha''_+};J\rangle|1\rangle\}, \quad (6.8)$$

with

$$N_\pm = \left\{1+\sin^2\Omega\tau_1 \pm 2\sin\Omega\tau_1\sin\Omega(\tau_4-\tau_3) \times\Re\left[e^{-i\omega_c\tau_4}\left(\cos\frac{\chi}{2}(\tau_3-\tau_2) + i\cos\theta_+\sin\frac{\chi}{2}(\tau_3-\tau_2)\right)^{2J}\right]\right\}^{1/2}, \quad (6.9)$$

$$\lambda_0 = e^{-i\omega_c\tau_4}e^{i\chi J(\tau_3-\tau_2)}\sin\Omega\tau_1\sin\Omega(\tau_4-\tau_3), \quad (6.10)$$

$$\lambda_1 = -ie^{-i\omega_c\tau_4}e^{i\chi J(\tau_3-\tau_2)}\sin\Omega\tau_1\cos\Omega(\tau_4-\tau_3). \quad (6.11)$$

From expressions (6.7) and (6.8), one concludes that the states of the system will attain the exact form of atomic Schrödinger cat, (2.14), if the following conditions on the strength parameters and application times of the different interactions are satisfied

$$\Omega\tau_1 = (2k_1+1)\frac{\pi}{2}, \quad k_1=0,1,2,\dots,$$

$$\chi(\tau_3-\tau_2) = (2k_2+1)\pi, \quad k_2=0,1,2,\dots,$$

$$\Omega(\tau_4-\tau_3) = (2k_3+1)\frac{\pi}{2}, \quad k_3=0,1,2,\dots, \quad (6.12)$$

where the sum k_1+k_3 must be equal to an even integer. These expressions are related to the time of flight through the cavity of atom 1 and atom 2, and the interval that the electric field yielding the Stark effect is applied to the system, which leads to the result $\overline{\alpha''_+}=-\overline{\alpha'_+}$. Once these conditions are established we find that

$$\omega_c\tau_4 = (2k_4+J)\pi, \quad k_4=0,1,2,\dots. \quad (6.13)$$

By means of (6.12) and (6.13), we can determine the time interval that the classical driving field will act to generate the Bloch state,

$$\tau_2-\tau_1 = \frac{\pi}{\omega_a}\left\{(2k_4+J)\frac{\omega_a}{\omega_c} - (k_1+k_3+1)\frac{\omega_a}{\Omega} - (2k_2+1)\frac{\omega_a}{\chi}\right\}. \quad (6.14)$$

If these conditions are satisfied the states $|\tilde{\psi}_{g_1,e_2}\rangle$ and $|\tilde{\psi}_{g_1,g_2}\rangle$ exactly correspond to the even and odd atomic Schrödinger cat states, respectively. These cat states depend on the angles

$$\overline{\theta'_+} = \theta_+,$$

$$\overline{\phi'_+} = \phi_+ + (2k_2+1)(\pi\omega'_a/\chi) + (2k_3+1)(\pi\omega_a/2\Omega). \quad (6.15)$$

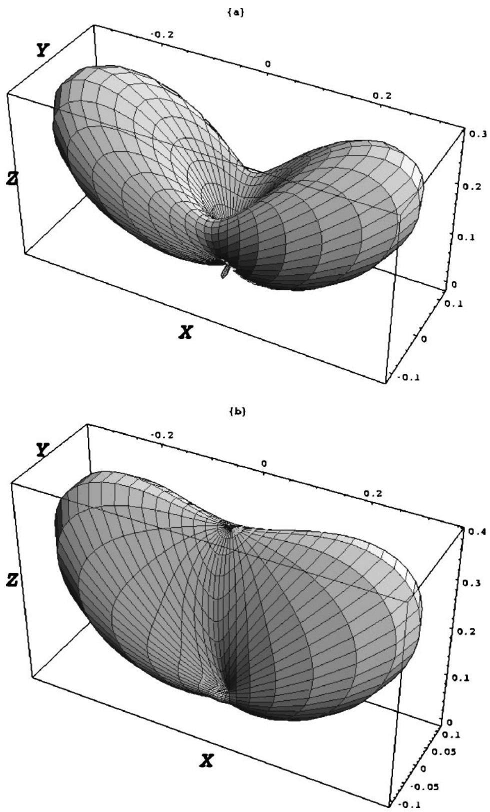

FIG. 5. The $Q^{\pm}(\theta,\phi)$ functions, associated to collective atomic Schrödinger cat states $\Psi_{\pm}(\pi/4,0)\rangle$, of five two level atoms are shown. The Q function for the even cat state is presented in (a) while for the odd cat state it is illustrated in (b).

The Q functions for even and odd atomic Schrödinger cat states are displayed in Figs. 5(a) and 5(b), respectively, which emphasize the influence of the interference terms.

VII. DISCUSSION

In this paper we have made a detailed mathematical analysis of a recent proposal to generate atomic Schrödinger cats. The relevance of some suggested characteristics of the atoms and light used for this purpose has been made evident. Thus it is crucial to detect the state of the first passing atom because unless it corresponds to the ground state no superposition of coherent atomic states will be obtained. Similarly, the time of flight of both atoms should be accurately selected to guarantee the success of the experiment. We may also emphasize some limitations of the procedure. For instance, maximally entangled states (2.16) cannot be produced by this mechanism because the Schrödinger cats generated here are superpositions of coherent states with different phases ϕ, while the entangled state (2.16) corresponds to superpositions with different θ. However, the accessible cat state $|\pi/2,0\rangle \pm |\pi/2,\pi\rangle$ exhibits a Q function similar to that of the state (2.16) and their spectroscopic characteristics should be studied.

The experimental feasibility of the proposal can be analyzed in terms of the parameters involved in current cavity quantum electrodynamics experiments that have been successfully carried out using procedures similar to those studied in this work. In this context, circular Rydberg atoms, as already mentioned by Gerry and Grobe [9], are perhaps the ideal physical realizations of both the two level atoms inside the cavity and the atoms that are sent through it. These kinds of atoms are characterized by a large principal quantum number n together with its corresponding maximum orbital $l=n-1$ and magnetic $m=l$ quantum numbers. These levels can be selectively detected by field ionization. In addition, they have an extremely long lifetime, 30 ms for $n=50$, and their coupling to millimiter wave radiation on a transition between neighboring circular states is very large. For instance, the dipole matrix element takes the value $d=1250$ a.u. for the transition with $\omega_a/2\pi=51.099$ Ghz between the $n=51$ and $n=50$ circular Rydberg states of Rubidium [22]. For this case, the cavity volume V necessary to achieve resonance is of the order of 0.7 cm^3, yielding a very strong atom coupling $\Omega/2\pi\sim 50$ kHz.

TABLE I. Possible application times of the interactions are illustrated in units of $[\mu s]$. In the first row the results correspond to $(2k_4+J)=3.1\times 10^6$ while all the other cases use $(2k_4+J)=10^7$.

	τ_1	$\tau_2-\tau_1$	$\tau_3-\tau_2$	$\tau_4-\tau_3$
$k_1=k_2=k_3=0$	5	1	20	5
$k_1=k_2=k_3=0$	5	70	20	5
$k_1=k_3=1,k_2=0$	15	50	20	15
$k_1=k_3=0,k_2=1$	5	30	60	5
$k_1=k_2=k_3=1$	15	10	60	15
$k_1=k_3=2,k_2=0$	25	30	20	25
$k_1=k_3=3,k_2=0$	35	10	20	35

Using circular Rydberg states and high Q cavities, the resonant atom-cavity entanglement has been experimentally demostrated [23]. In these type of experiments, we find that the atoms are sent through the cavity with velocities around 300 m/s. Different interaction times between the passing atom and the cavity mode can be obtained either by changing the velocity of the atoms or by Stark tuning the atomic transition in resonance during a fraction of the atom-cavity crossing time [24]. Finally the dispersive atom-cavity coupling arising from the nonresonant interaction has also been observed for low Q cavities through the measurement of the corresponding light shift [25]. The dispersive atom-cavity entanglement mentioned in Sec. V for which high Q cavities are necessary is under experimental study [22,26]

Thus most of the steps discussed in this work to obtain atomic Schrödinger cats have already been experimentally explored for circular Rydberg atoms passing through cavities. There is, however, an important point that requires further study: the N two-level atoms are assumed to be confined within the cavity. As a consequence, additional electromagnetic fields for trapping the atoms could be necessary and their effects on atomic levels should be carefully explored.

As an example, we consider Rydberg atoms like Rb. We find the following requirements. (i) The time of flight of atom 1 and atom 2 should be of the same order of magnitude, ranging from 5–35 μs. (ii) The additional electric field to yield the detuning of the N atoms of the cavity has to be applied during an interval of the order of $\tau_3 - \tau_2 \sim 20$–60 μs, for a detuning of 25 kHz. (iii) The single mode laser applied to the cavity to generate the Bloch states must be acting during an interval of the order $\tau_2 - \tau_1 \sim 1 - 70$ μs, which fixes a condition for the sum of the number of atoms inside the cavity and the integer k_4, that is $(2k_4 + J) > 3\times 10^6$. The specific values for the application times depend on the integers k_1, k_2, and k_3 appearing in the expressions (6.12). In Table I, we show some possible sets of conditions on the application times to successfully construct atomic Schrödinger cat states. In these examples the duration of the experiment is in the range $\sim$31–100 μs. Therefore, the cavity in which the experiment should be performed must have a damping time longer than this. In order to guarantee the cat state's formation, extreme care should be taken in the precision of the application times.

We conclude that the experimental realization of this proposal to construct atomic Schrödinger cat states seems to be near from present capabilities.

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