Comment on ‘‘Theory of detection in the micromaser’’

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In a recent paper [R. R. McGowan and W. C. Schieve, Phys. Rev. A **59**, 778 (1999)], the authors claim that the detection of the outcoming atoms has a back-action effect on the spectrum of the micromaser field. We point out that this assertion is wrong. In particular, we discuss the connection between atom statistics and field statistics, which has been stated to be the ultimate goal of the cited paper.

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Since the statistical properties of the quantized microwave field in a micromaser cavity cannot be directly measured, information about the field has to be gained by measurements on the outcoming atoms using state-selective detection. For this purpose different theoretical methods have been developed [2–9]. The method which is applied in Ref. [1] relies on a nonlinear master equation [2] for the (conditioned) evolution of the density operator of the cavity field between successive detector clicks. Another approach [3,6,7] is based on the unconditioned linear evolution of the density operator of the field, making use of a linear conditioned operator equation only for the calculation of waiting time distributions between the outcoming atoms [3]. In this Comment we follow the approach developed in Refs. [3,6,7] combining it with the scheme for phase-sensitive detection proposed in Ref. [8]. We point out that detecting the atoms after their interaction with the micromaser field has no influence on the properties of the field, in contrast to what is claimed in Ref. [1]. For this purpose we first give a very brief account of the theory of detection in the micromaser.

Due to the interaction with a single excited two-level atom crossing the cavity in the transit time τ, the reduced density operator ρ of the radiation field in the micromaser is changed according to $\rho(t+\tau)=M\rho(t)$. The superoperator M ensues from the complete solution of the atom-field interaction problem by taking the trace with respect to the atomic subsystem. When the excited atoms in the beam pumping the micromaser are uncorrelated, i.e., when they are injected into the micromaser cavity with the rate r according to Poissonian injection statistics, the density operator of the cavity field evolves as $\rho(t)=V(t)\rho(0)$, where

$$\dot{V}=r(M-1)V+LV. \tag{1}$$

As usual, L denotes the Liouvillian describing the coupling of the radiation field to a thermal heat bath. The superoperator M can be written as $M=A+B$ with A and B arising in a natural way when the trace is performed in a basis spanned by two arbitrarily chosen orthogonal states $|A\rangle$ and $|B\rangle$ of the two-level atom, which may be the energy eigenstates or superpositions thereof. When an outcoming atom is found to be in state $|A\rangle$, the density operator of the field is reduced according to $\rho\rightarrow A\rho$ (where normalization is neglected). In the following we consider the stationary regime of the micromaser. The probability per unit time of detecting an atom in state $|A\rangle$ is then given by [3] $P_1^A=rp_A\mathrm{Tr}(A\overline{\rho})$, where p_A denotes the efficiency of the detection and $\overline{\rho}=\lim_{t\rightarrow\infty}\rho(t)$. The multitime state-selective *coincidence probability densities* or *correlation functions*, respectively, for detecting k atoms in state $|A\rangle$ at the time instants $t_1,t_2,\ldots,t_k$ are found to be [3,6]

$$P_k^A(t_1,t_2,\ldots,t_k)=r^k(p_A)^k\mathrm{Tr}[AV(t_k-t_{k-1})\ldots \times AV(t_2-t_1)A\overline{\rho}]. \tag{2}$$

On the other hand, the state-selective *exclusive probability densities* referring to the case that exactly k atoms are detected in state $|A\rangle$ at the time instants $t_1,t_2,\ldots,t_k$ on the condition that no other atom is detected in this state in between, can be calculated by an expression which is analogous to Eq. (2) but in which the trace-conserving superoperator V is replaced by an operator V_0^A with [3]

$$\dot{V}_0^A=r[(1-p_A)A+B-1]V_0^A+LV_0^A. \tag{3}$$

The operator $V_0^A(t)\rho(0)$ has the meaning of a *conditioned* and non-normalized density operator for the radiation field in the cavity. We note that the right-hand side of Eq. (3) can be interpreted in a simplisitic way by stating that it describes the specific situation where the atoms either emerge in state $|A\rangle$ but escape detection due to the finite efficiency, or leave the cavity in state $|B\rangle$. Similarly, if we would like to consider an exclusive probability density which is not state selective, the evolution of the field between successive detector clicks had to be described by a non-normalized conditioned density operator $V_0(t)\rho(0)$ with

$$\dot{V}_0=r[(1-p_A)A+(1-p_B)B-1]V_0+LV_0. \tag{4}$$

Here p_B is the efficiency for detecting an atom in state $|B\rangle$. The operator V_0 refers to the evolution of the field under the condition that neither of the two state-selective detectors registers an atom. By using a normalized density operator instead of a non-normalized one to account for this conditioned evolution, we would arrive at the nonlinear master equation introduced in Ref. [2].

In order to evaluate experimental results, it is necessary to specify the superoperators A and B. For *phase-insensitive detection*, when the atoms are detected in the deexcited energy eigenstate state $|D\rangle$ or in the excited state $|E\rangle$, respectively, we have $A\equiv D$ and $B\equiv E$, where, in the photon number representation,

$$(D\rho)_{n,m}=\sin(g\tau\sqrt{n})\sin(g\tau\sqrt{m})\rho_{n-1,m-1}, \tag{5}$$

$$(E\rho)_{n,m}=\cos(g\tau\sqrt{n+1})\cos(g\tau\sqrt{m+1})\rho_{n,m} \tag{6}$$

with g being the atom-field coupling constant. For *phase-sensitive detection*, on the other hand, where the emerging atoms are exposed to a classical $\pi/2$ pulse before reaching the energy-selective detectors [8], we must put $A\equiv A'$ and $B\equiv B'$, where

$$A'=\frac{1}{2}(D+E-F),\quad B'=\frac{1}{2}(D+E+F) \tag{7}$$

with

$$(F\rho)_{n,m}=\cos(g\tau\sqrt{n+1})\sin(g\tau\sqrt{m})\rho_{n,m-1} +\sin(g\tau\sqrt{n})\cos(g\tau\sqrt{m+1})\rho_{n-1,m}. \tag{8}$$

According to the theory of stochastic processes [10], the state-selective statistics of the outcoming atoms with respect to any state $|A\rangle$ is completetely described by the whole set of the state-selective coincidence probability densities $P_k^A(t_1,t_2,\ldots,t_k)$ $(k=1,2,\ldots)$ referring to this state or, alternatively, by the whole set of exclusive probability densities. From these all other statistical quantities can be calculated applying standard methods. In particular, this procedure has been used in Ref. [3] for $A\equiv D$ to investigate the mean value and the variance of the number of deexcited atoms emerging from the cavity in a given time interval as well as the waiting-time distribution between deexcited atoms. The advantage of starting from the correlation functions (2) consists in the fact that the detector efficiencies drop out from the normalized quantities $P_k^A(t_1,t_2,\ldots,t_k)/(P_1^A)^k$ since the evolution equation (1) does not depend on them, in contrast to the conditioned equations (3) or (4). Finally, we remark that the preceding results can be generalized to be applicable for calculating the statistics of detector clicks in a wide class of micromasers pumped by a non-Poissonian beam where Eq. (1) does not hold and has to be replaced by a non-Markovian evolution equation. This has been done in Ref. [7] where for sub- and super-Poissonian pumping the correlation functions and waiting-time distributions for the detection of deexcited atoms have been calculated as well as the correlation functions for the photons in the cavity field.

The statistical properties of the radiation field in the micromaser cavity are completely characterized by the whole set of the normally ordered multitime correlation functions of the intensity (or the photon number, respectively) and of the electric field strength. By exploiting the properties of the evolution equation (1) it can be shown [3,4,6] that in the stationary micromaser regime at zero temperature there is an exact correspondence between the photon statistics of the field and the statistical distribution of the deexcited atoms, expressed by the equality

$$\frac{\langle a^\dagger(t_1))\cdots a^\dagger(t_k)a(t_k)\cdots a(t_1)\rangle}{\langle a^\dagger a\rangle^k}=\frac{P_k^D(t_1,\ldots,t_k)}{(P_1^D)^k}, \tag{9}$$

where $t_1\leqslant t_2\leqslant\cdots\leqslant t_k$, and a and $a^\dagger$ are the photon annihilation and creation operators of the cavity field. Equation (9) holds to an excellent approximation under present experimental conditions since the thermal photon number is negligibly small [6]. The steady-state normalized two-time correlation function of the electric field strength in the micromaser cavity is given by

$$g_1(t)\equiv\frac{\langle a^\dagger(t)a(0)\rangle}{\langle a^\dagger a\rangle}=\frac{\mathrm{Tr}[a^\dagger V(t)a\overline{\rho}]}{\mathrm{Tr}(a^\dagger a\overline{\rho})}, \tag{10}$$

where $V(t)$ obeys Eq. (1). Under certain operating conditions there exists an approximate equation which relates the function $g_1(t)$ to the two-time coincidence probabilities measured in phase-sensitive detection of the atoms and which reads [8]

$$g_1(t)\approx w(t)\equiv\frac{\mathrm{Tr}[(B'-A')V(t)(B'-A')\overline{\rho}]}{\mathrm{Tr}[(B'-A')^2\overline{\rho}]}, \tag{11}$$

where $|A'\rangle$ and $|B'\rangle$ are given by Eq. (7). As has been shown in Ref. [8], this approximation is justified when the steady-state photon-number distribution in the cavity has a single narrow peak at a large mean photon number. The micromaser spectrum is defined as $S(\omega)=\pi^{-1}\,\mathrm{Re}\int_0^\infty g_1(t)e^{-i(\omega-\nu)t}dt$ with ν being the resonance frequency of the cavity.

The main contents of the lengthy paper [1], ''Theory of detection in the micromaser,'' can be summarized as follows: Starting from the papers [2,8] the authors rederive a nonlinear master equation for the density operator of the micromaser field which has been derived already previously [2] and which describes the evolution of the field on the condition that neither of the two active detectors registers an atom. They emphasize that they consider the state vector of the entangled system, which consists of the atom, the field, and the detector. In order to apply the nonlinear density operator equation the authors develop a perturbative method with respect to the detector efficiencies and they use this method to calculate a function which they call the spectrum of the micromaser. As the result of their paper they find that the detection of the outcoming atoms has a back-action effect on the micromaser spectrum.

First we note that superpositions between different detector states decohere infinitely fast [11] since a detector is a macroscopic measuring device, and that the discussion in Ref. [1] concerning cross terms between detector states is therefore without any physical relevance.

Next we comment on the usefulness of developing a perturbative method for dealing with the nonlinear density operator equation. The authors stress that their perturbation theory, which is applicable for low detector efficiencies, effectively turns the nonlinear operator equation into a linear one. It should be remarked, however, that the nonlinearity of the differential equation describing the evolution of the conditioned density operator is solely due to its normalization. For calculating any physical quantity, on the other hand, it is sufficient to use the linear equation (4) describing the evolution of a non-normalized conditioned density operator. In fact, this procedure is even more appropriate, since the trace of the non-normalized conditioned density operator automatically corresponds to the probability that the conditions actually occur.

Finally, we emphasize that the micromaser spectrum is uniquely defined as the Fourier transform of Eq. (10) and does not depend in any way from the method applied to detect it or from the detector efficiencies. (We note in passing that in Ref. [1] the function (10) is denoted as the first-order photon-photon correlation function though, in contrast to the expression on the left-hand side of Eq. (9), it does not describe any correlations between photons.) To arrive at their assertion that the detection of the outcoming atoms has a back-action effect on the spectrum of the micromaser, the authors of Ref. [1] apply a procedure which is mathematically equivalent to replacing the evolution operator V obeying Eq. (1) by the conditioned operator V_0 evolving according to Eq. (4) when calculating the spectrum. Not surprisingly, the function evaluated this way depends on the detector efficiencies since the latter enter the evolution equation (4). However, this function yields no information about the true spectrum of the micromaser field and it is without any practical physical meaning. As for their physical reasoning, the authors of Ref. [1] obviously misunderstood the papers from which they took their starting equations. It is written there that if an atom is deteced in a specific state ''the photon field is, in effect, changed'' [2] or that in the phase-sensitive detection scheme ''the quantum-mechanical reduction of the photon state that is associated with the registration of a detector click imposes a phase on the photon state'' [8]. In this respect one has to keep in mind, however, that any state reduction corresponds to the selection of a specific quantum-mechanical subensemble and does not change the properties of the whole quantum-mechanical ensemble. In particular, the measurements on the atoms emerging from the micromaser cavity do not cause any back-action effect on the spectrum of the micromaser, or on its linewidth, respectively. Therefore the procedure of Ref. [1] is lacking a sound physical basis.

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