Transparency of a short laser pulse via decay interference in a closed V-type system

E. Paspalakis, N. J. Kylstra,* and P. L. Knight
Optics Section, Blackett Laboratory, Imperial College, London SW7 2BZ, United Kingdom
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We study the consequences of spontaneous emission interference in the propagation dynamics of a short laser pulse in a closed V-type medium. We show that, under specific conditions, the (otherwise opaque) medium becomes transparent to the laser pulse. The influence of incoherent processes on the transparency is also investigated.

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In the last decade there has been intensive interest in the study of quantum coherence phenomena in multilevel atomic systems. In particular, a number of fascinating effects have been discovered which occur as a result of the coherence introduced when a resonant pump laser is applied and the system is probed by a second laser [1–3]. The constructive role that dissipative processes can play in maintaining coherence in multilevel systems [4–19] has also been emphasized. These studies have modified considerably our understanding of the nature and consequences of quantum coherence on quantum and nonlinear optical processes in multilevel atoms. Moreover, it is now established that quantum coherence can be exploited to control the response of the laser-atom system.

In this paper we study the effects of spontaneous emission induced coherence in the propagation of a short laser pulse in a closed V-type medium. In our system two close-lying excited states are coupled to the ground state by a short laser pulse. Both upper levels decay spontaneously to the ground level. This system has been studied in the continuous wave steady-state limit, and phenomena such as transparency and amplification (or lasing) without inversion have been shown to occur [14–16]. The resonance fluorescence spectrum of this system has also been analyzed [17–19]. In our study we concentrate on the interaction of short, pulsed laser fields with the medium. We show that under specific conditions the pulse can propagate in the medium without absorption and without dispersion. The conditions for such behavior depend on the population trapping conditions and on the degree of adiabaticity of the laser-atom interaction.

We note that there has been recent theoretical interest in transparency, soliton wave propagation and lasing without inversion in closed V-type systems which are driven by two short, pulsed laser fields [20,21]. In these studies the propagation dynamics of a probe laser field are influenced by the application of a second (coupling) laser field, i.e., the necessary coherence is laser induced. However, coherence can also be created and preserved via internal (even dissipative) processes of the system, therefore requiring no coupling laser field [12,13].

In order to analyze the dynamics of the system we use the Maxwell-Bloch formalism. The dynamics of the atomic system is described using a density matrix approach. The Hamiltonian of the system in the interaction picture and in the rotating wave approximation is given by (we use units such that $\hbar=1$)

$$H=\sum_{m=0}^{2}\omega_m|m\rangle\langle m|+\sum_{\mathbf{k}}\omega_{\mathbf{k}}a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}+\Bigg[\sum_{m=1}^{2}\Omega_m f(z,t)e^{i(\omega t-kz)}|0\rangle\langle m|+\sum_{m=1}^{2}\sum_{\mathbf{k}}g_{m\mathbf{k}}|m\rangle\langle 0|a_{\mathbf{k}}+\text{H.c.}\Bigg], \quad (1)$$

Here, $\Omega_n=-\mu_{0n}\mathcal{E}$ $(n=1,2)$ is the Rabi frequency of the $|0\rangle\rightarrow|n\rangle$ transition. The corresponding transition matrix elements, which we take to be real, are $\mu_{0n}=\vec{\mu}_{0n}\cdot\hat{\boldsymbol{\epsilon}}=\mu_{n0}$ with $\hat{\boldsymbol{\epsilon}}$ being the polarization vector of the laser field and $\mathcal{E}$ being the peak electric field strength of the laser field. The laser pulse envelope is denoted $f(\zeta,\tau)$ and is normalized to unity. The laser detuning from resonance with the state $|n\rangle$ is $\Delta_n=\omega_{n0}-\omega$, where ω_{n0} is the transition frequency between the ground state and the excited state. Note that $\Delta_2-\Delta_1=\omega_{21}$, with ω_{21} being the energy separation of the excited levels. In addition, $a_{\mathbf{k}}$ $(a_{\mathbf{k}}^{\dagger})$ is the annihilation (creation) operator of the **k**th mode of the vacuum field having angular frequency $\omega_{\mathbf{k}}$ and $g_{m\mathbf{k}}$ is the coupling constant of the atomic transition $|m\rangle\leftrightarrow|0\rangle$ with the **k**th vacuum mode.

Using a generalized version of the Weisskopf-Wigner theory (see, for example Ref. [14]) we obtain the following equations of motion for the density matrix elements in the rotating wave approximation:

$$\frac{\partial\rho_{11}}{\partial\tau}=-\gamma_1\rho_{11}-p\frac{\sqrt{\gamma_1\gamma_2}}{2}(\rho_{12}+\rho_{21})-i\Omega_1 f^*(\zeta,\tau)\rho_{01}+i\Omega_1 f(\zeta,\tau)\rho_{10}, \quad (2)$$

$$\frac{\partial\rho_{22}}{\partial\tau}=-\gamma_2\rho_{22}-p\frac{\sqrt{\gamma_1\gamma_2}}{2}(\rho_{12}+\rho_{21})-i\Omega_2 f^*(\zeta,\tau)\rho_{02}+i\Omega_2 f(\zeta,\tau)\rho_{20}, \quad (3)$$

*Present address: Department of Physics, Science Laboratories, University of Durham, Durham DH1 3LE, United Kingdom.

$$\frac{\partial \rho_{01}}{\partial \tau} = \left(i\Delta_1 - \frac{\gamma_1}{2} - \gamma_{01} \right) \rho_{01} - p\frac{\sqrt{\gamma_1\gamma_2}}{2}\rho_{02} - i\Omega_1 f(\zeta,\tau)(\rho_{11}-\rho_{00}) - i\Omega_2 f(\zeta,\tau)\rho_{21}, \quad (4)$$

$$\frac{\partial \rho_{02}}{\partial \tau} = \left(i\Delta_2 - \frac{\gamma_2}{2} - \gamma_{02} \right) \rho_{02} - p\frac{\sqrt{\gamma_1\gamma_2}}{2}\rho_{01} - i\Omega_2 f(\zeta,\tau)(\rho_{22}-\rho_{00}) - i\Omega_1 f(\zeta,\tau)\rho_{12}, \quad (5)$$

$$\frac{\partial \rho_{12}}{\partial \tau} = \left(i\omega_{21} - \frac{\gamma_1+\gamma_2}{2} - \gamma_{12} \right) \rho_{12} - p\frac{\sqrt{\gamma_1\gamma_2}}{2}(\rho_{11}+\rho_{22}) + i\Omega_2 f(\zeta,\tau)\rho_{10} - i\Omega_1 f^*(\zeta,\tau)\rho_{02}. \quad (6)$$

Note that $\rho_{nm}=\rho^*_{mn}$ and $\Sigma^2_{n=0}\rho_{nn}=1$, as we study a closed atomic system, and that ρ_{nm} is a function of ζ and τ. These equations are written in the retarded (local) frame with $\tau=t-z/c$ and $\zeta=z$. The spontaneous decay rate of state $|n\rangle$ is γ_n, and interference occurs through the term $p\sqrt{\gamma_1\gamma_2}$ where $p=\vec{\mu}_{10}\cdot\vec{\mu}_{02}/(|\vec{\mu}_{10}||\vec{\mu}_{02}|)$ denotes the alignment of the two spontaneous emission dipole matrix elements. If these matrix elements are parallel (antiparallel) then $p=1(p=-1)$ and the system exhibits maximum quantum interference, while if $p=0$ no interference occurs. Finally, γ_{nm} denotes the dephasing rates, e.g., due to collisions. In the rest of this work we will consider the case of nondegenerate upper states ($\omega_{21}\neq 0$) and $p=1$. [22]

The propagation of the laser pulse in the medium is governed by Maxwell's wave equation, which in the slowly varying envelope approximation reduces to

$$\Omega_1\frac{\partial f(\zeta,\tau)}{\partial \zeta} = i\alpha_1\rho_{01}(\zeta,\tau) + i\sqrt{\alpha_1\alpha_2}\rho_{02}(\zeta,\tau), \quad (7)$$

where $\alpha_n = 2\pi\mathcal{N}\mu^2_{0n}\omega/c$ is the absorption coefficient of the medium having atomic density $\mathcal{N}$. Equations (2)–(7) must be solved self-consistently in order to study the propagation of the laser pulse.

Due to the complexity of Eqs. (2)–(7) general analytic solutions are not possible. However, several limits give valuable insight in the behavior of the system. Initially we assume that the dephasing rates are unimportant, and set $\gamma_{nm}=0$. Then, as has been discussed by several authors [17–19], the analysis of Eqs. (2)–(6) shows that population trapping is possible in the system if $\Delta_1=-\gamma_1\omega_{21}/(\gamma_1+\gamma_2)$. In this case the system can be characterized by an effective ''dark'' state

$$|\psi_{\rm dark}(\zeta,\tau)\rangle = N(\zeta,\tau)\left[\frac{\sqrt{\gamma_1\gamma_2}\omega_{21}}{\gamma_1+\gamma_2}|0\rangle + \Omega_2 f^*(\zeta,\tau)|1\rangle - \Omega_1 f^*(\zeta,\tau)|2\rangle\right], \quad (8)$$

where $N(\zeta,\tau)$ is the normalization factor. Therefore, under adiabatic excitation of the system (and if the trapping condition is satisfied), we obtain

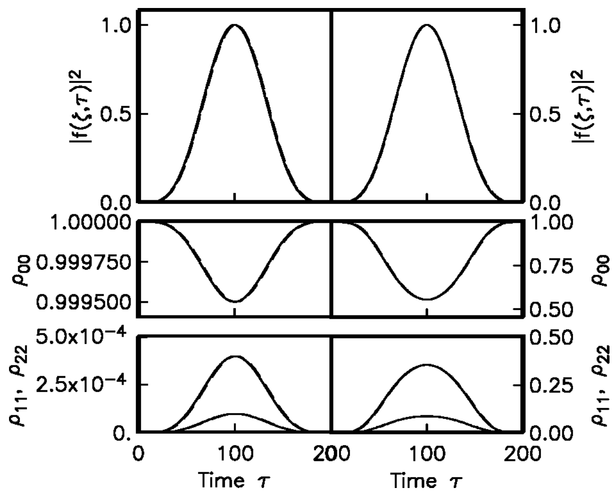

FIG. 1. The magnitude squared of the pulse envelope and the atomic populations as a function of τ for different values of ζ, with $\zeta=0$ (solid curves), $\zeta=\zeta_{\max}/2=50$ (dotted curves), and $\zeta=\zeta_{\max}=100$ (broken curves). For clarity the $\zeta=50$ results have been omitted in the population plots. The initial pulse was taken to be $f(\zeta=0,\tau)=\sin^2(\pi\tau/\tau_p)$, with τ_p being the pulse duration. The parameters used, in corresponding units of γ_1, were $\tau_p=200$, $\omega_{21}=50$, and $\delta_1=-10$. For the left-hand plots $\Omega_1=1/5$ and $\Omega_2=2/5$ while for the right-hand plots $\Omega_1=8$ and $\Omega_2=16$. Here and in the subsequent figures $\gamma_2=4$, $\alpha_1=1$, $\alpha_2=4$. In addition, note that in the lower plots here and in the following figures $\rho_{11}>\rho_{22}$.

$$\frac{\rho_{01}(\zeta,\tau)}{\rho_{02}(\zeta,\tau)} = -\frac{\Omega_2}{\Omega_1}. \quad (9)$$

Substituting this result into Eq. (7) and using the relation $\alpha_1/\alpha_2=\Omega_1^2/\Omega_2^2$, we find that

$$\frac{\partial}{\partial \zeta}f(\zeta,\tau)=0. \quad (10)$$

Therefore $|f(\zeta,\tau)|^2=|f(0,\tau)|^2$ for all ζ and hence, if the atomic medium evolves adiabatically in the laser field, the pulse propagates without absorption or dispersion. At this point we note that both the population trapping condition and the dark state given by Eq. (8) are the same as those obtained in the case of a four-level system with spontaneous emission interference [6,10,12].

The existence of transparency when the trapping conditions are satisfied is demonstrated in Fig. 1 where the magnitude squared of the pulse envelope at different positions in the medium as a function of τ is shown. Also shown are the corresponding populations of the field-free atomic states. The results were obtained by solving numerically Eqs. (2)–(7) for an adiabatically applied laser pulse for parameters that fulfill the trapping conditions. The results on the left-hand side of Fig. 1 show the propagation of a weak laser pulse, whereas on the right a strong laser pulse is considered. In both cases the propagation of the pulse is both loss- and dispersion-free, as predicted.

Approximate analytic solutions can also be derived for the case when the laser-atom interaction is weak. The problem can now be analyzed using a perturbative solution of Eqs.

(2)–(7). For weak fields, $\rho_{00}(\zeta,\tau)\approx 1$ and $\rho_{11}(\zeta,\tau)=\rho_{22}(\zeta,\tau)\approx 0$. Keeping terms to first order in $f(\zeta,\tau)$, Eqs. (4) and (5) reduce to

$$\left(\frac{\partial}{\partial\tau}-i\Delta_1+\frac{\gamma_1}{2}\right)\rho_{01}+\frac{\sqrt{\gamma_1\gamma_2}}{2}\rho_{02}=i\Omega_1 f(\zeta,\tau), \quad (11)$$

$$\frac{\sqrt{\gamma_1\gamma_2}}{2}\rho_{01}+\left(\frac{\partial}{\partial\tau}-i\Delta_2+\frac{\gamma_2}{2}\right)\rho_{02}=i\Omega_2 f(\zeta,\tau). \quad (12)$$

These equations are solved using a series expansion [12]. Keeping terms in the solution up to $\partial f/\partial\tau$ we obtain, under the population trapping condition,

$$\rho_{01}(\zeta,\tau)\approx\frac{(\gamma_1+\gamma_2)\Omega_2}{\sqrt{\gamma_1\gamma_2}\,\omega_{21}}f(\zeta,\tau)-\frac{(\gamma_1+\gamma_2)^4\Omega_2-2i(\gamma_1+\gamma_2)^2\Omega_2\gamma_2\omega_{21}}{2\gamma_1^{3/2}\gamma_2^{3/2}\omega_{21}^3}\frac{\partial f(\zeta,\tau)}{\partial\tau}, \quad (13)$$

$$\rho_{02}(\zeta,\tau)\approx-\frac{(\gamma_1+\gamma_2)\Omega_2}{\gamma_2\omega_{21}}f(\zeta,\tau)+\frac{(\gamma_1+\gamma_2)^4\Omega_2+2i(\gamma_1+\gamma_2)^2\Omega_2\gamma_1\omega_{21}}{2\gamma_1\gamma_2^2\omega_{21}^3}\frac{\partial f(\zeta,\tau)}{\partial\tau}. \quad (14)$$

Substituting Eqs. (13) and (14) into Eq. (7) results in

$$\left[\frac{\partial}{\partial\zeta}+\frac{1}{v}\frac{\partial}{\partial\tau}\right]f(\zeta,\tau)=0, \quad (15)$$

where $v=(\gamma_1^2\gamma_2\omega_{21}^2)/[\alpha_1(\gamma_1+\gamma_2)^3]$ is the group velocity of the laser pulse in the retarded frame. This means that as a first correction, nonadiabatic effects lead only to a reduction in the group velocity. In this case too, the result obtained here is the same as that obtained in the study of a four level system with spontaneous emission interference [12]. Numerical verification of this perturbative result is shown on the left-hand plots of Fig. 2. Stronger nonadiabatic effects will lead to both absorption and dispersion of the laser pulse. This is shown in Fig. 3. However, in this case the pulse will still propagate for many absorption lengths in the medium. We also note that as the pulse penetrates the medium the interaction of the pulse with the medium becomes more adiabatic and the rate of absorption of the pulse by the medium decreases.

To investigate the strong field and nonadiabatic regime, we resort again to numerical solutions of the Maxwell-Bloch equations. The results of the calculations are shown on the right-hand plot of Fig. 2. In this case, the front edge of the pulse is absorbed, however, afterwards adiabaticity is regained and the pulse then propagates in the medium without absorption. This behavior is similar to what occurs during the propagation of matched pulses [23].

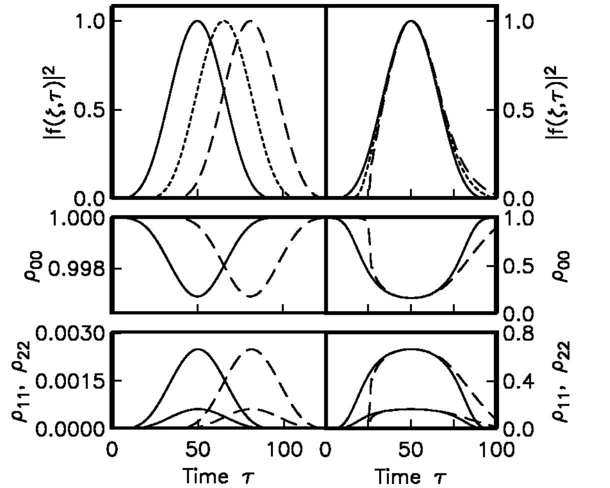

FIG. 2. Same as Fig. 1, however $\tau_p=100$, $\omega_{21}=20$, and $\delta_1=-4$. For the left-hand plots $\zeta_{\max}=400$ while on the right $\zeta_{\max}=200$.

Finally, we consider the influence of decoherence effects on transparency. Both for the weak and strong field cases, only numerical solutions of Eqs. (2)–(7) will be presented. In Fig. 4 we show the propagation of a pulse in a medium with the same parameters as in Fig. 1, but for an inhomogeneously broadened system. In this case, both in the weak and the strong field regimes, complete transparency does not occur. Due to the partial destruction of quantum coherence, absorption of the pulse by the medium can be seen. As the laser pulse intensity increases, and hence the Rabi frequencies become much larger than the dephasing rates, the pulse becomes more resilient to absorption. Similar results can be seen in Fig. 5 when the evolution of the system is strongly non-adiabatic.

In summary, we have investigated the effects of quantum coherence in a closed V-type medium, where two excited

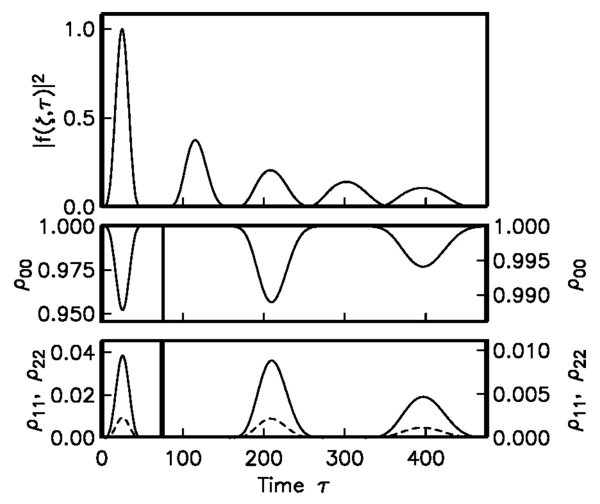

FIG. 3. The evolution of the system for a pulse duration $\tau_p=50$. The magnitude squared of the pulse envelope is shown for $\zeta=0$ to $\zeta_{\max}=300$ in steps of $\alpha\zeta_{\max}/4$. The populations are given in steps of $\alpha\zeta_{\max}/2$. The parameters used are $\Omega_1=1/5$, $\Omega_2=2/5$, $\omega_{21}=5$, and $\delta_1=-1$.

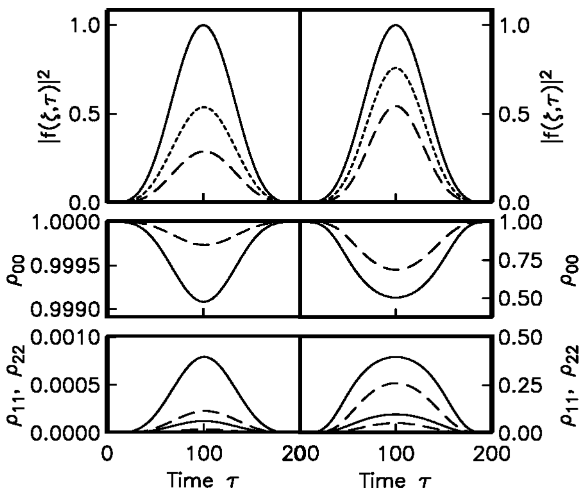

FIG. 4. The same as in Fig. 1, however, $\gamma_{01}=\gamma_{02}=\gamma_{12}=1/2$.

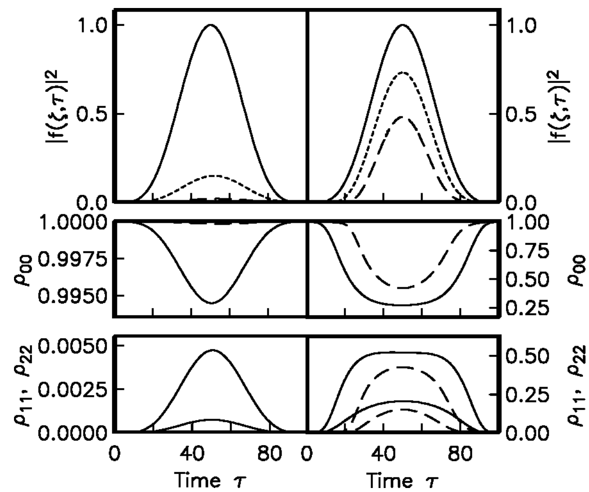

FIG. 5. The same as in Fig. 2, however, $\gamma_{01}=\gamma_{02}=\gamma_{12}=1/2$ and $\zeta_{\max}=50$.

states decay to the same ground state and the system is driven by a short laser pulse. We have shown that under specific conditions, namely, the conditions that lead to a dark state in the system, the medium can become transparent to the laser pulse when the evolution of the system is adiabatic. Nonadiabatic effects and decoherence effects have also been discussed. Experimental investigations of transparency in such V-type systems could be performed in several systems [24]. Examples include atomic media in dc-electric fields [5], molecular systems [7] and semiconductor quantum well systems [8].

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