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Operation of universal gates in a solid-state quantum computer based on clean Josephson junctions between *d*-wave superconductors

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The operation of a solid-state superconducting quantum computer based on clean Josephson junctions between two *d*-wave superconductors is considered. We show that freezing of passive qubits can be achieved using a dynamic global refocusing technique. Further, we demonstrate that a universal set of gates can be realized on this system, thereby proving its universality.

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Quantum computation algorithms that provide enormous speed in dealing with certain classes of problems [1,2] can only be realized if a quantum computing device is built on a scale of at least several thousand qubits. The inherent scalability of solid-state devices and a high level of expertise existing in industrial electronics and experimental mesoscopic physics make solid-state-based quantum computers an attractive choice [3,4]. The problem of quantum coherence preservation in such devices, in the presence of a macroscopic number of degrees of freedom, is difficult but at least theoretically solvable [3,4]. Moreover, in a recent experiment on a superconducting quantum dot [single-electron transistor, (SET)] [5] coherent quantum beats were demonstrated in this mesoscopic system, which proves its suitability as a qubit prototype. The coherent ground state and gapped excitation spectrum in superconductors make coherence preservation more achievable; there exist already several suggestions for quantum computers based on Josephson junctions and superconducting SETs [4,6,7]. In this paper we consider operation of quantum gates in a solid-state quantum computer based on clean Josephson junctions between *d*-wave superconductors [i.e., ballistic *d*-wave superconductor–normal conductor–*d*-wave superconductor (DND) or D (grain boundary) D junctions] [7]. Terminal *B* of the junction (Fig. 1) is formed by a massive *d*-wave superconductor; in a multiple-qubit system, *B* would be a common ''bus'' bar. Terminal *A* is small enough to allow, when isolated, quantum phase fluctuations. It is essentially the sign of the superconducting phase difference φ between the terminals *A* and *B* that plays the role of ''spin variable'' of quantum computing. The collapse of the wave function is achieved by connecting terminal *A* to the equilibrium electron reservoir (''ground'') through a parity key (superconducting SET), thus blocking phase fluctuations due to a phase-number uncertainty relation [8]. Other parity keys, with different parameters, are used to link adjacent qubits, allowing for controllable entanglement. (A parity key only passes Cooper pairs, and only at a certain gate voltage V_g [9,10].)

The dynamics of the device was considered in [7]. It is characterized by the phase difference φ between terminals *A* and *B*, which plays the role of the position of a quantum particle with mass $M \propto C$, C being the classical capacitance of the small terminal, in an effective two-well potential $U(\varphi)$ (Fig. 2). It is the crucial advantage of clean DXD junctions, that the equilibrium phase $\pm\varphi_0$ continuously depends on the angle between crystal lattices of *A* and *B* (and therefore on the *d*-wave order parameters in these terminals)

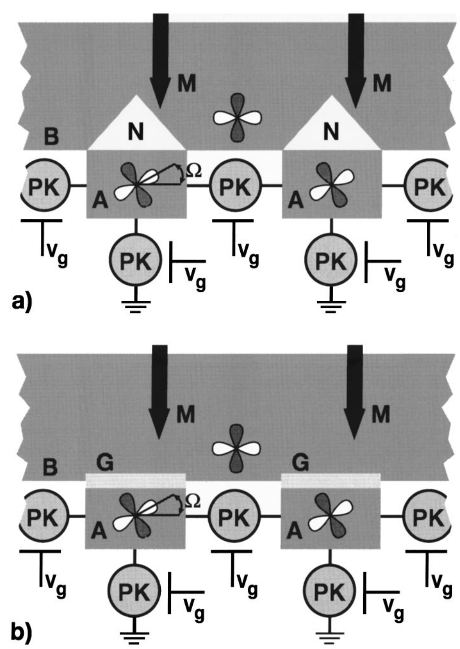

FIG. 1. (a) Superconducting DND qubits: *A*,*B* are *d*-wave superconductors, *N* normal conductor, PK parity key, *M* scanning tip, Ω the mismatch angle between the lattices of *A* and *B*. The cut in *B* is here along (110) and $(1\bar{1}0)$ directions. Positive lobes of *d*-wave order parameter are shaded. Two qubits are shown. (b) Version of (a) using grain boundary (*G*) junctions.

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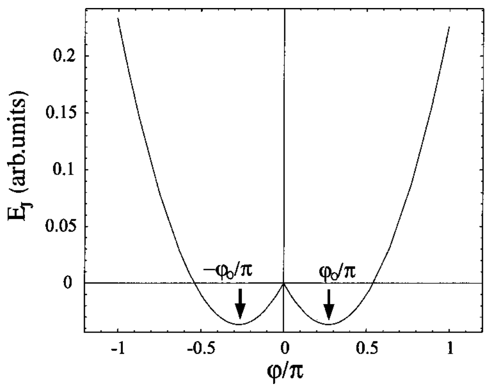

FIG. 2. Effective potential profile of the system. Minima at $\pm\varphi_0$ correspond to ''up'' and ''down'' pseudospin states of a qubit. The mismatch angle is $\Omega=\pi/8$.

in the interval $[0,\pi]$ allowing for exponentially wide tuning of the tunneling rate [7,11]. Moreover, due to time-reversal symmetry breaking in the system, states with $\varphi=-\varphi_0$ and $\varphi=\varphi_0$ are always degenerate and can be used as basic $|0\rangle$ and $|1\rangle$ states of a qubit [6,7].

The basic operations on a qubit are initialization, logical operations (quantum gates), and measurement. Measurement is a two-step procedure and can be performed simultaneously on all qubits or selectively on individual or groups of qubits. The first step, collapse of the wave function, is achieved by grounding terminal *A*. Readout is facilitated by the existence of small persistent currents and magnetic fluxes ($\ll\Phi_0$) that flow in opposite directions in the $|0\rangle$ and $|1\rangle$ states [7,11]. While too small to lead to unwanted inductive coupling between the qubits or decoherence, they can still be used to read out the state of the qubit once it was collapsed in one of the states with $\pm\varphi_0$, e.g., using a magnetic force microscope tip (which is removed during the computations). The estimated magnetic moment of order $(10^5\text{–}10^6)\mu_B$ is on the resolution limit of commercial magnetic force microscopes. The same property can be used to initialize individual qubits or whole registers, since this small coupling to an external field can put the qubit in a desired ($|0\rangle$ or $|1\rangle$) initial state.

Let us now describe how logical operations can be realized in this system. In order to maintain coherence, the qubit's electrodes *A* are isolated from the ground while performing logical operations. The basic one-qubit logical operations are rotations around the *x* and *z* axes, $X(\theta)$ and $Z(\phi)$:

$$X(\theta)=e^{-i\sigma_x\theta/2}; \tag{1}$$

$$Z(\phi)=e^{-i\sigma_z\phi/2}. \tag{2}$$

Operation $X(\theta)$, where $\theta=2t\Delta/\hbar$ and Δ are the tunneling matrix elements, is provided by natural quantum beats between the two basis states $|0\rangle$ and $|1\rangle$. On the other hand, an effective rotation around the *z* axis is realized by lifting the degeneracy of the basis states by an amount exceeding the tunneling width. Thereby tunneling between the basis states is suppressed and the natural oscillations between the basis states, $X(\theta)$, do not interfere with $Z(\phi)$ operations. The degeneracy between up/down states can be lifted in various ways. For example, it can be achieved by directly applying a localized magnetic field using a magnetic scanning tip. Other implementations will be discussed elsewhere.

As stated, the idle state of this system corresponds to the logical operation $X(\theta)$. For a single qubit ''computer'' this poses no problem as logical operations would be applied sequentially without waiting times (idle periods). In the case that an idle period is desired, one can choose this time to be a multiple of the oscillation period. Thus, using the above convention, this is equivalent to applying $X(2n\pi)=\mathbb{1}$, with *n* an integer and $\mathbb{1}$ the identity operator. The situation with more than one qubit is less straightforward. Here, we explicitly need passive qubits (qubits that undergo no logical operations) to be ''frozen'' during operation on the active qubits (qubits over which a logical operation is applied). For instance, if $Z(\phi)$ is applied on qubit one, the state of passive qubits must not change during this operation. Since the application time of logical gates will typically be incommensurate with the time required for $X(\theta)$ to be equal to $\mathbb{1}$, a scheme to freeze passive qubits is necessary. For this sake, it can be advantageous to have an idle state where the energy of $|0\rangle$ and $|1\rangle$ are degenerate and tunneling is coherently blocked. One way to do this would be to temporarily enlarge the capacitance of electrode *A* by linking it with an external circuit as it was suggested by Ioffe *et al.* in their ''quiet-qubit'' proposal [6]. However, such an approach brings the risk of losing coherence due to inelastic processes in the external normal circuits. On the other hand, making the external capacitor superconducting would bring uninvited evolution due to Josephson coupling between the external capacitor and electrode *A*. Our suggestion is to employ instead a technique of dynamic global refocusing closely related to refocusing methods of NMR [12,13] and strong focusing of accelerator physics [14]. It relies on periodic perturbation of the two-well potential with amplitude δE slightly exceeding the tunneling width. In this scheme, the energy shift between the basis states is periodically varied from $-\delta E$ to δE. Explicitly, this corresponds to the pulse sequence

$$\cdots Z(\delta E\tau/\hbar)Z(-\delta E\tau/\hbar)Z(\delta E\tau/\hbar)\cdots. \tag{3}$$

This results in a time-dependent angle of rotation around the *z* axes, which is given, in the ideal case, by a triangular function of period 2τ, the period of the refocusing sequence. The evolution operator for a single qubit is then given, without approximation, by the Magnus expansion [13]: $U(t)=\exp[-i\sigma_z\int_0^t dt'\,\phi'(t')/2]=\exp[-i\sigma_z(\delta E\tau-\delta E\tau+\delta E\tau-\cdots)/2\hbar]$ so that, in the worst case, it is equal to $\exp[\pm i\sigma_z\delta E\tau/2\hbar]$. For τ sufficiently small this reduces to $U(t)\approx\mathbb{1}$. Hence, this yields a true idle state as the information encoded by the qubits is not perturbed by tunneling nor by accumulation of relative phase between the basis states. The characteristic time scale of the refocusing pulse must be much less than the tunneling time (estimated in [7] as $\sim10^{-8}$ s).

It was recently demonstrated (Viola and Lloyd [15] using the spin-boson model; Viola, Knill, and Lloyd [16,17] under more general assumptions) that in the limit of *very* small τ, global refocusing leads to decoherence suppression in the σ_x and σ_y channels (phase decoherence) provided that $\phi=\pi$ and that delays between the refocusing pulses are smaller, or of the order of, the correlation time of the environment [18]. This correlation time is given by the inverse of a natural cutoff frequency $\tau_c \sim \omega_c^{-1}$ and determines the fastest time scale of the environment. In the case of semiconductor-based structures, where decoherence is due to phonons, τ_c is given by the inverse of the Debye frequency $\omega_c^{-1} \approx 10^{-13}$ s [16]. In the present situation, for τ to be very small requires $\tau \ll t_b$, where $t_b \sim l/v_f$ is the ballistic time (the time required for the formation of Andreev levels in the normal part of the system), l the size of the system, and v_f the Fermi velocity. Taking $l \sim 10^3$ Å and $v_f \sim 10^7$ cm/s [7], we arrive at $\tau \ll 10^{-12}$ s, a similar estimate as in [16]. Another potentially dangerous source of decoherence comes from the localized degrees of freedom (nuclear spins and paramagnetic impurities) [19]. The estimates based on the central spin model [19] show that the relevant energies correspond to much longer times, in excess of 10^{-8} s. (The same estimate can be made for the decoherence time from these subsystems.) On the other hand, the dynamics of a spin bath is much more complicated than the one of oscillator bath or spin-boson models, and its behavior under global refocusing should be a subject of special investigation. Logical gates can be performed simultaneously with global refocusing pulses. Indeed, because the refocusing pulses obviously commute with $Z(\phi)$, refocusing can be applied to all qubits (actives and passives) while performing $Z(\phi)$ on a qubit or in parallel on a group of qubits. The evolution of the active qubits is then given by $\exp\{-i\sigma_z[\int_0^t dt' \phi'(t')+\phi]/2\} \approx Z(\phi)$. As a result, application of $Z(\phi)$ on, e.g., the first qubit, in combination with the refocusing sequence yields the desired overall action on all qubits: $Z(\phi)\otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1}$. On the other hand, applying $X(\theta)$ reduces to stopping refocusing pulses on the active qubits for a determined period of time. This also yields the desired overall action [20].

In order to create entangled states, nonlocal gates are required. Such an entangling two-qubit operation is realized in this system by opening the parity key joining two adjacent qubits, Fig 1. With this parity key open, a Josephson current flows between states of opposite phases. Thus, the combination $|00\rangle$ and $|11\rangle$ carries no current while $|01\rangle$ and $|10\rangle$ do. As a result, states of opposite phase will differ from those of identical phase by a Josephson energy $E_J \sim 1-\cos(2\varphi_0)$. The evolution of a pair of qubits in this situation then corresponds to a conditional phase shift (CP) and, to an irrelevant phase factor, can be represented in the computational basis $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$ as

$$CP(\gamma)=\mathrm{Diag}(e^{i\gamma/2},e^{-i\gamma/2},e^{-i\gamma/2},e^{i\gamma/2}), \tag{4}$$

with $\gamma=E_J t/\hbar$. Because $CP(\gamma)$ is diagonal in the computational basis, it commutes with $Z(\phi)$. As a result, and under the assumption that the Josephson energy only weakly perturbs individual two-well potentials [21], the latter operation can be performed simultaneously with the global refocusing sequence. This condition can always be realized by tuning the gate voltage on the parity key, thus varying its transparency and Josephson energy.

Using the three basic operations defined above, it is possible to construct a controlled-NOT gate. This operation, denoted CN_{ij}, where i and j are the control and target qubits, respectively, acts as $CN_{12}|i,j\rangle=|i,i\oplus j\rangle$, with $\oplus$ denoting addition modulo 2. Using the above expressions for a one- and two-qubit gate, CN_{12} is realized in this system, up to an irrelevant global phase factor, by the following sequence

$$CN_{12}=e^{i5\pi/4}X_2(\pi/2)Z_2(\pi/2)X_2(\pi/2)Z_2(\pi/2)$$
$$Z_1(\pi/2)CP(\pi/2)X_2(\pi/2)Z_2(\pi/2)X_2(\pi/2). \tag{5}$$

In this expression, $X_i(\theta)$ $[Z_i(\phi)]$ applies $X(\theta)$ $[Z(\phi)]$ on the ith qubit while leaving the others unchanged [e.g., $Z_1(\phi)=Z(\phi)\otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1}$). In the setup of Fig. 1, it is possible to apply two-qubit gates only to adjacent qubits. It is therefore necessary to introduce a swap operator, denoted SW_{ij}, which exchanges the states of qubits i and j. A swap on two adjacent qubits is realized by the following combination of controlled-NOT gates

$$SW_{12}=CN_{12}CN_{21}CN_{12}. \tag{6}$$

Using this operator repeatedly, it is then possible to juxtapose any chosen pairs of qubits and, as a result, to apply controlled-not gates on any chosen pairs of qubits. Because of the commutation relations between the Pauli operators, combinations of rotations around the x and z axes generate SU(2), the group of 2×2 unitary matrices with determinant $+1$. Thus, it is possible to realize all one-qubit gates on this system. Furthermore, as has been shown by Barenco *et al.*, the set of all single-qubit gates and the controlled-not is complete for quantum computation [22]. It is therefore possible to generate all of SU(2^n) with proper sequences of gates in such an n-qubit DXD superconducting quantum computer. In conclusion, we have shown that a solid-state superconducting quantum computer suggested in [7] allows application of a complete set of quantum logical gates and is therefore a realization of a universal quantum computer.

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