Quantum intensity noise of laser diodes and nonorthogonal spatial eigenmodes

Jean-Philippe Poizat,* Tiejun Chang, and Philippe Grangier
Laboratoire Charles Fabry de l'Institut d'Optique, UMR 8501 du CNRS, Boîte Postale 147, F91403 Orsay Cedex, France

(Received 30 July 1999; published 13 March 2000)

Laser systems with nonorthogonal eigenmodes have a linewidth that is broader than the usual Schawlow-Townes value, by a factor that is known as the Petermann excess noise factor. In a recent quantum analysis, this excess noise was attributed to loss-induced coupling betwen the laser modes. Using the same approach, we show here that the Petermann excess noise also appears on the laser quantum intensity noise. The calculation is shown to be in good agreement with an experiment using a laser diode with two contributing transverse modes.

PACS number(s): 42.50.Lc, 42.55.Px, 42.60.Jf

I. INTRODUCTION

Laser diodes have proved in the past two decades to be a very powerful and convenient tool in the field of telecommunications [1], spectroscopy [2–5], and many other applications [6]. Their main advantages are compactness, energy efficiency, tunability, and low intensity noise. This last property has been brought into the quantum domain by Yamamoto and coworkers [7–9], who demonstrated that appropriate control of the driving current in laser diodes allows one to generate sub-Poissonian light through pump-noise suppression [10]. However, not all laser diodes are able to generate sub-Poissonian light (also referred to as squeezed light), and detailed investigation of the ''excess noise'' in laser diodes (i.e., the mechanisms that reduce or even destroy squeezing) has arisen great interest in recent years [11–21].

The main avenue that has been followed to understand this excess noise is the investigation of the influence of multimode effects. In principle, the total intensity noise of a multimode laser can be perfectly squeezed, provided that the gain medium is perfectly homogeneously broadened. The low total noise then rely on very strong anticorrelations (up to 40 dB) among modes that are individually very noisy [13–18]. However, small inhomogeneities, such as saturable losses [19,20], degrade slightly these anticorrelations, and the total intensity noise increases [15–20].

More recently, it was realized that even in lasers with a single lasing mode, excess noise may also arise owing to a multimode cavity structure. This type of excess noise is often referred to as Petermann excess noise, and is related to the appearance of nonorthogonal eigenmodes in the laser cavity [22–37]. In this paper, we analyze further the existence of nonzero correlations between lasing and nonlasing modes, associated to an excess noise in the lasing mode. We will show in more detail that this effect can be described as a contamination of the lasing mode by the noise of a subthreshold mode, through an effect that we called ''loss-induced coupling'' [36,37], and that is directly related to Petermann excess noise. In Sec. II, we present experimental observations realized with a semiconductor laser in a grating-extended external cavity configuration. In Sec. III, we introduce and then construct a theoretical model based on coupled Langevin equation, which allows us to study the effect of the loss-induced mode coupling on the laser noise. Finally, we present a comparison between theory and experiment (Sec. IV).

II. EXPERIMENT

A. Experimental set-up

The single-mode semiconductor laser that has been used is a Fabry-Perot, quantum well, index guided $Al_xGa_{1-x}As$ device (SDL 5411-G1) emitting at 810 nm. It is collimated by a high-numerical-aperture (0.65) aspherical lens. The semiconductor laser is stabilized by the 5% feedback of the first order reflection an external grating located at 10 cm of the laser, in a Littrow configuration. The zeroth order contains 90% of the input light. The gold-coated grating has 1200 groves/mm, and it is blazed for a wavelength of 250 nm. The relatively low feedback level allows for a large overall outcoupling efficiency of the system, and it is still large enough to lock efficiently the laser on the grating-extended cavity. The single mode operation is continuously checked using a scanning Fabry-Perot and an oscilloscope, and the mode frequency is controlled by adjusting the length of the external cavity via a PZT holding the external grating. As in Ref. [35], the transverse mode analysis is performed in the direction contained in the junction plane of the diode. The (horizontal) plane of incidence on the grating is perpendicular to this plane, so that the wavelength tuning of the grating is independent from the transverse mode analysis. The intensity profile of the beam has been checked to be a Gaussian. From previous analysis [35], the spatial behavior of this laser can be described by considering essentially two spatial modes, labeled TE_{00} and TE_{10}. Though the contribution from other higher order spatial modes is not strictly zero, it will be ignored in a first approach, which will make the measurements simpler and the discussion clearer.

The experimental setup (shown in Fig. 1) is similar to the one used in Ref. [35]. The noise analysis frequency is 12 MHz, but the frequency dependence of the noise is essentially flat within the bandwidth of our detectors (5–25 MHz). The laser beam is split into two channels. On channel 1, detector D_1 is a split photodiode (EGG C30822). An essential point in the experiment (see Ref. [35] and Sec. IV A

*Electronic address: jean-philippe.poizat@iota.u-psud.fr

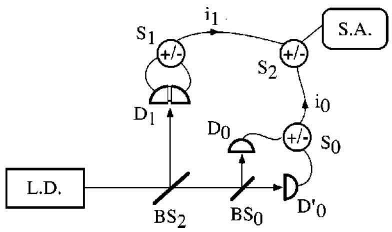

FIG. 1. Experimental setup for measuring the noise of lasing main mode TE_{00}, the noise of non-lasing mode TE_{10}, and the correlation between the two modes. D_0 and D_0' are ordinary photodiodes. BS_0 is the 50-50 beamsplitter of the balanced detection. D_1 is a split photodiode. $S_{0,1,2}$ are RF switchable (+/-) power combiners. S.A. is a spectrum analyzer.

below) is that when the switch S_1 is in position ''minus,'' the noise of the photocurrent i_1 gives the fluctuations of mode TE_{10}. On channel 0, detectors D_0 and D_0' are high efficiency p-i-n photodiodes (Silicon Sensor SSO PD20-7), used as a balanced detection. The switch S_0 is used for the shot noise calibration. When it is in position ''plus'' (respectively ''minus''), the photocurrent i_0 is proportional to the noise of mode TE_{00} (respectively to the beam shot noise) [38]. The correlation between the noises of mode TE_{00} (S_0 is in position ''plus'') and TE_{10} (S_1 in position ''minus'') is measured by recording the difference between the noise levels corresponding to S_2 in position ''plus'' and ''minus.'' We have checked that this method, which will be used throughout this work, gives the same results as a fitting procedure of the noise profile which was used in Ref. [35]. We note that the noise and the correlation values given below are corrected for the transmission of all the optical components located after the grating and for the detectors quantum efficiency.

When the driving current is increased, the beam of the free-running diode moves slightly, with a maximum angular deviation of about 3×10^{-4} rad. This is attributed to a thermally-induced motion of the laser chip with respect to the collimating objective, and this leads to a slight misalignement of the grating at high currents. In the following, the alignment procedure used for the grating will thus be critical. We note that in general terms, a misaligned grating will induce a coupling between the transverse modes of the diode.

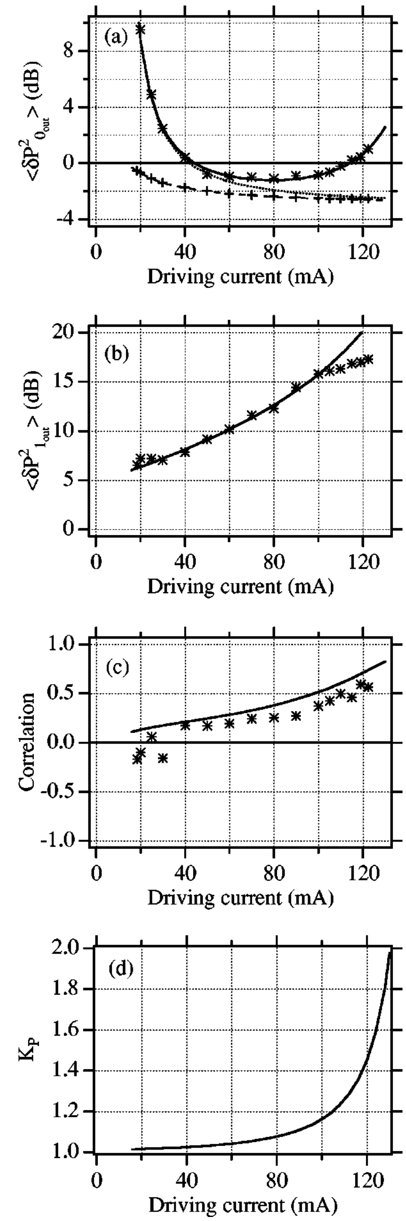

FIG. 2. All graphs are plotted versus the laser diode driving current. The external grating is not realigned for each current. The lines are the theoretical two-mode model presented in Sec. III. On curve (a), the * are the spectral variance $\langle \delta P^2_{0,out}\rangle$ of the lasing mode (in dB), and the + are the opposite of the quantum efficiency of the laser. The dotted line is the prediction of the model when the coupling is set to zero. On curve (b) is plotted the variance $\langle \delta P^2_{1,out}\rangle$ of the nonlasing mode (in dB), on curve (c) the normalized correlation, and on curve (d) the calculated value of the Petermann excess noise factor K_P.

B. Experimental results

1. Noise and correlations

Figures 2 and 3 show the experimental results of the noise of lasing mode TE_{00}, the noise of non-lasing spatial mode TE_{10}, and the correlation between the two modes, as a function of the driving current. It can be seen in Figs. 2(b) and 3(b) that the noise of nonlasing spatial mode TE_{10} increases with the current as expected for a subthreshold mode.

In the experimental results presented in Fig. 2, the external feedback is not realigned for each value of the current. The alignment of the external grating is obtained by minimizing the threshold current, and then kept the same for all currents. The grating will therefore find itself slightly misaligned at high current owing to the beam motion of the thermally induced motion of the laser chip. The most important point here is that the noise of the lasing mode is also increasing for high current. According to the single mode theoretical model for laser diode squeezing (see Sec. III below), this noise should be decreasing with current and reach a limit associated with quantum efficiency, as shown by the dotted line on Fig. 2(a). It can be observed in Fig. 2(c) that the excess noise of the lasing main mode is correlated with

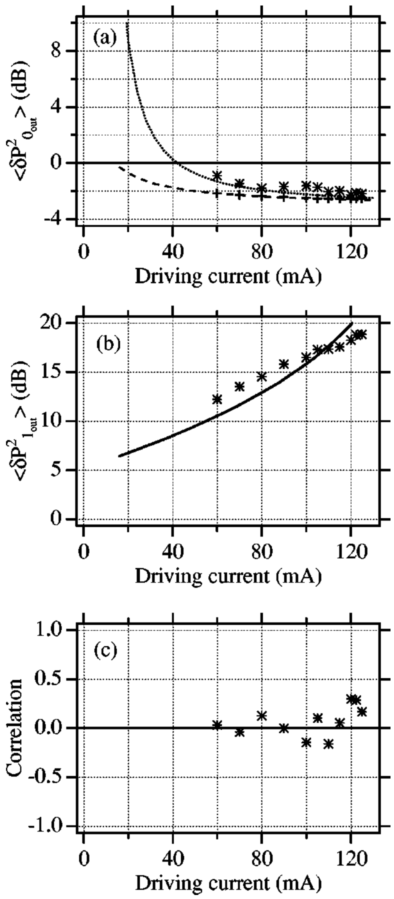

FIG. 3. All graphs are plotted versus the laser diode driving current. The external grating is realigned for each current, so that the correlation should in principle be zero. The lines come from the theoretical two-mode model presented in section III. On curve (a), the * are the spectral variance $\langle \delta P_{0,out}^2 \rangle$ of the lasing mode (in dB), and the + are the opposite of the quantum efficiency of the laser. The dotted line is the prediction of the model when the coupling is set to zero. On curve (b) is plotted the variance $\langle \delta P_{1,out}^2 \rangle$ of the non-lasing mode (in dB), and on curve (c) the normalized correlation.

the noise of the TE_{10} mode owing to the misalignement of the grating. The origin of the noise increase in the main mode and of the nonvanishing correlation is attributed to loss-induced coupling between the two modes [37]. In Sec. III we construct a theoretical model to explain this phenomenon.

On the other hand, the results displayed in Fig. 3 have been obtained by realigning the external grating for each current in order to minimize the intensity noise level of the lasing mode. It is to be mentionned that the alignement of the grating at high current is a delicate and hazardous operation, owing to the possible thermal damages caused by the large optical intensity not properly fed back into the laser waveguide. As can be seen in Fig. 3, the realignement of the grating at each current leads to a behavior of the main mode noise that follows the single-mode theoretical prediction, and to an almost vanishing correlation. The large dispersion in the correlation is attributed to the extreme sensitivity of the grating alignement.

2. Beam steering

In the case corresponding to Fig. 2 (grating misaligned), we have noticed that changing the external cavity length with the PZT on which the grating is glued gives rise to a steering (i.e., an angular deviation) of the output beam. This effect is attributed to the fact that the amplitude of the lasing mode has a small contribution from the TE_{10} mode. Then a modification of the external cavity length changes the relative phase between mode TE_{00} and mode TE_{10}. This appears as a change in the direction of the emitted beam, which at the first order remains Gaussian in shape [39]. Let us mention that this steering effect is intrinsic to the mode coupling effect, and is not directly related to the beam motion of the free-running diode discussed earlier: here, the beam steering effect appears for a given current, and its amplitude depends on the grating alignment. From the observed beam shape and steering, we have checked that the admixture of TE_{10} in the lasing mode remains small, typically less than 0.1 in amplitude.

We conclude from these observations that a significant mode-coupling effect is present in the extended cavity laser, and can be attributed to a grating misalignment. As we will see below, this effect has many generic features connected to the Petermann excess noise factor, which make it worthwhile to understand in detail. We will thus consider in the following section a general theoretical model for the coupling of laser modes in the presence of cavity losses (''loss-induced coupling'') [36,37].

III. THEORETICAL MODEL

A. Two-mode laser model

The purpose of this model is to describe a two-spatial mode laser, taking into account losses, gain, and coupling between the two modes. For a perfectly aligned ideal laser, the mode basis is made of the cavity eigenmodes TE_{00} and TE_{10}, which are orthogonal [40]. We will assume as usual that the lasing mode is the TE_{00} mode, and the nonlasing mode is the TE_{10} mode. When the alignment of the cavity (i.e., the external feedback in our experiment) is not perfect, the TE_{00} and TE_{10} modes are coupled, and in presence of losses it can be shown that the laser eigenmodes [40] are no longer orthogonal [36,37]. These nonorthogonal eigenmodes $\{u_0, u_1\}$, which are linear combinations of modes TE_{00} and TE_{10}, have been used in semiclassical calculations of the Petermann excess noise factor.

In order to build a quantum model, an orthogonal basis is required for quantization, as explained in [36,37] (see also appendix A). This orthogonal basis $\{w_0, w_1\}$ is built by first chosing $w_1 = u_1$, where u_1 is the eigenvector having the smallest eigenvalue, i.e., the nonlasing mode. Then w_0 is taken as a linear combination of u_0 and u_1, which is chosen orthogonal to w_1 (Schmidt orthonormalization procedure). A central point, which is demonstrated in Appendix A, is that mode w_1 will then appear in the roundtrip evolution of mode w_0 (the reverse is not true, since w_1 is a cavity eigenmode). In physical term, we shall say that the lasing mode w_0 is ''contaminated'' by the nonlasing mode $u_1 = w_1$. We may

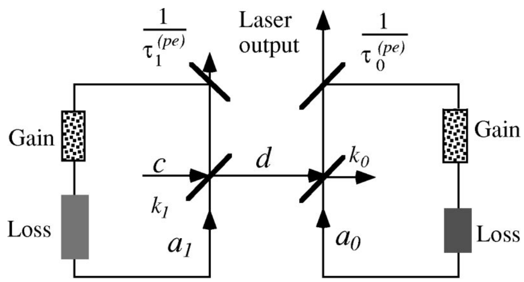

FIG. 4. The mode a_0 is lasing while the mode a_1 is below threshold. The mode a_1 is coupled with mode a_0 via a coupling mode *d*. Mode *c* is in the vacuum state.

thus describe the two-mode situation using the scheme shown in Fig. 4. In general, the lasing mode coherent amplitude is a linear combination of TE_{00} and TE_{10} modes, with coefficients that may vary with the driving current, as it has been experimentally observed (see Sec. II B 2 above). In the following sections, we will show explicitly that such a configuration leads to excess noise in the lasing mode.

We note that the generic situation where (many) nonlasing modes contaminate the lasing mode is directly related to the ''loss-induced coupling'' introduced in Ref. [37]. The Schmidt orthonormalization procedure which was used above to construct the two dimensional orthogonal basis $\{w_0, w_1\}$ can be generalized to an arbitrary, but finite number of modes (see Appendix A). Based on this picture, loss-induced coupling can then be considered as the basic mechanism for explaining Petermann excess noise [22–37].

B. Coupled Langevin equations

This theoretical model is based on coupled Langevin equations for the electromagnetic field operators of one lasing mode (labeled by the subscript $i=0$), one nonlasing mode (labeled by the subscript $i=1$), and the excited carrier population operator. In order to describe the amplitude fluctuations of the field, we will take the mean laser field as a real number, and we introduce the amplitude quadrature operator

$$P_i = a_i + a_i^\dagger . \tag{1}$$

For the lasing mode, the Langevin equation is given by

$$\frac{dP_0(t)}{dt} = \frac{1}{2}\left[-\left(\frac{1}{\tau_0^{(po)}} + \frac{1}{\tau_0^{(pe)}} + k_0 \right) + N_0(t)A_0 \right] P_0(t) + 2\gamma_0^{(po)}(t) + 2\gamma_0^{(pe)}(t) + 2\xi_0(t) + \sqrt{k_0}D(t), \tag{2}$$

and for the nonlasing mode

$$\frac{dP_1(t)}{dt} = \frac{1}{2}\left[-\left(\frac{1}{\tau_1^{(po)}} + \frac{1}{\tau_1^{(pe)}} + k_1 \right) + N_1(t)A_1 \right] P_1(t) + 2\gamma_1^{(po)}(t) + 2\gamma_1^{(pe)}(t) + 2\xi_1(t) + \sqrt{k_1}C(t), \tag{3}$$

where the parameter k_i is associated to the magnitude of the coupling between the two modes (see Fig. 4), and the intermediate ''loss'' mode is written as

$$D(t) = d(t) + d^\dagger(t) = \sqrt{k_1}P_1(t) - C(t), \tag{4}$$

where $C(t) = c(t) + c^\dagger(t)$, and mode c is in the vacuum state. In Eqs. (2) and (3), $1/\tau_i^{(pe)}$ is the photon decay rate due to the coupling mirror and it is identical for the two modes, $1/\tau_0^{(pe)} = 1/\tau_1^{(pe)}$, and $1/\tau_i^{(po)}$ is the photon decay rate due to intracavity optical losses. The lasing mode has smaller optical loss than the nonlasing mode, therefore we have $1/\tau_0^{(po)} < 1/\tau_1^{(po)}$. The coefficient A_i is the spontaneous emission rate and it is same for two modes, $A_0 = A_1$. The quantities $N_i(t)$ are the excited carrier numbers associated to the nonlasing and lasing modes, respectively.

In the Langevin equations for the two modes, the terms $\gamma_i(t)$ and $\xi_i(t)$ are Langevin noise operator terms, which represent the coupling of the field to heat baths. The $\gamma_i^{(pe)}(t)$ terms are associated with the output coupling, the $\gamma_i^{(po)}(t)$ terms correspond to the internal cavity losses, and the $\xi_i(t)$ terms concern the noise associated with the stimulated emission. Their correlation functions are given in Appendix B.

Above threshold, the oscillation condition for the lasing mode imposes

$$\langle N_0 \rangle A_0 = \frac{1}{\tau_0}, \tag{5}$$

whereas the carrier number $N_1(t)$ associated to the nonlasing mode verifies

$$\langle N_1 \rangle A_1 < \frac{1}{\tau_1}, \tag{6}$$

where $1/\tau_i = 1/\tau_i^{(po)} + 1/\tau_i^{(pe)} + k_i$ indicates the total cavity loss of the lasing and the nonlasing modes, respectively. The carrier numbers N_0 and N_1 have actually quite different roles. On one hand, N_0 is a dynamical variable that sets both the amplitude and the noise of the lasing mode, through the gain saturation mechanism. On the other hand, the only role of N_1 is to set the value of the gain for mode 1 which appears in Eq. (3), without any feedback mechanism. Since this gain is not known precisely, we will use for $\langle N_1 \rangle$ the following phenomenological expression

$$\langle N_1 \rangle = \mu p + \sigma \langle N_0 \rangle, \tag{7}$$

where μ and σ are constants depending on the gain distribution profile, with $\sigma < 1$. The parameter p is the pumping rate, $p = I/e$, where I is the driving current and e is the electron charge. Physically, the first term on the right-hand side of Eq. (7) is associated to the non saturated excited carrier on the two edges of the gain region. This carrier number is therefore increasing proportionaly to the pump rate. The second term corresponds to the gain in the center region, which is clamped by the lasing mode. The value of σ corresponds to the imperfect overlap of the amplitude profiles of the spatial modes.

The equation of motion for the excited carrier number $N_0(t)$ is then

$$\frac{dN_0(t)}{dt}=p-\frac{N_0(t)}{\tau_{sp}}-A_0(n_0(t)+1)N_0(t)+\Gamma^{(p)}(t)+\Gamma^{(sp)}(t)+\Gamma(t), \tag{8}$$

where τ_{sp} is the spontaneous electron lifetime, $n_0=a_0^\dagger a_0$ is the photon number operator of the lasing main mode in the cavity. The last three terms Eq. (8) are Langevin noise operators. The first one $\Gamma^{(p)}(t)$ is associated with the pump noise. The second and third one, $\Gamma^{(sp)}(t)$ and $\Gamma(t)$ are respectively associated to spontaneous noise and stimulated emission noise. Their correlations and cross-correlations are given in Appendix B. It can be noticed that the nonlasing mode has no contribution to the fluctuations of the excited carrier number

The noises of the two modes and their correlations are obtained after linearization around mean values. The detailed derivation for obtaining the various noise powers and correlations is given in Appendix C. The variance of the zero-frequency output amplitude fluctuations of the nonlasing mode is given by

$$\langle\delta P_{1,out}^2\rangle=1+\frac{8\langle N_1\rangle A_1}{\left(\dfrac{1}{\tau_1}-\langle N_1\rangle A_1\right)^2\tau_1^{(pe)}}=1+\langle:\delta P_{1,out}^2:\rangle, \tag{9}$$

where $\langle:\delta P_{1,out}^2:\rangle=\langle\delta P_{1,out}^2\rangle-1$ corresponds to the excess noise above the shot-noise level. The ‘‘:’’ means that normal ordering is used. As expected, when $\langle N_1\rangle=0$, i.e., without gain, this excess noise is zero.

The zero frequency noise power of the lasing mode can be written as the sum of two terms

$$\langle\delta P_{0out}^2\rangle=\langle\delta P_{0out}^2\rangle|_{k_1=0}+\langle\delta P_{0out}^2\rangle|_{k_0k_1}, \tag{10}$$

where $\langle\delta P_{0out}^2\rangle|_{k_1=0}$ is associated with the zero-coupling case $(k_1=0)$ and is given by

$$\langle\delta P_{0out}^2\rangle|_{k_1=0}=1+\left(\frac{\tau_0}{\tau_0^{(pe)}}\right)(-1+x+2x^2+\epsilon(1+x)), \tag{11}$$

where $x=1/(\tau_{sp}A_0n_0)$. This expression is the standard result from a single-mode theoretical model, which appears as a dotted line on Figs. 2 and 3. On the other hand, the noise leaking from nonlasing mode to lasing mode is directly proportional to the noise $\langle:\delta P_{1,out}^2:\rangle$ of the nonlasing mode and is written

$$\langle\delta P_{0out}^2\rangle|_{k_0k_1}=k_0k_1(1+x)^2\tau_0^2\frac{\tau_1^{(pe)}}{\tau_0^{(pe)}}\langle:\delta P_{1,out}^2:\rangle. \tag{12}$$

The correlation between the two modes is given by

$$C_{01}=\langle\delta P_{0out}\delta P_{1out}\rangle=\sqrt{k_0k_1}(1+x)\tau_0\sqrt{\frac{\tau_1^{(pe)}}{\tau_0^{(pe)}}}\langle:\delta P_{1out}^2:\rangle. \tag{13}$$

Equations (12) and (13) show that both the excess noise in the lasing mode and the correlation between the modes are proportional to the excess noise of the subthreshold mode $\langle:\delta P_{1out}^2:\rangle$, and depend on the magnitude of the coupling between the two modes.

We also note that the following relation is fulfilled (see discussion Sec. III C 2):

$$\langle\delta P_{0out}^2\rangle|_{k_0k_1}=\frac{C_{01}^2}{\langle:\delta P_{1out}^2:\rangle}. \tag{14}$$

C. Discussion

1. High current case

From the experiment, it appears that a large correlation between the lasing mode and the nonlasing mode, as well as a large excess noise in the lasing mode, are observed for high values of the driving current. In this case, the equations of noise and correlation can be simplified by using $A_0n_0\gg1/\tau_{sp}$ and $A_0\langle N_0\rangle=1/\tau_0$, and one obtains

$$\langle\delta P_{0out}^2\rangle|_{k_1=0}=1+(\epsilon-1)\frac{\tau_0}{\tau_0^{(pe)}}. \tag{15}$$

In the same way, due to $\tau_1^{(pe)}=\tau_0^{(pe)}$, we can write

$$\langle\delta P_{0out}^2\rangle|_{k_0k_1}=k_0k_1\tau_0^2\langle:\delta P_{1out}^2:\rangle. \tag{16}$$

For the case $\epsilon=0$, we have then

$$\langle\delta P_{0out}^2\rangle=1-\frac{\tau_0}{\tau_0^{(pe)}}+k_0k_1\tau_0^2\langle:\delta P_{1out}^2:\rangle, \tag{17}$$

and for the correlation, we obtain

$$C_{01}=\langle\delta P_{0out}\delta P_{1out}\rangle=\sqrt{k_0k_1}\tau_0\langle:\delta P_{1out}^2:\rangle. \tag{18}$$

Equations (16),(17), and (18) show in a compact form that, when the semiconductor laser is driven far above threshold, both the excess noise and the correlation are directly related to the noise of the subthreshold mode.

2. Minimum noise and correlation

Since the excess noises in the two modes are correlated, one may try to extract the noise in the nonlasing mode in order to correct the noise in the lasing one. The best result that can be obtained using such a procedure is equal to the so-called conditional variance of mode 0, given mode 1, which is

$$\langle\delta P_{0out}^2\rangle_{corr.}=\langle\delta P_{0out}^2\rangle_{uncorr.}(1-C_{01N}^2), \tag{19}$$

where the normalized correlation is given by

$$C_{01N}^2=\frac{C_{01}^2}{\langle\delta P_{0out}^2\rangle\langle\delta P_{1out}^2\rangle}. \tag{20}$$

One has thus

$$\langle\delta P_{0out}^2\rangle_{uncorr.}-\langle\delta P_{0out}^2\rangle_{corr.}=\frac{C_{01}^2}{\langle\delta P_{1out}^2\rangle}. \tag{21}$$

This expression, which was used for instance in Ref. [35], differs from the excess noise obtained above:

$$\langle\delta P_{0out}^2\rangle|_{k_0k_1}=\frac{C_{01}^2}{\langle:\delta P_{1out}^2:\rangle}=\frac{C_{01}^2}{(\langle\delta P_{1out}^2\rangle-1)}>\frac{C_{01}^2}{\langle\delta P_{1out}^2\rangle}. \tag{22}$$

These two expressions are different because a measurement of the noise in mode 1 will involve a contribution from shot noise, which brings no useful information. Thus the correction cannot be perfect, and a better result is obtained by suppressing directly the noise at its source, rather than attempting to correct it : this is the meaning of the difference between Eqs. (21) and (22). We note that in the case where $\langle\delta P_{1out}^2\rangle\gg 1$, the shot noise contribution is negligible and the two equations become the same.

3. Petermann factor

From the calculation given above, one can easily deduce the Petermann excess noise factor, defined as usual as the broadening of the laser linewidth with respect to the Schawlow-Townes value [37]. For doing that, we note that Eqs. (2) and (3) have just the same form when written for the phase quadratures $Q=(a-a^\dagger)/i$. The Petermann factor can then be calculated either from the ''spontaneous'' noise or from the ''vacuum'' noise, as explained in detail in Ref. [37]. We thus obtain

$$K_P=1+\frac{4k_0k_1\tau_0\langle N_1\rangle A_1}{\left(\frac{1}{\tau_1}-\langle N_1\rangle A_1\right)^2}=1+(k_0k_1\tau_0\tau_1^{(pe)}/2)\langle:\delta Q_{1,out}^2:\rangle. \tag{23}$$

with $\langle:\delta Q_{1,out}^2:\rangle=\langle:\delta P_{1,out}^2:\rangle$ for spontaneous emission noise. This value is plotted in Fig. 2(d). We note that the corresponding values of K_P are rather small, so that a direct measurement of the linewidth enhancement factor is possible, but would not be very easy to carry out.

In order to compare Eq. (23) with standard results, one must assume that both modes see the same gain [24], contrary to the more general situation that we considered before. Taking thus $\langle N_1\rangle A_1=1/\tau_0$, one obtains the usual result, involving only ''cold cavity'' parameters [36]:

$$K_P=1+\frac{4k_0k_1}{(1/\tau_1-1/\tau_0)^2}. \tag{24}$$

If one wishes to define a ''Petermann factor'' K_I for the excess intensity noise, one needs a reference value, that may be either the (squeezed) intensity noise of the truly single-mode laser, or the shot noise level (SNL). It is thus more convenient to rewrite Eq. (17) by using Eq. (23), so that for a noiseless pump the intensity noise far above threshold becomes

$$\begin{aligned}\langle\delta P_{0out}^2\rangle&=1-\frac{\tau_0}{\tau_0^{(pe)}}+2\frac{\tau_0}{\tau_0^{(pe)}}(K_P-1)\\&=1-\eta_L+2\,\eta_L(K_P-1),\end{aligned} \tag{25}$$

where $\eta_L=\tau_0/\tau_0^{(pe)}$ is the cavity quantum efficiency of the lasing mode. This equation shows clearly that the excess intensity noise of the lasing mode is directly related to (K_P-1), which is in turn proportional to the mode coupling coefficient k_0k_1, and to the spontaneous emission noise in mode 1. This emphasizes again that the excess intensity noise and phase noise have the same physical origin.

IV. COMPARISON BETWEEN THEORY AND EXPERIMENT

A. Results

From the theoretical analysis carried out in Ref. [35], it can be shown that when the switch S_1 of Fig. 1 is in position ''minus,'' the noise power of photocurrent i_1, normalized to the SNL, is

$$\langle\delta i_1^2\rangle=1+\eta_1\langle:\delta P_{1,out}^2:\rangle(\pi/2)\phi(a), \tag{26}$$

where η_i is the overall efficiency of the detection channel i, and $\phi(a)=[1-\mathrm{erf}(a)]\exp(4a^2)\sim 1$ takes into account the finite spacing between the two photodiodes of the split detector. This spacing, normalized to the size of the beam (half width at $1/e$), is $a=0.05$, and erf is the error function. The noise power of photocurrent i_0, normalized to its own shot noise, is

$$\langle\delta i_0^2\rangle=1+\eta_0\langle:\delta P_{0,out}^2:\rangle. \tag{27}$$

The experimental results together with the fitting results are shown in Figs. 2 and 3. The values of the fitting parameters used for these three figures are $\tau_{sp}=10^{-9}$s, $\tau_0^{(pe)}=\tau_1^{(pe)}=0.5\times10^{-11}$s, $\tau_0^{(po)}=1.22\tau_0^{(pe)}$, $I_{th}=14$ mA, which agrees with known parameters of the laser. The optical loss of nonlasing higher order spatial mode is larger than that of lasing main mode, we take $\tau_1^{(po)}=0.48\tau_0^{(pe)}$. The coupling of the non-lasing mode into the lasing mode is $k_0=0.13/\tau_0^{(pe)}$, which is 13% of the output coupling efficiency. The coefficients μ and σ are optimized in the fitting, and we take $\mu=\tau_{sp}/10.2$, and $\sigma=0.57$, which are compatible with an estimation based upon the spatial mode distribution. The pump noise is taken as $\epsilon=0$, which means a quiet pumping of the semiconductor laser.

From Fig. 2, we can see that the experiment is well fitted by a theoretical model assuming a constant value of $k_1=k_0$. The noise of nonlasing mode increases with driving current. As a result, more noise leaks into the lasing main mode and leads to an increase of its noise [see Fig. 2(a)]. The correlation between the two modes also increases with driving current accompanying the rising of the noise of nonlasing mode. The origin of the correlation, proportional to $\sqrt{k_0 k_1}$, is the loss-induced coupling between the modes.

As said above, the curves of Fig. 2 were obtained *without* readjusting the grating. On the contrary, if the grating is slightly readjusted for high values of the current, we did observe that both the noise of the main mode and the correlation decrease. This behavior is illustrated on Fig. 3 : by iteratively doing such readjustments for each current, it was possible to obtain values of $\langle \delta P_{0,out}^2 \rangle$ very close to the single-mode values shown by the dotted line on Fig. 2(a). This is in agreement with the behavior already reported in Ref. [35], though the required adjustements are more and more difficult to realize for high currents. This explains why the correlation is very noisy, though it is clearly smaller than on Fig. 2.

B. Possible improvements

A slight discrepancy can be noticed in the fitting for the correlation between the lasing and nonlasing modes [see Fig. 2(c)]: the theoretical value of the correlation is larger than the observed value. We discuss below possible ways to explain this missing correlation.

(i) It was said above that the relevant modes are not pure TE_{00} and TE_{10}. From beam intensity measurements, we estimated the admixture of mode TE_{10} in the lasing mode to be less than 0.1 (in amplitude). The resulting correction is too small to explain the discrepancy.

(ii) Other (even order) transverse modes may be present, but will not be detected by the split photodiode. We know from the fitting of the spatial noise distribution [35] that the mode TE_{20} has a contribution for the spatial noise, although it is much smaller than that of mode TE_{10}. As for mode TE_{10}, the loss-induced coupling between mode TE_{20} and the main mode also depends on the alignment of the external feedback. Since the spatial distribution is symmetrical, the misalignment might result from a non perfect adjustment of the collimating lens, or from geometrical aberrations. A direct proof of the existence of a correlation between the lasing mode and even order subthreshold modes might be obtained by using a spatial noise measurement scheme more sophisticated than a simple split photodiode.

(iii) In Fig. 2(a) we used a fixed value of the mode coupling coefficient $k_1 = 0.13/\tau_0^{(pe)}$. However, this parameter may also depend on the driving current, owing to the beam steering observed on the free-running diode. We have thus compared the experimental results with a model in which the mode coupling k_1 is linearly increasing from zero to its nominal value as a function of the driving current. We have found a small quantitative improvement of the fits (the curves remain qualitatively very similar with Fig. 2, and thus they are not shown here).

(iv) The coupling between transverse modes might not be the only effect of the misalignement of the grating. Effects such as the appearance of longitudinal side modes [15], or admixture of phase noise owing to phase amplitude coupling [41] cannot be completely excluded. These effects may clearly increase the noise, without contributing to the observed correlation between spatial modes.

The effects listed above were included for completeness, but from the good overall agreement shown on Fig. 2, they should in any case remain small.

V. CONCLUSION

In this paper, we have studied in detail the quantum intensity noise of a semiconductor laser stabilized by external cavity, which operates with a single longitudinal mode. Based on the experimental observations, we have constructed a theoretical model to describe both the noise and the correlations of the lasing and nonlasing spatial modes. This theoretical model fits the experiment quite well.

These results emphasize the role of higher order spatial mode, which were already known to play a central role in the spatial distribution of the intensity noise [35]. The new point here is the importance of the coupling between the nonlasing and lasing modes, that may be induced by a misalignment of the external feedback. Such a coupling can actually be obtained within the laser chip, due to defects in the waveguiding structure, or to intracavity scattered light, which do not rely on an external misalignement. This ''loss-induced'' coupling between cavity modes has an influence on the noise of the main mode, which is increased by an amount that depends both on the noise of the nonlasing mode, and on the coupling between the two modes. A non zero correlation between the modes is associated to the increase in the noise level of the lasing main mode. The physics of this effect is closely related to the so-called Petermann excess noise, which appears in lasers with a single-lasing mode.

ACKNOWLEDGMENTS

T.J.C. was supported by a ''Boursier du Gouvernement Français'' (BGF) fellowship. This work was completed as part of the ESPRIT Project Number 20029 ''Acquire'' and of the European TMR network ''Microlasers and Cavity QED.''

APPENDIX A: LOSS-INDUCED COUPLING BETWEEN LASER MODES

In this appendix are recalled the main results of Refs. [36] and [37].

Let us consider first the multimode cavity structure without the gain mechanism (''cold cavity'' situation). For quantum consistency, the round-trip equation should include not only the ''laser'' modes, which will see the gain, but also the ''vacuum'' modes that correspond to the various loss channels (output coupling, but also internal scattering, waveguide defects, grating misalignments . . .). In the general case, we introduce a set of $m+1$ normalized and orthogonal (classical) mode functions, which correspond to all input modes into the system. Any mode can be decomposed using this set as a basis, and will be written as a column vector $\{e_{in}\}$ (input modes) or $\{e_{out}\}$ (output modes). For instance, the nth basis vector is represented by a column with 1 on the nth line and 0 everywhere else. The general input-output transformation for a round trip in the cold cavity can then be written

$$\{e_{out}\}=S\{e_{in}\}. \tag{A1}$$

Since the set of mode functions will be used later on as a quantization basis, the scattering matrix S is by definition unitary, in order to insure that all operator commutation relations will be preserved in the input-output evolution. Since the modes can be split in two sets of ''laser'' and ''loss'' modes, it is convenient to introduce (Hermitian) projection operators P and Q, such as

$$P^2=P, \quad Q^2=Q, \quad P+Q=1, \tag{A2}$$

where P projects on the ''laser'' modes subset, and Q on the ''loss'' modes subset. One obtains, therefore,

$$P\{e_{out}\}=PS(P+Q)\{e_{in}\}=TP\{e_{in}\}+PSQ\{e_{in}\}, \tag{A3}$$

where $PSQ\{e_{in}\}$ corresponds to the contribution of the loss modes, while the ''truncated'' scattering matrix $T=PSP$ describes the input-output transformation for the laser modes only. In general, T is not unitary, and therefore cannot always be diagonalized in an orthogonal basis. (We note that a matrix can be diagonalized in an orthogonal basis if and only if it is *normal*, i.e., $T^\dagger T=TT^\dagger$. A unitary matrix is normal, but the reverse is not true.) In semi-classical theories [24], only T is considered, hence the name of ''non-normal resonator.'' In all cases, it is possible to diagonalize T in a nonorthogonal basis $\{u_n\}$ (with $0\leqslant n\leqslant m$). More precisely, the eigenvalues and eigenvectors of T can be written under the form [24]

$$TU=UG, \tag{A4}$$

where U corresponds to a matrix with columns formed by the $m+1$ normalized eigenvectors $\{u_n\}$ of T, and G to a diagonal matrix formed by the corresponding eigenvalues γ_n. In general, the eigenvectors of T are nonorthogonal, and therefore U is not unitary.

The usual approach for calculating the Petermann excess noise is then to introduce the matrix $V=(U^{-1})^\dagger$ with columns formed by the eigenvectors $\{v_n\}$ of $T^\dagger$ (''biorthogonal'' basis). Here we will use a different approach, which is the following. First, we order the eigenvectors $\{u_n\}$ by decreasing modulus of their eigenvalues, keeping thus as the last one the mode u_0 with the lowest losses (eigenvalue modulus closest to one). This mode will be called the ''lasing mode,'' since in a fully homogeneously broadened laser it will be the only one lasing. Then, starting from u_m and going down, we iteratively build an orthogonal basis $\{w_n\}$ by constructing mutually orthogonal linear combinations of the $\{u_n\}$ (Schmidt orthonormalization procedure). The lasing mode is thus the last one included in the procedure. Since T is diagonal in the $\{u_n\}$ basis, it is simple to show that it is triangular in the $\{w_n\}$ basis. It is then obvious that the lasing mode may include contributions from all other (subthreshold) modes, while no sub threshold mode will have a contribution from the lasing one. This is clearly required for consistency, since by definition a ''nonlasing mode'' cannot have a coherent field inside.

We thus obtain the very important result that it is always possible to choose an *orthogonal* set of modes in such a way that one lasing mode is coupled to excess noise, which comes from all other modes, which contain only amplified spontaneous emission. This justifies the ''one-way'' coupling used in the theoretical model. Physically and mathematically, the fact that the ''leakage'' from all subthreshhold modes in the lasing mode cannot be avoided is strictly equivalent to saying that the T matrix is non-normal, or that the semiclassical laser eigenmodes are nonorthogonal, or that there is ''loss-induced coupling'' between the laser modes (the loss modes being the ones which are in S, but not in T).

According to the calculations done in Ref. [37], the total amount of excess noise which is brought in the lasing mode is just given by the Petermann factor, and thus yields the correct value for the excess laser linewidth. The present approach gives a quantum-mechanically consistent picture of the origin of this noise, which can now easily be included in laser equations in order to take into account other relevant effects such as saturation and sub-Poissonian pump noise.

APPENDIX B: LANGEVIN NOISE CORRELATIONS

When the heat baths exhibit broad frequency spectra and therefore allow the dissipation processes to be considered as Markovian, their correlation function corresponding to the field Eqs. (2) and (3) are given as

$$\langle \gamma_i^{(po)}(t)\gamma_i^{(po)}(t')\rangle=\frac{1}{4\tau_i^{(po)}}\delta(t-t'), \tag{B1}$$

$$\langle \gamma_i^{(pe)}(t)\gamma_i^{(pe)}(t')\rangle=\frac{1}{4\tau_i^{(pe)}}\delta(t-t'), \tag{B2}$$

$$\langle \xi_i(t)\xi_i(t')\rangle=\frac{A_i\langle N_i(t)\rangle}{4}\delta(t-t'). \tag{B3}$$

For the amplitude of vacuum field, we have

$$\langle C(t)C(t')\rangle=\delta(t-t'). \tag{B4}$$

The correlations and nonzero cross correlations corresponding to the excited carrier equation of motion [Eq. (8)] are given by

$$\langle \Gamma^{(p)}(t)\Gamma^{(p)}(t')\rangle=\epsilon p\,\delta(t-t'), \tag{B5}$$

with $\epsilon=0$ for a pump-noise-suppressed laser, and $\epsilon=1$ for a laser driven by a Poissonian pump,

$$\langle \Gamma^{(sp)}(t)\Gamma^{(sp)}(t')\rangle=\frac{\langle N_0\rangle}{\tau_{sp}}\delta(t-t'), \tag{B6}$$

$$\langle \Gamma(t)\Gamma(t')\rangle=A_0\langle N_0\rangle\langle n_0\rangle\,\delta(t-t'). \tag{B7}$$

Finally, due to their same physical origin, the noise terms associated with the stimulated gain for the photons and the stimulated emission for the electrons are perfectly anticorrelated and have cross correlations

$$\langle \Gamma(t)\xi_0(t')\rangle = -A_0\langle N_0\rangle \sqrt{\frac{\langle n_0\rangle}{4}}\delta(t-t'). \quad \text{(B8)}$$

APPENDIX C: EXPLICIT CALCULATION OF THE NOISES AND CORRELATIONS

We present in this appendix the explicit derivation of the expressions given in the main text for the noise power of the nonlasing mode [Eq. (9)], for the noise power of the main mode [Eqs. (10),(11), and (12)], and for their correlation [Eq. (13)].

The stationary solutions are obtained after taking $dP_i(t)/dt=0$ and $dN_0(t)/dt=0$. From Eq. (8), the fluctuations of the excited carrier number is

$$\delta N_0 = \frac{-A_0 N_0\sqrt{n_0}}{\frac{1}{\tau_{sp}}+A_0 n_0}\delta P_0 + \frac{\Gamma^{(p)}(t)+\Gamma^{(sp)}(t)+\Gamma(t)}{\frac{1}{\tau_{sp}}+A_0 n_0}. \quad \text{(C1)}$$

For the nonlasing mode, $\langle P_1\rangle = 0$ and $P_1=\delta P_1$. From Eq. (3), we get the amplitude fluctuations as

$$\delta P_1 = \frac{4\gamma_1^{(po)}(t)+4\gamma_1^{(pe)}(t)+4\xi_1(t)+2\sqrt{k_1}C(t)}{\frac{1}{\tau_1}-\langle N_1\rangle A_1}. \quad \text{(C2)}$$

It can be seen that the fluctuations of the excited carrier number has no contribution to the amplitude fluctuations of the nonlasing mode.

Therefore, the variance of the zero-frequency intracavity amplitude fluctuations of the nonlasing mode is given by

$$\langle \delta P_1^2\rangle = \frac{4\left(\frac{1}{\tau_1}+\langle N_1\rangle A_1\right)}{\left(\frac{1}{\tau_1}-\langle N_1\rangle A_1\right)^2}. \quad \text{(C3)}$$

The corresponding output amplitude is obtained using the usual input-output relations,

$$P_{i,out} = \frac{1}{\sqrt{\tau_i^{(pe)}}}P_i - 2\sqrt{\tau_i^{(pe)}}\gamma_i^{(pe)}. \quad \text{(C4)}$$

The variance of the zero-frequency output amplitude fluctuations of the nonlasing mode is given by

$$\langle \delta P_{1,out}^2\rangle = 1 + \frac{8\langle N_1\rangle A_1}{\left(\frac{1}{\tau_1}-\langle N_1\rangle A_1\right)^2\tau_1^{(pe)}} = 1 + \langle :\delta P_{1,out}^2 :\rangle, \quad \text{(C5)}$$

where $\langle :\delta P_{1,out}^2 :\rangle = \langle \delta P_{1,out}^2\rangle - 1$ corresponds to the excess noise above the shot noise level. The ‘‘:’’ means that normal ordering is used. As expected, when $\langle N_1\rangle = 0$, ie without gain, this noise is zero.

From the equation of motion of the lasing mode [Eq. (2)], we can also get the fluctuation of the total excited carrier number as

$$\delta N_0 = -\frac{1}{A_0\sqrt{n_0}}\sqrt{k_0k_1}\delta P_1 - \frac{1}{A_0\sqrt{n_0}}[2\gamma_0^{(po)}(t)+2\gamma_0^{(pe)}(t)+2\xi_0(t)-\sqrt{k_0}C(t)]. \quad \text{(C6)}$$

From this equation it can be seen that the contribution of the nonlasing mode to the fluctuations of the total excited carrier number is via the coupling between the two modes. By comparing Eqs. (C1) and (C6), we get the amplitude fluctuation of lasing main mode as

$$\delta P_0 = \frac{\frac{1}{\tau_{sp}}+A_0 n_0}{A_0^2 N_0 n_0}[\sqrt{k_0k_1}\delta P_1 + 2\gamma_0^{(po)}(t)+2\gamma_0^{(pe)}(t) + 2\xi_0(t) - \sqrt{k_0}C(t)] + \frac{1}{A_0N_0\sqrt{n_0}}[\Gamma^{(p)}+\Gamma^{(sp)}(t)+\Gamma]. \quad \text{(C7)}$$

Using the nonzero correlations and input-output relation, we can obtain the noise of the lasing mode as

$$\begin{aligned}\langle \delta P_{0out}^2\rangle = {} & \frac{1}{\tau_0^{(pe)}}\left(\frac{\frac{1}{\tau_{sp}}+A_0n_0}{A_0^2\langle N_0\rangle n_0\sqrt{\tau_0^{(pe)}}}-\sqrt{\tau_0^{(pe)}}\right)^2 \\ & +(\epsilon-1)\frac{\frac{1}{\tau_{sp}}+A_0n_0}{A_0^2\langle N_0\rangle n_0\tau_0^{(pe)}} + \left(\frac{\frac{1}{\tau_{sp}}+A_0n_0}{A_0^2\langle N_0\rangle n_0\sqrt{\tau_0^{(pe)}}}\right)^2 \\ & \times\left(\frac{1}{\tau_0^{(po)}}+A_0\langle N_0\rangle + k_0\right) + \left(\frac{\frac{1}{\tau_{sp}}+A_0n_0}{A_0^2\langle N_0\rangle n_0\sqrt{\tau_0^{(pe)}}}\right)^2 \\ & \times\left(k_0k_1\langle\delta P_1^2\rangle - \frac{4k_0k_1}{\frac{1}{\tau_1}-A_1\langle N_1\rangle}\right). \end{aligned} \quad \text{(C8)}$$

In order to show the contamination effect clearly, the noise of the lasing mode is written as two terms: one is associated with the zero-coupling case ($k_1=0$) and the other corresponds to the contamination coming from the nonlasing mode ($k_0k_1\neq 0$)

$$\langle \delta P_{0out}^2\rangle = \langle \delta P_{0out}^2\rangle|_{k_1=0} + \langle \delta P_{0out}^2\rangle|_{k_0k_1}, \quad \text{(C9)}$$

where

$$\langle \delta P_{0out}^2\rangle|_{k_1=0}=\frac{1}{\tau_0^{(pe)}}\left(\frac{\frac{1}{\tau_{sp}}+A_0n_0}{A_0^2\langle N_0\rangle n_0\sqrt{\tau_0^{(pe)}}}-\sqrt{\tau_0^{(pe)}}\right)^2 +(\epsilon-1)\frac{\frac{1}{\tau_{sp}}+A_0n_0}{A_0^2\langle N_0\rangle n_0\tau_0^{(pe)}} +\left(\frac{\frac{1}{\tau_{sp}}+A_0n_0}{A_0^2\langle N_0\rangle n_0\sqrt{\tau_0^{(pe)}}}\right)^2 \times\left(\frac{1}{\tau_0^{(po)}}+A_0\langle N_0\rangle+k_0\right) =1+\left(\frac{\tau_0}{\tau_0^{(pe)}}\right)[-1+x+2x^2+\epsilon(1+x)], \tag{C10}$$

where $x=1/(\tau_{sp}A_0n_0)$. This expression is the standard result from a single-mode theoretical model, which appears as a dotted line on Figs. 2 and 3. On the other hand, the noise leaking from nonlasing mode to lasing mode is

$$\langle \delta P_{0out}^2\rangle|_{k_0k_1}=k_0k_1\left(\frac{\frac{1}{\tau_{sp}}+A_0n_0}{A_0^2\langle N_0\rangle n_0\sqrt{\tau_0^{(pe)}}}\right)^2 \times\left(\langle \delta P_1^2\rangle-\frac{4}{\frac{1}{\tau_1}-A_1\langle N_1\rangle}\right). \tag{C11}$$

From Eq. (C5), we get

$$\langle \delta P_{0out}^2\rangle|_{k_0k_1}=k_0k_1(1+x)^2\tau_0^2\,\frac{\tau_1^{(pe)}}{\tau_0^{(pe)}}\langle :\delta P_{1,out}^2:\rangle. \tag{C12}$$

The correlation between the two modes is given by

$$C_{01}=\langle \delta P_{0out}\delta P_{1out}\rangle =\sqrt{\frac{k_0k_1}{\tau_0^{(pe)}\tau_1^{(pe)}}}\left(\frac{\frac{1}{\tau_{sp}}+A_0n_0}{A_0^2\langle N_0\rangle n_0}\right)(\langle \delta P_1^2\rangle -2\tau_1^{(pe)}\langle \delta P_1\gamma_1^{(pe)}(t)\rangle-\langle \delta P_1C(t)\rangle/\sqrt{k_1}) =\sqrt{k_0k_1}(1+x)\tau_0\sqrt{\frac{\tau_1^{(pe)}}{\tau_0^{(pe)}}}\langle :\delta P_{1out}^2:\rangle. \tag{C13}$$

[1] See for example, J.-C. Bouley, and G. Destefanis, IEEE Comm. Mag. **32**, 54 (1994).
[2] C.E. Wieman and L. Hollberg, Rev. Sci. Instrum. **62**, 1 (1991).
[3] D.C. Kilper, A.C. Schaefer, J. Erland, and D.G. Steel, Phys. Rev. A **54**, R1785 (1996).
[4] F. Marin, A. Bramati, V. Jost, and E. Giacobino, Opt. Commun. **140**, 146 (1997).
[5] S. Kasapi, S. Lathi, and Y. Yamamoto, Opt. Lett. **22**, 478 (1997); J. Opt. Soc. Am. B **15**, 2626 (1998).
[6] Y. Li, P. Lymnam, M. Xiao, and P.J. Edwards, Phys. Rev. Lett. **78**, 3105 (1997).
[7] Y. Yamamoto, S. Machida, and O. Nilsson, Phys. Rev. A **34**, 4025 (1986).
[8] S. Machida, Y. Yamamoto, and Y. Itaya, Phys. Rev. Lett. **58**, 1000 (1987).
[9] W.H. Richardson, S. Machida, and Y. Yamamoto, Phys. Rev. Lett. **66**, 2867 (1991).
[10] Yu.M. Golubev, I.V. Sokolov, Zh. Éksp. Teor. Phys, **87**, 804 (1984) [Sov. Phys. JETP **60**, 234 (1984)].
[11] H. Wang, M.J. Freeman, and D.G. Steel, Phys. Rev. Lett. **71**, 3951 (1993).
[12] J. Kitching, R. Boyd, A. Yariv, and Y. Shevy, Opt. Lett. **19**, 1331 (1994); J. Kitching, A. Yariv, and Y. Shevy, Phys. Rev. Lett. **74**, 3372 (1995).
[13] S. Inoue, H. Ohzu, S. Machida, and Y. Yamamoto, Phys. Rev. A **46**, 2757 (1992).
[14] D.C. Kilper, D.G. Steel, R. Craig, and D.R. Scifres, Opt. Lett. **21**, 1283 (1996).
[15] F. Marin, A. Bramati, E. Giacobino, T.-C. Zhang, J.-Ph. Poizat, J.-F. Roch, and P. Grangier, Phys. Rev. Lett. **75**, 4606 (1995).
[16] T.-C. Zhang, J.-Ph. Poizat, P. Grelu, J.-F. Roch, P. Grangier, F. Marin, A. Bramati, V. Jost, M.D. Levenson, and E. Giacobino, Quantum Semiclassic. Opt. **7**, 601 (1995).
[17] J.-Ph. Poizat and Ph. Grangier, J. Opt. Soc. Am. B **14**, 2772 (1997).
[18] C. Becher, E. Gehrig, and K.-J. Boller, Phys. Rev. A **57**, 3952 (1998).
[19] S. Lathi and Y. Yamamoto, Phys. Rev. A **59**, 819 (1999).
[20] S. Lathi, K. Tanaka, T. Morita, S. Inoue, H. Kan, and Y. Yamamoto, IEEE J. Quantum Electron. **QE 35**, 387 (1999).
[21] F. Jérémie, C. Chabran, and P. Gallion, J. Opt. Soc. Am. B **16**, 460 (1999).
[22] K. Petermann, IEEE J. Quantum Electron. **15**, 566 (1979).
[23] H.A. Haus and S. Kawakami, IEEE J. Quantum Electron. **21**, 63 (1985).
[24] A.E. Siegman, Phys. Rev. A **39**, 1253 (1989); **39**, 1264 (1989).
[25] A.E. Siegman, *Lasers* (University Science Books, Mill Valley, CA, 1986).
[26] W.A. Hamel and J.P. Woerdman, Phys. Rev. A **40**, 2785 (1989); Phys. Rev. Lett. **64**, 1506 (1990).
[27] G. Yao, Y.C. Chen, C.M. Harding, S.M. Sherrick, R.J. Dalby, R.G. Waters, and C. Largent, Opt. Lett. **17**, 1207 (1992).
[28] Y.J. Cheng, C.G. Fanning, and A.E. Siegman, Phys. Rev. Lett. **77**, 627 (1996).
[29] M.A. van Eijkelenborg, Å.M. Lindberg, M.S. Thijssen, and

J.P. Woerdman, Phys. Rev. Lett. **77**, 4314 (1996).

[30] A.M. van der Lee, N.J. van Druten, A. Mieremet, M.A. van Eijkelenborg, Å.M. Lindberg, M.P. van Exter, and J.P. Woerdman, Phys. Rev. Lett. **79**, 4357 (1997).

[31] M. Brunel, G. Ropars, A. Le Floch, and F. Bretenaker, Phys. Rev. A **55**, 4563 (1997).

[32] O. Emile, M. Brunel, F. Bretenaker, and A. Le Floch, Phys. Rev. A **57**, 4889 (1998).

[33] M.A. van Eijkelenborg, M.P. van Exter, and J.P. Woerdman, Phys. Rev. A **57**, 571 (1998).

[34] A.M. van der Lee, M.P. van Exter, A.L. Mieremet, N.J. van Druten, and J.P. Woerdman, Phys. Rev. Lett. **81**, 5121 (1998).

[35] J.-Ph. Poizat, T. Chang, O. Ripoll, and Ph. Grangier, J. Opt. Soc. Am. B **15**, 1757 (1998).

[36] Ph. Grangier and J.-Ph. Poizat, Eur. Phys. J. D **1**, 97 (1998).

[37] Ph. Grangier and J.-Ph. Poizat, Eur. Phys. J. D **7**, 99 (1999).

[38] H.P. Yuen and V.W.S. Chan, Opt. Lett. **8**, 177 (1983).

[39] M.F C. Schermmann, C.J. van der Poel, B.A.H. van Bakel, H.P.M. M. Ambrosius, A. Valster, J.A.M. van den Heijkant, and G.A. Acket, Appl. Phys. Lett. **66**, 920 (1995).

[40] An eigenmode of a system is a mode that reproduces itself after a round trip in the system. Its amplitude is multiplied by a complex coefficient (the eigenvalue) after a round trip (see Ref. [25] for more details).

[41] T. Chang, J.-Ph. Poizat, and Ph. Grangier, Opt. Commun. **148**, 180 (1997).