

Spontaneous emission between two parallel plates, one or both infinitely permeable

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(Received 9 August 1999; published 11 February 2000)

We compute the influence on the spontaneous-emission rate for a two-level atom when it is located between two parallel plates of a different nature, namely a perfectly conducting plate ($\epsilon \rightarrow \infty$) and an infinitely permeable one ($\mu \rightarrow \infty$). We also discuss the case of two infinitely permeable plates. We compare our results with those found in the literature for the case of two perfectly conducting plates.

PACS number(s): 12.20.-m, 32.80.-t

Using thermodynamic arguments and assuming that thermal equilibrium between matter and radiation is always achieved, Einstein [1] was able to demonstrate that, other than stimulated emission, excited atoms must also decay spontaneously. Even an “isolated” excited atom in the vacuum must inevitably decay to the ground state. In other words, an excited stationary state of an atom is not actually a stationary state and we can say that spontaneous emission is in fact not a property of an isolated atom, but of an atom-vacuum system [2]. In the context of quantum electrodynamics (QED), we can say that the ultimate reason for spontaneous emission of excited atoms is the interaction of the atom with the quantized electromagnetic field of the vacuum state. As a consequence, any modification in the vacuum electromagnetic field caused, for instance, by cavities, can in principle modify the radiative properties of atomic systems. We can say that the presence of material walls in the vicinity of atomic systems renormalizes their transition frequencies as well as the widths of their spectral lines. The former effect corresponds to the influence of boundary conditions (BC) on the analog of the Lamb shift, while the latter corresponds to the change in the spontaneous-emission rate of excited atoms. The branch of physics that is concerned with the influence of the environment of an atomic system on its radiative properties is generically called cavity QED and the above examples are only two among many others [for a review see for instance Refs. [3,4]. Here we shall be concerned with one of the above effects, namely, the influence of BC imposed on the radiation field on the spontaneous-emission rate of a two-level atom. It is worth mentioning that cavity QED was born precisely from the observation of Purcell [5] half a century ago when he realized that a spontaneous-emission process associated with nuclear magnetic moment transitions at radio frequencies could be enhanced if the system were coupled to a resonant external electric circuit placed in the vicinity of the system. However, we can say that the first detailed papers on this subject were those written by Casimir and Polder [6] in which, among other things, forces between polarizable atoms and metallic walls were treated, and by Casimir in his

seminal work that brought about the Casimir effect [7]. Since then, cavity QED has attracted the attention of many physicists, both theoretical and experimentalists. Particularly, the effects of the proximity of plane walls to atomic systems have been investigated: for instance, Morawitz [8] discussed both classically and quantum mechanically the influence of a plane mirror on the spontaneous-emission rate of a two-level atom. A few years later, Drexhage [9] observed experimentally the oscillatory behavior of the lifetime on the distance to the mirror. The QED of charged particles between two parallel mirrors was discussed extensively by Barton [10,11], who was the first to compute explicitly the influence of two parallel perfect conducting plates on the spontaneous-emission rate for a spherically averaged atomic transition [10]. Barton’s result was rederived by Philpott [12] with a similar method and by Milonni and Knight [13] in the context of the image method. An interesting feature of the modified spontaneous-emission rate between two conducting mirrors is the fact that for the case of a transition dipole moment parallel to the plates there must be a strong suppression for $2L/\lambda_0 < 1$, where L is the distance between the plates and λ_0 , the transition wavelength (see for instance Ref. [14]). This inhibited spontaneous emission has been observed experimentally by Hulet, Hilfer, and Kleppner [15]. Many other interesting experiments have been done and we suggest for the interested reader the reviews by Haroche and Kleppner [2] and Hinds [16] and references therein. It is worth mentioning that outside the context of QED, toy models can be very useful in the understanding of the main features of the influence of boundary conditions on the radiative properties of atomic systems; see, for example Ref. [17].

In this paper we compute the spontaneous-emission rate for a two-level atom when it is located between two parallel plates of different nature ($\epsilon \rightarrow \infty$ and $\mu \rightarrow \infty$) and between two infinitely permeable plates ($\mu \rightarrow \infty$), and then we compare our results with those found in the literature [10,12,13] for the case of two perfectly conducting plates. Though analogous, the results are different, since when we change the boundary conditions on the photon field, the vacuum field modes also change. The unusual pair of plates described above were first employed by Boyer [18] in the context of random electrodynamics, a version of classical electrodynamics that includes the zero-point electromagnetic oscillations. These plates provide the simplest example where we

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can find Casimir forces [7] of a repulsive nature at work. Recently, Boyer's result and its thermal corrections were re-derived in the context of quantum field theory [19]. Also recently, Boyer's boundary conditions were employed by Hushwater [20] as a counterexample in order to prove that the naive interpretation of the Casimir attraction force as due to a difference between the number of vacuum modes in the region between the plates and the region outside the plates is not correct. These boundary conditions were also employed in connection with the Scharnhorst effect [21] where they provide one more example in which it is shown that the speed of light in regions where the electromagnetic vacuum is confined is altered with respect to the situation in which the propagation is in unconfined vacuum [22,23]. As expected, a strong suppression also occurs for both cases treated here. However, curious as it may seem, this suppression occurs when the transition dipole moment is perpendicular to the plates, in contrast to the suppression when the dipole moment is parallel to the plates which occurs for the two perfectly conducting plates.

Our starting point is the general expression for the spontaneous-emission rate of a transition $2 \rightarrow 1$ of a two-level atom, which is given by

$$A_{21}(\mathbf{r}) = \frac{4\pi^2\omega_0^2}{\hbar} \sum_{\alpha} \frac{1}{\omega_{\alpha}} |A_{\alpha}(\mathbf{r}) \cdot \mathbf{d}_{12}|^2 \delta(\omega_{\alpha} - \omega_0), \quad (1)$$

where ω_0 corresponds to the transition frequency, \mathbf{d}_{12} is the transition dipole moment, and each mode $A_{\alpha}(\mathbf{r})$ of the vacuum field is characterized by a wave vector \mathbf{k} and a given polarization (see for instance Ref. [14]).

The first setup we will consider consists of two infinite parallel surfaces (the plates), one of which will be considered to be a perfect conductor ($\epsilon \rightarrow \infty$) while the other is supposed to be perfectly permeable ($\mu \rightarrow \infty$). Also, we will choose Cartesian axes in such a way that the axis OZ is perpendicular to both surfaces. The perfectly conducting surface will be placed at $z=0$ and the permeable one, at $z=L$. The electro-

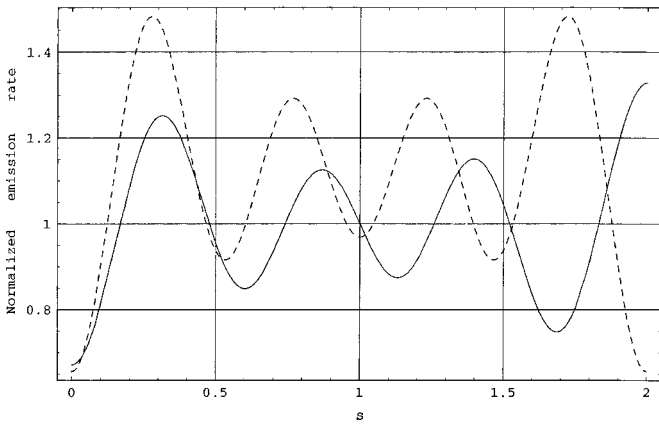


FIG. 1. The ratio A_{21}/A_{21}^0 as a function of the dimensionless variable $s = z/\lambda_0$ for the case of two perfectly conducting plates (dashed curve) and the case of a perfectly conducting plate and an infinitely permeable plate (solid curve). The range of the variable s is a typical one.

magnetic fields must satisfy the following boundary conditions: (a) the tangential components E_x and E_y of the electric field as well as the normal component B_z of the magnetic field must vanish on the metallic plate at $z=0$. (b) The tangential components B_x and B_y of the magnetic field must vanish on the permeable plate at $z=L$. It is convenient to work with the vector potential $\mathbf{A}(\mathbf{r}, t)$ in the Coulomb gauge in which $\nabla \cdot \mathbf{A}(\mathbf{r}, t) = 0$, $\mathbf{E}(\mathbf{r}, t) = -\partial \mathbf{A}(\mathbf{r}, t) / \partial t$, and $\mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t)$. With this choice of gauge, the above boundary conditions can be written as conditions imposed on the vector potential components,

$$A_x(x, y, 0, t) = A_y(x, y, 0, t) = \frac{\partial A_z}{\partial z}(x, y, 0, t) = 0, \quad (2)$$

$$\frac{\partial A_x}{\partial z}(x, y, L, t) = \frac{\partial A_y}{\partial z}(x, y, L, t) = A_z(x, y, L, t) = 0. \quad (3)$$

The mode functions for this case are [23]

$$\mathbf{A}_{k1}(\mathbf{r}) = \left(\frac{2}{V}\right)^{1/2} (\hat{\mathbf{k}}_{\parallel} \times \hat{\mathbf{z}}) \sin\left[\left(n + \frac{1}{2}\right) \frac{\pi z}{L}\right] e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}_{\parallel}} \quad (4)$$

and

$$\mathbf{A}_{k2}(\mathbf{r}) = \left(\frac{1}{k}\right) \left(\frac{2}{V}\right)^{1/2} e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}_{\parallel}} \left\{ k_{\parallel} \hat{\mathbf{z}} \cos\left[\left(n + \frac{1}{2}\right) \frac{\pi z}{L}\right] - i \frac{\pi}{L} \left(n + \frac{1}{2}\right) \hat{\mathbf{k}}_{\parallel} \sin\left[\left(n + \frac{1}{2}\right) \frac{\pi z}{L}\right] \right\}. \quad (5)$$

The contributions for the spontaneous-emission rate associated with \mathbf{d}_{12}^{\perp} and $\mathbf{d}_{12}^{\parallel}$ are given, respectively, by

$$A_{21}^{\perp}(z) = \frac{3\pi}{k_0 L} A_{21}^{0\perp} \sum_{n=0}^N \left[1 - \frac{\left(n + \frac{1}{2}\right)^2}{k_0^2 L^2} \pi^2 \right] \cos^2\left[\left(n + \frac{1}{2}\right) \frac{\pi z}{L}\right] \quad (6)$$

and

$$A_{21}^{\parallel}(z) = \frac{3\pi}{2k_0 L} A_{21}^{0\parallel} \sum_{n=0}^N \left[1 + \frac{\left(n + \frac{1}{2}\right)^2}{k_0^2 L^2} \pi^2 \right] \times \sin^2\left[\left(n + \frac{1}{2}\right) \frac{\pi z}{L}\right], \quad (7)$$

where $A_{21}^{0\perp}$ and $A_{21}^{0\parallel}$ are the corresponding contributions for the spontaneous-emission rate in unbounded (free) space, namely,

$$A_{21}^{0\parallel} = \frac{4|\mathbf{d}_{12}^{\parallel}|^2 \omega_0^3}{3\hbar c^3} \quad \text{and} \quad A_{21}^{0\perp} = \frac{4|\mathbf{d}_{12}^{\perp}|^2 \omega_0^3}{3\hbar c^3} \quad (8)$$

and N is the greatest integer part of $k_0 L / \pi - 1/2$. The total emission coefficient is given by $A_{21} = A_{21}^{\perp} + A_{21}^{\parallel}$. Recall that Einstein's coefficient for spontaneous emission is simply given by

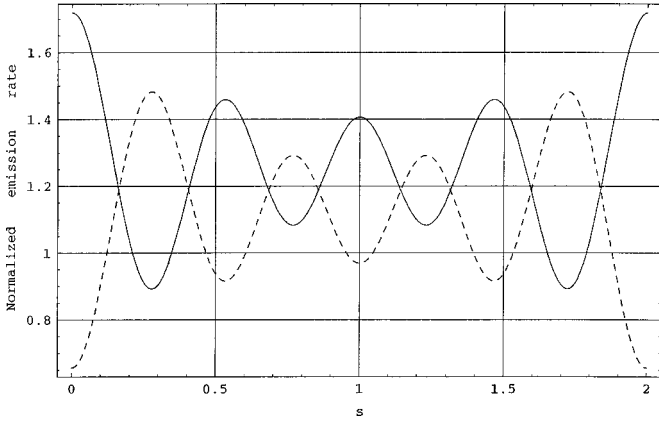


FIG. 2. The ratio A_{21}/A_{21}^0 as a function of the dimensionless variable $s=z/\lambda_0$ for the case of two perfectly conducting plates (dashed curve) and the case of two infinitely permeable plates (solid curve). The range of the variable s is a typical one.

$$A_{21}^0 = A_{21}^{0\parallel} + A_{21}^{0\perp} = \frac{4|d_{12}|^2 \omega_0^3}{3\hbar c^3}. \quad (9)$$

The graph displayed in Fig. 1 shows the ratio between A_{21} and A_{21}^0 as a function of the dimensionless variable $s:=z/\lambda_0$ for the case of two conducting plates (dashed line) and the case of a conducting plate and a permeable plate (solid line). Although the two curves are analogous in the sense that both present oscillations with s , they are different curves since the mode functions of the vacuum field in each case are not the same. It is worth emphasizing the lack of symmetry of the latter curve around the point that is equidistant from the plates. This was expected because in this case the two plates correspond to distinct electromagnetic media with different properties.

The second example we shall be concerned with consists of two perfectly permeable plates. The boundary conditions for this case can be cast into the form

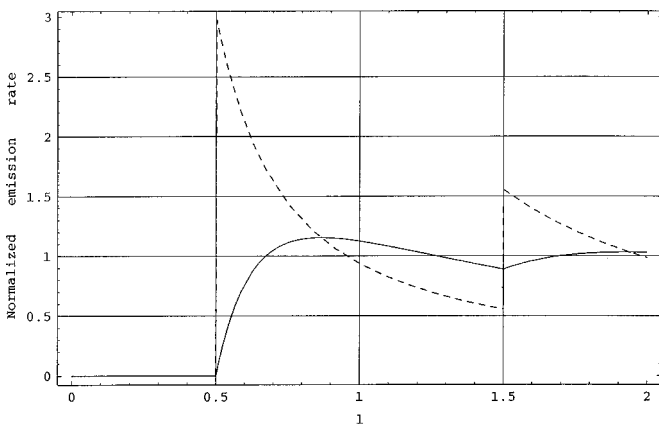


FIG. 3. The ratio $A_{21}^{\parallel}/A_{21}^{0\parallel}$ for the case of two perfectly conducting plates (dashed curve) and the ratio $A_{21}^{\perp}/A_{21}^{0\perp}$ for the case of two infinitely permeable plates (solid curve) as functions of the dimensionless variable $l=L/\lambda_0$.

$$\frac{\partial A_x}{\partial z}(x,y,0,t) = \frac{\partial A_y}{\partial z}(x,y,0,t) = A_z(x,y,0,t) = 0, \quad (10)$$

$$\frac{\partial A_x}{\partial z}(x,y,L,t) = \frac{\partial A_y}{\partial z}(x,y,L,t) = A_z(x,y,L,t) = 0. \quad (11)$$

The corresponding mode functions are

$$\mathbf{A}_{k1}(\mathbf{r}) = \left(\frac{2}{V}\right)^{1/2} (\hat{\mathbf{k}}_{\parallel} \times \hat{\mathbf{z}}) \cos\left(n \frac{\pi z}{L}\right) e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}_{\parallel}} \quad (12)$$

and

$$\mathbf{A}_{k2}(\mathbf{r}) = \left(\frac{1}{k}\right) \left(\frac{2}{V}\right)^{1/2} e^{i\mathbf{k}_{\parallel} \cdot \mathbf{r}_{\parallel}} \left\{ k_{\parallel} \hat{\mathbf{z}} \sin\left(n \frac{\pi z}{L}\right) + i \left(\frac{n\pi}{L}\right) \hat{\mathbf{k}}_{\parallel} \cos\left(n \frac{\pi z}{L}\right) \right\}. \quad (13)$$

The contributions for the spontaneous-emission rate associated with \mathbf{d}_{12}^{\perp} and $\mathbf{d}_{12}^{\parallel}$ are given, respectively, by

$$A_{21}^{\perp}(z) = \frac{3\pi}{k_0 L} A_{21}^{0\perp} \sum_{n=1}^N \left(1 - \frac{n^2 \pi^2}{k_0^2 L^2}\right) \sin^2\left(\frac{n\pi z}{L}\right) \quad (14)$$

and

$$A_{21}^{\parallel}(z) = \frac{3\pi}{2k_0 L} A_{21}^{0\parallel} \left\{ 1 + \sum_{n=1}^N \left(1 + \frac{n^2 \pi^2}{k_0^2 L^2}\right) \cos^2\left(\frac{n\pi z}{L}\right) \right\}, \quad (15)$$

where N is the greatest integer part of $k_0 L/\pi$.

Figure 2 shows the ratio between A_{21} and A_{21}^0 as a function of $s=z/\lambda_0$ for the case of two conducting plates (dashed line) and the case of two permeable plates (solid line). The curve for this latter case also presents oscillations in the spontaneous-emission rate as the distance from the atom to each plate varies and is also symmetric with respect to the equidistant point to the plates. However, there is a remarkable difference between these two curves: whenever there is an enhancement in the spontaneous-emission rate of the former, there will be a depletion for the latter and vice versa. Particularly, their behavior near the plates are quite different.

The strong suppression that occurs in the case of two conducting plates for A_{21}^{\parallel} has its counterpart in the two cases discussed previously, as we shall see. However, we should emphasize that in the case of two permeable plates as well as in the case of a conducting plate and a permeable one, the suppression occurs for A_{21}^{\perp} , in contrast with the case of two conducting plates. For simplicity, let us just fix the atom at a point equidistant from the parallel infinite plates in both setups and vary the distance L between the plates. Also, for convenience, in the remaining figures of this paper we shall plot the graphs of the ratios $A_{21}^{\parallel}/A_{21}^{0\parallel}$ and $A_{21}^{\perp}/A_{21}^{0\perp}$ as functions of the dimensionless parameter $l:=L/\lambda_0$. Figure 3 shows jointly the suppression of A_{21}^{\perp} for the case of two permeable plates (solid line) and the suppression of A_{21}^{\parallel} for the case of two conducting plates (dashed line). Observe that

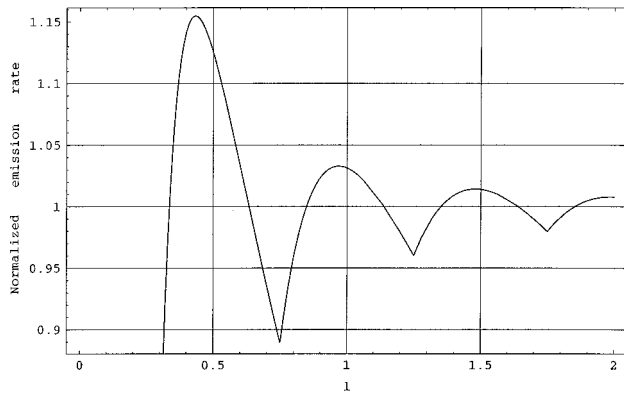


FIG. 4. The ratio $A_{21}^{\perp}/A_{21}^{0\perp}$ for the case of one perfectly conducting and one infinitely permeable plate. Suppression occurs at $l = L/\lambda_0 = \frac{1}{4}$.

both occur for the same value of the distance between the plates. Though not obvious, this result is quite reasonable, since for the case of two permeable plates the mode functions of the vacuum field are also symmetric with respect to $s = l/2$. In this sense, for the case of a conducting plate and a permeable one, for which mixed boundary conditions are used, it is natural to expect that the suppression will occur for a different value of l . This is indeed what happens and as is shown separately in Fig. 4, the suppression occurs for a value of l which is smaller than the value found for the other cases (shown in Fig. 3). Concerning Figs. 3 and 4, a last comment is in order: observe that in Fig. 3 the distances

between two successive peaks for $A_{21}^{\parallel}/A_{21}^{0\parallel}$ (dashed curve) or discontinuities in the derivative of $A_{21}^{\perp}/A_{21}^{0\perp}$ (continuous curve) are $\Delta l = \lambda_0$, in contrast with the value $\Delta l = \lambda_0/2$ observed for the discontinuities in the derivative of $A_{21}^{\perp}/A_{21}^{0\perp}$ in Fig. 4, since for the situation described by Fig. 4 there are no nodes for the vacuum modes at the midpoint between the two plates.

It is interesting to notice that though the Casimir energy density for the case of two perfectly parallel conducting plates is exactly the same as that for two infinitely permeable parallel plates, the influence of these two different surroundings on radiative properties of an atomic system (like the spontaneous-emission rate of an atom) can be quite different. In other words, though the Casimir effect is “blind” to the change of the two conducting plates by the two infinitely permeable ones, the atom is not. The reason for that is simply because only the possible field frequencies enter in the calculation of the Casimir energy density, while the atom interacts directly with each vacuum field mode. It probes locally the vacuum field. Though highly idealized, the situations that we have examined here can be considered as a first step towards more realistic situations in which the influence of a finite magnetic permeability on the spontaneous-emission rate would be taken into account.

The authors acknowledge P.A.M. Neto and C. Lenz for valuable suggestions. D.T.A. would like to thank CAPES and UFPA for financial support and C.F. would like to thank CNPq for partial financial support.

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