Spin monopoles with Bose-Einstein condensates

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We study the feasibility of preparing a Bose-Einstein condensed sample of atoms in a macroscopic quantum state that resembles a *spin monopole*. In this state, the atomic internal spins lie in the x-y plane and point along the radial direction. The stability and dynamics of this structure are studied analytically in some cases and numerically in the more general situation. We find these structures to be stable objects giving rise to a nontrivial state of a Bose-Einstein condensate.

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I. INTRODUCTION

After the successful generation of Bose-Einstein condensation of alkali-metal atoms, the creation and manipulation of macroscopic quantum states remains as one of the fundamental goals in the field of atomic physics [1]. During recent years, several ways to create vortices [2,3] and solitons [2,4] with atomic Bose-Einstein condensates (BECs) have been proposed. These nonlinear states have been observed in many other contexts, such as condensed matter or nonlinear optical systems, and are characteristic of systems described by nonlinear partial differential equations, like the Gross-Pitaevskii equation (GPE) giving the evolution of an atomic condensate. In this last system, it has been shown that solitons and vortices may become dynamically stable [5], which has motivated a strong experimental effort to generate them and verify some of their properties.

On the other hand, the first experiments in which atoms were condensed in two or more internal states [6] opened up a new field for the theoretical analysis of Bose-Einstein condensates [13,14]. In particular, the experiments offer the possibility of creating macroscopic quantum states incorporating different kinds of topological defects. In multicomponent systems there is a vector degree of freedom that leads to new structures sometimes called vector structures or vectorial defects when they are related in some way to singularities of any of the components of the vector field. In the JILA experiments the components of the vector states are characterized by the value of the internal atomic state (spin) at each point of space and cannot easily be created experimentally in other systems. There is a vast literature [7] on theoretical properties of those kind of states, although only a few of them have been observed. One of the few examples found in BECs are the recently obtained vortices in multicomponent systems [8]. However, other cases are known in other fields, such as the vortex vector solitons [9] and dipole-mode solitons of nonlinear optics [10] and the vectorial defects and patterns in laser physics [11].

In this paper we study a macroscopic quantum state for a multicomponent BEC that we call a *spin monopole*. It is a state in which the atomic spin points in the radial direction in the x-y plane [Fig. 1(a)]. This has certain (mostly graphical) analogies with the never found magnetic monopoles that motivate the choice of the name ''spin monopole.'' We must stress that the same name has been used to describe different but somewhat related nontrivial objects found in BECs [12]. We will show that a spin monopole is stable under realistic conditions and analyze a method to generate it that requires only current experimental technology.

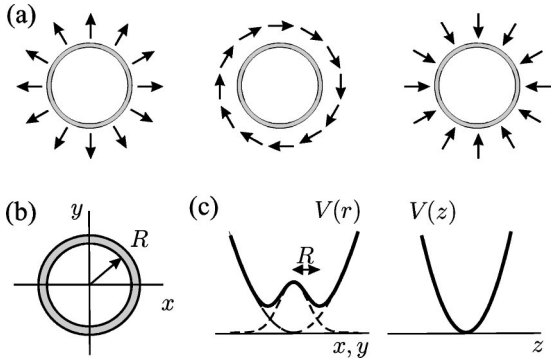

FIG. 1. (a) Spin monopole in two dimensions. For $\mu \neq 0$ the spins rotate as a function of time. (b) Ring trap. (c) Potential $V(r,z)$.

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II. SPIN MONOPOLES IN A RING

A. Model

We consider a Bose-Einstein condensed sample of N atoms within the mean field approximation, i.e., in the limit in which it can be described by Gross-Pitaevskii models. The atoms have two internal levels, $|\uparrow\rangle$ and $|\downarrow\rangle$, with N_1 and N_2 atoms in each. The atomic motion is confined by an external trap with a ring shape [see Fig. 1(b)] [15]. If the motion along the radial (r) and axial (z) direction is frozen, the dynamics of the motional state depends only on the polar angle (θ). We write the wave function as

$$|\Psi(\theta,\tau)\rangle=\sqrt{N_1}\phi_1(\theta,\tau)|\uparrow\rangle+\sqrt{N_2}\phi_2(\theta,\tau)|\downarrow\rangle, \qquad (1)$$

where $\phi_{1,2}$ are the motional wave functions corresponding to the internal states $|\uparrow,\downarrow\rangle$, respectively, and fulfill the coupled GPE

$$i\frac{\partial}{\partial\tau}\phi_1=\left(-\frac{\partial^2}{\partial\theta^2}+\delta_1+u_{11}|\phi_1|^2+u_{21}|\phi_2|^2\right)\phi_1, \quad (2a)$$

$$i\frac{\partial}{\partial\tau}\phi_2=\left(-\frac{\partial^2}{\partial\theta^2}+\delta_2+u_{12}|\phi_1|^2+u_{22}|\phi_2|^2\right)\phi_2, \quad (2b)$$

with normalization

$$\frac{1}{2\pi}\int_0^{2\pi}d\theta|\phi_{1,2}(\theta,\tau)|^2=1. \qquad (3)$$

In this model, $\delta_{1,2}$ denotes the energy of the two internal states and $u_{ij}=u_{ji}$ $(i,j=1,2)$ describe their mutual interactions. All the quantities in Eqs. (2) have been rescaled so that they are dimensionless. In particular, the u_{ij} are proportional to the number of atoms $N_{1,2}$, the corresponding scattering length, and the square of the ring radius [see Eq. (11a) below]. The normalization of each component is preserved during evolution since the type of coupling present in Eq. (2) does not lead to transfer of particles between spin polarizations.

We can find stationary solutions [16] of Eqs. (2) such that

$$\phi_1^{\rm mp}(\theta,\tau)=e^{-i\mu_1\tau}, \quad \phi_2^{\rm mp}(\theta,\tau)=e^{i\theta}e^{-i\mu_2\tau}, \qquad (4)$$

with $\mu_j=j-1+u_{1j}+u_{2j}+\delta_j$ $(j=1,2)$. Defining the Pauli operator $\vec{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ as usual, we see that the state (1) with (4) is an eigenstate of the operator $\vec{\sigma}\cdot\vec{n}$ with

$$\vec{n}=\begin{pmatrix}2\sqrt{N_1N_2}\cos(\theta-\mu\tau)\\ 2\sqrt{N_1N_2}\sin(\theta-\mu\tau)\\ N_1-N_2\end{pmatrix} \qquad (5)$$

and $\mu\equiv\mu_2-\mu_1$. This means that for $\mu=0$ the projections of the spins are always pointing outward in the x-y plane, whereas for other conditions they rotate as shown in Fig. 1(a).

B. Stability

To analyze the stability of the ring spin monopole we have considered a small perturbation around the monopole solution, such that $\phi_{1,2}(\theta,\tau)=\phi_{1,2}^{\rm mp}(\theta,\tau)+\epsilon\alpha_{1,2}(\theta,\tau)$. Inserting this expression into Eq. (2) and expanding up to first order in ϵ we obtain a linear set of coupled equations for $\alpha_{1,2}$ and $\alpha_{1,2}^*$. Writing $\alpha_{1,2}(\theta,\tau)=\Sigma_{n=-\infty}^{\infty}\alpha_{1,2}^{(n)}(\tau)e^{in\theta}$ and decoupling independent terms, we obtain an infinite family of ordinary differential equations

$$i\frac{\partial}{\partial\tau}\vec{\alpha}^{(n)}=E\mathcal{H}_n\vec{\alpha}^{(n)}, \qquad (6)$$

where the variable is a column vector:

$$\vec{\alpha}^{(n)}=[\alpha_1^{(n)},\alpha_1^{(-n)*},\alpha_2^{(n+1)},\alpha_2^{(-n+1)*}]^{\rm T}. \qquad (7)$$

Here, $\mathcal{H}_n\equiv K_n+H^{\rm int}$ rules the evolution, and is decomposed into 4×4 matrices,

$$E={\rm diag}(1,-1,1,-1), \qquad (8a)$$

$$K_n={\rm diag}[n^2,n^2,(n+1)^2-1,(n-1)^2-1], \qquad (8b)$$

$$H^{\rm int}=\begin{pmatrix}u_{11}&u_{21}\\ u_{12}&u_{22}\end{pmatrix}\otimes\begin{pmatrix}1&1\\ 1&1\end{pmatrix}, \qquad (8c)$$

where $\otimes$ denotes the tensor product.

The stability analysis can be carried out by diagonalizing the matrices $E\mathcal{H}_n$. Complex eigenvalues $\lambda_r\pm i\lambda_i$ correspond to exponentially growing solutions $[\alpha(\tau)=\alpha(0)e^{i\lambda_r\tau}e^{\lambda_i\tau}]$ and thus correspond to stationary solutions that are *dynamically unstable*. Real eigenvalues λ lead to small oscillatory solutions $[\alpha(\tau)=\alpha(0)e^{\pm i\lambda\tau}]$, which may correspond to higher $(\lambda>0)$ or lower $(\lambda<0)$ values of the Gross-Pitaevskii energy functional with respect to the stationary solution. In the first case, the stationary solution is *stable*, whereas in the latter the system is *energetically unstable*; this means that if energy can be drained out from the system at a given rate, then the stationary solution is unstable on that time scale. Finally, when one of the matrices is not simple (i.e., nondiagonalizable), there are solutions that grow at most polynomically with time [for example, for a 2×2 Jordan matrix corresponding to a degenerate eigenvalue $\lambda,\alpha(\tau)=\alpha(0)(1+\lambda\tau)$]. This can be regarded as a *dynamical instability with a much slower time scale*.

On the other hand, there is a simple sufficient condition for the stability of the system, which involves the positivity of $\mathcal{H}_n$: *If all matrices $\mathcal{H}_n\geqslant0$ and the projector operator P_0^n on the kernel of $\mathcal{H}_n$ commutes with E for all n, then the stationary solution is stable*. The proof is as follows: first, the positivity of $\mathcal{H}_n$ ensures the positivity of the Gross-Pitaevskii energy functional and therefore there cannot be energetic instabilities; secondly, all eigenvalues of $E\mathcal{H}_n$ are also eigenvalues of $\mathcal{H}_n^{1/2}E\mathcal{H}_n^{1/2}$, which is Hermitian (has real eigenvalues) and therefore there cannot be dynamical instabilities; finally, if $[P_0^n,E]=0$ one can easily prove that if a

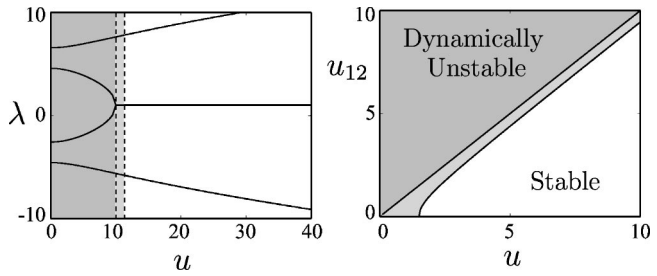

FIG. 2. Left panel: Excitation frequencies as a function of u for $u_{12}=10$. Right panel: Stability diagram.

vector belongs to the kernel (range) of $\mathcal{H}_n$, then it also belongs to the kernel (range) of $E\mathcal{H}_n$, and therefore this last matrix is simple.

We will apply the preceding considerations to our system. We distinguish two situations. The first one is $u_{11}u_{22}>u_{12}^2$. Given that $K_n \geqslant 0$ for $|n|>2$, the theorem above allows us to concentrate on $|n|=1$. Furthermore, since the first three diagonal minors of $\mathcal{H}_n$ are positive, we only have to impose that the determinant be positive to get absolute stability. The conclusion is that if $u_{11}u_{22}-u_{12}^2+u_{22}/2-3u_{11}/2-3/4>0$ the monopole from Eq. (4) is stable. Let us investigate which kind of instabilities occur in the opposite case. We will again restrict ourselves to $|n|=1$ and find the eigenvalues of the matrix $E\mathcal{H}_1$. In the relevant case, which is $u_{11}=u_{22}\equiv u$ (see below), this can be done analytically. We obtain the result that the eigenvalues are always real, either positive or negative:

$$\lambda = 1 \pm [2(1+u \pm \sqrt{1+2u+u_{12}^2})]^{1/2}. \tag{9}$$

The second case is $u_{11}u_{22}<u_{12}^2$. With this type of parameter, a numerical inspection of the matrices shows that at least a pair of the eigenvalues is complex. Indeed, if we assume once more that $u_{ii}=u$, then the eigenvalues of $E\mathcal{H}_1$ have the same analytic expression (9), which has nonreal roots. Therefore, in this case the system is dynamically unstable.

Summing up, the scenario is as follows:

$$u_{12}^2 \leqslant u^2-u-3/4, \quad \text{stable},$$
$$u^2-u-3/4<u_{12}^2<u^2, \quad \text{energetically unstable},$$
$$u^2<u_{12}^2, \quad \text{dynamically unstable}.$$

The eigenvalues as well as the stability diagram are shown in Fig. 2. For $u>u_{12}$ and when the interaction energy becomes more important than the (rotational) kinetic energy ($u \gg 1$), the solution is stable as long as $u-u_{12} \gtrsim 1$. Since both u and u_{12} are proportional to the number of atoms N, one can completely stabilize the spin monopole by increasing N. As a side note, an interesting situation occurs for $u=u_{12}$; in that case the matrix $E\mathcal{H}_1$ is not simple, which implies that the perturbation grows only linearly with time. Finally, we must say that the dynamical instability occurring for $u<u_{12}$ also appears for the homogeneous stationary solution and therefore it simply corresponds to a *phase separation* of the two components (internal states), as could be expected.

III. SPIN MONOPOLE IN THREE DIMENSIONS

A. Model

In order to establish whether the monopole remains stable in a realistic three–dimensional situation, we consider a trapping potential of the form

$$V(r,z)=\tfrac{1}{2}m\omega_z^2 z^2+\tfrac{1}{2}m\omega_r^2 r^2+V_0 e^{-r^2/(2\sigma^2)} \tag{10}$$

[Fig. 11(c)]. This corresponds to a standard dipole trap with an off-resonant Gaussian laser beam propagating along the z direction [15]. V_0 gives the ac Stark shift at the center of the trap and determines the equilibrium point R along the radial direction, which is given by $R=2\sigma^2 \ln[V_0/(m\omega_r^2\sigma^2)]$. In order to have a truly ring trap, we choose the parameters such that $R \gg \Delta r$, where Δr is the typical size of the atomic cloud along the z and r directions. Writing $|\Psi(r,z,\theta)\rangle = \psi_1(r,z,\theta)|\uparrow\rangle + \psi_2(r,z,\theta)|\downarrow\rangle$, the Gross-Pitaevskii equations governing the evolution become

$$i\hbar\frac{\partial}{\partial t}\psi_1=\left(-\frac{\hbar^2\nabla^2}{2m}+V+\hbar\tilde{\delta}_1+\tilde{u}_{11}|\psi_1|^2+\tilde{u}_{12}|\psi_2|^2\right)\psi_1,$$

$$i\hbar\frac{\partial}{\partial t}\psi_2=\left(-\frac{\hbar^2\nabla^2}{2m}+V+\hbar\tilde{\delta}_2+\tilde{u}_{21}|\psi_1|^2+\tilde{u}_{22}|\psi_2|^2\right)\psi_2.$$

Here $\tilde{\delta}_{1,2}$ are two constant offsets, the nonlinearity is parametrized by $\tilde{u}_{ij}=4\pi\hbar^2 a_{ij}/m$, and $a_{ij}=a_{ji}$ are the s-wave scattering lengths corresponding to the different collisions. Finally, the functions $\psi_{1,2}$ are normalized to the number of atoms $N_{1,2}$ in each internal state.

Let us first make the connection between the three-dimensional (3D) model and the ring. We consider the simple situation in which the motions along the radial and z direction are frozen. The conditions of validity of this situation will be discussed below. If the number of atoms in each internal level is the same, $N_1=N_2\equiv N/2$, we can reduce the full three-dimensional problem to the ring case studied above by writing $\psi_{1,2}(r,z,\theta,t)=[N_{1,2}/(4\pi)]^{1/2}f_{1,2}(r,z)\phi_{1,2}(\theta,t)$, multiplying the coupled GPE by f_i^*, and integrating in r and z. We obtain the result that $\phi_{1,2}$ satisfy the Eqs. (2) with

$$u_{ij}=2\sqrt{N_i N_j}R^2 a_{ij}\int_{-\infty}^{\infty}dz\int_0^{\infty}r\,dr|f_i(r,z)|^2|f_j(r,z)|^2, \tag{11a}$$

$$\delta_i=\frac{2mR^2}{\hbar^2}(\hbar\tilde{\delta}_i+\epsilon_i), \quad \tau=\frac{\hbar}{2mR^2}t, \tag{11b}$$

where ϵ_i is the expectation value of the kinetic plus potential energy with the wave function f_i.

In general, we look for stationary solutions with a singularity on the z axis:

$$\psi_1(r,z,\theta,t)=\sqrt{N_1/2}f_1(r,z)e^{-i\tilde{\mu}_1 t}, \tag{12a}$$

$$\psi_2(r,z,\theta,t)=\sqrt{N_2/2}f_2(r,z)e^{i\theta}e^{-i\tilde{\mu}_2 t}. \tag{12b}$$

Here $f_{1,2}$ are normalized real functions satisfying $\tilde{L}_{1,2}f_{1,2}=0$, where

$$\tilde{L}_n=-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{d^2}{dz^2}-\frac{(n-1)^2}{r^2}\right)+V(r,z)-\hbar\tilde{\delta}_n-\hbar\mu_n+\tilde{u}_{1n}f_1^2+\tilde{u}_{2n}f_2^2. \quad (13)$$

B. Stability

We will concentrate on the case $\tilde{u}_{11}=\tilde{u}_{22}\equiv\tilde{u}$ (equivalently, $a_{11}=a_{22}\equiv a$), which is the relevant experimental situation described below. In the limit $R\gg\Delta r$ the centrifugal term in $\tilde{L}_2$ can be approximated by a constant $\hbar^2/(2mR^2)$. If there is no phase separation ($a>a_{12}$) we find $f_1(r,z)=f_2(r,z)\equiv f(r,z)$ and $\tilde{\mu}\equiv\tilde{\mu}_2-\tilde{\mu}_1=\hbar/(2mR^2)+\tilde{\delta}_2-\tilde{\delta}_1$. Solution (12) is again an eigenstate of $\vec{n}\cdot\vec{\sigma}$ with $\vec{n}=[\cos(\theta-\tilde{\mu}t),\sin(\theta-\tilde{\mu}t),0]$, being the eigenvalue proportional to $|f(r,z)|^2$. Thus it represents a state where the atoms have their spins in the x-y plane forming a 2D monopole. The length of the spin at each point depends on the corresponding local density. We now analyze the stability of such a solution. As before, we linearize around the solution (12) by adding a small quantity $\alpha_{1,2}$ and expanding it in powers of $e^{i\theta}$. We obtain Eq. (6) where now $\mathcal{H}_n=\tilde{L}+\tilde{K}_n+\tilde{H}^{\rm int}$, a 4×4 matrix operator with $\tilde{L}=\mathrm{diag}(\tilde{L}_1,\tilde{L}_1,\tilde{L}_2,\tilde{L}_2)$, $\tilde{K}_n=\hbar^2/(2mr^2)K_n$, and $\tilde{H}^{\rm int}=f^2H^{\rm int}$. Again using $R\gg\Delta r$, we have $L_1=L_2$ and therefore $\tilde{L}=L_1$ times the 4×4 identity matrix.

Since we are only interested in the stability, we analyze the positivity of $\mathcal{H}_n$. As before, we just have to study the cases with $|n|<2$. Let us distinguish two situations.

Weak interactions. In the limit $N(a+a_{12})/R\ll1$ the interaction energy $N\tilde{u}/[2\pi R(\Delta r)^2]$ is much smaller than the harmonic oscillator quantum $\hbar\omega$. In that case, $\Delta r\simeq a_0$ where $a_0=[\hbar/(m\omega)]^{1/2}$ is the size of the harmonic potential ground state. The spectrum of $\mathcal{H}_{0,1}$ is dominated by $\tilde{L}$. The radial and z dependence give rise to excitation energies $k\hbar\omega$ (k integer). The lowest excitations $k=0$ correspond to $\alpha(r,z)\propto f(r,z)$, and therefore we obtain $\tilde{\lambda}=\hbar^2/(2mR^2)\lambda$ (much less than $\hbar\omega$ in absolute value), where λ is given in Eq. (9) with u and u_{12} given in Eq. (11a). Thus, for excitation energies lower than $\hbar\omega$ the problem fully reduces to the ring case.

Strong interactions. The opposite limit is the Thomas-Fermi regime, where $\Delta r=a_0[32N(a+a_{12})/R]^{1/4}$. Now one cannot simply separate radial and z excitations from ring excitations. The excitation spectrum of $\mathcal{H}_{0,1}$ is dominated by $\tilde{L}+\tilde{H}^{\rm int}$. It is convenient to diagonalize $\tilde{H}^{\rm int}$ and consider the eigenfunctions separately.

As a first case we can consider $\vec{\alpha}^{(n)}=(g_1,g_2)\otimes(1,-1)$: here $\tilde{H}^{\rm int}$ is zero, and therefore the excitation frequencies correspond to those of $\tilde{L}$, which are of the order of $k\hbar^2/[2m(\Delta r)^2]\gg\hbar^2/(2mR^2)$. A second possibility is twofold, with $\vec{\alpha}^{(n)}=(g,g)\otimes(1,1),H^{\rm int}$ giving $(\tilde{u}+\tilde{u}_{12})f^2$, and $\vec{\alpha}^{(n)}=(g,-g)\otimes(1,1)$ giving $(\tilde{u}-\tilde{u}_{12})f^2$. This means that the lowest energy will be of the order of $N(\tilde{u}-\tilde{u}_{12})/[2\pi R(\Delta r)^2]$.

As long as this energy is larger than $\hbar^2/(2mR^2)$ we can consider both cases separately, treating K_n as a perturbation; in both the correction is positive, i.e., the monopole is stable. If the energy is smaller than $\hbar^2/(2mR^2)$, one has to be more careful in the perturbation analysis, as the excitation energies may become negative. In the end we obtain a necessary condition for stability, $N(\tilde{u}-\tilde{u}_{12})/[2\pi R(\Delta r)^2]\gg\hbar^2/(2mR^2)$. Using Eq. (11a) we can write this condition as $u-u_{12}>1$, which coincides with the basic stability condition derived for the ring. It means that the interactions have to be sufficiently strong to stabilize the monopole.

IV. DISCUSSION

In order to be specific, we will propose a particular configuration to create the spin monopole. We consider an alkali-metal atom in a ground $F=1$ hyperfine state. We will assume that the energy of the $m_F=0$ level is made higher, so that it is not involved in the dynamics. This can be achieved by using an off-resonant laser or a resonant radio-frequency field and the fact that the Clebsch-Gordan coefficients are different for the $m=0$ than for the $m=\pm1$ states. In this case we can identify $|\uparrow\rangle=|F=1,m_F=1\rangle$ and $|\downarrow\rangle=|F=1,m_F=-1\rangle$, the collisions do not change spin, and $\tilde{u}_{11}=\tilde{u}_{22}\equiv\tilde{u}$.

In order to generate the 2D spin monopole (12) we propose to use an off-resonant Raman beam. The atoms are initially condensed in the internal $|\uparrow\rangle$ state. A Raman laser that connects the states $|\uparrow\rangle$ and $|\downarrow\rangle$ is then switched on. It should have the appropriate spatial dependence so that the angular momentum in the z direction is changed by one unit [17].

Denoting by $\Omega(z,r,\theta)$ the effective Rabi frequency, the evolution equations are the above Gross-Pitaevskii equations (11) but with a coupling term between ψ_1 and ψ_2 proportional to Ω; the Raman detuning is incorporated in the definition of $\tilde{\delta}_{1,2}$. Initially, one takes $\tilde{\delta}_1\gg\tilde{\delta}_2$, so that the laser does not affect the internal atomic state, since it is effectively out of resonance. Then, $\tilde{\delta}_2-\tilde{\delta}_1$ is changed adiabatically until $\tilde{\delta}_2-\tilde{\delta}_1\simeq0$.

The method is more robust than the one proposed in Ref. [2] to generate vortices, since the spatial wave functions $f_{1,2}(r,z)$ remain almost constant with our setup. Actually, in this case one can simply use a $\pi/2$ laser pulse, taking $\tilde{\delta}_1=\tilde{\delta}_2$. This allows one to generate the monopole state on a much faster time scale, which will be of the order of several inverse trap frequencies.

In order to evaluate our proposal, we have performed a three-dimensional numerical simulation of the Gross-Pitaevskii equations in the presence of the laser for the creation of the spin monopole. We have used an optimized three-dimensional collocation Fourier method with typically $80\times80\times40$ collocation points and integrated in time with a symmetrized split-step operator technique. The results are

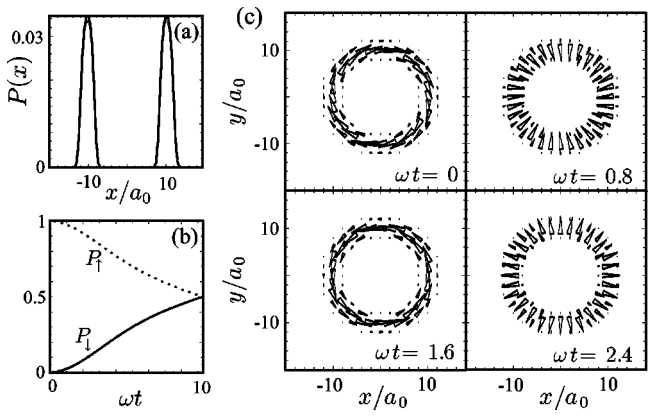

FIG. 3. Preparation of the 2D spin monopole. Trap parameters are $\omega_z=2\omega_r$, $R=10a_0$, $\sigma=5a_0$, $V_0=200\hbar\omega_r$, where $a_0=(\hbar/m\omega)^{1/2}$; interactions are $\tilde{u}_{11}=\tilde{u}_{22}=\tilde{u}_{12}/0.9=3600\hbar\omega_r a_0^3$; laser parameters are $\Omega(r)=\Omega_0[\sin(kx)+i\sin(ky)]$ with $\Omega_0=0.28\omega_r$ and $k=\pi/(6R)$. All plots are in adimensional units. (a) Final density distribution $|\psi_1|^2$ (dashed line) and $|\psi_2|^2$ (solid line) at $y=z=0$ as a function of x. (b) Evolution of the population of the $|\uparrow\rangle$ (dashed line) and $|\downarrow\rangle$ (solid line) states. (c) Evolution of the spin density after the preparation with $\delta_2-\delta_1=2\omega_r$; the triangles point along the expectation value of $\langle\sigma\rangle$ and are proportional to the local density.

shown in Fig. 3. In Fig. 3(a) we have plotted the final density profile along the x axis at the end of the generation process, whereas in Fig. 3(b) we have plotted the evolution of the population of the internal levels until the monopole is generated. After this process, we switch off the laser and apply an internal energy shift so that the spins start precessing, as is shown in Fig. 3(c). For this figure we have taken realistic parameters. For example, taking Na with $\omega_r=100$ Hz, $a_0=2$ μm, and $a=52a_B$, we have the result that the number of atoms is of the order of 2×10^5. In order to ensure stability, we have evolved the formed state in imaginary time (renormalizing the state after each evolution step) for about 20 trap oscillation times, without noticing any instability.

V. SUMMARY

We have proposed a complex state of a spinor Bose-Einstein condensate, which we have called a *spin monopole*. We have provided analytical and numerical evidence that the spin monopoles are stable in a ring trap when the interactions are sufficiently strong and also in a fully three-dimensional configuration. Finally, we have also shown a way to prepare such a state, and verified with a full 3D numerical simulation that it can indeed be prepared with current experimental parameters following the guidelines of our proposal.

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[16] We say these states are *stationary* in the sense that the evolution of the system can be factored out as an exponential of a sum of generators of the group of symmetries of the system. In the language of dynamical systems, these states are periodic solutions of the model equations.

[17] As proposed in Ref. [2], this can be achieved by using two standing waves propagating along the x and y axes with a phase diference, i.e., an effective electric field of the form $\sin(kx)+i\sin(ky)\simeq k(x+iy)$ if k is sufficiently small. This last requirement can be achieved using pairs of beams forming the appropriate angle.