Squeezing in the Kerr nonlinear coupler via phase-space representation

Abdel-Baset M. A. Ibrahim, B. A. Umarov, and M. R. B. Wahiddin
Institute of Mathematical Sciences, Faculty of Science, University of Malaya, 50603, Kuala Lumpur, Malaysia
(Received 16 June 1999; revised manuscript received 14 September 1999; published 10 March 2000)

Quantum-statistical properties of light propagating in a coupler with third order nonlinearity are investigated. The stochastic equations describing the dynamics of the system are derived in positive-P and Wigner representations. The possibility to generate quadrature-squeezed states is shown numerically.

PACS number(s): 42.65.Ky

INTRODUCTION

The investigation of the nonclassical properties of light propagating in nonlinear optical systems is currently the subject of much theoretical and experimental efforts in quantum optics [1]. The third order nonlinearity (Kerr nonlinearity) of silica glass fibers was among the first proposed for generation of squeezed light and for quantum nondemolition measurements [2]. The nonlinearity of the glass is small compared with other materials, but its cumulative effect is enhanced by the possibility of one-dimensional propagation with power confinement and low attenuation in modern optical fibers. Also the Kerr effect does not require phase matching, and nonlinear response is very fast. All these properties make the optical fiber very attractive both for optical communications and for producing and studying nonclassical light properties. The quadrature squeezing of continuous wave in the fiber was observed in the pioneering work [3]. Also the quantum solitons propagation in optical fiber have been intensively investigated theoretically and experimentally [4,5]. The possibility of quadrature squeezing and soliton photon-number noise reduction have been shown [6] and implementation of quantum solitons for quantum nondemolition measurements have been discussed [7].

The two-mode nonlinear directional coupler (NLDC) consists of a pair of linearly coupled Kerr fibers. Since their introduction by Jensen [8] and Maier [9] NLDC have become one of the primary topics in the field of photonics due to their enormous potential for various applications (e.g., for all-optical switching), and also because they are relatively simple optical systems in which many subtle nonlinear effects can be realized (see the review [10]). Recently the quantum properties of NLDC have attracted particular interest [11,12], because they can be useful for generation and transmission of nonclassical light. The interesting quantum phenomena of collapses and revivals of photon number oscillations have been studied both numerically and using perturbation theory [11]. Also this phenomena has been considered by the solution of Heisenberg equations in rotating wave approximation [12], where in addition squeezing has been calculated.

In the case of quantum noise of arbitrary strength however more adequate description of quantum optical system can be achieved within the framework of an exact nonlinear treatment of quantum fluctuations via the solution of appropriate stochastic equations derived using P-positive representation of the density operator [1,13]. It is the aim of this paper to present the results of the investigation of light propagation in a quantum NLDC. We solved numerically the stochastic equations of P-positive representation and calculated different physical quantities. For large nonlinearities and large evolution distances instabilities may appear in numerical implementation of stochastic equations in P-positive representation [14]. The reason for this problem is the doubled number of variables in comparison with the classic equations so we also consider the ''truncated Wigner'' method [1,15], which is an approximate but robust method.

I. DESCRIPTION OF THE MODEL AND BASIC EQUATIONS

The system under consideration is the NLDC consisting of two fibers with third order nonlinearity. Exchange of the energy between two coupled modes is realized by means of evanescent fields. The Hamiltonian for this system can be written in the next form [11],

$$\hat{H}=\hbar\omega\hat{a}_1^\dagger\hat{a}_1+\hbar\omega\hat{a}_2^\dagger\hat{a}_2+\hbar\omega g\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1+\hbar\omega g\hat{a}_2^\dagger\hat{a}_2^\dagger\hat{a}_2\hat{a}_2+\hbar k(\hat{a}_1^\dagger\hat{a}_2+\hat{a}_2^\dagger\hat{a}_1), \tag{1}$$

where $\hbar$ is the Planck constant, ω is the frequency common for both wave guides, and g is the coupling constant proportional to the third order susceptibility $\chi^{(3)}$ responsible for the self-action process (we do not consider the nonlinear coupling or the cross action between the two modes), k is the linear exchange coupling coefficient between wave guides, and $\hat{a}_1,\hat{a}_2$ are the photon annihilation operators in the first and second wave guides, respectively. This Hamiltonian describes the behavior of continuous wave (CW) fields in NLDC in coupled-mode approximation. By application of standard techniques for the Hamiltonian (1) we obtain the master equation for the density matrix of the system, which in positive P representation [13] can be converted to the Fokker-Planck equation for the quasiprobability distribution function $P(\alpha_1,\alpha_2,\beta_1,\beta_2)$

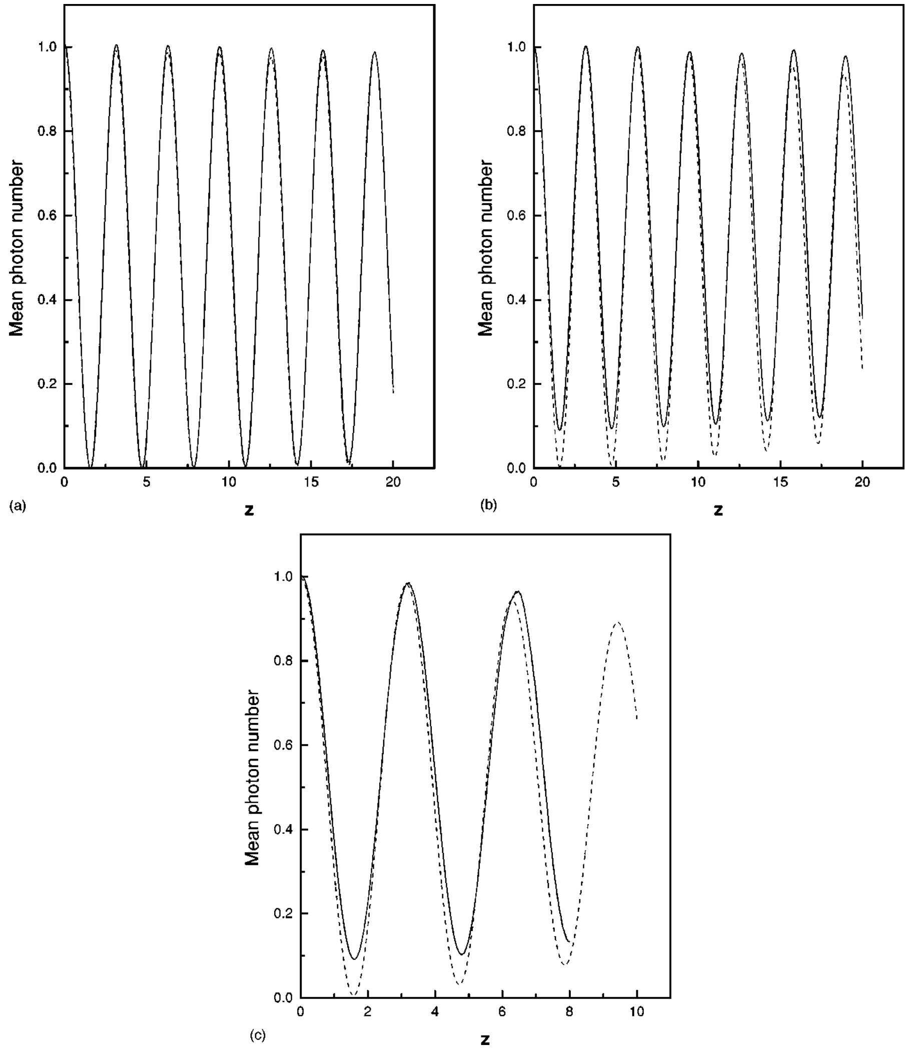

FIG. 1. The mean photon number in the first channel N_1 as a function of distance z for $k=1$, $\omega=1$, $\alpha_{c,1}=1$, $\alpha_{c,2}=0$. Solid line positive P and dashed line Wigner representations. (a) $g=0.01$, (b) $g=0.02$, (c) $g=0.05$.

$$\begin{aligned}\frac{\partial P}{\partial z}=&\frac{\partial}{\partial\alpha_1}(i\omega\alpha_1+2ig\beta_1\alpha_1^2+ik\alpha_2)-\frac{\partial}{\partial\beta_1}(i\omega\beta_1\\&+2ig\alpha_1\beta_1^2+ik\beta_2)+\frac{\partial}{\partial\alpha_2}(i\omega\alpha_2+2ig\beta_2\alpha_2^2+ik\alpha_1)\\&-\frac{\partial}{\partial\beta_2}(i\omega\beta_2+2ig\alpha_2\beta_2^2+ik\beta_1)+\frac{1}{2}\frac{\partial^2}{\partial\alpha_1^2}(-2ig\alpha_1^2)\\&+\frac{1}{2}\frac{\partial^2}{\partial\beta_1^2}(2ig\beta_1^2)+\frac{1}{2}\frac{\partial^2}{\partial\alpha_2^2}(-2ig\alpha_2^2)\\&+\frac{1}{2}\frac{\partial^2}{\partial\beta_2^2}(2ig\beta_2^2).\end{aligned}$$

Here the evolution parameter z, has the meaning of the normalized distance i.e a coordinate in the direction of propagation of radiation, and the normalization parameter is $z_0=1/k$. α_i,β_i are the complex valued functions of z. Using the Ito rules we can obtain from the Fokker-Planck equation the Langevin stochastic equations for the α_i and β_i variables as

$$\frac{d\alpha_1}{dz}=-i(\omega\alpha_1+2g\beta_1\alpha_1^2+k\alpha_2)+\alpha_1\sqrt{-2ig}\,\eta_1(z), \tag{2}$$

$$\frac{d\beta_1}{dz}=i(\omega\beta_1+2g\beta_1^2\alpha_1+k\beta_2)+\beta_1\sqrt{2ig}\,\eta_2(z), \tag{3}$$

$$\frac{d\alpha_2}{dz}=-i(\omega\alpha_2+2g\beta_2\alpha_2^2+k\alpha_1)+\alpha_2\sqrt{-2ig}\,\eta_3(z), \tag{4}$$

$$\frac{d\beta_2}{dz}=i(\omega\beta_2+2g\beta_2^2\alpha_2+k\beta_1)+\beta_2\sqrt{2ig}\,\eta_4(z). \tag{5}$$

η_i are the independent real Langevin sources of the noise with the following nonzero correlation functions:

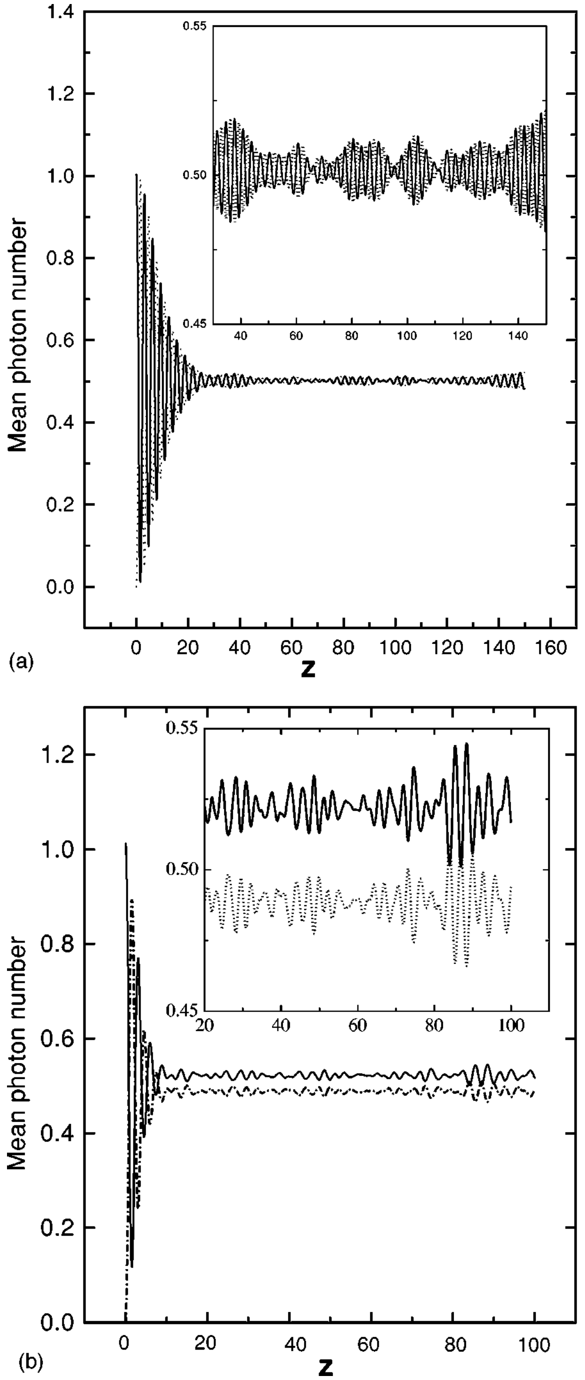

FIG. 2. The mean photon number in the first channel (solid line), and second channel (dashed line) as functions of distance z for $k=1$, $\omega=1$, $\alpha_{c,1}=1$, $\alpha_{c,2}=0$, $g=0.1$. Inset: The magnified version of mean photon number. (b) The mean photon number in the first channel (solid line) and second channel (dashed line) as functions of distance z for $k=1$, $\omega=1, \alpha_{c,1}=1, \alpha_{c,2}=0, g=0.3$. Inset: The magnified version of mean photon number.

$$\langle \eta_i^2 \rangle = 1. \tag{6}$$

The initial conditions are deterministic for coherent initial fields and $\alpha_i(0)=\alpha_{c,i}, \beta_i(0)=\alpha_{c,i}^*$, where $|\alpha_{c,i}|^2$ is the initial mean photon number. Here $i=1,2$ determines the channel in the NLDC. In positive P representation the quantum averages are found as moments of the P distribution function that correspond to normally ordered expectation values

$$\langle \hat{a}_1^{\dagger n} \hat{a}_1^m \hat{a}_2^{\dagger p} \hat{a}_2^q \rangle_{Qv} = \int d^2\alpha_1 d^2\alpha_2 d^2\beta_1 d^2\beta_2 (\beta_1^n \alpha_1^m \beta_2^p \alpha_2^q P) = \langle \beta_1^n \alpha_1^m \beta_2^p \alpha_2^q \rangle. \tag{7}$$

Here $\langle \cdots \rangle_{Qv}$ denotes the quantum averaging and $\langle \cdots \rangle$ denotes the classical phase space, stochastic averaging. So any quantum expectation value may be found by ensemble averaging over many trajectories using the stochastic Eqs. (2)–(5).

Another approach to quantum optical nonlinear problems is the Wigner distribution function $W(\alpha_1, \alpha_2)$ method [1]. The master equation can be transformed to an equivalent partial differential equation for the Wigner distribution, which contains third and fourth order derivatives and so cannot be directly converted to Langevin stochastic equations. The simplest approximation [15] is just to neglect third and higher order derivatives. Then we have for our system the following stochastic equations in Wigner representation:

$$\frac{d\alpha_1}{dz} = -i(\omega\alpha_1 + 2g(|\alpha_1|^2 - 1)\alpha_1 + k\alpha_2), \tag{8}$$

$$\frac{d\alpha_2}{dz} = -i(\omega\alpha_2 + 2g(|\alpha_2|^2 - 1)\alpha_2 + k\alpha_1). \tag{9}$$

Equations (8),(9) do not contain the explicit noise sources, but nevertheless the vacuum or quantum noise is included in the initial state for the Wigner equations [16]. Other noise sources such as thermal noise will appear in the second and higher order derivatives. The justification for the approximation is that this should have a small effect on the dynamics, which seems to be the case here, but is not always true. If the initial fields are coherent states specified by α_{ci} and $\delta\alpha_i$ are the initial displacements from α_{ci} (at $z=0$) for one member of an ensemble of trajectories governed by Eqs. (8) and (9) then

$$\langle (\delta\alpha_i)^2 \rangle = 0, \quad \langle (|\delta\alpha_i|)^2 \rangle = 0.5.$$

In the Wigner representation the moments of the distribution function W represent symmetrically ordered combinations of field operators

$$\langle (\hat{a}_1^{\dagger n} \hat{a}_1^m \hat{a}_2^{\dagger p} \hat{a}_2^q)_{sym} \rangle_{Qv} = \int d^2\alpha_1 d^2\alpha_2 (\alpha_1^{*n} \alpha_1^m \alpha_2^{*p} \alpha_2^q W) = \langle \alpha_1^{*n} \alpha_1^m \alpha_2^{*p} \alpha_2^q \rangle_W. \tag{10}$$

II. RESULTS OF NUMERICAL SIMULATIONS

Let us now discuss the results of our numerical simulations of the systems (2)–(5) and Eqs. (8) and (9). The incident fields are assumed to be in the coherent states. The following physical quantities have been calculated. The mean number of photons in single modes

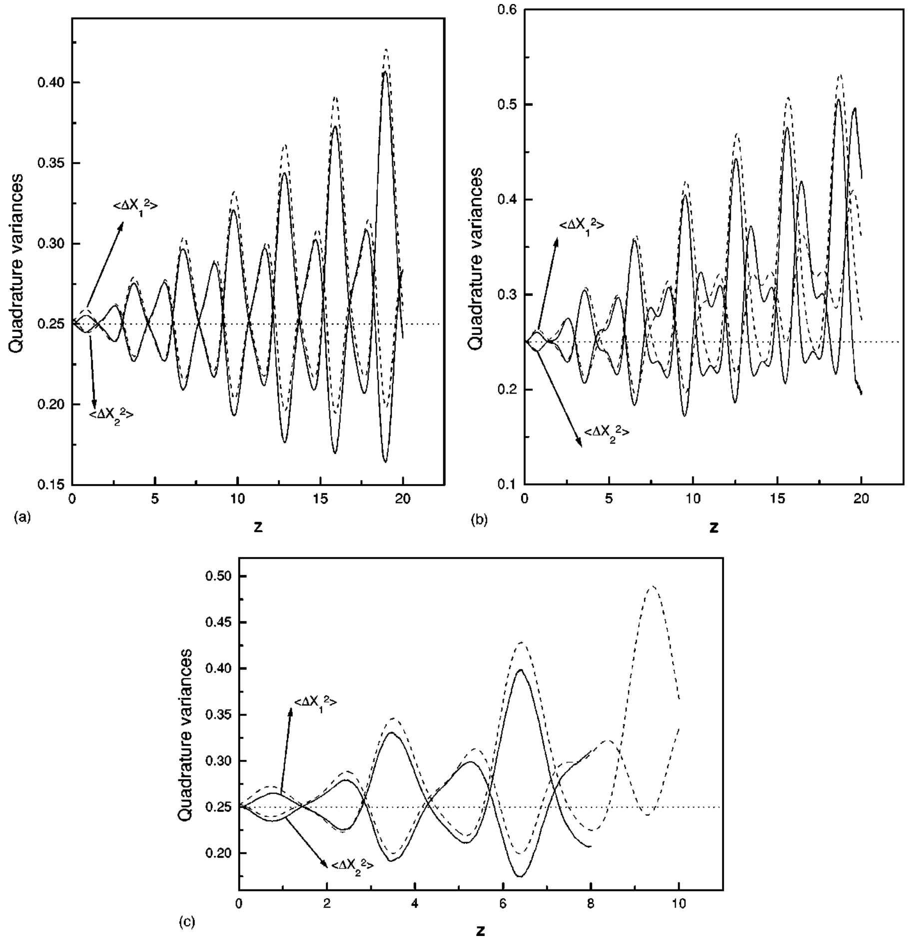

FIG. 3. The quadrature variances as functions of distance z in first channel for $k=1$, $\omega=1$, $\alpha_{c,1}=1$, $\alpha_{c,2}=0$. Solid line positive P and dashed line Wigner representations. Dotted line shows the shot noise level. (a) $g=0.01$, (b) $g=0.02$, (c) $g=0.05$.

$$N_i(\tau)=\langle \hat{a}_i^\dagger \hat{a}_i\rangle_{Qv}, \quad (11)$$

and variances $\langle \Delta\hat{X}_{1,i}^2\rangle$, $\langle \Delta\hat{X}_{2,i}^2\rangle$ of the quadrature operators $\hat{X}_{1,i}$ and $\hat{X}_{2,i}$ of the single modes, where

$$\hat{X}_{1,i}=\frac{1}{2}(\hat{a}_i+\hat{a}_i^\dagger), \quad (12)$$

$$\hat{X}_{2,i}=-\frac{i}{2}(\hat{a}_i-\hat{a}_i^\dagger). \quad (13)$$

In Fig. 1 we plot the evolution of mean photon number versus distance in the first channel for different values of nonlinearity parameter g. The initial conditions are the coherent state in one mode $\alpha_{c,1}=1$, and vacuum state in another $\alpha_{c,2}=0$. The linear coupling parameter is chosen as $k=1$. For small values of g we observe the periodic exchange of energy between two modes, and the truncated Wigner representation agrees well with the exact positive P description. The maximal distance in calculations is determined by the appearance of instabilities in positive P representation [14]. For higher g the positive P trajectories become unstable for very short distances. The results we obtained for mean photon number are also qualitatively very close to the results of the papers [11,12]. The calculation by the Wigner method as in Fig. 2(a) show the collapses and revivals for mean photon number oscillations in a long NLDC. We should note that it will be difficult to observe this phenomena in real couplers because of unavoidable fluctuations in linear coupling parameter. If we continue to increase the nonlinearity parameter g the light is localized in one channel. Although the collapses and revivals are more pronounced via the analytical solution given in Ref. [12], but to date only approximate methods have been used, and therefore it is difficult to decide which method is most reliable. In Fig. 2(b) we plot the evolution of the mean photon number in both channels to exhibit the self-trapping effect comparable to Fig. 4(d) in Ref. [11]. In conclusion, the Wigner results can also reproduce the different mean photon number behavior reported in Refs. [11,12].

Now we proceed with the main objective of our study, i.e., the calculations of the quadrature variances of single modes from which we can determine the possibility of squeezing for light propagating in a NLDC. In Fig. 3 the results of our calculations in positive P and Wigner representation are presented. The nonclassical behavior is clearly seen. The squeezing is observed for all calculated values of the nonlinear parameter. In Fig. 3 we show the results for the first channel but the evolution of quadrature variances is ac-

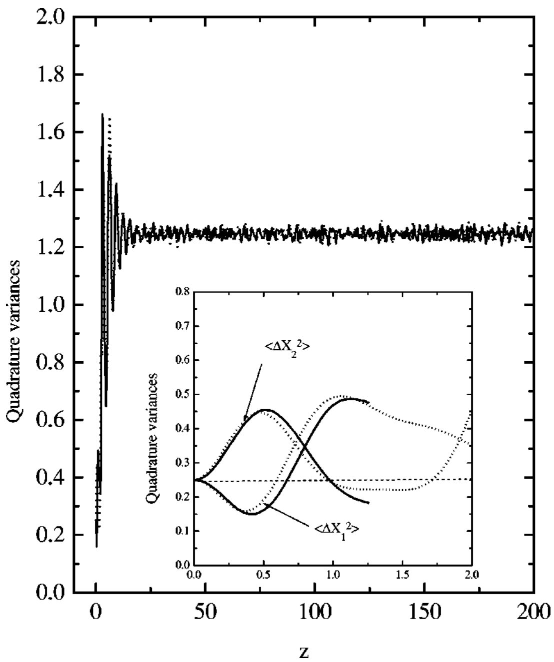

FIG. 4. The Wigner results for the quadrature variances as functions of distance z in first channel for $k=1$, $\omega=1$, $\alpha_{c,1}=2$, $\alpha_{c,2}=0$. The solid line corresponds to $\langle\Delta X_1^2\rangle$ and dashed line to $\langle\Delta X_2^2\rangle$. Inset: The corresponding Wigner and positive P results for small z.

tually very similar for both channels. Note that in Ref. [12] squeezing is only predicted in one of the channels (the one with coherent light input) for parameters we consider here. The variances of quadrature amplitudes periodically depend on the distance and it can be less than the shot noise level. Hence, light initially in a coherent state exhibits quadrature squeezing which has an oscillating behavior. The maximal value of squeezing increase with the increasing of g. It is interesting to note that local minima and maxima in quadrature variances depend on the distance. Further, as g increases the range of z exhibiting squeezing decreases. In Fig. 4 we also plot the behavior of the quadrature variances in the first channel (the quadrature variances evolution are very similar in the second channel) using the same parameters in Fig. 3 of Ref. [12]. We note that the initial squeezing obtained is very similar in both cases. However, squeezing does not recur as in the latter's case when $z=80$; there is no squeezing reported in Ref. [12] in the second channel. The Wigner method based results agree quite well with the positive P approach ones but for longer distances the discrepancies between approximate and exact methods become clearer. The results we obtained by the positive P representation here exactly describes the quantum NLDC.

In conclusion we have modeled the quantum NLDC by a set of stochastic differential equations derived using the positive P and Wigner representations. The mean photon number and quadrature variances for single modes have been calculated. The periodical exchange of energy between two channels and collapses and revivals of mean photon number oscillations are observed in simulations. The possibility of generating squeezed light in a NLDC is shown.

ACKNOWLEDGMENTS

This work was partially funded by Malaysia IRPA R&D Grant No. 09-02-03-0337. B.A. Umarov acknowledges the support of the Malaysia Ministry of Science and Technology.

[1] D.F. Walls and G.J. Milburn, *Quantum Optics* (Springer-Verlag, Berlin, 1995); C.W. Gardiner, *Quantum Noise* (Springer-Verlag, Berlin, 1991).

[2] Phys. Rev. A **35**, 4443 (1987); V.B. Braginsky and F.Ya. Khalili, *Quantum Measurements* (Cambridge University Press, Cambridge, England, 1992).

[3] R.M. Shelby, M.D. Levenson, S.H. Perlmutter, R.G. De Voe, and D.F. Walls, Phys. Rev. Lett. **57**, 691 (1986).

[4] P.D. Drummond, R.M. Shelby, S.R. Friberg, and Y. Yamamoto, Nature (London) **365**, 307 (1993).

[5] A. Sizmann, Appl. Phys. B: Lasers Opt. **65**, 745 (1997).

[6] P.D. Drummond and S.J. Carter, J. Opt. Soc. Am. B **4**, 1565 (1987).

[7] S.R. Friberg, S. Mashida, and Y. Yamamoto, Phys. Rev. Lett. **69**, 3165 (1992).

[8] S.M. Jensen, IEEE J. Quantum Electron. **QE-18**, 1580 (1982).

[9] A.M. Maier, Kvant. Elektron. (Moscow) **9**, 2996 (1982).

[10] M. Romagnoli, S. Trillo, and S. Wabnitz, Opt. Quantum Electron. **24**, S1237 (1992).

[11] A. Cheefles and S.M. Barnett, J. Mod. Opt. **43**, 709 (1996).

[12] M. Korolkova and J. Perina, Opt. Commun. **136**, 135 (1997).

[13] P.D. Drummond and C.W. Gardiner, J. Phys. A **13**, 2353 (1980).

[14] A. Gilchrist, P.D. Drummond, and C.W. Gardiner, Phys. Rev. A **55**, 3014 (1997).

[15] M.J. Werner and P.D. Drummond, J. Comput. Phys. **132**, 312 (1997).

[16] M.J. Steel, M.K. Olsen, L.I. Plimak, P.D. Drummond, S.M. Tan, M.J. Collett, D.F. Walls, and R. Graham, Phys. Rev. A **58**, 4824 (1998).