Limits to sympathetic evaporative cooling of a two-component Fermi gas

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We find a limit cycle in a quasiequilibrium model of evaporative cooling of a two-component trapped fermion gas. The existence of such a limit cycle represents an obstruction to reaching the quantum ground state evaporatively. We show that evaporatively $\beta\mu \sim O(1)$. We speculate that one may be able to cool an atomic Fermi gas further by photoassociating dimers near the bottom of the Fermi sea.

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I. INTRODUCTION

The spectacular successes of laser-cooling techniques in creating Bose-Einstein condensation (BEC) in trapped dilute alkali-metal vapors [1–3] has stimulated efforts to form dilute, nearly degenerate atomic fermion gases. Such systems undoubtedly have unique phenomenology, and since one can control the composition, densities, and even scattering lengths in principle, they furnish a window to familiar phenomena (such as superconductivity, etc.) in unusual parameter regimes [4–8]. A critical step in achieving BEC in dilute alkali-metal vapors is evaporative cooling. In this paper we describe limitations to the use of evaporative cooling for a harmonically trapped two-component Fermi system.

Overall antisymmetry of the final-state wave function forces the *s* wave scattering amplitude for two-body collisions in a single-component polarized Fermi gas to vanish identically. However, in a system composed of two or more Fermi species there can still be appreciable *s*-wave scattering amplitudes at low energies. Recent experimental observation of quantum-statistical effects reducing the scattering frequency at low temperature has been reported in Ref. [9]. Sympathetic evaporative cooling in two-component Bose systems has been experimentally verified [10]. Aspects of the dynamics of sympathetic cooling in a two-component Fermi system have been discussed theoretically [11,12], and recently achieved experimentally [13].

We will show in a robust model that sympathetic cooling of trapped fermions is intrinsically limited to $(\mu/T) \sim O(1)$, where T is the temperature and μ is the chemical potential. Largely independent of trap and atomic parameters, this limit indicates that sympathetic evaporative cooling alone cannot achieve occupation probabilities in the trap single-particle ground state characteristic of typical degenerate Fermi systems (e.g., atomic nuclei and typical metals).

In summary, we model evaporation as a succession of quasiequilibrium states. This shows that evaporation moves the chemical potential toward saturation at a fixed (nonzero) fraction of the evaporation energy. Lowering the evaporation energy in an attempt to further cool simply causes both the Fermi surface and the temperature to recede, thus not substantially increasing the occupation probability of the lowest single-particle state. Rather than end on a pessimistic note, we conclude by speculating on a possible method for surmounting the difficulty of cooling an atomic Fermi gas.

II. MODEL

We aim to explain the general features of the evaporative cooling of a two-component fermion gas without recourse to the details of dynamics and transport. In particular, trap lifetime and other time scales will play almost no role in our model.

As described above, we focus on a two-fermion system cooling sympathetically by evaporation through interspecial two-body collisions only. Aspects of the thermodynamics of dilute, harmonically trapped fermions without an energy cutoff are described in Ref. [5]. Furthermore, we ignore the effect of any other environmental fields (for example, trap fields) and assume them to be constant over the lifetime of the system. The average effect of the interactions between the different species is absorbed into a mean-field term that we sweep into μ (see, for example, Ref. [18]). We further assume that particle number is not communicated between the species, and so their individual μ can differ. Interspecial scattering processes do communicate energy between species, and so we assume that both species are always at a common temperature.

Consider evaporating both species at the same energy cutoff E_{evap}. We complete the analysis for this case and then turn to the opposite extreme where only one species evaporates. If both fermion species a and b are (nearly) the same mass, then crossing symmetry equates the total rate for scattering into final state $|a,b\rangle$ with that of $|b,a\rangle$. This means that for the same evaporative cutoff, the rate at which particles of one species evaporates equals that of the other.

Finally, for simplicity, we model the evaporative process as one that always reduces the particle number by 1 and removes energy E_{evap}. In the model we develop we will ignore the contribution to the cooling that results from the interspecial mean field [18]. In the cases of interest in current experiments this interspecial mean-field energy is expected to be very small compared with the other relevant energy scales (for example, the Fermi energy). Although this model of evaporation is a gross simplification, it becomes a progressively better approximation as the temperature drops, and we are confident it captures the main features of the evaporative process.

We approximate each component's scaled number and energy by cutoff equilibrium distribution functions

$$N = \int_0^1 \frac{x^d dx}{e^{\beta(x-\mu)}+1}, \quad E = \int_0^1 \frac{x^{d+1} dx}{e^{\beta(x-\mu)}+1}, \tag{1}$$

where β and μ are, respectively, the inverse temperature and the chemical potential both made dimensionless by factors of E_{evap}. The exponent d depends on the actual spectrum of the trap, and for a three-dimensional isotropic harmonic trap, d is 2. We will keep the discussion rather general with respect to d, but use $d=2$ in all the graphs and particular conclusions below. We find that reducing d (by, for example, significantly changing the aspect ratio of the trap) makes sympathetic evaporative cooling generally less effective. Finally, in Eq. (1), E is dimensionless (i.e., in units of E_{evap}) and we have suppressed some overall factors that depend on E_{evap} and trap frequency.

We model evaporative cooling by simply following Eq. (1) through flow along

$$\begin{pmatrix} dN \\ dE \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} dN. \tag{2}$$

By using symmetries of the scattering for fermion species of nearly equal mass for the case where both fermion species are being evaporated at the same energy E_{evap}, we find that the net effect on the individual distribution functions is encapsulated in Eqs. (1) and (2) for each species separately (and so in what follows for this case we suppress indices).

The resulting differential equations for β and μ along this evaporative trajectory read

$$\frac{d\beta}{dN} = \frac{1}{\det(M)} \left(-\left.\frac{\partial E}{\partial \mu}\right|_{\beta} + \left.\frac{\partial N}{\partial \mu}\right|_{\beta} \right), \tag{3}$$

$$\frac{d\mu}{dN} = \frac{1}{\det(M)} \left(\left.\frac{\partial E}{\partial \beta}\right|_{\mu} - \left.\frac{\partial N}{\partial \beta}\right|_{\mu} \right), \tag{4}$$

where the determinant $\det(M)$ is given via

$$\det(M) = \left.\frac{\partial N}{\partial \beta}\right|_{\mu} \left.\frac{\partial E}{\partial \mu}\right|_{\beta} - \left.\frac{\partial N}{\partial \mu}\right|_{\beta} \left.\frac{\partial E}{\partial \beta}\right|_{\mu}, \tag{5}$$

which, from the quasiequilibrium distribution functions of Eq. (1), we find $\det(M)>0$ for all β, d, and μ. This positivity may be understood on general grounds via the connection between $\det(M)$ and the specific heat, c_V at constant N,

$$c_V = \frac{\beta^2 \det(M)}{(\partial N/\partial \mu)|_{\beta}}, \tag{6}$$

and by the fact that $(\partial N/\partial \mu)|_{\beta}>0$ as a consequence of, for example, Eq. (1). The $\det(M)$ vanishes in the low-temperature limit as $\sim \pi^2 \mu^4/3\beta^3$. See Fig. 1 for an example of this behavior (for $d=2$ and $\mu=2/3$).

Starting far from degeneracy, the net effect of the evaporative process is to alter μ while increasing β. Note that using the equilibrium distributions Eq. (1) implies by Eq. (3) that β increases monotonically along the flow. We see that μ's evolution does not share this property. Instead, we find that the system, Eqs. (1), (3), and (4), has a limit cycle at $(\partial E/\partial \beta)|_{\mu} - (\partial N/\partial \beta)|_{\mu} = 0$ in the (β,μ) plane. This limit cycle is at an intermediate value of $\mu=\mu^*(\beta)$ for all temperatures. Figure 2 is a graph of μ^* as a function of temperature for $d=2$.

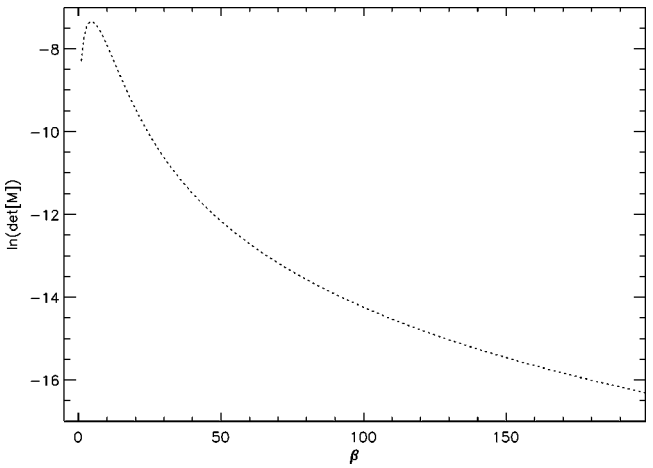

FIG. 1. $\det(M)$ for $\mu=2/3$, as a function of β.

For $d=2$, the low-temperature limit of μ^* is 2/3. We reiterate that for values of (β,μ) below the curve, the evaporative process increases μ, whereas for (β,μ) values above the curve, they *reduce* it. Thus as one tries to lower the temperature by reducing E_{evap} (thus increasing μ), evaporation causes the Fermi surface $\mu^* E_{\mathrm{evap}}$ to also drop away. Of course, as one drops E_{evap}, the scaled inverse temperature β also drops trivially.

A figure of merit measuring how close one is to the quantum many-body ground state is the occupation probability in the trap single-particle ground state. This is a function of the ratio of the chemical potential to the temperature, in our notation $\beta\mu$. This combination is independent of E_{evap} in our simple model. We now show that this product is limited by the total scaled atom number N [which in the normalization of Eq. (1) is limited to be at most $1/(d+1)$].

We consider two cooling methodologies that we refer to alternatively as ''passive'' and ''active.'' They refer, respectively, to holding the E_{evap} fixed or suddenly reducing it.

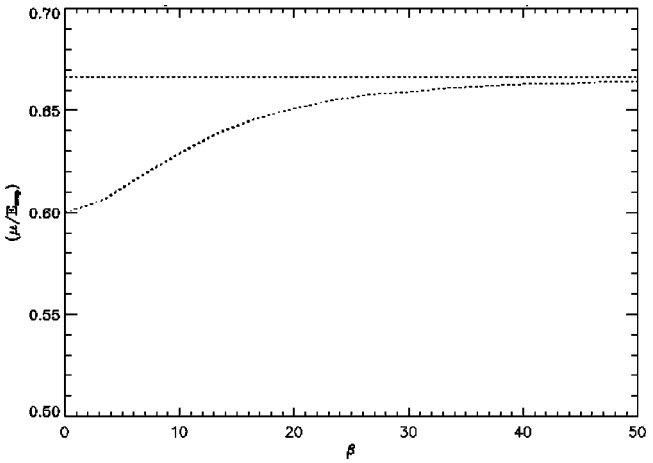

FIG. 2. The evaporative limit cycle with the low-temperature asymptote.

Unfortunately, in actual experiments underway [13], E_{evap} is varied continuously, so these two cooling methodologies are probably not good models of current experiments (see, however, Ref. [20]). These choices of cooling methodologies are advantageous analytically since they allow evaluation of the effects of evaporation without recourse to any dynamical time scales.

In the passive method, the system is held at a fixed E_{evap} and allowed to cool indefinitely by evaporation. Dynamically, since the emerging Fermi surface at μ^* is always a fraction of E_{evap}, the cooling rate is limited by the overall rate of escape, which (at low temperature) is Boltzmann-suppressed by a factor of $e^{-\beta(1-\mu^*)}$ and the trap lifetime. However, we reiterate that the ''kinetic'' model we employ analytically encapsulates limits to sympathetic evaporative cooling of the two-component fermion system without including ''dynamical'' effects (such as collisional or trap time scales).

In the active model, on the other hand, we abruptly lower E_{evap} to a value at or below our initial $\mu^* E_{\mathrm{evap}}$. Call this new evaporation energy E'_{evap}. The distribution function is invariant under such a change in E_{evap}. The μ and β values will trivially jump by factors of the evaporative energy scale ratio. Of course, the overall scaled phase-space constants that we suppressed in Eq. (1) do go as positive powers of E_{evap}. Dropping E_{evap} to $E'_{\mathrm{evap}}<E_{\mathrm{evap}}$ has the immediate effect of dropping N, for example. This corresponds precisely to the statement that all the particles with $E>E'_{\mathrm{evap}}$ leave immediately.

Since $\mu\beta$ measures our progress towards the ground state and is invariant under a sudden drop in E_{evap}, we see that only subsequent evaporation (and rethermalization) of the remaining fermions can increase the $\mu\beta$ product. We now show that evaporation after the drop in E_{evap} does not lead the system substantially closer to the quantum ground state.

The evaporation equations above can be integrated numerically for $\beta(\mu)$ by eliminating N from Eqs. (3) and (4). In doing so, one finds that for values of $\mu\neq\mu^*$, the change in the dimensionless temperature ratio β is generally relatively small, on the order of β itself. To get to the nearly degenerate Fermi system starting far away from the ground state, we need a cooling regime in which much larger temperature drops are achievable. In numerical simulations, one finds that the only large temperature changes happen evaporatively when the system is at a μ very near μ^*, basically within 10% of that value. Figure 3 is the integral $\beta(\mu)$ of Eqs. (3) and (4) for two initial conditions, $(\mu_i,\beta_i)=(0,2.5)$ and (2.8,4.5). Trajectories that start at higher initial temperatures (lower β_i) remain substantially lower in Fig. 3 throughout the entire evaporative trajectory, but do eventually wind along the limit cycle at μ^* toward large β. Recall that in the active method we are actually starting generally at smaller β_i than shown.

Curiously, note that for $\mu\gg\mu^*$, the $\mu\beta$ product can actually initially *decrease* along the evaporative trajectory. In the Conclusion we comment on a heuristic way of understanding such counterintuitive behavior. At any rate, it is clear from Fig. 3 that achieving a condensed Fermi system

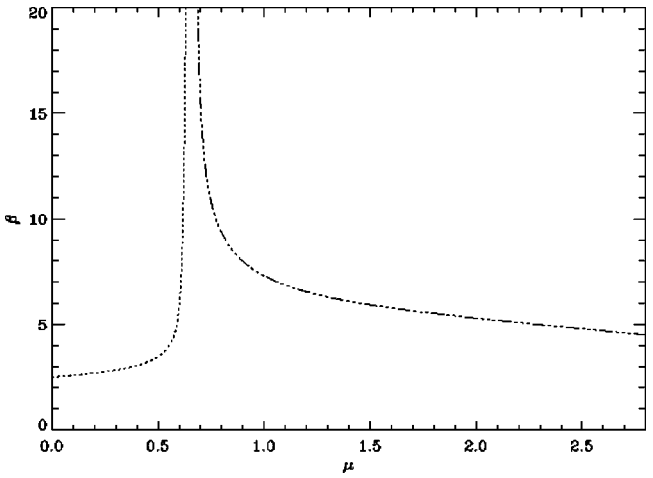

FIG. 3. Two typical cooling trajectories, one starting above and one below μ^*.

requires evolving μ close to μ^*. We now investigate some liberal bounds on how well evaporation can achieve that goal.

In Fig. 3 we integrated Eqs. (3) and (4) for $\beta(\mu)$ by eliminating N, not taking into account that N must always be positive and strictly decreasing. We now study how far along these $\beta(\mu)$ trajectories we may progress evaporatively until we substantially run out of particles.

Analytically, at low temperature, we find that the flow equation for μ near μ^* is independent of β entirely, and reads

$$\frac{d\mu}{dN}=\frac{\pi^2}{6}\frac{2-3\mu}{\mu^3}+\cdots, \tag{7}$$

which can be integrated for all μ to read

$$\left[-\frac{8}{27}\ln(\mu-2/3)-\mu^2\frac{\mu+1}{3}-\frac{4\mu}{9}\right]_{\mu_i}^{\mu_f}=\frac{\pi^2}{2}\Delta N. \tag{8}$$

The number evaporated ΔN must of course be less than the total number of particles in the trap. Using Eq. (1), we see that there are indeed stringent limits on the right-hand side (RHS) of Eq. (8). Since we know that appreciable cooling in this scheme does not occur until one is close to μ^* (equal to 2/3 at low temperature), we know that the best one can do is to evaporate all the particles in excess of the ground state at μ^*. For the scenario in which we start at a μ above μ^* (for example, as may be created in the active method) this limits the RHS substantially. For $\mu<\mu^*$, the system evaporatively evolves toward μ^* *but can never reach it* because there are simply not enough particles to evaporate. This is one reason to use the active method in a phase of the cooling, since it can raise the initial μ above the μ^*. However, raising μ by this means is also self-limiting for two reasons. First, if it is raised substantially above 1, too many particles are lost from the trap and there are too few remaining to evaporate back to degeneracy. Second, as described earlier, it reduces β_i by the same factor it increases μ_i, indicating the need to get even

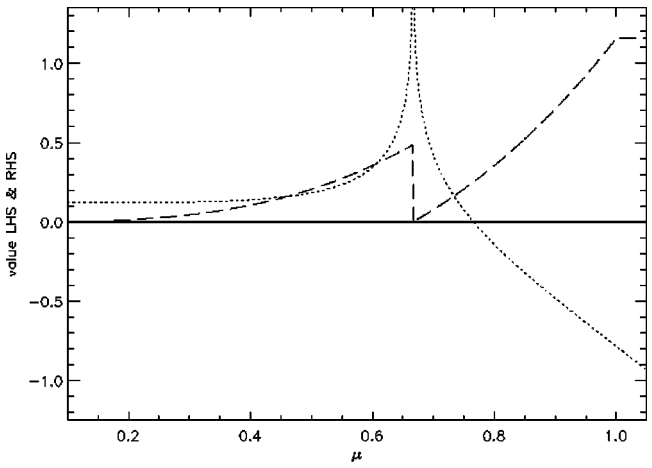

FIG. 4. Graph of Eq. (8).

closer to μ^* to wind along the limit cycle and recover large β. We explain this is more detail quantitatively below.

We graphically describe the consequences of Eq. (8) in Fig. 4. All the discussion here is in the ''best-case'' scenario, in that we imagine starting with a system at already relatively large β [so Eq. (8) applies] and ask how well evaporative cooling can further increase β and thus $\beta\mu$. Figure 4 is a graph of Eq. (8). The dashed line represents a maximum possible RHS at that initial μ. As per the preceding discussion, we have plotted the contribution from the total number of particles for $\mu<\mu^*$, and plot the excess *only* for $\mu>\mu^*$. The light dotted trace is the left-hand side (LHS) of Eq. (8). Thus to estimate the maximum possible increase or decrease in μ, starting at some initial μ_i, use the height of the dashed line to estimate how much of a change in the height of the light dotted line you may achieve. The resulting position at that height on the light dotted curve then is an (over)estimate of the largest μ achievable.[1]

A study of the graph indicates that the process allows one to get within perhaps 10% of μ^* at best. As a very crude estimate, we can see that this occasions at most a roughly threefold increase in β. Cooling from a typical active-method initial state of $(\beta,\mu)\sim(1,1)$, we find that one cannot achieve $\beta\mu$ products in excess of roughly 3. For such a gas, the typical occupation probability of the lowest energy single-particle state in the trap is roughly 90%. Typical, nearly degenerate quantum Fermi systems such as nucleons in nuclei and electrons in typical metallic systems have much larger $\beta\mu$ products.

III. A SECOND MODEL

We now study the scenario in which the evaporative thresholds for the two species *a* and *b* are very different. We forgo a detailed quantitative analysis specific to this case, and instead reduce to and reason from the simpler model we have already discussed. For simplicity, we assume that the masses and the trap potential of the two species are identical. Thus the quasiequilibrium expectation values of $N_{a,b}$ and $E_{a,b}$ are given by the obvious doubling and indexing of Eq. (1). However, to include the fact that the evaporative threshold for species *a* is so much larger than that of species *b*, we replace the upper limit ''1'' of the integrals in Eq. (1) by ''∞.'' For species *a*, we then scale β, μ_a, and μ_b by the evaporative threshold of the *b* species. Subsequently, $E_{\rm evap}$ refers to the evaporative threshold for species *b* only.

The condition $dN_a=0$ now becomes a linear constraint in the space (μ_a,μ_b,β). All these variables are dimensionless, scaled by the appropriate factors of $E_{\rm evap}$.

That linear constraint reduces the evaporative evolution of this two-component Fermi system again to a two-dimensional dynamical system (i.e., at fixed $E_{\rm evap}$ and N_a, μ_a is really a function of β). In particular, scattering events that lead to evaporation of a particle of species *b* at energy $E_{\rm evap}$ will, on average, remove a net E_a from the total energy of species *a*, and a net E_b from species *b*, where $E_a+E_b=E_{\rm evap}$, and with ratio E_a/E_b depending on (μ_a,μ_b,β). In equations, $dN_a=0$ implies that

$$\frac{dE_a}{d\beta}=-\frac{\det(M_a)}{(\partial N_a/\partial\mu_a)|_\beta} \tag{9}$$

[compare to Eq. (6)], where $\det(M_a)$ is the determinant of the matrix of partial derivatives for the *a* system only. Note that this determinant $\det(M_a)$ is now computed with the integrals extending to ∞, and so is singular at high temperatures but still looks like the rest of the graph in Fig. 1 for low temperatures. The analogous function for the *b* species, $\det(M_b)$, is precisely the same as for Eq. (5) with Eq. (1) (that is, using integration limits [0:1]).

Energy conservation implies (in scaled dimensionless quantities) that the evaporative trajectory is along

$$\begin{pmatrix} dN_b \\ d(E_a+E_b)\end{pmatrix}=\begin{pmatrix}-1\\-1\end{pmatrix}dN_b\,. \tag{10}$$

The differential relations between E_b, N_b, μ_b, and β are exactly the same as for the matrix system analyzed in the first model. We now use Eq. (9) to rewrite the dE_b term in Eq. (10) in terms of $d\beta$ and to rewrite the two-dimensional system for the evolution of μ_b and β in terms of dN_b. Recall that μ_a also changes, but is given parametrically in terms of β (and N_a, which is held fixed). We find

$$\frac{d\beta}{dN_b}=\frac{1}{\det(\mathcal{M})}\left(-\left.\frac{\partial E_b}{\partial\mu_b}\right|_\beta+\left.\frac{\partial N_b}{\partial\mu_b}\right|_\beta\right), \tag{11}$$

$$\frac{d\mu_b}{dN_b}=\frac{1}{\det(\mathcal{M})}\left(\left.\frac{\partial E_b}{\partial\beta}\right|_{\mu_b}-\left.\frac{\partial N_b}{\partial\beta}\right|_{\mu_b}-\frac{\det(M_a)}{\partial N_a/\partial\mu_a|_\beta}\right). \tag{12}$$

The denominator is

[1]Note that although $(\partial N/\partial\mu)|_\beta$ is always greater than zero, the total derivative $(dN/d\mu)|<0$ for $\mu<\mu^*$ because $(\partial N/\partial\beta)|_\mu(d\beta/d\mu)$ is negative.

$$\det\left(\mathcal{M}=\det(M_b)+\det(M_a)\left(\frac{(\partial N_b/\partial\mu_b)|_\beta}{(\partial N_a/\partial\mu_a)|_\beta}\right)\right) \qquad (13)$$

and is again strictly positive.

We can now use the intuition gained by studying the first model to constrain cooling the *a* species in this scenario. The measure of how close to the ground state we are for that species is again the product $\beta\mu_a$. The most promising initial (high-temperature) state in this case seems to be $N_a \sim N_b$ (that is, $\mu_a \sim \mu_b$).

Looking now at Eq. (12), we see that the limit cycle still exists and occurs at a value μ_b^* higher (that is, closer to the evaporative threshold) than in the first model we discussed. Also note that the β evolution equation in this scenario differs only by the prefactor $\det(\mathcal{M})$, from the first model we considered. Due to positivity of the individual $\det(M_{a,b})$, we expect that factor to *reduce* the evolution of β as compared with the first model. Indeed, in the high-temperature limit the suppression is through powers of the ratio of the evaporative energy scales of the two species, but for low temperatures, $\det(\mathcal{M}) \sim 2\ \det(M_b)$. Thus, as may have been expected due to the larger thermal inertia of the entire system compared with that of an individual species, the actual overall evaporative cooling efficiency is suppressed at low temperatures relative to the first model we considered.

This means that if we cool in the active mode, μ_b will be large initially and thus the temperature of the entire system will not drop dramatically as one evaporates. Efficient cooling could occur if we are able to get to $\mu \sim \mu^*$, but it is difficult to reach that regime because, although we can evaporate *all* of the *b* species [thereby having a continuous curve for the dashed curve in Fig. 4 representing the upper bound for the RHS of Eq. (8)], in the low-temperature limit the cooling of the whole system proceeds slower, and so the light dotted curve in Fig. 4 is roughly twice as steep. Thus the obstruction to reaching the quantum ground state in the two-component system in this case (unequal $E_{\rm evap}$) can be understood from the considerations and qualitative behavior of the first case (equal $E_{\rm evap}$).

IV. CONCLUSION

There exists a limit cycle in evaporative cooling a two-component fermion system that has the consequence of severely limiting the approach to the quantum degenerate ground state. One heuristic way to understand this result is that, since there is always a gap between the putative Fermi surface and the evaporative threshold, evaporation can actually ''heat up'' (that is, disorder) the distribution function at the Fermi surface. This is one way to understand the rather counterintuitive finding that starting at $\mu \gg \mu^*$, the $\mu\beta$ product actually initially *decreases* during evaporation.

There have been many proposals for surmounting the difficulty of achieving a degenerate Fermi ground state, and it is not the purpose of this paper to review these many inventive ideas. They include condensing Fermi-Bose mixtures (a difficult technical feat) [14–17,19] and using various perturbing fields on pure Fermi systems. What this paper suggests is that proposed cooling techniques relying exclusively on evaporating fermions may be constrained by a limit cycle of the type we have described.

We would like to end with a brief speculative proposal for reaching lower $\beta\mu$ products in a trapped atomic Fermi gas. Consider photoassociating Fermi dimers into states just below the trap single-particle ground state. With fermions there is no stimulated atomic channel back to the trap as there would be in the case of photoassociating dimers from a Bose condensate; instead, Pauli blocking and the Fermi energy both push the system toward dimerization. The remaining fermions then scatter off the dimers. In a sense, photoassociating has enhanced the three-body collision rate, which, even for identical fermions, is not suppressed by statistics at low energies. Every time a dimer breaks in collision, as long as the trapping potential is high enough, the fermions go back into the trap; the net effect of creating and breaking dimers in this proposed scheme is to use the difference in the dimer pump beams to cool the fermion system ''from below'' (near the single-particle ground state of the trap) instead of evaporatively ''from above'' (that is, above the Fermi surface). In that sense this scheme has the flavor of Bose-Fermi mixture schemes, but might be simpler technically. Also, this cooling proposal does not *a priori* require a two-fermion mixture, though we imagine that photoassociating dimers composed of dissimilar fermions is likely to be easier than photoassociation of identical fermions. It remains to be seen whether such a technique can be practically implemented in a polarized atomic Fermi system.

Finally, it would be of great interest to compare the predictions of this simple evaporative model directly with experiment. One step in that direction is to generalize the model to include a time-dependent $E_{\rm evap}$ and trap lifetime effects. One use of such a direct comparison would be to further test how large quantum-statistical effects are in current experiments that are far from degeneracy. Investigations of such ''dynamical'' effects are underway [20] but clearly beyond the ''kinematic'' scope and spirit of this paper.

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