# Atom localization via resonance fluorescence

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We propose a simple scheme of atom localization based on resonance fluorescence from a standing-wave field. The Rabi frequency is position dependent and therefore the spontaneously emitted photon carries the information of the atomic center-of-mass motion. This leads to atom localization even during the flight through the standing-wave field.

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Precision position measurement of atoms has a vast history of interest due to its involvement in many applications like laser cooling, Bose-Einstein condensation, and atom lithography. The experimental progress in using light forces to manipulate the motion of atoms [1] make it more desirable to get high resolution position measurement of atoms with optical techniques.

In recent years, several schemes have been considered for the localization of an atom using the standing optical light field. For example, Storey *et al.* [2] and Marte and Zoller [3] proposed the idea of a virtual ''optical slit.'' In that scheme, the atom is localized by measuring the phase shift of the optical field in a cavity due to the spatially varying atom-field coupling. The localization in position space based on the phase-shift measurement on the field is further investigated via homodyne detection [4,5] by using the method of quantum trajectories [6]. A related technique for the position measurement of the atom is used by Kunze *et al.* [7] in which the phase shift of the atomic dipole, rather than the light field is used. Kien *et al.* further investigated this method and showed that a coherent cavity field substantially enhances the resolution as compared to a classical field [8]. In a recent experiment, Kunze *et al.* [13] demonstrated how the entanglement between the atom's position and its internal state allows one to localize the atom without *directly* affecting the particle's spatial wave function. They reported the possibility of producing narrow localization structures with widths below $\lambda/20$.

Other techniques such as atom imaging methods are proposed by Thomas and co-workers [9–11]. These methods are based on resonance imaging, i.e., a spatially varying potential shifts the resonance frequency of an atomic transition. Therefore the resonance frequency is position dependent and position distribution is determined by spectroscopic methods. They achieved a spatial resolution of 1.7 $\mu$m for the atomic position measurement by using the technique of atom imaging in high magnetic field gradients. Thomas *et al.* further demonstrated that a suboptical wavelength localization can be achieved by using light-shift gradient for atom imaging [12].

In this article we suggest a simple scheme to localize an atom inside the standing wave during its motion. This scheme utilizes the idea that the frequency of the spontaneously emitted photon carries the information about the position of the atom due to its dependence on the position-dependent Rabi frequency of the driving field. Therefore an atom is localized as soon as the spontaneously emitted photon is detected. This scheme presents a simple method for the localization of an atom using a simple two-level atom interacting with the classical standing-wave field. In the presence of the driving field, dynamic Stark splitting of the atomic levels takes place and we get a three-peak resonance fluorescence spectrum. The splitting is directly proportional to the position-dependent Rabi frequency. Our scheme exploits this fact and by measuring the frequency of the spontaneously emitted photon we can localize the atom during its motion through the standing field. It is worthwhile to mention that such a scheme, along with a similar scheme for atom localization based on Autler-Townes spectroscopy [14], affords a direct method to obtain information about the quantum state of the radiation field without any major numerical computations [15,16].

We consider a two-level atom $A$ with energy levels $|a\rangle$ and $|b\rangle$ and transition frequency $\omega_{ab}$ that is described by a center-of-mass wave function $f(x)$. The atom is moving along the $z$ axis and interacts with a resonant standing-wave light field of wave vector $\kappa=\omega_{ab}/c$ aligned along the $x$ direction as shown in Fig. 1. The velocity component of the atom along the $z$ axis is considered large enough so that the motion in this direction is treated classically. The driven atom radiates spontaneously and one of the modes of the scattered light interacts with the detector atom $B$, initially in its ground state. The detector atom consists of the ground level $|\beta\rangle$ and a set of excited levels $|\alpha_{\mathbf{k}}\rangle$. We assume that the scattered light of wave vector $\mathbf{k}_0$ is absorbed by the detector atom and is excited to an appropriate energy level $|\alpha_{\mathbf{k}_0}\rangle$. Our aim is to find the conditional position distribution of the atom $A$, i.e., the conditional probability $W(x;t|\alpha_{\mathbf{k}_0})$ of finding the atom $A$ at position $x$ at time $t$ when the detector atom $B$ is excited to the level $|\alpha_{\mathbf{k}_0}\rangle$.

We assume that the center-of-mass momentum of the atom $A$ along $x$ axis does not change appreciably during its passage through the standing wave. We can then neglect the kinetic energy term for the atom in the Raman-Nath approximation. The interaction Hamiltonian for the atom $A$, in the dipole and rotating-wave approximations, is therefore given by

 

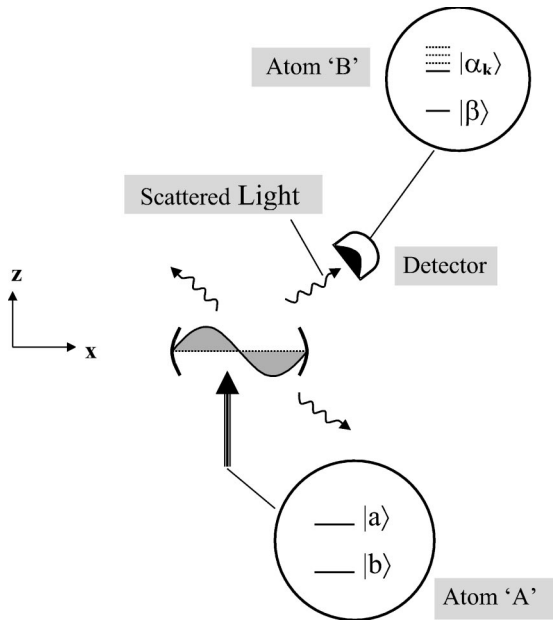


FIG. 1. Two-level atom *A* moving along the *z* axis and interacting with a resonant standing-wave light field of wave vector $\kappa = \omega_{ab}/c$ aligned along the *x* axis. The driven atom *A* radiates spontaneously in all directions. The detector atom *B*, consisting of the ground level $|\beta\rangle$ and a set of excited levels $|\alpha_{\mathbf{k}}\rangle$, absorbs the emitted photon in mode $\mathbf{k}$.

$$H(t)=\hbar g(x)[|a\rangle\langle b|+|b\rangle\langle a|]+\hbar\sum_{\mathbf{k}}\,[g_{\mathbf{k}}(x)|a\rangle \times\langle b|e^{-i(\nu_k-\omega_{ab})t}b_{\mathbf{k}}+g^{*}_{\mathbf{k}}(x)|b\rangle\langle a|e^{i(\nu_k-\omega_{ab})t}b^{\dagger}_{\mathbf{k}}], \tag{1}$$

where $g(x)=G\sin(\kappa x)$ is the position-dependent Rabi frequency, the operators $b_{\mathbf{k}}$ and $b^{\dagger}_{\mathbf{k}}$ are the annihilation and creation operators for the photons in the reservoir modes with frequency $\nu_k=c|\mathbf{k}|$, and $g_{\mathbf{k}}(x)$ is the coupling constant between the atom and the vacuum mode $\mathbf{k}$. The state vector for the complete atom-field system is

$$|\Psi(t)\rangle=\int dx\, f(x)|x\rangle\sum_{n_{\mathbf{q}}}\,[C_{a,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta}(x;t)|a,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta\rangle + C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta}(x;t)|b,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta\rangle + C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}(x;t)|b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}\rangle], \tag{2}$$

where $C_{i,0_{\mathbf{k}_0},n_{\mathbf{q}},\beta}(x;t)$ is the position-dependent probability amplitude with the atom *A* being in the level $|i\rangle$ $(i=a,b)$ with no spontaneously emitted photon present in the mode $\mathbf{k}_0$ and *n* photons present in the mode $\mathbf{q}$, while the detector atom remains in the ground level $|\beta\rangle$. Similarly $C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}(x;t)$ is the probability amplitude for the atom *A* to be in the level $|b\rangle$ after emitting one photon in the $\mathbf{k}_0$th mode and having *n* photons in the mode $\mathbf{q}$; the emitted photon is absorbed by the detector atom *B* exciting it to the state $|\alpha_{\mathbf{k}_0}\rangle$ with no photon left in the $\mathbf{k}_0$th mode.

Our scheme utilizes the fact that the frequency of the spontaneously emitted photon is directly related to the *x*-dependent Rabi frequency of the driving field. We now see how the spectrum of the spontaneously emitted photons or scattered light mimic the position probability of the center-of-mass motion of an atom. The conditional probability $W(x;t|\alpha_{\mathbf{k}_0})$ of finding the atom *A* at position *x* at time *t* when the detector atom excites to the level $|\alpha_{\mathbf{k}_0}\rangle$ is

$$W(x;t|\alpha_{\mathbf{k}_0})=\sum_{n_{\mathbf{q}}}\,|\langle x|\psi_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}\rangle|^2, \tag{3}$$

where

$$|\psi_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}\rangle=\mathcal{N}\langle\alpha_{\mathbf{k}_0}|\langle n_{\mathbf{q}}|\langle 0_{\mathbf{k}_0}|\langle b|\Psi(t)\rangle =\mathcal{N}\int dx\, f(x)C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}(x;t)|x\rangle. \tag{4}$$

Here $\mathcal{N}$ is a normalization factor. Thus the conditional position probability is given by

$$W(x;t|\alpha_{\mathbf{k}_0})\equiv W(x)=|f(x)|^2P(\omega,x,t), \tag{5}$$

with $\omega=|\mathbf{k}_0|/c$ and

$$P(\omega,x,t)=|\mathcal{N}|^2\sum_{n_{\mathbf{q}}}\,|C_{b,0_{\mathbf{k}_0},n_{\mathbf{q}},\alpha_{\mathbf{k}_0}}(x;t)|^2. \tag{6}$$

Here $P(\omega,x,t)$ is the filter function which is directly proportional to the excitation probability of the detector atom. The problem therefore reduces to finding the excitation probability $P(\omega,x,t)$ for a single photon detection.

The detector atom is interacting with the scattered light due to the decay of atom *A*. The interaction picture Hamiltonian for the interaction between the detector atom located at position vector $\mathbf{r}$ and the scattered field $E^{-}(\mathbf{r},t)$, in the rotating-wave approximation, is

$$H_d=-\sum_{\mathbf{k}}\,[\wp_{\alpha_{\mathbf{k}}\beta}\sigma_{\alpha_{\mathbf{k}}\beta}E^{+}(\mathbf{r},t)e^{i\omega t} +\wp^{*}_{\alpha_{\mathbf{k}}\beta}\sigma_{\beta\alpha_{\mathbf{k}}}E^{-}(\mathbf{r},t)e^{-i\omega t}]. \tag{7}$$

For the detector atom initially in its ground state $|\beta\rangle$ and the field in some state $|f\rangle$, the state of the atom-field system at time *t* is given by

$$|\Psi(t)\rangle=U_I(t)|\beta\rangle|f\rangle. \tag{8}$$

We then have

$$|\Psi(t)\rangle=\left[1-\frac{i}{\hbar}\int_{t_0}^{t}dt'H_d(t')\right]|\beta\rangle|f\rangle. \tag{9}$$

The probability of exciting the detector atom to level $|\alpha_{\mathbf{k}_o}\rangle$ is found by calculating the expectation value of the projection operator $|\alpha_{\mathbf{k}_0}\rangle\langle\alpha_{\mathbf{k}_0}|$, i.e.,

$$P(\omega,x,t)=\langle\Psi(t)|\alpha_{\mathbf{k}_0}\rangle\langle\alpha_{\mathbf{k}_0}|\Psi(t)\rangle. \tag{10}$$

The resulting expression for the excitation probability is

$$P(\omega,x,T)=\frac{\wp^2}{\hbar^2}\int_{t_0}^{T}dt_1\int_{t_0}^{T}dt_2\langle E^-(\mathbf{r},t_1)E^+(\mathbf{r},t_2)\rangle \times e^{-i\omega(t_1-t_2)}. \tag{11}$$

This excitation probability $P(\omega,x,T)$ is therefore proportional to the power spectrum of the scattered light [17] emitted from the atom $A$. In the steady state ($T\gg\Gamma^{-1}$) the field emitted by the atom is statistically stationary, i.e., the field correlation function $\langle E^-(\mathbf{r},t_1)E^+(\mathbf{r},t_2)\rangle$ depends only on the time difference $\tau=t_1-t_2$. We then obtain

$$P(\omega,x,\infty)=\frac{1}{\pi}\mathrm{Re}\int_0^\infty d\tau\langle E^-(\mathbf{r};t)E^+(\mathbf{r};t+\tau)\rangle e^{+i\omega\tau}. \tag{12}$$

As the field operators $E^-(\mathbf{r},t)$, $E^+(\mathbf{r},t)$ are proportional to the atomic operators $\sigma_+(t)$, $\sigma_-(t)$, respectively, we obtain

$$\langle E^-(\mathbf{r};t)E^+(\mathbf{r};t+\tau)\rangle=I_0(\mathbf{r})\langle\sigma_+(t)\sigma_-(t+\tau)\rangle, \tag{13}$$

where $I_0(\mathbf{r})$ is a constant. The two-time correlation function of atomic dipole operator $\langle\sigma_+(t)\sigma_-(t+\tau)\rangle$ can be calculated by using the quantum regression theorem. It follows on taking the Fourier transform of $\langle\sigma_+(t)\sigma_-(t+\tau)\rangle$ that the power spectrum of fluorescence light is [18]

$$P(\omega,x,\infty)=\frac{I_0(r)}{4\pi}\left(\frac{4g^2(x)}{\Gamma^2+8g^2(x)}\right)\left[\frac{4\pi\Gamma^2}{\Gamma^2+8g^2(x)}\delta(\Delta)+\frac{\Gamma}{\Delta^2+(\Gamma/2)^2}+\frac{c_+}{(\Delta+\mu)^2+(3\Gamma/4)^2}+\frac{c_-}{(\Delta-\mu)^2+(3\Gamma/4)^2}\right], \tag{14}$$

where $\Delta=\omega_{ab}-\omega$, $\mu=\sqrt{4g^2(x)-\Gamma^2/16}$, and

$$c_\pm=\frac{3\Gamma}{4}\left[\frac{8g^2(x)-\Gamma^2}{8g^2(x)+\Gamma^2}\right]\pm(\Delta\pm\mu)\frac{\Gamma}{4\mu}\left[\frac{40g^2(x)-\Gamma^2}{8g^2(x)+\Gamma^2}\right]. \tag{15}$$

The expression for $P(\omega)$ simplifies considerably when the Rabi frequency is much larger than the decay rate of the atom i.e., $g(x)\gg\Gamma$. The resulting expression for the spectrum is

$$P(\omega,x,\infty)=\frac{I_0(r)}{8\pi}\left[\frac{3\Gamma/4}{[\Delta+2g(x)]^2+(3\Gamma/4)^2}+\frac{\Gamma}{\Delta^2+(\Gamma/2)^2}+\frac{3\Gamma/4}{[\Delta-2g(x)]^2+(3\Gamma/4)^2}\right]. \tag{16}$$

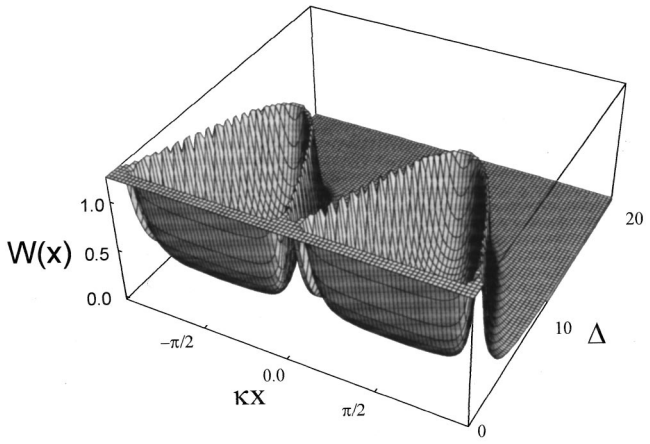


FIG. 2. Conditional position probability distribution $W(x)$ as a function of normalized position $\kappa x(0\leqslant\kappa x\leqslant 2\pi)$, and detuning $\Delta$, for $g(x)\gg\Gamma$. For $\Delta=0$ there is a uniform position probability distribution over the wavelength domain of the standing wave. By an increase in $\Delta$, maxima corresponding to atom localization at different positions inside the standing wave (initially near the nodes of the standing wave) for four different values of $g(x)$, are observed. These maxima move away from the nodes with increasing detuning. For $\Delta=\pm 2G$, four maxima merge into two and lie on the antinodes of the standing wave and for $|\Delta|>2G$ no resonances exist and a flat position distribution over the wavelength domain is obtained.

This is the well-known three-peak Mollow spectrum, the only difference being the position dependence of the Rabi frequency.

In the resonance fluorescence spectrum we have three peaks centered at $\Delta=0$ and $\Delta=\pm 2g$. In our scheme of localization of an atom we replace the Rabi frequency $g$ with the position-dependent Rabi frequency $g(x)=G\sin(\kappa x)$. The peaks are now $x$ dependent and are located at $\Delta=0$ and $\Delta=\pm 2G\sin(\kappa x)$. The atom now undergoes a different Rabi oscillation at a different position in a standing wave and we get maxima in the position distribution corresponding to these Rabi frequencies. In Fig. 2 we show a three-dimensional plot of the conditional position distribution $W(x)$ for an initially broad wave packet as a function of the normalized position $\kappa x$ and detuning $\Delta$. We note that for zero detuning there is a uniform position probability distribution over the wavelength domain of the standing wave. This is due to the fact that the atom exhibits a peak at $\Delta=0$ for any value of Rabi frequency, and hence for all values of $\kappa x$. The heights of the peaks for all values of position are the same and we therefore obtain a uniform position distribution. Thus the conditional position distribution provides no information about the atom localization for $\Delta=0$. An increase in detuning corresponds to the localization of the atom at different positions inside the standing wave, depending on the value of the position-dependent Rabi frequency $g(x)$. We obtain four maxima of same heights and widths in the region $0\leqslant\kappa x\leqslant 2\pi$ located at $\kappa x=\pm\sin^{-1}(\Delta/2G)\pm n\pi$ ($n=0,\pm 1$). For small values of $\Delta$, these maxima are located near the nodes of the standing wave. However, with the increased detuning these peaks move towards the antinodes of the standing wave. For $\Delta=\pm 2G$, four maxima merge into two

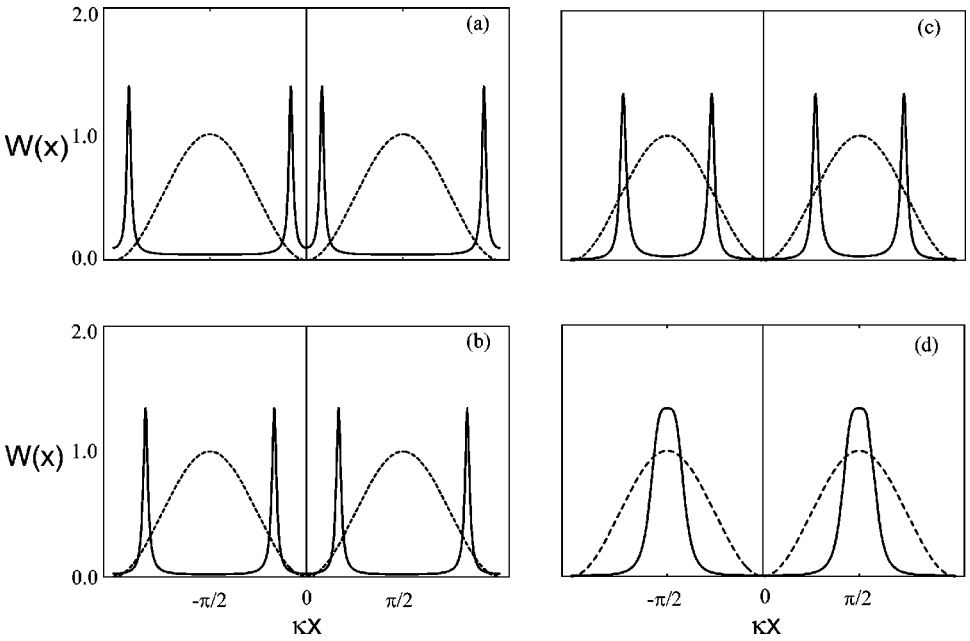


FIG. 3. Conditional position distribution $W(x)$ with $G/\Gamma=10$ for (a) $\Delta/\Gamma=5$, (b) $\Delta/\Gamma=10$, (c) $\Delta/\Gamma=15$, and (d) $\Delta/\Gamma=20$, clearly shows the dependence of position information on the detuning $\Delta$. The solid line corresponds to the conditional position distribution $W(x)$ and the dotted line corresponds to the standing wave. Hence position information is available in the subwavelength domain of the standing-light field.

and lie on the antinodes of the standing wave. There are noresonances for $|\Delta|>2G$ and we obtain a flat position distribution over the wavelength domain.

These results indicate a strong correlation between the detuning of the scattered light and the position of the atom. The measurement of a particular frequency corresponds to the localization of the atom in a subwavelength domain of the standing wave.

A clearer picture of the dependence of the localization scheme of an atom on the position-dependent Rabi frequency and detuning is demonstrated in Figs. 3(a)–3(d), where we show two-dimensional plots of the conditional position distribution $W(x)$ as a function of normalized position $\kappa x$ (ranging from $-\pi\rightarrow\pi$) for four different values of detuning, i.e., $\Delta/\Gamma=5,10,15,20$. The amplitude of the position-dependent Rabi frequency is taken to be $G/\Gamma=10$. It is clear from these plots that the best resolved peak is obtained at $\Delta=G$ for which the signal-to-background ratio is maximum. We get a partial overlap of the adjacent peaks for the ranges $0<|\Delta|<G$ and $G<|\Delta|<2G$. This causes an enhancement of the background. The strength of these overlaps and, consequently, the signal-to-background ratio depends on how much the detuning deviates from the maximum value of $G$. However, a complete overlap is observed for $\Delta=0$ and $\pm 2G$, which corresponds to the node and antinode, respectively.

We also investigate the dependence of the width of the best resolved peaks, for which the signal-to-background ratio is maximum, on the amplitude of the position-dependent Rabi frequency $g(x)=G\sin(\kappa x)$ (Fig. 4). It is noted that the width decreases with the increase in the amplitude $G$ of the position-dependent Rabi frequency. The figure shows that the decrease in the width of the peak is very sharp for the values of $G/\Gamma$ ranging from $2\rightarrow 20$. Outside this limit the width decreases slowly to a certain minimum value and stays practically asymptotic for $G/\Gamma>100$. This happens because, in this regime, the amplitude of the Rabi frequency is very large and the effect of the spontaneous emission, i.e., the linewidth, is minimized.

The spatial resolution in our scheme depends on the ratio of $G/\Gamma$ and we must get a better spatial resolution for high Rabi frequency $G$ as compared to the decay rate $\Gamma$. Apart from the periodicity of the standing wave which results four peaks in a conditional position distribution within a unit wavelength, a spatial resolution of $\approx\lambda/60$ can be achieved for a ratio of $G/\Gamma=10$. This is a reasonable approximation for the ratio $G/\Gamma$ because recent experiments in the optical region on the realization of single atoms in the cavity QED reported a ratio of $G/\Gamma$ of 8 [19] and in a more recent work it is enhanced to approximately 20 [20].

Here we mention again that the above power spectrum gives the conditional position distribution, i.e., the position information is conditioned on the measurement of the frequency of the emitted light. The frequency $\omega$ of the spontaneously emitted photon is related to the detuning parameter $\Delta$, as $\omega=\omega_{ab}-\Delta$, where $\Delta=\pm 2g(x)$. Hence the detection of the spontaneously emitted photon gives the immediate information about the position of the atom inside the opti-

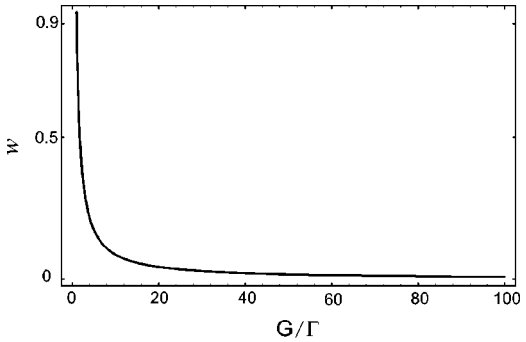


FIG. 4. Plot of width ($w\approx\kappa\Delta x$) versus $G/\Gamma$ for the best resolved peaks ($\Delta=G$) in the conditional position distribution $W(x)$. The plot shows a strong dependence of $w$ on the amplitude of the position-dependent Rabi frequency $G$.

calfield. Although the spontaneous-emission process is isotropic in nature and would require the use of $4\pi$ detectors in principle, for practical purposes it is not necessary to measure every atom.

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[1] See the special issue on atom optics, edited by J. Mlynek, V. Balykin, and P. Meystre, Appl. Phys. B: Photophys. Laser Chem. **54**, 319 (1992).
[2] P. Storey, M. Collett, and D. F. Walls, Phys. Rev. Lett. **68**, 472 (1992); P. Storey, M. Collett, and D. F. Walls, Phys. Rev. A **47**, 405 (1993).
[3] M. A. M. Marte and P. Zoller, Appl. Phys. B: Photophys. Laser Chem. **54**, 477 (1992).
[4] R. Quadt, M. Collett, and D. F. Walls, Phys. Rev. Lett. **74**, 351 (1995).
[5] A. M. Herkommer, H. J. Carmichael, and W. P. Schleich, Quantum Semiclassic. Opt. **8**, 189 (1996).
[6] H. J. Carmichael, *An Open Systems Approach to Quantum Optics*, Lecture Notes in Physics (Springer-Verlag, Berlin, 1993).
[7] S. Kunze, G. Rempe, and M. Wilkens, Europhys. Lett. **27**, 115 (1994).
[8] F. L. Kien, G. Rempe, W. P. Schleich, and M. S. Zubairy, Phys. Rev. A **56**, 2972 (1997).
[9] J. E. Thomas, Opt. Lett. **14**, 1186 (1989); J. E. Thomas, Phys. Rev. A **42**, 5652 (1990).
[10] K. D. Stokes, C. Schnurr, J. R. Gardner, M. Marable, G. R. Welch, and J. E. Thomas, Phys. Rev. Lett. **67**, 1997 (1991).
[11] J. R. Gardner, M. L. Marable, G. R. Welch, and J. E. Thomas, Phys. Rev. Lett. **70**, 3404 (1993).
[12] For a comprehensive report on exisiting schemes and some suggested methods for precision position measurement of moving atoms in beams and traps, see J. E. Thomas and L. J. Wang, Phys. Rep. **262**, 311 (1995). This report also describes the applications of atom imaging to measure atomic interference patterns and potential methods for correlated position measurement of multiple atoms.
[13] S. Kunze, K. Dieckmann, and G. Rempe, Phys. Rev. Lett. **78**, 2038 (1997).
[14] A. M. Herkommer, W. P. Schleich, and M. S. Zubairy, J. Mod. Opt. **44**, 2507 (1997).
[15] M. S. Zubairy, Phys. Lett. A **222**, 91 (1996).
[16] M. S. Zubairy, Phys. Rev. A **57**, 2066 (1998).
[17] J. H. Eberly and K. Wodkiewicz, J. Opt. Soc. Am. **67**, 1252 (1977).
[18] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997).
[19] Q. A. Turchette, R. A. Thompson, and H. J. Kimble, Appl. Phys. B: Lasers Opt. **60**, S1 (1995).
[20] C. J. Hood, M. S. Chapman, T. W. Lynn, and H. J. Kimble, Phys. Rev. Lett. **80**, 4157 (1998).