Spontaneous-emission enhancement and population oscillation in photonic crystals via quantum interference

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Spontaneous emission from a V-type three-level atom in a photonic crystal is investigated. Quantum interference between the two atomic transitions affects the constructor of dressed states, and leads to an interesting behavior of the populations in the two upper levels: antitrapping, periodic oscillation, and no population inversion. Those properties depend strongly on the relative position of the upper levels from the forbidden gap and the initial state of the atom, and differ from that of a two-level atom in a photonic crystal. The emitted field, which is composed of localized mode(s) and propagating mode(s), is also studied. Quantum interference can enhance or reduce the energy of the localized field.

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I. INTRODUCTION

Quantum interference is one of the basic features of quantum mechanics. In a multilevel atomic system, quantum interference can lead to many unexpected effects, for example, absorption reduction and cancellation and spontaneous emission reduction and cancellation [1–3].

In recent years, there has been increasing interest in the spontaneous emission from an excited atom embedded in photonic crystals [4–8]. A photonic crystal is a manmade periodic dielectric structure designed to influence the propagation of electromagnetic waves [9–11]. Periodic dielectric structures can exhibit one (or more than one) full photonic band gap (frequency region) in which propagating electromagnetic waves are forbidden in all directions [9–11]. The deformations of the dispersion characteristics of waves traveling in a photonic crystal and the mode density of the electromagnetic field could lead to a number of distinctive quantum electrodynamics effects, which can be incorporated into designs of optoelectronic devices [12]. An excited atom in a photonic crystal can form a photon-atom bound dressed state when the atomic resonant frequency lies near the photonic band gap, which results in a fractional steady-state population in the excited state. Spontaneous emission from the atom displays quasioscillatory behavior instead of a simple exponential decay if the atom is in a vacuum. The spontaneous emission from a Λ three-level system was investigated by John and Quang in Ref. [5], and the emission spectrum for an Ξ three-level atomic system was studied by Bay and Lambropoulos in Ref. [7]. In these studies, the authors assumed one transition to be near the band edge and the other to be near a flat background of radiation modes (i.e., the vacuum). In Ref. [6], coherent control of spontaneous emission from a Λ or V three-level system was discussed. There one transition frequency was assumed to be far inside the band gap, so that the related spontaneous emission was ignored. Recently, a V three-level atomic system with two upper levels emitting photons into the same continuum was discussed [8]. The effect of quantum interference between two transitions leads to quasiperiodic oscillations of the population between two upper levels with large amplitudes [8]. However, only a special case was considered in which the position of the cutoff frequency of the band edge is at the center of the two upper levels. We will show that atomic populations on the upper levels are constants or display a periodic oscillation as time goes to infinity depending on the relative position of cutoff to the two upper levels. There is a special position at which the populations are from constant to oscillation.

In the present paper, we investigate spontaneous emission from a three-level atom, which has two upper levels coupled by the same modes to a lower level and is embedded in a photonic crystal, and we study how quantum interference affects the spontaneous emission process. The dependences of the interference on the relative position of the upper levels the band gap and initial conditions of the atom are discussed. Due to quantum interference between the two atomic transitions, the two upper levels split into dressed states in a combining fashion. The emitted field has been calculated in detail, and the emitted field is composed of two parts: a localized field and a traveling field. The localization distance of the localized field, the energy velocity, and the phase velocity of the propagating field are given. It is found that quantum interference can enhance or reduce the energy of the localized field. The populations trapped in the two upper levels display some interesting behaviors: (1) *Antitrapping*. The quantum interference between the two decay processes may transfer all the energy of the localized field into the propagating field, and the population in the ground level cannot jump back to the two upper levels without a localized field. (2) *Periodic oscillation*. The quantum interference between two localized modes may lead to population exchange between the two upper levels, and the populations display a periodic oscillatory behavior. (3) *No population inversion*. Due to the quantum interference, the population in the lower level may come back to both upper levels by absorbing the emitted photon. As the frequency of the localized field is closer to the second upper level, more population jumps back to this level, and inversion cannot occur.

This paper is organized as follows: In Sec. II, the basic theory to investigate the spontaneous emission is given. In Sec. III, the quantum interference effects on the atomic split-

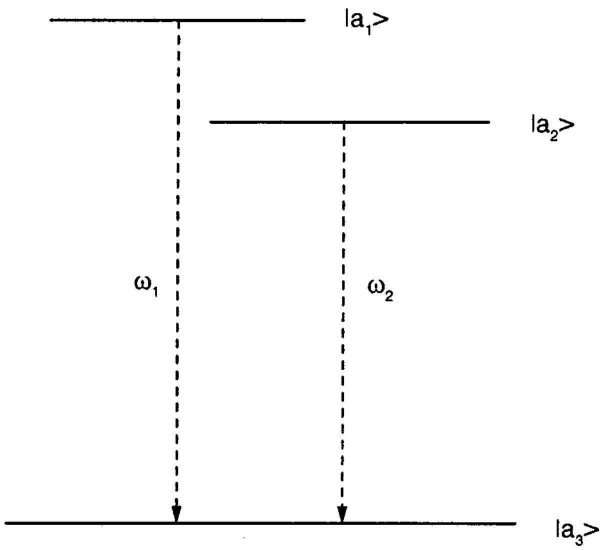

FIG. 1. A three-level atom.

ting are studied. In Sec. IV, we pay attention to the radiated field of the excited atom. The populations in the upper levels are studied in Sec. V.

II. BASIC THEORY

We consider a three-level atom as shown in Fig. 1. It is placed in a photonic crystal, and has two upper levels $|a_1\rangle$ and $|a_2\rangle$ which are coupled by the same vacuum modes to the lower level $|a_3\rangle$. The resonant transition frequencies between levels $|a_1\rangle$, $|a_2\rangle$, and $|a_3\rangle$ are ω_1 and ω_2, and are assumed to be near the edge of a band. There is, therefore, a strong quantum interference between two transitions from each upper level to the lower level. The Hamiltonian of the system decrying the spontaneous emission of the excited atom can be written as

$$\hat{H}=H_0+H_I,$$

$$H_0=\hbar\omega_1|a_1\rangle\langle a_1|+\hbar\omega_2|a_2\rangle\langle a_2|+\sum_k \hbar\omega_k b_k^\dagger b_k, \quad (1)$$

$$H_I=i\hbar\sum_k [g_k^{(1)}b_k^\dagger|a_3\rangle\langle a_1|+g_k^{(2)}b_k^\dagger|a_3\rangle\langle a_2|-g_k^{(1)}b_k|a_1\rangle \times\langle a_3|-g_k^{(2)}b_k|a_2\rangle\langle a_3|].$$

Here, b_k and $b_k^\dagger$ are the annihilation and creation operators for the kth vacuum mode with frequency ω_k. $g_k^{(1,2)}$ are the coupling constants between the kth vacuum mode and the atomic transitions from $|a_1\rangle$ and $|a_2\rangle$ to $|a_3\rangle$, and are assumed to be real: $g_k^{(i)}=(\omega_i d_i/\hbar)(\hbar/2\epsilon_0\omega_k V)^{1/2}\vec{e}_k\cdot\vec{u}_i$. k represents both the momentum and polarization of the vacuum mode. $\vec{u}_i$ are the unit vectors of the atomic dipole moments. The energy of the lower level has been set to zero. The state vector of the system at time t may be written as

$$|\psi(t)\rangle=[A_1(t)e^{-i\omega_1 t}|a_1\rangle+A_2(t)e^{-i\omega_2 t}|a_2\rangle]|0\rangle_f +\sum_k B_k(t)e^{-i\omega_k t}|a_3\rangle|1_k\rangle_f, \quad (2)$$

where the state vectors $|a_1\rangle|0\rangle_f$ and $|a_2\rangle|0\rangle_f$ describe the atom in its excited states $|a_1\rangle$ and $|a_2\rangle$, with no photons present in any vacuum mode, and the state vector $|a_3\rangle|1_k\rangle_f$ describes the atom in its ground state and a single photon in the kth mode with frequency ω_k. We assume the atom is initially in the upper levels, i.e., $|A_1(0)|^2+|A_2(0)|^2=1$ and $B_k(0)=0$. In a photonic crystal, the dispersion characteristics of radiation waves are deformed [4,5]. Near the band edge, they may be expressed approximately by

$$\omega_k=\omega_c+A(k-k_0)^2, \quad (3)$$

where A is constant coefficient, $A=\omega_c/k_0^2$. From the Schrödinger equation, we can obtain the following first-order differential equations for the amplitudes:

$$\frac{\partial}{\partial t}A_{1,2}(t)=-\sum_k g_k^{(1,2)}B_k(t)e^{-i(\omega_k-\omega_{1,2})t}, \quad (4a)$$

$$\frac{\partial}{\partial t}B_k(t)=g_k^{(1)}A_1(t)e^{i(\omega_k-\omega_1)t}+g_k^{(2)}A_2(t)e^{i(\omega_k-\omega_2)t}. \quad (4b)$$

With the help of the Laplace transform, we can solve the above two equations. The Laplace transforms $A^{(1,2)}(s)$ for the amplitudes $A^{(1,2)}(t)$ are found:

$$A_1(s)=\frac{A_1(0)(s-i\omega_{12}+\Gamma_{22})-A_2(0)\Gamma_{12}}{(s+\Gamma_{11})(s-i\omega_{12}+\Gamma_{22})-(\Gamma_{12})^2}, \quad (5a)$$

$$A_2(s-i\omega_{12})=\frac{A_2(0)(s+\Gamma_{11})-A_1(0)\Gamma_{12}}{(s+\Gamma_{11})(s-i\omega_{12}+\Gamma_{22})-(\Gamma_{12})^2}. \quad (5b)$$

Here $\Gamma_{mn}=\Sigma_k((g_k^{(m)}g_k^{(n)})/[s-i(\omega_1-\omega_k)])$, $(m,n=1,2)$ and $\omega_{12}=\omega_1-\omega_2$. Using the dispersion relation, and converting the mode sum over transverse plane waves into an integral and performing the integral, we have

$$\Gamma_{11}=\frac{\beta_1^{3/2}}{i\sqrt{-is-\omega_{1c}}}, \quad (6a)$$

$$\Gamma_{22}=\frac{\beta_2^{3/2}}{i\sqrt{-is-\omega_{1c}}}, \quad (6b)$$

$$\Gamma_{12}=\begin{cases}\dfrac{(\beta_1\beta_2)^{3/4}}{i\sqrt{-is-\omega_{1c}}} & \text{(parallel)}\\ -\dfrac{(\beta_1\beta_2)^{3/4}}{i\sqrt{-is-\omega_{1c}}} & \text{(antiparallel)}\\ 0 & \text{(orthogonal)},\end{cases} \quad (6c)$$

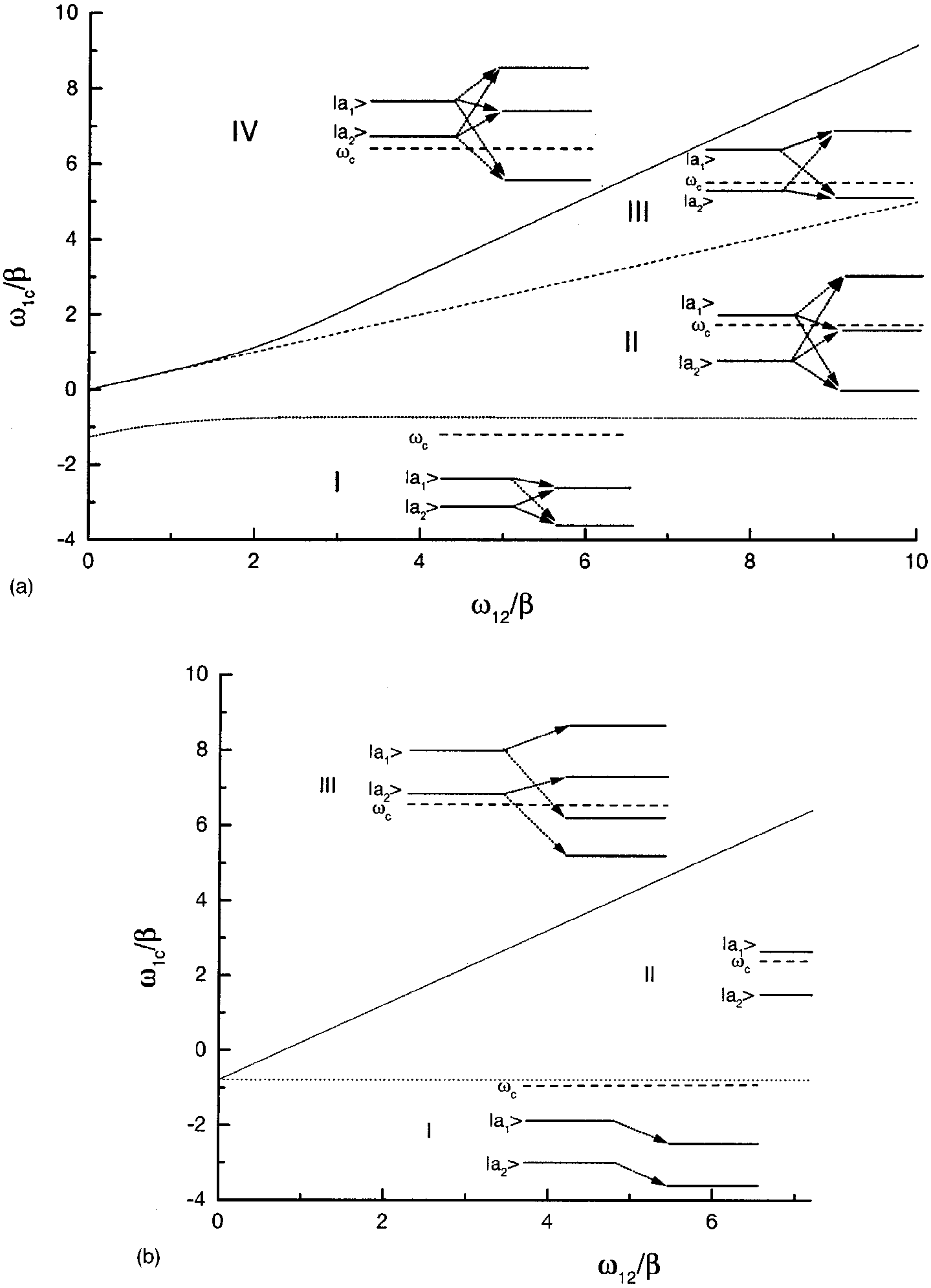

FIG. 2. The region for these roots and atomic splitting; (a) the parallel case; (b) the orthogonal case.

where $\beta_j^{3/2}=[(\omega_j d_j)^2/6\pi\epsilon_0\hbar](k_0^3/\omega_c^{3/2})$ $(j=1,2)$, $\omega_{1c}=\omega_1-\omega_c$, and $\omega_{2c}=\omega_2-\omega_c$ (see Appendix A). Note that the phase angle of $\sqrt{-is-\omega_{1c}}$ in $\Gamma_{m,n}$ has been defined $-\pi/2<\arg(\sqrt{-is-\omega_{1c}})<\pi/2$.

In the following discussion, we assume that the atomic dipole moments of two transitions $|a_{1,2}\rangle\rightarrow|a_3\rangle$ are parallel to each other, and $g_k^{(1)}=g_k^{(2)}=g_k$ for simplicity. So we have $\beta_1=\beta_2=\beta$ and $\Gamma_{11}=\Gamma_{22}=\Gamma_{12}=\Gamma$. If the two dipole moments are antiparallel to each other, the corresponding formulas are similar to changing $A_2(0)$ to $-A_2(0)$, and the same results will be obtained. If they are orthogonal to each other, the three-level system is a simple combination of two two-level systems [5]. In the parallel or antiparallel cases there is quantum interference between the two transitions, but for the orthogonal case the quantum interference between the two transitions does not exist.

The amplitudes $A_1(t)$ and $A_2(t)$ can be calculated by means of the inverse Laplace transforms

$$A_1(t)=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}A_1(s)e^{st}ds, \tag{7a}$$

$$A_2(t)=\frac{e^{-i\omega_{12}t}}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}A_2(s-i\omega_{12})e^{st}ds, \tag{7b}$$

where σ is a real constant that exceeds the real part of all the singularities of $A(s)$.

In Appendix B, we show that the poles of the complex integral functions [Eq. (7)] are important in the calculations of $A_{1,2}(t)$ and $B_k(t)$, and are directly related to the populations of two upper levels and the emitted field. So we need to discuss the poles. How many poles do the functions have? What are their values? What are the effects of these poles on the emitted field? According to Appendix B, the poles we consider are (1) the roots of the equation

$$s(s-i\omega_{12})+\frac{(2s-i\omega_{12})\beta^{3/2}}{i\sqrt{-is-\omega_{1c}}}=0 \tag{8a}$$

in the region $(\mathrm{Im}(s)>\omega_{1c}$ or $\mathrm{Re}(s)>0)$; and (2) the roots of the equation

$$s(s-i\omega_{12})+\frac{(2s-i\omega_{12})\beta^{3/2}}{\sqrt{is+\omega_{1c}}}=0 \tag{8b}$$

in the region $(\mathrm{Im}(s)<\omega_{1c}$, and $\mathrm{Re}(s)<0)$.

With the help of numerical calculations, we have found there are at least two roots, and at most three roots. Those roots can be classified into two types: (i) pure imaginary roots, which are the roots of Eq. (8a) with their imaginary parts larger than ω_{1c}, which correspond to localized modes in the emitted field [8]; and (ii) complex roots, which are the roots of Eq. (8b) with a negative real part and an imaginary part smaller than ω_{1c} which correspond to a propagating mode in the emitted field [5,8]. The number of the roots and their values depend on the relative position of the atomic upper levels from the band edge (ω_{1c} and ω_{2c}). For the pure imaginary roots of Eq. (8a), it has been proven analytically that we can have one pure imaginary root or two pure imaginary roots. If $\omega_{1c}\geqslant\omega_{12}/2$, we have one and only one pure imaginary root $ib_1^{(1)}$, which is in the range

$$b_1^{(1)}>\max(\omega_{1c},\omega_{12}). \tag{9a}$$

If $\omega_{1c}<\omega_{12}/2$, there are two pure imaginary roots $ib_1^{(1)}$ and $ib_2^{(1)}$, which are in the ranges

$$b_1^{(1)}>\omega_{12} \tag{9b}$$

$$\frac{\omega_{12}}{2}>b_2^{(1)}>\max(0,\omega_{1c}). \tag{9c}$$

According to the numbers and the values of the roots, we can have four cases, as shown in Fig. 2(a). In region I, only two pure imaginary roots exist. In region II, one complex root and two pure imaginary roots exist. In region III, there is one complex root and one pure imaginary root. One pure imaginary root and two complex roots exist in region IV.

After having $A_{1,2}(t)$, the populations on the two excited states can be obtained by

$$P_1=|A_1(t)e^{i\omega_1 t}|^2=|A_1(t)|^2, \tag{10a}$$

$$P_2=|A_2(t)e^{-i\omega_2 t}|^2=|A_2(t)|^2. \tag{10b}$$

$B_k(t)$ can be also obtained from Eqs. (4) and (7). Certainly, such calculations are complicated, and detailed steps and the results are given in Appendix B.

The amplitude of the radiated field at a particular space point r is [13]

$$E(r,t)=\frac{\omega_1 d_1\sin\eta}{8\pi^2\epsilon_0 ri}\int_0^\infty\frac{B_k(t)}{g_k}ke^{-i(\omega_k t-kr)}dk, \tag{11}$$

where η is the angle between the atomic dipole vector and the $\vec{r}$ vector. Equation (11) is valid in the far field. It can be seen that in the far-zone approximation, the radiated field is polarized in the x direction.

III. ATOMIC SPLITTING

With the help of residues at poles of a complex function (see Appendix B), the amplitude can be rewritten as

$$A_1(t)=\sum_j\frac{f_1(x_j^{(1)})}{G'(x_j^{(1)})}e^{ib_j^{(1)}t}+\sum_j\frac{f_2(x_j^{(2)})}{H'(x_j^{(2)})}\times e^{(a_j^{(2)}+ib_j^{(2)})t}-R_1(t), \tag{12a}$$

$$A_2(t)=e^{-i\omega_{12}t}\left[\sum_j\frac{f_3(x_j^{(1)})}{G'(x_j^{(1)})}e^{ib_j^{(1)}t}+\sum_j\frac{f_4(x_j^{(2)})}{H'(x_j^{(2)})}e^{(a_j^{(2)}+ib_j^{(2)})t}\right]-R_2(t), \tag{12b}$$

where $x_j^{(1)}=ib_j^{(1)}$ represents the pure imaginary root, and $x_j^{(2)}=a_j^{(2)}+ib_j^{(2)}$ is the complex root with a negative real part. The functions $f_j(x)$, $G'(x)$, and $H'(x)$ are defined in Appendix B, and the functions $f_j(x)$ are related to both $A_1(0)$ and $A_2(0)$. The functions $R(t)$ are defined as follows:

$$R_1(t)=\frac{e^{i\omega_{1c}t}\beta^{3/2}}{\pi\sqrt{i}}\int_0^\infty\frac{\sqrt{x}(x-i\omega_{2c})[A_1(0)(x-i\omega_{2c})+A_2(0)(x-i\omega_{1c})]}{x(x-i\omega_{1c})^2(x-i\omega_{2c})^2-i[2x-i(\omega_{1c}+\omega_{2c})]^2\beta^3}e^{-xt}dx,$$

$$R_2(t)=\frac{e^{i\omega_{2c}t}\beta^{3/2}}{\pi\sqrt{i}}\int_0^\infty\frac{\sqrt{x}(x-i\omega_{1c})[A_1(0)(x-i\omega_{2c})+A_2(0)(x-i\omega_{1c})]}{x(x-i\omega_{1c})^2(x-i\omega_{2c})^2-i[2x-i(\omega_{1c}+\omega_{2c})]^2\beta^3}e^{-xt}dx.$$

$R_1(t)$ and $R_2(t)$ come from the integration along the cut of the single-valued branches, and they decay to zero for large t.

From Eqs. (12a) and (12b), we can see that the upper levels $|a_1\rangle$ and $|a_2\rangle$ split into dressed states (due to the strong interaction between the atom and its own radiation field [5,8]) in a combinating fashion due to the interference between the two transitions from two upper levels to the lower level. It is very clear that each dressed state is a combination of both upper levels; see the inset schemes in Fig. 2(a).

If the two dipole moments are orthogonal to each other, the amplitudes can be calculated by

$$A_j^T(t)=A_j(0)\left\{\frac{e^{s_j^{(1)}t}}{G_j^T(s_j^{(1)})}+\frac{e^{s_j^{(2)}t}}{H_j^T(s_j^{(2)})}-R_j^T\right\} \quad (j=1,2) \tag{13}$$

where functions $G_j^T(s)$, $H_j^T(s)$, and R_j^T are defined as

$$G_j^T(s)=1+\frac{\beta^{3/2}}{2(\sqrt{-is-\omega_{jc}})^3},$$

$$H_j^T(s)=1-\frac{i\beta^{3/2}}{2(\sqrt{is+\omega_{jc}})^3},$$

$$R_j^T=\frac{\beta^{3/2}e^{-i\omega_{jc}t}}{\pi i^{1/2}}\int_0^\infty\frac{\sqrt{s}e^{-st}}{s(-s+i\omega_{jc})^2-i\beta^{3/2}}ds.$$

$s_j^{(1)}$ is the pure imaginary root of the equation $s+(\beta^{3/2}/i\sqrt{-is-\omega_{jc}})=0$ in the range ($\mathrm{Im}(s)>\omega_{jc}$) and always exists. $s_j^{(2)}$ is the complex root of the equation $s+(\beta^{3/2}/\sqrt{is+\omega_{jc}})=0$ in the region ($\mathrm{Im}(s)<\omega_{jc}$ and $\mathrm{Re}(s)<0$), and exists only when $\omega_{jc}>-0.7937\beta$ [5]. From Eqs. (4) and (13) we know that the upper levels $|a_j\rangle$ ($j=1,2$) will also split into two dressed states when $\omega_{jc}\rangle-0.7937\beta$, but there is no mixing of the two upper levels. The details of the splitting are plotted in Fig. 2(b).

Comparing Eqs (12a) and (12b) with Eq. (13), we can find that $A_j(t)$ is related to both $A_1(0)$ and $A_2(0)$, but $A_j^T(t)$ is related only to $A_j(0)$. The distinction comes from the quantum interference between the two transitions. For the parallel dipoles, the population in one upper level can decay to the lower level, and then jump to the either upper level by absorbing the photon emitted in the previous decay process; however, for the case of orthogonal dipole moments, it can only jump back to the level where it originates.

The quantum interference between the two transitions strongly affects the atomic splitting (formation of the dressed states). The effects on the splitting are plotted in Figs. 2(a) and 2(b) (see the insets for the parallel and orthogonal cases, respectively). Without interference (orthogonal case) the splitting is separated. As contrasted with the case of parallel dipole moments, each dressed state consists only of one of the two upper levels. In the parallel case, both upper levels make contributions to each dressed state due to the interference.

IV. EMITTED FIELD

From Appendixes B and C, the radiated field can be written as

$$E(r,t)=E_1^{(1)}+E_2^{(1)}+E_1^{(2)}+E_2^{(2)}+E^{(0)}. \tag{14}$$

$E_j^{(1)}$ ($j=1,2$) comes from the pure imaginary roots. If there is no second pure imaginary root, $E_2^{(1)}$ will be zero; $E_j^{(2)}$ ($j=1,2$) comes from the complex roots. If these complex roots do not exist, the relevant $E_j^{(2)}$ will vanish. $E^{(0)}$ is some integration; the exact result cannot be found. From their expression formulas in Appendixes B and C, it can be proven that E_0 decays to zero as time goes to infinity.

In region I of Fig. 2(a), there are only two pure imaginary roots $x_1^{(1)}=ib_1^{(1)}$ and $x_2^{(1)}=ib_2^{(1)}$ (no complex root), and the amplitude of the emitted field can be expressed as $E(r,t)=E_1^{(1)}+E_2^{(1)}+E^0$, where $E_1^{(1)}$ and $E_2^{(1)}$ come from the pure imaginary roots $ib_1^{(1)}$ and $ib_2^{(1)}$, respectively. According to Appendix C, $E_j^{(1)}$ can be written as

$$\begin{aligned}E_j^{(1)}(r,t)=&-\frac{\omega_1 d_1\sin\eta}{8\pi\epsilon_0 r}\frac{f_5(x_j^{(1)})}{G'(x_j^{(1)})}\frac{k_0^2}{\omega_c}\\&\times\left(\sqrt{\frac{\omega_c}{\omega_c-(\omega_1-b_j^{(1)})}}+i\right)e^{-i(\omega_1-b_j^{(1)})t+ik_0r}\\&\times\exp\left(-k_0r\sqrt{\frac{\omega_c-(\omega_1-b_j^{(1)})}{\omega_c}}\right)\\&\times\Theta\left(\sqrt{\frac{\omega_c-(\omega_1-b_j^{(1)})}{\omega_c}}t-\frac{k_0r}{2\omega_c}\right),\end{aligned} \tag{15}$$

where $\Theta(x)$ is the step function:

$$\Theta(x)=\begin{cases}0, & x<0\\ 1, & x\geqslant 0.\end{cases}$$

$E_j^{(1)}$ ($j=1,2$) is localized near the atom, and do not travel away from the atom. The $E_j^{(1)}$'s represent two localized modes, which form the localized field. Their frequencies are $\omega_1-b_1^{(1)}$ and $\omega_1-b_2^{(1)}$, which both are within the band gap. The amplitude of $E_j^{(1)}(r,t)$ ($j=1$ or 2) does not decay in time, and is proportional to

$$\frac{\sin\eta}{r}\left(\sqrt{\frac{\omega_c}{\omega_c-(\omega_1-b_j^{(1)})}}+i\right),$$

and drops exponentially as e^{-r/l_j}, where l_j is the localization distance:

$$l_j=\frac{1}{k_0}\sqrt{\frac{\omega_c}{\omega_c-(\omega_1-b_j^{(1)})}}.$$

The two pure imaginary roots can be written as $ib_1^{(1)}$ and $ib_2^{(1)}$, with $b_1^{(1)}>b_2^{(1)}$, and consequently the localization distances l_1 and l_2 satisfy $l_1<l_2$.

In region II, there are two pure imaginary roots $x_1^{(1)} = ib_1^{(1)}$ and $x_2^{(1)} = ib_2^{(1)}$, and one complex root $x_1^{(2)} = a_1^{(2)} + ib_1^{(2)}$. The field $E(r,t)$ can be rewritten as $E(r,t) = E_1^{(1)} + E_2^{(1)} + E_1^{(2)} + E^{(0)}$. $E_1^{(1)}$ and $E_2^{(1)}$ come from the two pure imaginary roots, and do not decay in time. They are two localized modes of the localized field, and take the form of Eq. (15). $E_1^{(2)}$ comes from the complex root, and can be written as

$$E_1^{(2)} = -\frac{\omega_1 d_1 \sin\eta}{8\pi\epsilon_0 r}\frac{f_5(x_1^{(2)})}{H'(x_1^{(2)})}\frac{ik_0^2}{\omega_c} \times\left(\sqrt{\frac{\omega_c}{\omega_1 - b_1^{(2)} - \omega_c + ia_1^{(2)}}} + 1\right) \times\exp\left(-i(\omega_1 - b_1^{(2)})t + ik_0 r + a_1^{(2)} t + ik_0 r\sqrt{\frac{\omega_1 - b_1^{(2)} - \omega_c + ia_1^{(2)}}{\omega_c}}\right) \times\Theta\left((\mathrm{Im}+\mathrm{Re})\sqrt{\frac{\omega_1 - b_1^{(2)} - \omega_c + ia_1^{(2)}}{\omega_c}}t - \frac{k_0 r}{2\omega_c}\right). \tag{16}$$

The frequency of $E_1^{(2)}(r,t)$ is $\omega_1 - b_1^{(2)}$, which is within transmitting band, and it represents a propagating mode, and travels away from the atom in the form of traveling pulse.

In region III, we have one pure imaginary root $ib_1^{(1)}$ and one complex root $a_1^{(2)} + ib_1^{(2)}$, and $E(r,t)$ can be obtained as $E(r,t) = E_1^{(1)} + E_1^{(2)} + E^{(0)}$, one localized mode $E_1^{(1)}$, and one propagating mode $E_1^{(2)}$. In region IV, there are two complex roots and one pure imaginary root. That is to say we have two propagating modes and one localized mode.

If the imaginary part of a root is b, the frequency of the field emitted by the atom is $\omega_1 - b$ [see Eqs. (15) and (16)]. For the pure imaginary root $ib_j^{(1)}$, the emitted field is a localized mode with frequency $\omega_1 - b_j^{(1)} < \omega_c$ in the gap. For the complex root $a_j^{(2)} + ib_j^{(2)}$, the emitted field is a propagating mode with a frequency $\omega_1 - b_j^{(2)} > \omega_c$ in the band.

Thus the radiated field may be characterized by three components: the localized field, the propagating field, and a decaying field. There are two modes localized in the localized field as $\omega_{1c} < -\omega_{2c}$, and one mode localized as $\omega_{1c} \geqslant -\omega_{2c}$. In the propagating field, there are two propagating modes for region IV, one mode for regions II and III, and no propagating mode for region I. If we change the relative position of the upper levels of the atom and the forbidden gap from region I to regions II and III, and then region IV, we can switch the emission from no traveling pulse to one traveling pulse and then two traveling pulses. Such a property may be used to design active optoelectronic switches.

V. POPULATIONS IN THE TWO UPPER LEVELS

The spontaneous decay from the two upper levels and the evolution of the atom population are greatly influenced by the interference. This influence can be observed by examining the populations in the two upper levels, which are $P_j = |A_j(t)|^2$ $(j=1,2)$ for the parallel case and $P_j = |A_j^T(t)|^2$ for the orthogonal case.

Let us analyze the roles of the quantum interference in the steady-state atomic populations P_{10} and P_{20} in upper levels with various initial states and with different relative positions of the upper levels from the forbidden gap. As time goes to infinity, only the first terms in Eqs. (12a) and (12b) contribute to the populations, and other terms can be neglected.

$$P_{10} = \left|\sum_{j=1,2}\frac{f_1(x_j^{(1)})e^{x_j^{(1)}t}}{G'(x_j^{(1)})}\right|^2, \quad P_{20} = \left|\sum_{j=1,2}\frac{f_3(x_j^{(1)})e^{x_j^{(1)}t}}{G'(x_j^{(1)})}\right|^2. \tag{17}$$

Using the relation

$$x_j^{(1)}(x_j^{(1)} - i\omega_{12}) + \frac{(2x_j^{(1)} - i\omega_{12})}{i\sqrt{-ix_j^{(1)} - \omega_{1c}}} = 0,$$

we can write the functions f_1 and f_3 as

$$f_1(x_j^{(1)}) = \frac{x_j^{(1)} - i\omega_{12}}{2x_j^{(1)} - i\omega_{12}}[A_1(0)(x_j^{(1)} - i\omega_{12}) + A_2(0)x_j^{(1)}], \tag{18}$$

$$f_3(x_j^{(1)}) = \frac{x_j^{(1)}}{2x_j^{(1)} - i\omega_{12}}[A_1(0)(x_j^{(1)} - i\omega_{12}) + A_2(0)x_j^{(1)}]. \tag{19}$$

There are two types of interference: (1) the interference between two upper levels, which is proportional to $A_1(0)A_2(0)^* x_1^{(1)*}(x_1^{(1)} - i\omega_{12})$ [see Eqs. (18) and (19)], and (2) the interferences between two dressed states, which are proportional to $f_1(x_1^{(1)})e^{x_1^{(1)}t}$ and $f_1^*(x_2^{(1)})e^{x_2^{(1)*}t}$ when there are two dressed states [see Eq. (17)]. These two types of interference could lead to some new phenomena which are different from what we have obtained in a two-level system.

A. Complete decay of populations

It is well known for a two-level system that some population is trapped in the upper level due to the existence of a localized field emitted by the two-level system [5]. In the present three-level system, the localized field can be enhanced or reduced by the interference. The energy in the localized field depends on the interference. The populations in the two upper levels are proportional to $|A_1(0)(b_1^{(1)} - \omega_{12}) + A_2(0)b_1^{(1)}|^2$ for $\omega_{1c} \geqslant -\omega_{2c}$ [see Eqs. (17)–(19); $x_1^{(1)} = ib_1^{(1)}$ is the pure imaginary root]. Because of the first type of interference, a complete interference can result in $|A_1(0)(b_1^{(1)} - \omega_{12}) + A_2(0)b_1^{(1)}|^2 = 0$ as $[A_1(0)/A_2(0)] = -[b_1^{(1)}/(b_1^{(1)} - \omega_{12})]$. In this case, the populations in the two upper levels can both be equal to zero, which means a complete decay of the upper levels (see Fig. 3).

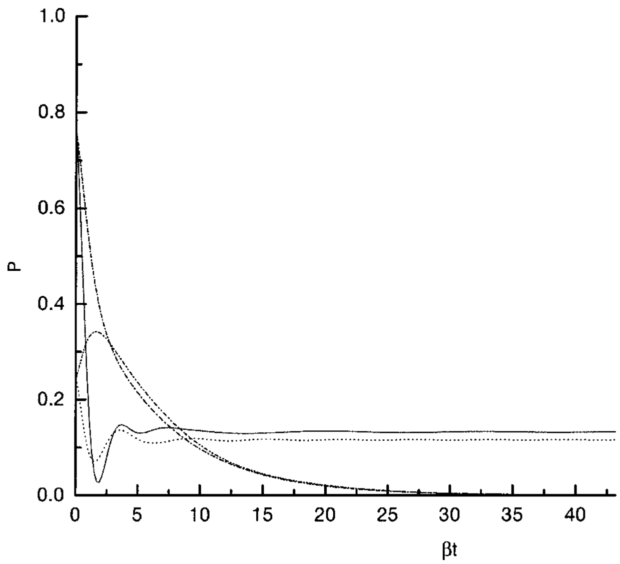

FIG. 3. The time evolution of the upper-level populations $P_1(t)$ and $P_2(t)$, with $\omega_{12}=\beta$, $\omega_{1c}=0.9\beta$, and $|\Psi(0)\rangle=0.8693|a_1\rangle-0.4942|a_2\rangle$. $P_1(t)$ and $P_2(t)$ in the parallel case (long-short-dashed curve and long-short-short-dashed curve, respectively), and $P_1(t)$ and $P_2(t)$ in the orthogonal case (short-dotted curve and dotted curve, respectively).

In fact, the energy of localized field is also proportional to $|A_1(0)(b_1^{(1)}-\omega_{12})+A_2(0)b_1^{(1)}|^2$ [see Eq. (15)]. We can see, when the initial phase difference is in the region $0\leqslant|\phi_1-\phi_2|<\pi/2$ $[A_1(0)=|A_1(0)|e^{i\phi_1},A_2(0)=|A_2(0)|e^{i\phi_2}]$, that the quantum interference enhances the energy of the localized field; when the initial phase difference is in the region $\pi/2<|\phi_1-\phi_2|\leqslant\pi$, the quantum interference reduces the energy of the localized field. When $[A_1(0)/A_2(0)]=-[b_1^{(1)}/(b_1^{(1)}-\omega_{12})]$, the interference will transfer all energy of the localized field into the propagating field. When the populations in the upper levels have decayed to a lower level, the atom cannot jump back to the upper levels without a localized field. When the energy of the localized field is reduced to zero by quantum interference, the population will stay in the lower level forever. Therefore, there is no population in the upper levels in the steady state.

Obviously, such a result is significantly different from the orthogonal case (or two-level case), where quantum interference between two transitions does not exist. There is always a localized field for each upper level. Thus the population in one upper level can decay to a lower level, and then can jump back to the same upper level due to the localized field. From Eq. (13), we can obtain the population P_{j0}^T for the orthogonal case; that is, $P_{j0}^T=|[A_j(0)/G_j^T(s_j^{(1)})]|^2$. $A_1(0)$ and $A_2(2)$ are not equal to zero at the same time, so there are fractionalized steady-state atomic population in the excited states (shown in Fig. 3).

B. No population inversion

For the orthogonal case, the ratio of the final populations in the upper levels is

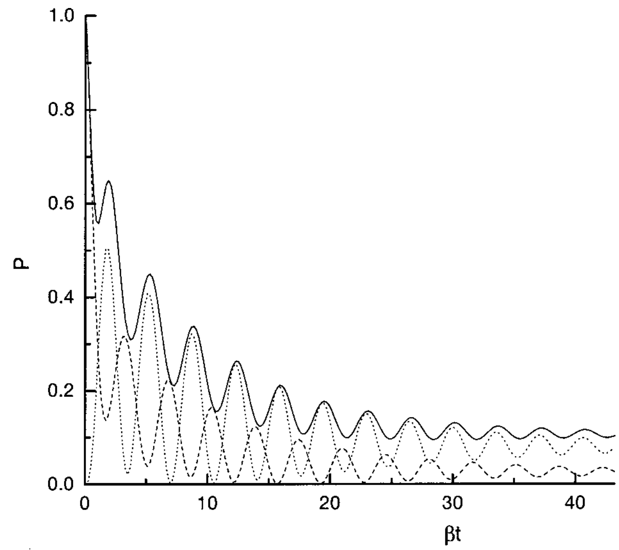

FIG. 4. The time evolution of the upper-level populations for the parallel case with $\omega_{12}=\beta$, $\omega_{1c}=0.8\beta$, and $|\Psi(0)\rangle=|a_1\rangle$.

$$\frac{P_{10}^T}{P_{20}^T}=\left|\frac{A_1(0)}{A_2(0)}\,\frac{G_2^T(s_2^{(1)})}{G_1^T(s_1^{(1)})}\right|^2.$$

For any relative position ω_{1c} or ω_{2c}, we can always obtain the final population inversion between the two upper levels through initial population inversion [for example, $A_1=1$ or $A_2(0)=0$].

For the parallel case, the quantum interference affects the populations in the upper level, and leads to no final population inversion between the two upper levels. When $\omega_{1c}\geqslant-\omega_{2c}$, the ratio of the final populations in the upper levels is independent of the initial condition, and can be obtained from Eqs. (17)–(19),

$$\frac{P_{10}}{P_{20}}=\left|\frac{b_1^{(1)}-\omega_{12}}{b_1^{(1)}}\right|^2.$$

From range (9a) of $b_1^{(1)}$, we know that $P_{10}/P_{20}<1$. That is to say, the population inversion does not exist in the case of $\omega_{1c}\geqslant-\omega_{2c}$, even when the initial conditions are $A_1(0)=1$ and $A_2(0)=0$ (see Fig. 4).

This is due to the quantum interference. Due to the existence of the localized field, the population in the lower level can jump back to the upper level. Due to quantum interference, the population in the lower level can come back to both upper levels by absorbing the emitted photon. The population in one upper level decays to the lower level, and then jumps in part back to the upper level $|a_1\rangle$ and in part back to the upper level $|a_2\rangle$. As the frequency level of the localized field is closer to level $|a_2\rangle$ than to level $|a_1\rangle$, the localized field leads to more population to the upper level $|a_2\rangle$. Consequently, the final population in $|a_2\rangle$ is always larger than that in $|a_1\rangle$.

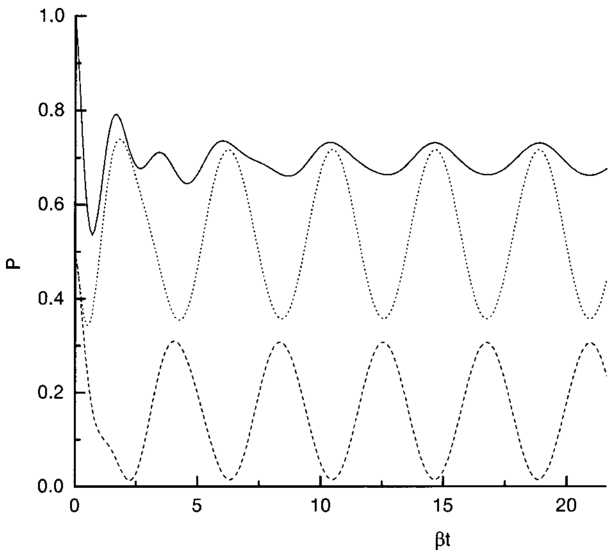

FIG. 5. The time evolution of the upper-level populations for the parallel case with $\omega_{12}=\beta$, $\omega_{1c}=-\beta$, and $|\Psi(0)\rangle=(1/\sqrt{2})(|a_1\rangle+|a_2\rangle)$.

C. Periodic oscillation

When $\omega_{1c}<-\omega_{2c}$, there are two dressed states with no decay. The quantum interference between the two dressed states leads to periodic oscillatory behaviors of the population trapping in the upper two levels for large time *t*. The amplitudes of the periodic oscillations do not decrease in time (shown in Fig. 5). From Eqs. (17)–(19), we obtain the following results: (1) The period of the oscillations for both populations is $2\pi/(b_1^{(1)}-b_2^{(1)})$. (2) The amplitudes K_1 (for $|a_1\rangle$) and K_2 (for $|a_2\rangle$) of the two periodic oscillations depend on the initial state. But the ratio of the amplitudes is independent of the initial state, that is

$$\frac{K_1}{K_2}=\frac{(b_1^{(1)}-\omega_{12})(\omega_{12}-b_2^{(1)})}{b_1^{(1)}b_2^{(1)}}<1.$$

(3) The phase difference of the two periodic oscillations is π, and the phase angle of the total population in the excited states is the same as that of the upper level $|a_2\rangle$. The periodic oscillation and phase-difference property mean that population exchange between the two upper levels always exists.

Actually, such a period oscillation of the populations is due to the existence of two different population exchanges between two upper levels, which are caused by the quantum interferences and two localized field. When $\omega_{1c}<-\omega_{2c}$, two pure imaginary roots $ib_1^{(1)}$ and $ib_2^{(1)}$ exist. In the above discussion we know that the population exchange, which is caused by the localized field related to $ib_1^{(1)}$ leads to more population trapping in the upper level $|a_2\rangle$. But the population exchange, which is caused by the localized field related to $ib_2^{(1)}$, will lead to more population trapping in the upper level $|a_1\rangle$. The influence between two population exchanges results in the periodic oscillation.

When the two dipole moments are orthogonal, there is no quantum interference between the two transitions, and the population exchange between the upper levels is not present. The fractional steady-state atomic populations are trapped in the two upper states.

From the above discussion, we can also see that the populations on the upper levels for large time are constants if $\omega_{1c}\geqslant-\omega_{2c}$, and display a periodic oscillation if $\omega_{1c}<-\omega_{2c}$. Therefore, the position corresponding to $\omega_{1c}=-\omega_{2c}$ is the critical position from constant to oscillation.

VI. CONCLUSION

We studied spontaneous emission from a *V*-type three-level atom in a photonic crystal. If the dipole moments of the two transitions are parallel, we have strong quantum interference between the two transitions. The dependence of the atomic population on the interference has been discussed. We found that the complete decay of the upper-level population, the impossibility of population inversion for the atom, and the periodic oscillation of the population are clearly proof of quantum interference effects. We also calculated the radiation field emitted by the atom. There is surely one localized mode (possibly two). The radiation field may contain zero or one or two traveling pulse(s) depending on the relative position between the upper levels and the band edge. This might be used to design an active microsized optical switch.

ACKNOWLEDGMENTS

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APPENDIX A: CALCULATION OF $\Gamma_{m,n}$

The coupling constants $g_k^{(j)}(j=1,2)$ may be written in the form

$$g^{(j)}=\frac{\omega_j d_j}{\hbar}\left(\frac{\hbar}{2\epsilon_0\omega_k V}\right)^{1/2}\vec{e}_k\cdot\vec{u}_j, \tag{A1}$$

where $\vec{u}_j$ are the unit vectors of the atomic dipole moments, and $\vec{e}_k\equiv\vec{e}_{k,\sigma}$ are the two transverse unit vectors.

We first calculate $\Gamma_{1,1}$:

$$\Gamma_{11}=\sum_k\frac{g_k^{(1)}g_k^{(1)}}{s-i(\omega_1-\omega_k)}$$

$$=\frac{(\omega_1 d_1)^2}{2\epsilon_0\hbar V}\sum_k\frac{\vec{u}_1\cdot\vec{u}_1-\dfrac{(\vec{k}\cdot\vec{u}_1)(\vec{k}\cdot\vec{u}_1)}{k^2}}{\omega_k[s-i(\omega_1-\omega_k)]}. \tag{A2}$$

Assuming $\vec{k}=(k\sin\theta\cos\phi,k\sin\theta\sin\phi,k\cos\theta)$, $\vec{u}_1=(0,0,1)$, we obtain

$$\Gamma_{11}=\frac{(\omega_1 d_1)^2}{2\epsilon_0\hbar V}\sum_k \frac{\sin^2\theta}{\omega_k[s-i(\omega_1-\omega_k)]}$$
$$=\frac{(\omega_1 d_1)^2}{6\epsilon_0\hbar\pi^2}\int_0^\infty \frac{k^2dk}{\omega_k[s-i(\omega_1-\omega_k)]}. \tag{A3}$$

Here we have replaced the sum by an integral via $\Sigma_k\rightarrow[V/(2\pi)^3]\int d^3k$. Using the dispersion characteristics $\omega_k=\omega_c+(\omega_c/k_0^2)(k-k_0)^2$, we can obtain

$$\Gamma_{11}=\frac{(\omega_1 d_1)^2}{6\epsilon_0\hbar\pi^2}\int_0^\infty \frac{k^2dk}{\left(\omega_c+\omega_c\frac{(k-k_0)^2}{k_0^2}\right)\left[s-i(\omega_1-\omega_c)+i\omega_c\frac{(k-k_0)^2}{k_0^2}\right]}$$
$$\simeq\frac{(\omega_1 d_1)^2}{6\epsilon_0\hbar\pi^2}\frac{k_0^2}{\omega_c}\int_{-\infty}^\infty \frac{dk}{s-i(\omega_1-\omega_c)+i\omega_c\frac{k^2}{k_0^2}}$$
$$=\frac{\beta_1^{3/2}}{i\sqrt{-is-(\omega_1-\omega_c)}}, \tag{A4}$$

where $\beta_1^{3/2}=[(\omega_1 d_1)^2/6\pi\epsilon_0\hbar](k_0^3/\omega_c^{3/2})$, and the phase angle is defined as $-\pi/2<\arg\sqrt{-is-(\omega_1-\omega_c)}<\pi/2$.

Similarly, Γ_{22} can be worked out:

$$\Gamma_{22}=\frac{\beta_2^{3/2}}{i\sqrt{-is-(\omega_1-\omega_c)}}, \tag{A5}$$

where $\beta_2^{3/2}=[(\omega_2 d_2)^2/6\pi\epsilon_0\hbar]k_0^3/\omega_c^{3/2}$.

When the two dipole moments of the two transitions are parallel to each other, i.e., $\vec{u}_1=\vec{u}_2$:

$$\Gamma_{12}=\sum_k \frac{g_k^{(1)}g_k^{(2)}}{s-i(\omega_1-\omega_k)}$$
$$=\frac{\omega_1 d_1\omega_2 d_2}{2\epsilon_0\hbar V}\sum_k\frac{(\vec{e}_k\cdot\vec{u}_1)(\vec{e}_k\cdot\vec{u}_2)}{\omega_k[s-i(\omega_1-\omega_k)]}$$
$$=\frac{(\beta_1\beta_2)^{3/4}}{i\sqrt{-is-(\omega_1-\omega_c)}}. \tag{A6}$$

When the two dipole moments are antiparallel to each other, i.e., $\vec{u}_1=-\vec{u}_2$:

$$\Gamma_{12}=-\frac{(\beta_1\beta_2)^{3/4}}{i\sqrt{-is-(\omega_1-\omega_c)}}. \tag{A7}$$

When the two dipole moments are orthogonal to each other, i.e., $\vec{u}_1=(0,0,1)$ and $\vec{u}_2=(1,0,0), \Gamma_{12}$ will be zero:

$$\Gamma_{12}=\frac{\omega_1 d_1\omega_2 d_2}{2\epsilon_0\hbar V}\sum_k\frac{(\vec{e}_k\cdot\vec{u}_1)(\vec{e}_k\cdot\vec{u}_2)}{\omega_k[s-i(\omega_1-\omega_k)]}$$
$$=-\frac{\omega_1 d_1\omega_2 d_2}{16\pi^3\epsilon_0\hbar}\int_0^\infty\int_0^\pi\int_0^{2\pi}\frac{k^2\sin^2\theta\cos\theta\cos\phi\, dk\, d\theta\, d\phi}{\omega_k[s-i(\omega_1-\omega_k)]}$$
$$=0. \tag{A8}$$

Hence Γ_{mn} can be written as follows:

$$\Gamma_{11}=\frac{\beta_1^{3/2}}{i\sqrt{-is-(\omega_1-\omega_c)}},$$
$$\Gamma_{22}=\frac{\beta_2^{3/2}}{i\sqrt{-is-(\omega_1-\omega_c)}}, \tag{A9}$$
$$\Gamma_{12}=\begin{cases}\dfrac{(\beta_1\beta_2)^{3/4}}{i\sqrt{-is-(\omega_1-\omega_c)}} & (\text{parallel})\\ -\dfrac{(\beta_1\beta_2)^{3/4}}{i\sqrt{-is-(\omega_1-\omega_c)}} & (\text{antiparallel})\\ 0 & (\text{orthogonal}).\end{cases}$$

APPENDIX B: CALCULATION OF $A^{(1)}(T)$, $A^{(2)}(T)$, AND $B_k(T)$

We now define some functions as follows:

$$f_1(x)=A_1(0)(x-i\omega_{12})+\frac{(A_1(0)-A_2(0))\beta^{3/2}}{i\sqrt{-ix-\omega_{1c}}},$$

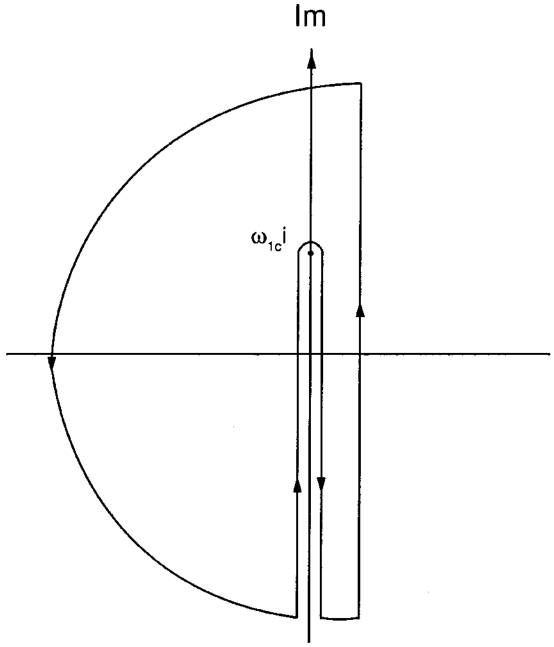

FIG. 6. The integration contours for Eq. (B1).

$$f_2(x)=A_1(0)(x-i\omega_{12})+\frac{(A_1(0)-A_2(0))\beta^{3/2}}{\sqrt{ix+\omega_{1c}}},$$

$$f_3(x)=A_2(0)x+\frac{(A_2(0)-A_1(0))\beta^{3/2}}{i\sqrt{-ix-\omega_{1c}}},$$

$$f_4(x)=A_2(0)x+\frac{(A_2(0)-A_1(0))\beta^{3/2}}{\sqrt{ix+\omega_{1c}}},$$

$$f_5(x)=(A_1(0)+A_2(0))x-iA_1(0)\omega_{12},$$

$$G(x)=x(x-i\omega_{12})+\frac{(2x-i\omega_{12})\beta^{3/2}}{i\sqrt{-ix-\omega_{1c}}},$$

$$H(x)=x(x-i\omega_{12})+\frac{(2x-i\omega_{12})\beta^{3/2}}{\sqrt{ix+\omega_{1c}}}.$$

For $G(x),H(x)$, we have

$$G'(x)=2x-i\omega_{12}+\frac{2\beta^{3/2}}{i\sqrt{-ix-\omega_{1c}}}+\frac{(2x-i\omega_{12})\beta^{3/2}}{2(\sqrt{-ix-\omega_{1c}})^3},$$

$$H'(x)=2x-i\omega_{12}+\frac{2\beta^{3/2}}{\sqrt{ix+\omega_{1c}}}-\frac{(2x-i\omega_{12})i\beta^{3/2}}{2(\sqrt{ix+\omega_{1c}})^3}.$$

Using the inverse Laplace transform, we obtain

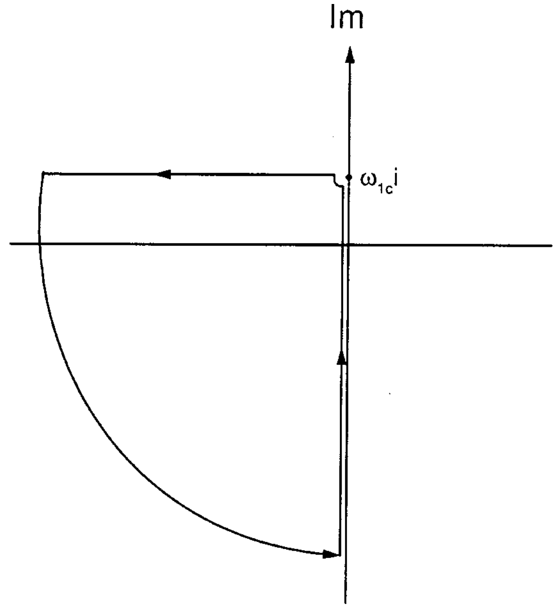

FIG. 7. The integration contours for Eq. (B2).

$$\begin{aligned}A_1(t)&=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}A_1(s)e^{st}ds\\&=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{f_1(x)}{G(x)}e^{xt}dx\\&=\sum_j\frac{f_1(x_j^{(1)})}{G'(x_j^{(1)})}e^{x_j^{(1)}t}\\&\quad-\frac{1}{2\pi i}\left[\int_{-\infty i-0}^{\omega_{1c}i-0}+\int_{\omega_{1c}i+0}^{-\infty i+0}\right]\frac{f_1(x)}{G(x)}e^{xt}dx,\end{aligned}\tag{B1}$$

where $x_j^{(1)}$ are the roots of the equation $G(x)=0$, the real number σ is chosen so that $x=\sigma$ lies to the right of all the singularity $x_j^{(1)}$. The integration contours are shown in Fig. 6:

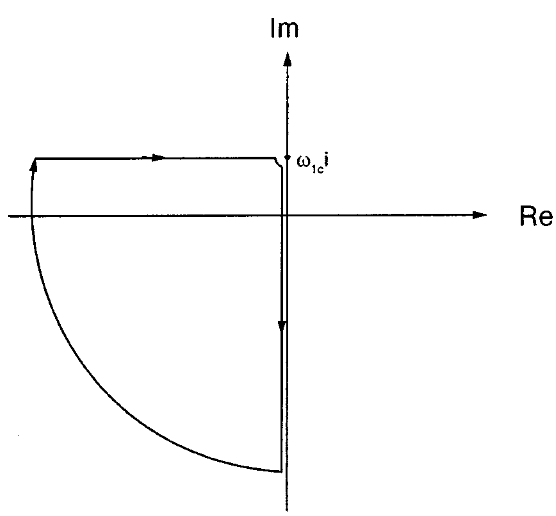

FIG. 8. The integration contours for Eq. (B3).

$$\frac{1}{2\pi i}\int_{-\infty i-0}^{\omega_{1c}i-0}\frac{f_1(x)}{G(x)}e^{xt}dx$$
$$=\sum_j \frac{f_1(x_j^{(1)})}{G'(x_j^{(1)})}e^{x_j^{(1)}t}-\frac{e^{i\omega_{1c}t}}{2\pi i}\int_0^\infty \frac{A_1(0)\sqrt{x}(x-i\omega_{2c})+(A_1(0)-A_2(0))\sqrt{i}\beta^{3/2}}{\sqrt{x}(x-i\omega_{1c})(x-i\omega_{2c})+[2x-i(\omega_{1c}+\omega_{2c})]\sqrt{i}\beta^{3/2}}e^{-xt}dx. \quad \text{(B2)}$$

Note that here $x_j^{(1)}$ are the roots of $G(x)=0$ in the region $\text{Im}(x)<\omega_{1c}$ and $\text{Re}(x)<0$. The second integration contours are shown in Fig. 7,

$$\frac{1}{2\pi i}\int_{\omega_{1c}i+0}^{-\infty i+0}\frac{f_1(x)}{G(x)}e^{xt}dx=\frac{1}{2\pi i}\int_{\omega_{1c}i}^{-\infty i}\frac{f_2(x)}{H(x)}e^{xt}dx$$
$$=-\sum_j \frac{f_2(x_j^{(2)})}{H'(x_j^{(2)})}e^{x_j^{(2)}t}+\frac{e^{i\omega_{1c}t}}{2\pi i}$$
$$\times\int_0^\infty \frac{A_1(0)\sqrt{x}(x-i\omega_{2c})-(A_1(0)-A_2(0))\sqrt{i}\beta^{3/2}}{\sqrt{x}(x-i\omega_{1c})(x-i\omega_{2c})-[2x-i(\omega_{1c}+\omega_{2c})]\sqrt{i}\beta^{3/2}}e^{-xt}dx, \quad \text{(B3)}$$

where $x_j^{(2)}$ are the roots of $H(x)=0$, and satisfy $\text{Im}(x_j^{(2)})<\omega_{1c}$ and $\text{Re}(x_j^{(2)})<0$. The integration contours are shown in Fig. 8.

Substituting Eqs. (B2) and (B3) into Eq. (B1), we have

$$A_1(t)=\sum_j \frac{f_1(x_j^{(1)})}{G'(x_j^{(1)})}e^{x_j^{(1)}t}+\sum_j \frac{f_2(x_j^{(2)})}{H'(x_j^{(2)})}e^{x_j^{(2)}t}$$
$$-\frac{e^{i\omega_{1c}t}\beta^{3/2}}{\pi\sqrt{i}}\int_0^\infty \frac{\sqrt{x}(x-i\omega_{2c})[A_1(0)(x-i\omega_{2c})+A_2(0)(x-i\omega_{1c})]}{x(x-i\omega_{1c})^2(x-i\omega_{2c})^2-i[2x-i(\omega_{1c}+\omega_{2c})]^2\beta^3}e^{-xt}dx. \quad \text{(B4)}$$

In Eq. (B4), $x_j^{(1)}$ are the roots of $G(x)=0$ in the region $\text{Im}(x_j^{(1)})>\omega_{1c}$ or $\text{Re}(x_j^{(1)})>0$; $x_j^{(2)}$ are the roots of $H(x)=0$ in the region $\text{Im}(x_j^{(2)})<\omega_{1c}$ and $\text{Re}(x_j^{(2)})<0$.

Similarly, $A_2(t)$ takes the following form:

$$A_2(t)=e^{-i\omega_{12}t}\left[\sum_j \frac{f_3(x_j^{(1)})}{G'(x_j^{(1)})}e^{x_j^{(1)}t}+\sum_j \frac{f_4(x_j^{(2)})}{H'(x_j^{(2)})}e^{x_j^{(2)}t}\right]$$
$$-\frac{e^{i\omega_{2c}t}\beta^{3/2}}{\pi\sqrt{i}}\int_0^\infty \frac{\sqrt{x}(x-i\omega_{1c})[A_1(0)(x-i\omega_{2c})+A_2(0)(x-i\omega_{1c})]}{x(x-i\omega_{1c})^2(x-i\omega_{2c})^2-i[2x-i(\omega_{1c}+\omega_{2c})]^2\beta^3}e^{-xt}dx. \quad \text{(B5)}$$

From Eqs. (4), (B4), and (B5), we can obtain

$$B_k(t)=g_k\sum_j\left(\frac{f_5(x_j^{(1)})}{G'(x_j^{(1)})}\cdot\frac{e^{i(\omega_k-\omega_1)t+x_j^{(1)}t}-1}{i(\omega_k-\omega_1)+x_j^{(1)}}\right)+g_k\sum_j\left(\frac{f_5(x_j^{(2)})}{H'(x_j^{(2)})}\cdot\frac{e^{i(\omega_k-\omega_1)t+x_j^{(2)}t}-1}{i(\omega_k-\omega_1)+x_j^{(2)}}\right)$$
$$-\frac{g_k\beta^{3/2}}{\pi\sqrt{i}}\int_0^\infty \frac{e^{i(\omega_k-\omega_c)t-xt}-1}{i(\omega_k-\omega_c)-x}\,\frac{\sqrt{x}[A_1(0)(x-i\omega_{2c})+A_2(0)(x-i\omega_{1c})][2x-i(\omega_{1c}+\omega_{2c})]}{x(x-i\omega_{1c})^2(x-i\omega_{2c})^2-i[2x-i(\omega_{1c}+\omega_{2c})]^2\beta^3}dx. \quad \text{(B6)}$$

APPENDIX C: CALCULATION OF THE RADIATED FIELD

The amplitude of the radiated field at a particular space point $\vec{r}$ is

$$\vec{E}(\vec{r},t)=\sum_k \sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}e^{-i(\omega_k t-\vec{k}\cdot\vec{r})}B_k(t)\vec{e}_k$$
$$=\frac{\omega_1 d_1}{16\pi^3\epsilon_0}\int_0^\infty k^2\frac{B_k(t)}{g_k}e^{-i\omega_k t}dk\int_0^{2\pi}d\phi\int_0^\pi \sin\theta e^{i\vec{k}\cdot\vec{r}}\left(\vec{u}-\frac{\vec{k}(\vec{k}\cdot\vec{u})}{k^2}\right)d\theta,$$

where we have replaced the sum by an integral via $\Sigma_k \to (V/2\pi^3)\int d^3k$. We assume that the electric dipole is in the x-z plane, and $\vec{r}$ is parallel to the z axis. The vectors $\vec{k}$ and $\vec{u}$ can be defined in polar coordinates by

$$\vec{k}=k(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta),$$

$$\vec{u}=(\sin\eta,0,\cos\eta),$$

where η is the angle that the dipole makes with the $\vec{r}$ vector. We consider the three components of $\vec{E}$, and obtain

$$E_x=\frac{\omega_1 d_1 \sin\eta}{4\pi^2\epsilon_0 r}\int_0^\infty k\sin kr e^{-i\omega_k t}\frac{B_k(t)}{g_k}dk+O\left(\frac{1}{r^2}\right),$$

$$E_y=0,$$

$$E_z\sim O\left(\frac{1}{r^2}\right).$$

In the far-field region, the terms proportional to $O(1/r^2)$ can be neglected. Therefore, the y and z components of the electric field vanish, while the x component remains. Ignoring the incoming wave contribution, we have

$$E(r,t)=\frac{\omega_1 d_1 \sin\eta}{8\pi^2\epsilon_0 r i}\int_0^\infty \frac{B_k(t)}{g_k}ke^{-i(\omega_k t-kr)}dk. \tag{C1}$$

From Eq. (B6), we can see that $B_k(t)$ is composed of the contributions of these poles and an integral,

$$B_k(t)=g_k\left(\sum_j B_j^{(1)}+\sum_j B_j^{(2)}+B_0^{(0)}\right), \tag{C2}$$

where

$$B_j^{(1)}=\frac{f_5(x_j^{(1)})}{G'(x_j^{(1)})}\frac{e^{i(\omega_k-\omega_1)t+x_j^{(1)}t}-1}{i(\omega_k-\omega_1)+x_j^{(1)}},$$

$$B_j^{(2)}=\frac{f_5(x_j^{(2)})}{H'(x_j^{(2)})}\frac{e^{i(\omega_k-\omega_1)t+x_j^{(2)}t}-1}{i(\omega_k-\omega_1)+x_j^{(2)}},$$

$$B_j^{(3)}=-\frac{1}{\pi\sqrt{i}}\int_0^\infty \frac{e^{i(\omega_k-\omega_c)t-xt}-1}{i(\omega_k-\omega_c)-x}K(x)dx.$$

In these above formulas, $x_j^{(1)}$ are the pure imaginary roots, and $x_j^{(2)}$ are the complex roots, and

$$K(x)=\frac{\sqrt{x}[A_1(0)(x-i\omega_{2c})+A_2(0)(x-i\omega_{1c})][2x-i(\omega_{1c}+\omega_{2c})]\beta^{3/2}}{x(x-i\omega_{1c})^2(x-i\omega_{2c})^2-i[2x-i(\omega_{1c}+\omega_{2c})]^2\beta^3}.$$

(1) For the pure imaginary root $x^{(1)}=ib^{(1)}$, we have $\omega_1-b^{(1)}<\omega_c$:

$$E^{(1)}=\frac{\omega_1 d_1\sin\eta}{8\pi^2\epsilon_0 r}\frac{f_5(x^{(1)})}{G'(x^{(1)})}\int_0^\infty k\frac{e^{-i(\omega_1-b^{(1)})t+ikr}-e^{-i\omega_k t+ikr}}{\omega_k-(\omega_1-b^{(1)})}dk$$
$$=-\frac{\omega_1 d_1\sin\eta}{8\pi^2\epsilon_0 r}\frac{f_5(x^{(1)})}{G'(x^{(1)})}(E_a^{(1)}-E_b^{(1)}), \tag{C3}$$

$$E_a^{(1)}=\int_0^\infty \frac{ke^{-i(\omega_1-b^{(1)})t+ikr}}{\omega_k-(\omega_1-b^{(1)})}dk$$
$$\simeq\frac{\pi k_0^2}{\omega_c}\left(\sqrt{\frac{\omega_c}{\omega_c-(\omega_1-b^{(1)})}}+i\right)\exp\left(-i(\omega_1-b^{(1)})t+ik_0r-k_0r\sqrt{\frac{\omega_c-(\omega_1-b^{(1)})}{\omega_c}}\right), \tag{C4}$$

$$E_b^{(1)}=\int_0^\infty \frac{ke^{-i\omega_k t+ikr}}{\omega_k-(\omega_1-b^{(1)})}dk$$
$$=e^{-i\omega_c t+ik_0r+i(k_0^2r^2/4\omega_c t)}\int_{-k_0-(k_0^2r/2\omega_c t)}^\infty \frac{\left(k+\frac{k_0^2r}{2\omega_c t}+k_0\right)e^{-i(\omega_c/k_0^2)k^2t}}{\frac{\omega_c}{k_0^2}\left(k+\frac{k_0^2r}{2\omega_c t}\right)^2+\omega_c-(\omega_1-b^{(1)})}dk$$
$$=e^{-i\omega_c t+ik_0r+i(k_0^2r^2/4\omega_c t)}(E_{b1}^{(1)}+E_{b2}^{(2)}), \tag{C5}$$

$$E_{b1}^{(1)}=\int_{-k_0-(k_0^2r/2\omega_ct)}^{0}\frac{\left(k+\frac{k_0^2r}{2\omega_ct}+k_0\right)e^{-i(\omega_c/k_0^2)k^2t}}{\frac{\omega_c}{k_0^2}\left(k+\frac{k_0^2r}{2\omega_ct}\right)^2+\omega_c-(\omega_1-b^{(1)})}dk$$

$$=\frac{\pi k_0^2}{\omega_c}\left(\sqrt{\frac{\omega_c}{\omega_c-(\omega_1-b^{(1)})}}+i\right)\exp\left(i(\omega_c-(\omega_1-b^{(1)}))t-\frac{ik_0^2r^2}{4\omega_ct}-k_0r\sqrt{\frac{\omega_c-(\omega_1-b^{(1)})}{\omega_c}}\right)$$

$$\times\Theta\left(\frac{k_0r}{2\omega_ct}-\sqrt{\frac{\omega_c-(\omega_1-b^{(1)})}{\omega_c}}\right)-\int_0^{\infty}\frac{e^{(3\pi/4)i}\left(\rho e^{(3\pi/4)i}+\frac{k_0^2r}{2\omega_ct}+k_0\right)e^{-(\omega_c/k_0^2)\rho^2t}}{\frac{\omega_c}{k_0^2}\left(\rho e^{(3\pi/4)i}+\frac{k_0^2r}{2\omega_ct}\right)^2+\omega_c-(\omega_1-b^{(1)})}d\rho,$$

$$E_{b2}^{(1)}=\int_0^{\infty}\frac{\left(k+\frac{k_0^2r}{2\omega_ct}+k_0\right)e^{-i(\omega_c/k_0^2)k^2t}}{\frac{\omega_c}{k_0^2}\left(k+\frac{k_0^2r}{2\omega_ct}\right)^2+\omega_c-(\omega_1-b^{(1)})}dk$$

$$=\int_0^{\infty}\frac{e^{-(\pi/4)i}\left(\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_ct}+k_0\right)e^{-(\omega_k/k_0^2)\rho^2t}}{\frac{\omega_c}{k_0^2}\left(\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_ct}\right)^2+\omega_c-(\omega_1-b^{(1)})}d\rho.$$

From Eqs. (C3)–(C5), we can obtain

$$E^{(1)}=-\frac{\omega_1d_1\sin\eta}{8\pi\epsilon_0r}\frac{f_5(x^{(1)})}{G'(x^{(1)})}\frac{k_0^2}{\omega_c}\left(\sqrt{\frac{\omega_c}{\omega_c-(\omega_1-b^{(1)})}}+i\right)e^{-i(\omega_1-b^{(1)})t+ik_0r}\exp\left(-k_0r\sqrt{\frac{\omega_c-(\omega_1-b^{(1)})}{\omega_c}}\right)$$

$$\times\Theta\left(\sqrt{\frac{\omega_c-(\omega_1-b^{(1)})}{\omega_c}}-\frac{k_0r}{2\omega_ct}\right)+\frac{\omega_1d_1\sin\eta}{8\pi^2\epsilon_0r}\frac{f_5(x^{(1)})}{G'(x^{(1)})}\exp\left[-i\omega_ct+ik_0r+i\frac{k_0^2r^2}{4\omega_ct}-\frac{\pi}{4}i\right]$$

$$\times\left[\int_0^{\infty}\frac{\left(\rho e^{(3\pi/4)i}+\frac{k_0^2r}{2\omega_ct}+k_0\right)e^{-(\omega_c/k_0^2)\rho^2t}}{\frac{\omega_c}{k_0^2}\left(\rho e^{(3\pi/4)i}+\frac{k_0^2r}{2\omega_ct}\right)^2+\omega_c-(\omega_1-b^{(1)})}d\rho+\int_0^{\infty}\frac{\left(\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_ct}+k_0\right)e^{-(\omega_c/k_0^2)\rho^2t}}{\frac{\omega_c}{k_0^2}\left(\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_ct}\right)^2+\omega_c-(\omega_1-b^{(1)})}d\rho\right],\quad\text{(C6)}$$

where $\Theta(x)$ is the step function for $x\geqslant 0,\Theta(x)=1$ and $x<0,\Theta(x)=0$. The first term represents a localized field at frequency $\omega_1-b^{(1)}$. The size of the localized photon mode is $(k_0\sqrt{[\omega_c-(\omega_1-b^{(1)})/\omega_c]})^{-1}$. The second term will be zero as time $t\to\infty$.

(2) For the complex root $x^{(2)}=a^{(2)}+ib^{(2)}$, we have $a^{(2)}<0,\ \omega_1-b^{(2)}>\omega_c$:

$$E^{(2)}=-\frac{\omega_1d_1\sin\eta}{8\pi^2\epsilon_0r}\cdot\frac{f_5(x^{(2)})}{H'(x^{(2)})}\int_0^{\infty}k\frac{\exp(-i[(\omega_1-b^{(2)})t-kr]+a^{(2)}t)-e^{-i(\omega_kt-ikr)}}{\omega_k-(\omega_1-b^{(2)})-ia^{(2)}}dk$$

$$=-\frac{\omega_1d_1\sin\eta}{8\pi^2\epsilon_0r}\frac{f_5(x^{(2)})}{H'(x^{(2)})}(E_a^{(2)}-E_b^{(2)}),\quad\text{(C7)}$$

$$E_a^{(2)}=\int_0^\infty \frac{k\exp(-i[(\omega_1-b^{(2)})t-kr]+a^{(2)}t)}{\omega_k-(\omega_1-b^{(2)})-ia^{(2)}}dk$$

$$=e^{-i(\omega_1-b^{(2)})t+a^{(2)}t+ik_0r}\int_{-k_0}^\infty \frac{(k+k_0)e^{ikr}}{\omega_c-(\omega_1-b^{(2)})-ia^{(2)}+\frac{\omega_c}{k_0^2}k^2}dk$$

$$\simeq-\frac{i\pi k_0^2}{\omega_c}\left(\sqrt{\frac{\omega_c}{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}}-1\right)\exp\left(-i(\omega_1-b^{(2)})t+a^{(2)}t+ik_0r-ik_0r\sqrt{\frac{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}{\omega_c}}\right), \quad \text{(C8)}$$

$$E_b^{(2)}=\int_0^\infty \frac{ke^{-i\omega_k t+ikr}}{\omega_k-(\omega_1-b^{(2)})-ia^{(2)}}dk$$

$$=\exp\left(-i\omega_c t+ik_0r+i\frac{k_0^2r^2}{4\omega_c t}\right)\int_{-k_0-(k_0^2r/2\omega_c t)}^\infty \frac{\left(k+\frac{k_0^2r}{2\omega_c t}+k_0\right)e^{-i(\omega_c/k_0^2)k^2t}}{\frac{\omega_c}{k_0^2}\left(k+\frac{k_0^2r}{2\omega_c t}\right)^2+\omega_c-(\omega_1-b^{(2)})-ia^{(2)}}dk$$

$$=\exp\left(-i\omega_c t+ik_0r+i\frac{k_0^2r^2}{4\omega_c t}\right)(E_{b1}^{(2)}+E_{b2}^{(2)}), \quad \text{(C9)}$$

$$E_{b1}^{(2)}=\int_{-k_0-(k_0^2r/2\omega_c t)}^0 \frac{\left(k+\frac{k_0^2r}{2\omega_c t}+k_0\right)e^{-i(\omega_c/k_0^2)k^2t}}{\frac{\omega_c}{k_0^2}\left(k+\frac{k_0^2r}{2\omega_c t}\right)^2+\omega_c-(\omega_1-b^{(2)})-ia^{(2)}}dk$$

$$\simeq-\frac{i\pi k_0^2}{\omega_c}\left(\sqrt{\frac{\omega_c}{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}}-1\right)\exp\left(-i(\omega_1-b^{(2)}-\omega_c+ia^{(2)})t-\frac{ik_0^2r^2}{4\omega_c t}-ik_0r\sqrt{\frac{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}{\omega_c}}\right)$$

$$-\int_0^\infty \frac{e^{(3\pi/4)i}\left(\rho e^{(3\pi/4)i}+\frac{k_0^2r}{2\omega_c t}+k_0\right)e^{-(\omega_c/k_0^2)\rho^2t}}{\frac{\omega_c}{k_0^2}\left(\rho e^{(3\pi/4)i}+\frac{k_0^2r}{2\omega_c t}\right)^2+\omega_c-(\omega_1-b^{(2)})-ia^{(2)}}d\rho,$$

$$E_{b2}^{(2)}=\int_0^\infty \frac{\left(k+\frac{k_0^2r}{2\omega_c t}+k_0\right)e^{-i(\omega_c/k_0^2)k^2t}}{\frac{\omega_c}{k_0^2}\left(k+\frac{k_0^2r}{2\omega_c t}\right)^2+\omega_c-(\omega_1-b^{(2)})-ia^{(2)}}dk$$

$$=-\frac{i\pi k_0^2}{\omega_c}\left(\sqrt{\frac{\omega_c}{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}}+1\right)\exp\left(-i(\omega_1-b^{(2)}-\omega_c+ia^{(2)})t-\frac{ik_0^2r^2}{4\omega_c t}+ik_0r\sqrt{\frac{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}{\omega_c}}\right)$$

$$\times\Theta\left((\mathrm{Im}+\mathrm{Re})\sqrt{\frac{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}{\omega_c}}-\frac{k_0r}{2\omega_c t}\right)+\int_0^\infty \frac{e^{-(\pi/4)i}\left(\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_c t}+k_0\right)e^{-(\omega_c/k_0^2)\rho^2t}}{\frac{\omega_c}{k_0^2}\left(\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_c t}\right)^2+\omega_c-(\omega_1-b^{(2)})-ia^{(2)}}d\rho.$$

From Eqs. (C7)–(C9), we can obtain

$$E^{(2)}=-\frac{\omega_1 d_1 \sin\eta}{8\pi\epsilon_0 r}\frac{f_5(x^{(2)})}{H'(x^{(2)})}\frac{ik_0^2}{\omega_c}\left(\sqrt{\frac{\omega_c}{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}}+1\right)\exp\left(-i(\omega_1-b^{(2)})t+ik_0r+a^{(2)}t\right.$$
$$\left.+ik_0r\sqrt{\frac{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}{\omega_c}}\right)\Theta\left((\mathrm{Im}+\mathrm{Re})\sqrt{\frac{\omega_1-b^{(2)}-\omega_c+ia^{(2)}}{\omega_c}}-\frac{k_0r}{2\omega_c t}\right)$$
$$-\frac{\omega_1 d_1 \sin\eta}{8\pi^2\epsilon_0 r}\frac{f_5(x^{(2)})}{H'(x^{(2)})}\exp\left(-i\omega_c t+ik_0r+i\frac{k_0^2r^2}{4\omega_c t}+\frac{3\pi}{4}i\right)$$
$$\times\left[\int_0^\infty\frac{\left(\rho e(3\pi/4)i+\frac{k_0^2r}{2\omega_c t}+k_0\right)e^{-(\omega_c/k_0^2)\rho^2 t}}{\frac{\omega_c}{k_0^2}\left(\rho e^{(3\pi/4)i}+\frac{k_0^2r}{2\omega_c t}\right)^2+\omega_c-(\omega_1-b^{(1)})-ia^{(2)}}d\rho+\int_0^\infty\frac{\left(\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_c t}+k_0\right)e^{-(\omega_c/k_0^2)\rho^2 t}}{\frac{\omega_c}{k_0^2}\left(\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_c t}\right)^2+\omega_c-(\omega_1-b^{(1)})-ia^{(2)}}d\rho\right]. \tag{C10}$$

In the above equation, the second term decays to zero and can be neglected as time $t\to\infty$. The first term is a pulse.

(3) Similarly, we can obtain the contribution of $B^{(0)}$:

$$E^{(0)}=\frac{\omega_1 d_1\sin\eta}{8\pi^2\epsilon_0 ri}\int_0^\infty dk\left(-\frac{k}{\pi\sqrt{i}}\right)e^{-i\omega_k t+ikr}\int_0^\infty dx\,K(x)\frac{e^{i(\omega_k-\omega_c)t-x\beta t}-1}{i(\omega_k-\omega_c)-x\beta}$$
$$=-\frac{\omega_1 d_1\sin\eta}{8\pi^3\epsilon_0 ri\sqrt{i}}\exp\left(-i\omega_c t+ik_0r+\frac{ik_0^2r^2}{4\omega_c t}-\frac{3\pi}{4}i\right)\int_0^\infty L(\rho)e^{-(\omega_c t/k_0\rho^2}d\rho, \tag{C11}$$

where

$$L(\rho)=\int_0^\infty dx\,K(x)\left[\frac{\rho e^{(3\pi/4)i}+\frac{k_0^2r}{2\omega_c t}+k_0}{\frac{\omega_c}{k_0^2}\left(\rho e^{(3\pi/4)i}+\frac{k_0^2r}{2\omega_c t}\right)^2+ix\beta}+\frac{\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_c t}+k_0}{\frac{\omega_c}{k_0^2}\left(\rho e^{-(\pi/4)i}+\frac{k_0^2r}{2\omega_c t}\right)^2+ix\beta}\right].$$

The modulus of the function $L(\rho)$ is a limiting value, so $E^{(0)}$ decays to zero as time $t\to\infty$.

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